Algebraic Topology (Fall 2023): Homework 3

- You **must email your submission** as a **PDF file** to kbala@wsu.edu. You are welcome to write answers by hand, and scan or take photos of the writings. Put all the images on a PDF file, though.
- Your file name should identify you **in the following manner**. For instance, if you are Ike McCormick, you should name your submission IkeMcCormick_Hw3.pdf (and **NOT** "Ike McCormick*pdf" or "Ike **mcc**ormick_*pdf or ...). Please avoid white spaces in the file name :-).
- **Begin the SUBJECT of your email submission with the same `FirstnameLastname`, e.g., "IkeMcCormick Hw3 submission".**
- The book of Munkres (Elements of Algebraic Topology) is denoted [M].
- **This homework is due by 11:59 PM on Thursday, September 28**.

1. (30) This problem is similar to the first part of Problem 9 in Page 14 of [M]. Let K be an infinite simplicial complex in $\mathbb{R}^d$ (for finite d). Recall the two topologies defined on $|K|$ (discussed in Lecture 3)—one taking it directly as a subspace of $\mathbb{R}^d$ along with the topology *induced* from $\mathbb{R}^d$, and the other defined by taking each simplex separately with its natural topology in $\mathbb{R}^d$, and then topologizing their union (some times called the *identification* topology). Assume that these two topologies are the same for K. Further, assume that $|K|$ is a closed subset of $\mathbb{R}^d$. Show that each $\mathbf{x} \in \mathbb{R}^d$ has a neighborhood that intersects at most a finite number of simplices of K. *Hint: Show that K is locally finite, and then use this fact to identify such a neighborhood.*

2. (10) [M] Problem 1 (c), Page 19. The word "describe" means you should identify what space this abstract simplicial complex represents, e.g., a sphere, torus, Möbius strip, etc.

3. (25) [M] Problem 2, Page 20. Same interpretation of the word "describe" as in the previous problem.

4. (20) [M] Problem 4, Page 20.

5. (35) [M] Problem 3, Page 26.