EXERCISE 2(a)

$$\theta^5 (1-\theta)^2 = P(D|\theta)$$

(b) WE'RE LOOKING FOR:

$$\hat{\theta} = \arg\max_{\theta} P(D|\theta)$$

WE DO THIS BY FINDING THE MINIMUM OF $l(\theta; D)$

WHERE $l(\theta; D) = -\sum_{n=1}^{7} \log p(x_n|\theta)$

$$= -\log \theta^5 (1-\theta)^2 = -5 \log \theta - 2 \log 1-\theta$$

SOLVING BY DIFFERENTIATION YIELDS:

$$0 = \nabla l(\theta; D) = -\frac{5}{\theta} + \frac{2}{1-\theta}$$

$$\Rightarrow \hat{\theta} = \frac{5}{7}$$

AS FOR THE NEXT TWO TOSSES:

$$P(x_8 = \text{head}, x_9 = \text{head} \mid \hat{\theta}) = \hat{\theta}^2 = \frac{25}{49}$$

(c) FOR THE POSTERIOR ~~PROBABILITY~~ DISTRIBUTION:

$$p(\theta|D) = \frac{p(D|\theta)\, p(\theta)}{P(D)}$$

WITH $P(D|\theta)$ AS IN a), $p(\theta)$ AS GIVEN AND:

$$p(D) = \int_\theta p(D, \theta)\, d\theta = \int_\theta p(D|\theta)\, p(\theta)\, d\theta = \int_0^1 d\theta\, \theta^5 (1-\theta)^2$$

$$= \frac{1}{168}$$

TURN ⇒

CONTINUATION OF EXERCISE 2(C)...

HENCE, $p(\theta \mid D) = \begin{cases} 168\ \theta^5 (1-\theta)^2 & \text{IF } 0 \leq \theta \leq 1 \\ 0 & \text{ELSE} \end{cases}$

AND THE PROBABILITY THAT THE NEXT TWO TOSSES ARE HEAD IS:

$$\int_0^1 \theta^2 \cdot 168\ \theta^5 (1-\theta)^2\, d\theta = 168 \int_0^1 d\theta\ \theta^7 (1-\theta)^2$$

$$= \frac{168}{360} = \frac{7}{15}$$