Numerical Methods for Differential Equations
Assignment 10
Upload solutions until 11 July 2022, 3pm

Exercise 10.1 (Multi-Step Methods and Butcher tableau)   **(20 points)**
Consider the initial value problem

$$y'(t) = -ty(t), \qquad y(0) = 2,$$

with real-valued functions $y(t)$ and $y'(t)$.

(a) Compute the analytical solution of this IVP.

(b) Determine the numerical solution $y(4)$ (and the path to it from $y(0)$) by implementing the following methods in `Matlab`:

   (i) two step Adams-Moulton method and

   (ii) the method described by the following Butcher tableau

$$
\begin{array}{c|cccc}
0 & 0 & 0 & 0 & 0 \\
\frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\
\frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\
1 & 0 & 0 & 1 & 0 \\
\hline
 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6}
\end{array}
$$

with step size $h = \frac{1}{10}$. For the two-step method, first approximate $y(h)$ using the corresponding one-step method, and then continue with the two step method. Plot the paths obtained from the methods together with the analytical solution in the domain $[0, 4]$.