Algebraic Topology (Fall 2023): Homework 1

- You **must email your submission** as a **PDF file** to kbala@wsu.edu. You are welcome to write answers by hand, and scan or take photos of the writings. Put all the images on a PDF file, though.
- Your file name should identify you in the following manner. If you are Eric Cartman, you should name your submission EricCartman_Hw1.pdf. Please avoid white spaces in the file name :-).
- The total points (given in parentheses) add up to 80.
- **This homework is due by 5 PM on Thursday, Aug 31**.

1. (20) Meet with me briefly—in person or on Zoom. You can do so during the scheduled Check-In hours, or set up another time with me. You are *not* required to ask me questions about this homework, but you are welcome to.

2. (15) Show that the closed unit interval ($[0, 1]$) and the closed upper unit semicircle are homeomorphic by explicitly defining the homeomorphism from the former to the latter.

3. (25) Assuming the definition of a topological space given in terms of neighborhoods (termed **Definition I** or **Def I** in Lecture 1), verify that the definition given in Lecture 2 in terms of open sets (termed **Def II**) is indeed correct. In other words, starting with **Def II**, check that all the axioms given as part of **Def I** hold.

4. (20) Let $\{\mathbf{a}_0, \ldots, \mathbf{a}_n\}$ be geometrically independent (GI), and let P be the n-plane spanned by these vectors. If $\mathbf{w} \notin P$, then show that $\{\mathbf{w}, \mathbf{a}_0, \ldots, \mathbf{a}_n\}$ is also GI.