Technische Universität Braunschweig
Institute for Partial Differential Equations
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Summer term 2022

Numerical Methods for Differential Equations

Assignment 9

Upload solutions until 04 July 2022, 3pm

Exercise 9.1 (Euler methods in $\mathbb{R}^n$) **(10 points)**

Consider an initial value problem

$$y'(t) = -Ay(t), \qquad y(0) = y_0$$

with $A \in \mathbb{R}^{n\times n}$, $y(t) \in \mathbb{R}^n$ and $y'(t) \in \mathbb{R}^n$.
Write two `Matlab` function files `explicitEulerVector` and `implicitEulerVector` which take as input the stiffness matrix A of the initial value problem, the initial value y_0, the end time t_{end} and some time step h and compute a numerical solution of the IVP at time t_{end} using explicit Euler's method and implicit Euler's method respectively. The output of the `Matlab` functions should not only be the solution of the problem but also the intermediate values (path to the solution at t_{end} from the initial time $t_0 = 0$).

Exercise 9.2 (Euler methods in $\mathbb{R}^2$) **(10 points)**

Using the two above written function files compute a numerical solution of the following initial value problem

$$y'(t) = \begin{pmatrix} -4 & 6 \\ 31 & -189 \end{pmatrix} y(t), \qquad y(t) = \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix}, \qquad y(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

with the end time $t_{end} = 2$ and step size $h = \{0.01, 0.02\}$.
For each step size, plot the path of $y_1(t)$ and $y_2(t)$ separately. Each plot should include the paths from both the methods together with the analytical solution in the interval [0, 2]. Observe how the paths vary. Of the given step sizes, why does/doesn't a step size work for the two methods? Make a comment in your script. Based on your observations, what can you say about the nature of the problem? Justify your answer and make a comment in your script.

Hint: Mind the minus sign in the IVP!