Technische Universität Braunschweig
Institute for Partial Differential Equations
Prof. Dr. Carmen Gräßle
Jannis Marquardt

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Numerical Methods for Differential Equations

Assignment 6

Upload solutions until 13 June 2022, 3pm

Exercise 6.1 (Banach fixed-point iteration) (10 points)

Consider the nonlinear equation $x^3 + x = 1$. First, write a `Matlab` script to solve the equation using the inbuilt `Matlab` command `fsolve`, starting with an initial guess $x_0 = 0.7$. Compare the result with the analytical solution

$$x^* = \frac{1}{6^{2/3}}\left(\sqrt[3]{2\left(9+\sqrt{93}\right)} - \sqrt[3]{2\left(\sqrt{93}-9\right)}\right).$$

Now, write a `Matlab` script that uses the Banach fixed-point interation to solve the equation with the same initial guess. For the iteration procedure, use the following two rules:

(i) $x_{k+1} = g_1(x_k)$ with $g_1(x) = \frac{1}{1+x^2}$,

(ii) $x_{k+1} = g_2(x_k)$ with $g_2(x) = 1 - x^3$.

Set the first 20 iteration steps as a termination criterion for each iteration procedure. Plot both the iterative processes in the form of a Cobweb plot. Thereafter, compare the two obtained numerical values with the numerical value of the analytical solution x^*. Make a comment in your script (just 2-3 lines) explaining the behaviour of both iterative processes.

Hint: For information about Cobweb plots, please consult the following Wikipedia page: https://en.wikipedia.org/wiki/Cobweb_plot

Exercise 6.2 (Newton's method) (10 points)

Write a `Matlab` function file `newton1d` which takes as input a univariate function f and a staring point x_0 and gives as output a zero of the function $f(x)$ computed using Newton's method. Again, set the first 20 iteration steps as a termination criterion for the iteration procedure.
Now, using this function file `newton1d`, solve the nonlinear equation from the previous task with the same initial guess $x_0 = 0,7$. Compare your result with all the numerical solutions obtained in the previous exercise in this assignment. Make a comment in the script based on your observations.

Hint: The `newtown1d` *function file also has to compute the derivative of the function f which is required for Newton's method. This can be done using symbolic* `Matlab` *variables.*
