Computational Tests on CKPs

We used a modification of PROCEDURE CKP given in the main manuscript to generate ten instances of CKP with the structure illustrated by Example 2 (KP4), i.e., with $\mathbf{a}_1 = \mathbf{p}_1 M_1 + \mathbf{p}_2 M_2 + \mathbf{p}_3 M_3 + \mathbf{r}$, for $n = 50$. To keep the knapsack coefficients relatively small, the entries of $\mathbf{p}_1, \mathbf{p}_2$, and $\mathbf{p}_3$ are chosen randomly from $\{1, 2, 3\}$ and those of $\mathbf{r}$ from $\{-1, 0, 1\}$, such that no two a_j's are identical. We tried to solve the original formulations of the CKP, the CKP with $\mathbf{p}_1\mathbf{x}$ fixed, and the CKP with $\mathbf{p}_1\mathbf{x}$ and $\mathbf{p}_2\mathbf{x}$ fixed. All calculations are done on a Intel PC with 8 cores and a 2.33 GHz CPU. As an MIP solver, we used CPLEX 12.6.3.0. For feasibility versions of integer programs, the sum of the variables is used as the dummy objective function. Ideally, these problems are expected to become easier when $\mathbf{p}_1\mathbf{x}$, and then $\mathbf{p}_2\mathbf{x}$, are fixed. At the same time, all the subproblems obtained by fixing $\mathbf{p}_1\mathbf{x} = k_1 + 1$ and then $\mathbf{p}_2\mathbf{x} = k_2 + 1$ still remain relatively hard—they all remain unsolved after one hour of computational time. For the record, the number of B&B nodes examined within this time for all the runs was 41 $\pm$ 12 million (mean $\pm$ std. dev.). The details of the computations are provided in Table 1. Notice that the CKP instances remain relatively hard for ordinary B&B even after branching on both the hyperplanes defined by $\mathbf{p}_1\mathbf{x}$ and $\mathbf{p}_2\mathbf{x}$.

Table 1: Statistics for CKP instances of size $n = 50$ with $\mathrm{iwidth}(\mathbf{p}_1|\mathrm{CKP}) = 2$, $\mathrm{iwidth}(\mathbf{p}_2|(\mathrm{CKP}) \wedge \mathbf{p}_1\mathbf{x} = k_1 + 1) = 2$ and $\mathrm{iwidth}(\mathbf{p}_3|(\mathrm{CKP}) \wedge \mathbf{p}_1\mathbf{x} = k_1 + 1 \wedge \mathbf{p}_2\mathbf{x} = k_2 + 1) = 0$. $a_{\min}$ and $a_{\max}$ give the smallest and largest knapsack coefficients. w_i gives $\mathrm{width}(\mathbf{p}_i, \mathrm{CKP})$ for $i = 1, 2, 3$. w_{j1} gives $\mathrm{width}(\mathbf{p}_j, (\mathrm{CKP}) \wedge \mathbf{p}_1\mathbf{x} = k_1 + 1)$ for $j = 2, 3$. w_{312} gives $\mathrm{width}(\mathbf{p}_3, (\mathrm{CKP}) \wedge \mathbf{p}_1\mathbf{x} = k_1 + 1 \wedge \mathbf{p}_2\mathbf{x} = k_2 + 1)$. For each instance, the original CKP, the CKP after fixing $\mathbf{p}_1\mathbf{x} = k_1 + 1$, as well as the CKP after fixing both $\mathbf{p}_1\mathbf{x} = k_1 + 1$ and $\mathbf{p}_2\mathbf{x} = k_2 + 1$ were all unsolved after the one hour time limit. CBR gives the number of B&B nodes examined to solve the CBR-based reformulation, which was integer infeasible for every instance. The times taken to obtain the CBR-reformulation and to solve it were negligible—each step took less than 1 second.

	CKP numbers				CKP widths						CBR
#	$a_{\min}$	$a_{\max}$	β'	β	w_1	w_2	w_3	w_{21}	w_{31}	w_{312}	BB
1	13354	26674	424920	424921	2.005	42.02	41.89	2.443	38.92	0.946	7
2	12251	24467	367732	367733	2.073	40.90	43.07	2.421	38.84	0.944	11
3	14456	28877	416513	416514	2.050	42.91	40.35	2.083	37.37	0.944	8
4	14549	28490	461490	461491	2.033	42.95	42.17	2.037	38.57	0.941	2
5	15375	30716	457004	457005	2.007	42.01	41.82	2.011	39.60	0.943	10
6	11234	21946	326617	326618	2.077	38.93	40.86	2.449	39.02	0.943	20
7	11306	22578	336621	336622	2.077	38.97	39.74	2.369	37.73	0.943	16
8	14696	29358	437657	437658	2.054	44.96	41.80	2.161	38.74	0.943	10
9	15722	31407	453190	453191	2.050	44.91	42.20	2.141	37.95	0.947	12
10	15145	30255	466572	466573	2.036	42.95	39.77	2.011	37.61	0.944	14

All these instances of (CKP) are integer infeasible. They have an integer width of 2 along $\mathbf{p}_1$, and for one of the two subproblems created by branching on the hyperplane defined by $\mathbf{p}_1\mathbf{x}$, the integer width along $\mathbf{p}_2$ is 2 as well. The other subproblem is not guaranteed to have this property. In four of the instances, the other subproblem is solved by branching on the hyperplane defined by $\mathbf{p}_2\mathbf{x}$ (integer width along $\mathbf{p}_2$ is zero), while for the remaining six instances, the integer width along $\mathbf{p}_2$ is 1. For all ten instances, the integer infeasibility of the subproblems created by branching on the hyperplane defined by $\mathbf{p}_2\mathbf{x}$ (if any) is proven by branching on the hyperplane defined by $\mathbf{p}_3\mathbf{x}$ in the next level. As mentioned previously, this modified version of PROCEDURE CKP cannot be guaranteed to work for every choice of problem parameters. Still, we used the modified procedure as a guideline to search for appropriate parameters that generated instances with the desired structure. As part of the computational tests, we tried to solve the original CKP, the original CKP with $\mathbf{p}_1\mathbf{x}$ fixed, and also the original CKP with $\mathbf{p}_1\mathbf{x}$ and $\mathbf{p}_2\mathbf{x}$ fixed.

The instances are available online at http://www.math.wsu.edu/faculty/bkrishna/CKP/.