

$$⑥ \quad x = (0, \frac{\ln 3}{3}, \frac{\ln 4}{4}, \frac{\ln 5}{5}, \dots)$$

$$x = \begin{cases} 0, & k=1 \\ \frac{\ln k+1}{k+1}, & k \geq 2 \end{cases}$$

$$\sum_{k=1}^{\infty} |x_k|^p = \sum_{k=2}^{\infty} \left| \frac{\ln k+1}{k+1} \right|^p + 0^p = \sum_{k=2}^{\infty} \left| \frac{\ln k+1}{k+1} \right|^p$$

$$\left(\frac{\ln k+1}{k+1} \right)^p \sim \left(\frac{\ln k}{k} \right)^p$$

при больших k $(\ln k)^p$ возрастает медленнее, чем $k^p \Rightarrow$

$$x_k^p \sim \frac{1}{k^p}$$

$$\sum_{k=2}^{\infty} \frac{1}{k^p} \text{ сходится при } p > 1 \Rightarrow p_{\min} = 2$$

$$\sum_{k=2}^{\infty} \frac{\ln^2(k+1)}{(k+1)^2} - \text{сходится по интегральному признаку}$$

$$\int_2^{\infty} \frac{\ln^2(k+1)}{(k+1)^2} dk = \lim_{x \rightarrow \infty} \int_2^x \frac{\ln^2(k+1)}{(k+1)^2} dk$$

$$\int \frac{\ln^2(k+1)}{(k+1)^2} dk = \int \frac{\ln^2 u}{u^2} du = -\frac{\ln^2 u}{u} + \int \frac{2 \ln u}{u^2} du = -\frac{\ln^2 u + 2 \ln u - 2}{u} = -\frac{\ln^2(k+1) + 2 \ln(k+1) - 2}{k+1}$$

$$\lim_{x \rightarrow \infty} \left(-\frac{\ln^2(k+1) + 2 \ln(k+1) - 2}{k+1} \right) \Big|_2^x = \lim_{x \rightarrow \infty} -\frac{\ln^2(x+1) + 2 \ln(x+1) - 2}{x+1} -$$

$$\lim_{x \rightarrow \infty} -\frac{\ln^2 3 + 2 \ln 3 - 2}{3} = 0 - \left(-\frac{\ln^2 3 + 2 \ln 3 - 2}{3} \right) = \frac{\ln^2 3 + 2 \ln 3 - 2}{3}$$

$$\Rightarrow \sum_{k=2}^{\infty} \frac{\ln^2(k+1)}{(k+1)^2} \approx \frac{\ln^2 3 + 2 \ln 3 - 2}{3}$$

• Ответ: $p_{\min} = 2$