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Report on the
Discrete mathematical model's laboratory task No. 2

"Model Sensitivity and Uncertainty Analysis"

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Goal

Perform a sensitivity analysis for a demographic model with respect to a set of parameters: fertility rate, boys/girls ratio, «survival» rate for different age groups (not all can be taken). Model output: number of inhabitants for a given year. Test on the final forecast values for 10, 20, 50, 100 years. Also perform uncertainty analysis for the same model on different forecast values.

Formulation of the problem

Sensitivity analysis is the study of how the uncertainty in the output of a mathematical model or system can be correlated with various sources of uncertainty in its input and / or parameters. Uncertainty analysis – is a field of science that quantifies and reduces uncertainties in both computational and real-world applications. This type of analysis attempts to determine how likely certain results are if certain aspects of the system are not precisely known.

Solution method

All programming implemented with the Python programming language.

To perform the sensitivity analysis with Sobol model, we need to perform several tasks:

1. Define the model, it`s input and output variables.
2. Define the “problem” for the Sobol SA model.
3. Calculate the bounds of parameters for all the input variables from the historical data (1950-2000).
4. Calculate the coefficients for different forecast range.

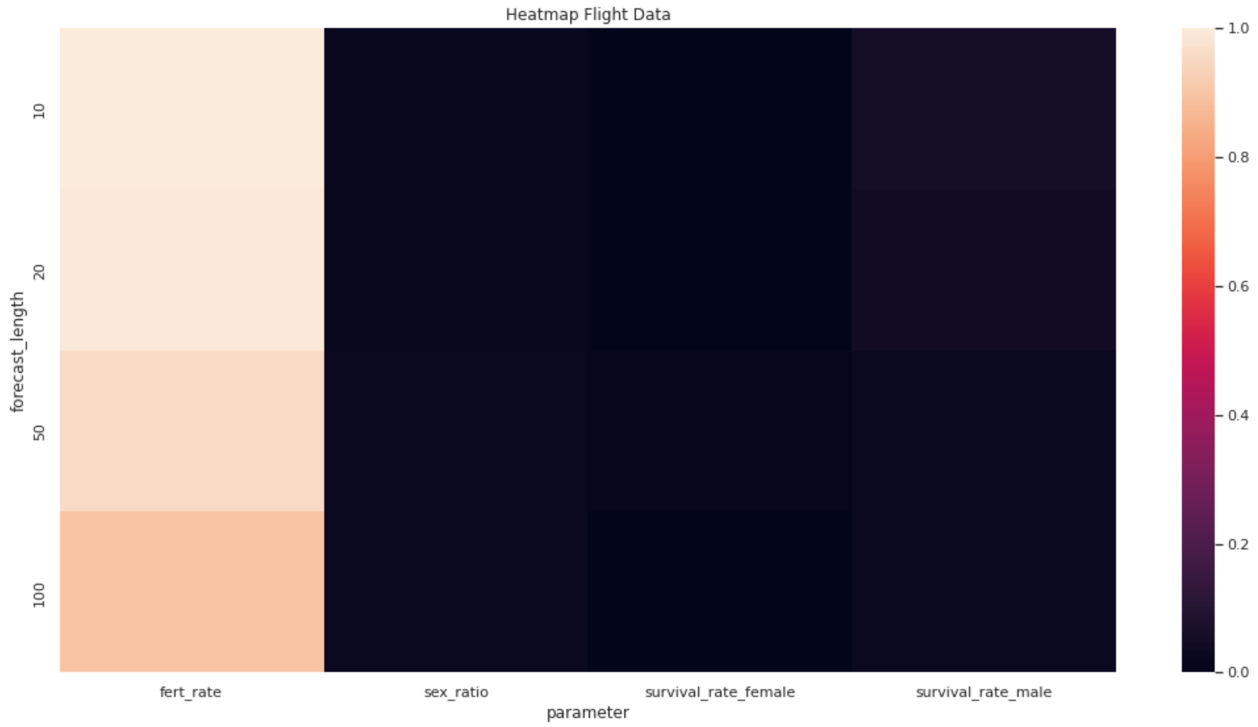
The necessary input variable was calculated from history data and bounds determined. For this bounds the parameters for model with saltelli function were generated. Then the Sobol model indexes were calculated for different forecast years – 10, 20, 50 and 100 years.

The sensitivity analysis code is presented at Listing 1.

