Integer Optimization (Spring 2025): Homework 4

- **You must email your submission as a Zipped folder to kbala@wsu.edu**. Include the **PDF file with answers to proof problems** as well as all **files for the AMPL problems** in this folder.
- **Your folder name should identify you in the following manner. If you are Prince Of Canada, you should name your submission folder PrinceCanada_Hw4. If you want to add more bits to the title, e.g., Math567, you could name it PrinceCanada_Math567_Hw4, for instance. But start the folder name with PrinceCanada; and NOT "Prince Canada" or "Prin_Canada" or ...**
- Name your **PDF file** as **PrinceCanada_Hw4.pdf**.
- **Begin the SUBJECT of your email submission with the same `FirstnameLastname`, e.g., "PrinceCanada Hw4 submission".**
- The total points (given in parentheses) add up to 120. You will be graded for 115 points.
- **This homework is due by 11:59 PM on Friday, March 7**.

1. (10) Write down the sharp formulation for the set $S = \{\mathbf{x} \in \mathbb{R}^2 \mid (x_1 = 0,\ x_2 \geq 0) \vee (x_1 \geq 0,\ x_2 = 0)\}$, which actually violates Assumption 2 about having same recession cones. What is the set you have actually represented?

2. (10) In general, what is the set represented by the system in the sharp formulation (given as ($\divideontimes$-sharp) in Page 6 of Lecture 4) if we ignore Assumption 2?

3. (20) Prove that the formulation obtained from the CNF of the $\Leftarrow$ implication of the statement in Problem 4 (b) of Homework 2 is sharp. (The statement of interest here is $J_1 \wedge \cdots \wedge J_n \Rightarrow L_1 \vee \cdots \vee L_m$.)

4. (30) **[A]** We want to assign digits (or numbers) to the letters $W, D, O, T, G, L, E, C, M$ from out of 0–9 so that the following subtraction is true:

$$\begin{array}{cccccc} W & W & W & D & O & T - \\ G & O & O & G & L & E = \\ \hline D & O & T & C & O & M \end{array}$$

   There are nine letters, and each letter must be assigned a different number. Formulate this problem as an integer program and solve it using AMPL+Gurobi

5. (50) **[A]** Write an AMPL model to solve the Sudoku puzzle. Your model should be generic, i.e., it should work for any *standard* Sudoku puzzle of size $n^2 \times n^2$ for any n. Go to http://www.websudoku.com, download and solve one 9×9 puzzle in each of the four levels—Easy, Medium, Hard, and Evil. Record the running times and the number of branch-and-bound nodes taken to solve each problem. In your data files, note the *number* (or ID) of the corresponding puzzle.