Technische Universität Braunschweig
Institute for Partial Differential Equations
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Summer term 2022

Numerical Methods for Differential Equations

Assignment 8

Upload solutions until 27 June 2022, 3pm

Exercise 8.1 (Explicit one-step methods) (5+5+5+2+3=20 points)

Consider the initial value problem

$$y'(t) = f(t, y(t)), \quad y(0) = y_0.$$

(a) Write a `Matlab` function file `explicitEuler` which takes as input the right-hand side f of the initial value problem, the initial value y_0, the end time t_{end} and step size h and computes a numerical solution of the IVP at time t_{end} using explicit Euler method. The output of the `Matlab` function should not only be the solution of the problem but should also include intermediate values for each step (path to the solution at t_{end} from the initial time $t_0 = 0$).

(b) Write a `Matlab` function file `improvedEuler` which meets the same requirements as `explicitEuler` in (a), but uses the improved Euler method instead of the explicit Euler method.

(c) Write a `Matlab` function file `EulerHeun` which meets the same requirements as `explicitEuler` in (a), but uses the Euler-Heun method instead of the explicit Euler method.

For the rest of this exercise, consider the initial value problem

$$y'(t) = 2t(1 + y(t)), \quad y(0) = 0$$

(d) Compute an analytic solution $y(t)$.
Hint: You may use separation of variables or simply guess a solution. You don't have to hand in your computation steps. A comment in the Matlab file from (e) is enough.

(e) Use the function files from (a)-(c) to compute numerical solutions for the given IVP at $t_{\text{end}} = 2$ as well as the paths which lead to the solution.
Therefore, consider step sizes $h \in \{1, 0.5, 0.1, 0.01\}$. Plot for each choice of h the paths together with the analytic solution from (d) in a common plot over the interval $[0, 2]$.