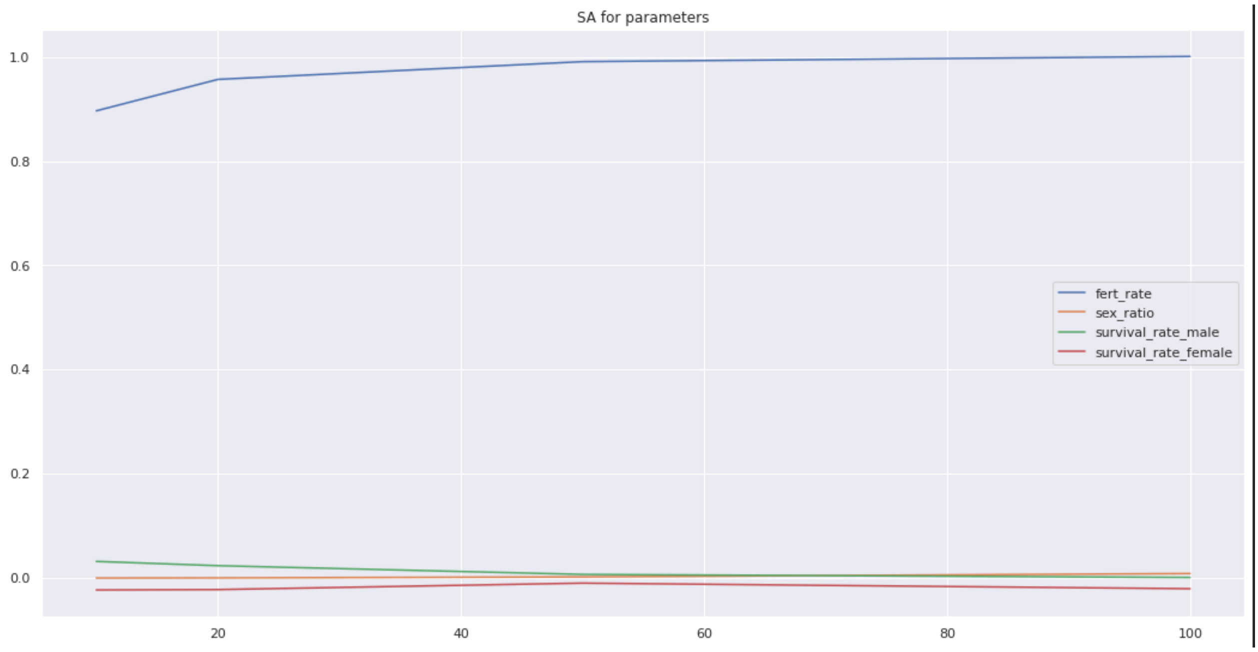
To perform uncertainty analysis the model was analyzed for all set of parameter samples generated for SA. For this set of models, the quantiles, mean and other statistics were calculated and analyzed.

Results

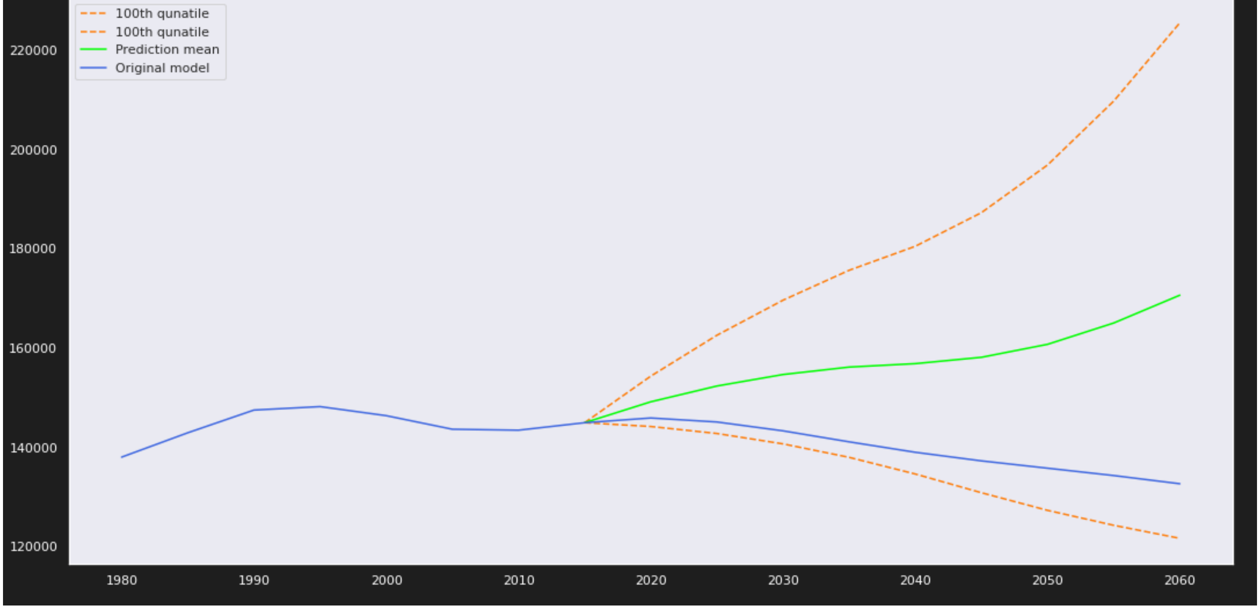
For defined model the forecast was made for 10, 20, 50 and 100 years. Then sensitivity analysis was performed, and the results plotted in form of heatmap and simple chart. From the results we can see that “fertility rate” parameter matters the most of all. Which is understandable because it has the biggest bounds variety and heavily impact forecasting results.



