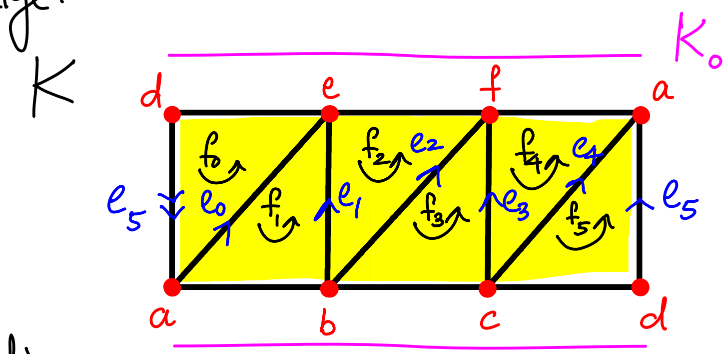
MATH 524 – Lecture 30 (12/07/2023)

Today: * relative cohomology on Möbius strip
* duality results

2. Möbius strip modulo its edge.



Relative 1- and 2-cochains:

f_i^* $(i=0,\dots,5)$ are all relative 2-cochains. They are all relative 2-cocycles (trivially).

$f_0^*,\dots,f_5^*$ form a basis for $Z^2(K,K_0)$.

Similarly, $e_0^*,\dots,e_5^*$ are relative 1-cochains.

$e_0^*,\dots,e_5^*$ form a basis for $C^1(K,K_0)$.

It is convenient to use f_0^*, $f_i^*-f_{i+1}^*$, $i=0,\dots,4$ as a basis for $Z^2(K,K_0)$, and $e_0^*,\dots,e_4^*$, $e_0^*+\dots+e_5^*$ as a basis for $C^1(K,K_0)$. Then

$$\delta e_i^* = f_i^* - f_{i+1}^*, \quad i=0,\dots,4, \quad \text{and}$$

$$\delta(e_0^*+\dots+e_5^*) = 2f_0^*$$

f_0^* is not a coboundary, but $2f_0^*$ is!

$$\Rightarrow H^2(K,K_0) \simeq \mathbb{Z}/2, \quad \{f_0^*\} \text{ is a generator.}$$

$H^1(K,K_0)=0$, as there are no relative 1-cocycles ($\delta e_i^* \neq 0 \ \forall i$).

Duality

§65 in [M]

<u>Poincaré duality</u> Let X be a compact triangulated <u>homology n-manifold</u>. generalizes the topological manifold If X is orientable, then for each p there exists an isomorphism

$$H^p(X;G) \simeq H_{n-p}(X;G)$$

where G is an arbitrary coefficient group.

If X is nonorientable, then for each p, there exists an isomorphism

$$H^p(X;\mathbb{Z}/2) \simeq H_{n-p}(X;\mathbb{Z}/2).$$

<u>Alexander duality</u> §71 in [M]

Let A be a proper nonempty subset of S^n. Suppose (S^n, A) is triangulable. Then there is an isomorphism

$$\tilde{H}^k(A) \simeq \tilde{H}_{n-k-1}(S^n - A)$$