Technische Universität Braunschweig
Institute for Partial Differential Equations
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Summer term 2022

Numerical Methods for Differential Equations

Assignment 12

Upload solutions until 25 July 2022, 3pm

Exercise 12.1 (Heat equation) (20 points)

The temperature $U(x,t)$ along a bar of **unit length** is governed by the equation

$$\frac{\partial U(x,t)}{\partial t} = \frac{\partial^2 U(x,t)}{\partial x^2}.$$

The ends of the bar are kept cooled at 0K the whole time, and the **initial temperature** of the bar is 10K.

$$U(0,t) = 0K \rightarrow \boxed{\text{Bar of unit length}} \leftarrow U(1,t) = 0K$$

$$\uparrow$$

$$U(x,0) = 10K$$

Write a `Matlab` script to evaluate numerically the temperature of the bar at the end of 0.25 seconds.
To do this, discretize the differential w.r.t x first. Use the finite difference method with step size $\Delta x = 0.01$ and take the boundary conditions into account. Thereafter, you can obtain an ODE of the form

$$\mathbf{U}'(t) = \frac{1}{\Delta x^2}\mathbf{A}\mathbf{U}(t).$$

Choosing the highest possible stable step size Δt for time variable, solve this ODE using the explicit Euler method (*which you have already implemented in Assignment 9*).
Make a plot showing how the temperature along the bar varies from the initial value to the obtained numerical solution.

Hint: You can verify your solution by keeping in mind the physics behind the problem!