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Numerical Methods for Differential Equations
Tutorial 9

Topics: Runge-Kutta methods, stability

Exercise 9.1:

Set the Runge-Kutta method that is described by the following Butcher tableau.

$$\begin{array}{c|ccc}
0 & & & \\
\frac{1}{3} & \frac{1}{3} & & \\
\frac{2}{3} & 0 & \frac{2}{3} & \\
\hline
 & \frac{1}{4} & 0 & \frac{3}{4}
\end{array}$$

Exercise 9.2:

Consider renewed the ordinary differential equation of motion for the one-mass oscillator from Task 7.1 ii) b), namely,

$$\dot{\mathbf{q}}(t) = \begin{pmatrix} 0 & 1 \\ -\frac{401}{400} & -\frac{1}{10} \end{pmatrix} \mathbf{q}(t), \qquad \mathbf{q}(0) = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}.$$

For which step sizes does

(a) the explicit Euler method
(b) the implicit Euler method

deliver a qualitatively correct solution?

Supplementary Exercise 9.3:

(a) Consider the one-step method from Exercise 8.2. Write the Butcher tableau corresponding to this method.

(b) Convert each Butcher tableau into an algorithm

$$\begin{array}{c|cc}
0 & & \\
\frac{1}{2} & \frac{1}{2} & \\
\hline
 & 0 & 1
\end{array}
\qquad\qquad
\begin{array}{c|ccc}
0 & & & \\
1 & 1 & & \\
\frac{1}{2} & \frac{1}{4} & \frac{1}{4} & \\
\hline
 & \frac{1}{6} & \frac{1}{6} & \frac{2}{3}
\end{array}$$

Execute the first step of the algorithms with the step size $h = 1$ to obtain a numerical solution of $y' = -2ty^2, y(0) = 1$.