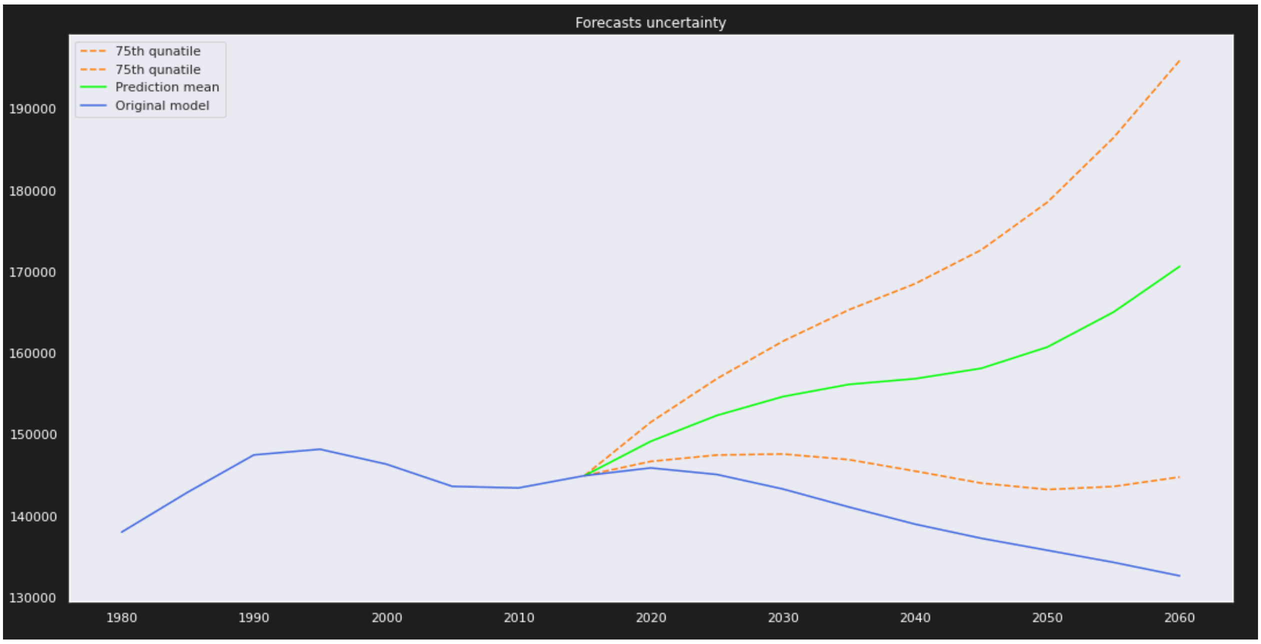
As we can see also fertility rate matters the most the longer forecast years is, which is explainable by the model definition.



The uncertainty analysis was performed for population model as well. The sample of parameters equal the sample of parameters defined in sensitivity analysis. The results were plotted in form of chart with quantiles and other statistics.



Plot of 75^{th} quantile:



Conclusions

In this laboratory task for discrete mathematical model of the population the sensitivity and uncertainty analyses were performed, and results analyzed. From the results we can say that the most important input parameter for the model is “fertility rate” with a huge gap. As for the uncertainty analyses, the base model of the forecast lies closer to the lower bound of the prediction interval (around 10^{th} quantile), which is understandable because most of parameter samples has higher fertility rates.

Appendix

Listing of code and charts in form of Python notebook: GitHub

Listing 1

```
problem = {
    "num_vars": 42,
    "names": ["fert_rate", "sex_ratio",
              [str("surv_rate_male_"+key) for key, value in AGE_GRPS_DICT.items()
if key != "100+"],
              [str("surv_rate_fmale_"+key) for key, value in
AGE_GRPS_DICT.items() if key != "100+"]],
    "bounds": [fertility_bound, sex_ratio_bound, *surv_rate_male_bounds,
*surv_rate_fmale_bounds]
}

param_values = saltelli.sample(problem, 32)

sobol_idx = []

for i in (10, 20, 50, 100):

    FORECAST_YEARS = i
    # Generate samples

    Y = Evaluate(param_values)

    # Analyzing
    Si = sobol.analyze(problem, Y, print_to_console=False)

    # Print the first-order sensitivity indices
    print("__________________")
    print(Si["S1"])

    sobol_idx.append({FORECAST_YEARS:Si})
```

Listing 2

```
# Uncertainty analysis
import statistics

def forecast(param_values, year_end = 2030):
    Y = []
    year_start = int(2015)

    for params in param_values:
        res = []

        fert_rate, sex_ratio, *surv_rates = params
        surv_rate_male = surv_rates[:20]
```

```
        surv_rate_fmale = surv_rates[20:]
        pop_data = rus_data[rus_data["Time"].isin(list(range(year_start-5,
year_start+1, 1)))]
        rus_pop = Population(pop_data, year_start, fert_rate, surv_rate_male,
surv_rate_fmale, sex_ratio)

        curr_year = 2015

        while curr_year <= year_end:
            res.append(rus_pop.get_total_pop())

            rus_pop.simulate_period_constant()
            curr_year += 5

        Y.append(res)
    return Y

param_values = saltelli.sample(problem, 32)
forecast_data = forecast(param_values, 2060)
```