Technische Universität Braunschweig
Institute for Partial Differential Equations
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Summer term 2022

Numerical Methods for Differential Equations

Assignment 4

Upload solutions until 23 May 2022, 3pm

Exercise 4.1(Summed Kepler's barrel rule) **(4+4+2=10 points)**

Consider the integral $\int_a^b f(x)\,dx$.

(a) Write a `Matlab` function which subdivides $[a, b]$ into equidistant subintervals and applies Kepler's barrel rule to each subinterval. The function should take four inputs: `f`, `a`, `b` and `J`, where `J` denotes the number of subintervals.

(b) We now aim to iteratively call the function from (a) in order to compute the integral of the function $f(x) = \sin(x)$ for the interval $[a, b] = [0, \pi]$. Each time, the number of used intervals J should be increased by one. The number of intervals should be increased as long as the value of the summed quadratures keeps adjusting. More precisely: Iterate as long as the computed values vary more than 10^{-4} between iterations. Stop the iteration if the difference between the computed values for the J-th step and the $(J + 1)$-th step is smaller than 10^{-4} for more than 10 iterations.
Also build in a safeguard: Let your program terminate after a maximal number of steps with an error message.

(c) Extend your program in the following way: Store the value of the integral for each number of intervals J in a suitable way. Create a plot with the number of intervals on the x-axis and the distance between the value in the J-th step and the last computed value (for the largest number of intervals) on the y-axis.

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Exercise 4.2 (Numerical differentiation) **(3+4+3 = 10 points)**

(a) Write a function file (a ".m"-file) `diff_quot`, which calculates the difference quotient of a given function $f(x)$ for a certain granularity h. Your function should take four inputs: The function `f`, the position `x` where the derivative shall be computed, a granularity `h` and a string `method`. The `method`-string may assume the values "`forward`", "`backward`" or "`central`" and `diff_quot` should return the value of the forward, backward or central difference quotient accordingly.

(b) Put a grid with $N = 2, 3, 4, ...$ nodes on the interval $[0, 2\pi]$. Consider the function $f(x) = \sin(x)$. Compute the forward, backward and central difference quotient for each of the grid points. Hereby, the granularity `h` shall be the distance between two nodes. Plot the results in a coordinate system and connect the dots of the forward/backward/central difference quotient by a line (Hence, you should get three polygonal chains for each N).
Create an animation, where each frame depicts the plot described above for a certain value of N. Start with $N = 1$ in the first frame and continue till you reach $N = 100$.
Hint: You may utilise the `Matlab` *command* `pause` *to create such a plot.*

(c) Consider $f(x) = \sin(x)$ again. Use the function `diff_quot` from (a) in order to compute (affine) linear functions

$$T_i(x) = m_i x + b_i,$$

such that $T_i(x)$ touches $f(x)$ tangential at the points $x_1 = 0, x_2 = \pi$ and $x_3 = \frac{3\pi}{2}$. Use central differential quotients. Plot the three tangents $T_i(x)$ in a common plot with $f(x)$.
