

MATH 273 - Lecture 1 (08/26/2014)

(11)

Calculus III - Section 2

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See the syllabus on the course web page. All info and documents relevant for this course will be available from the web page. In particular, no paper versions will be handed out in class.

Functions of several variables (Section 13.1 from the book).

Def Let D be a set of n -tuples of real numbers (x_1, \dots, x_n) . A real-valued **function** f on D is a rule that assigns a unique real number

$$w = f(x_1, \dots, x_n)$$

to each element in D .

D is the **domain** of f . The set of real values w taken by f is its **range**.

f is a function of the **independent variables** x_1, \dots, x_n .
 w is the **dependent variable**.

Another set of related terminology calls x_1, \dots, x_n the **input variables**, and w the **output variable**.

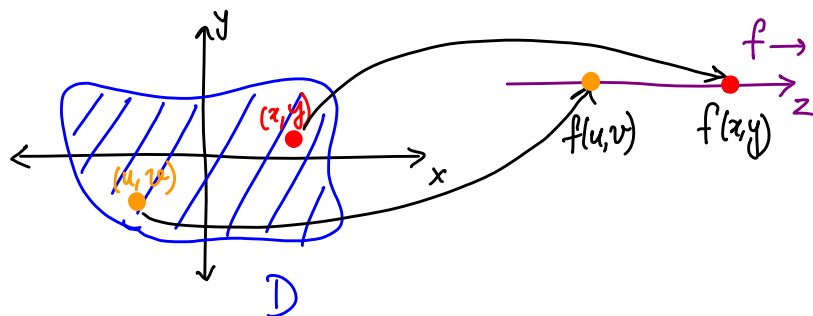
One could think of "putting in" x_1, x_2, \dots, x_n into the "box" that is f , and getting out w . For a given tuple (x_1, \dots, x_n) , the output w is unique.

Notation

In 2D, we write $z = f(x, y)$.

In 3D, we write $w = f(x, y, z)$.

Indeed, we will "draw" the 2D function in 3D using (x, y, z) coordinates!



Evaluating functions \rightarrow plug in the values of x and y !

Prob 2b (pg 692). $f(x, y) = \sin(xy)$

$$(b) \underset{x}{f}(-3, \underset{y}{\frac{\pi}{2}}) = \sin\left(-3 \times \frac{\pi}{2}\right) = \sin\left(-\frac{3\pi}{2}\right) = -\sin\left(\frac{\pi}{2}\right) = -\frac{1}{\sqrt{2}}.$$

Domains and Ranges (of functions)

We extend the ideas used to define the domain of functions of a single variable to those of several variables. As in the 1D case, we look to avoid division by zero and negative values inside square roots.

Def The **domain** of f is the largest set of variable values that generate real numbers as output values.
The **range** of f is the set of all real output values.

Notice that both the domain and the range are sets.

Prob 5 (pg 692)

$f(x,y) = \sqrt{y-x-2}$. Find and describe the domain of f .

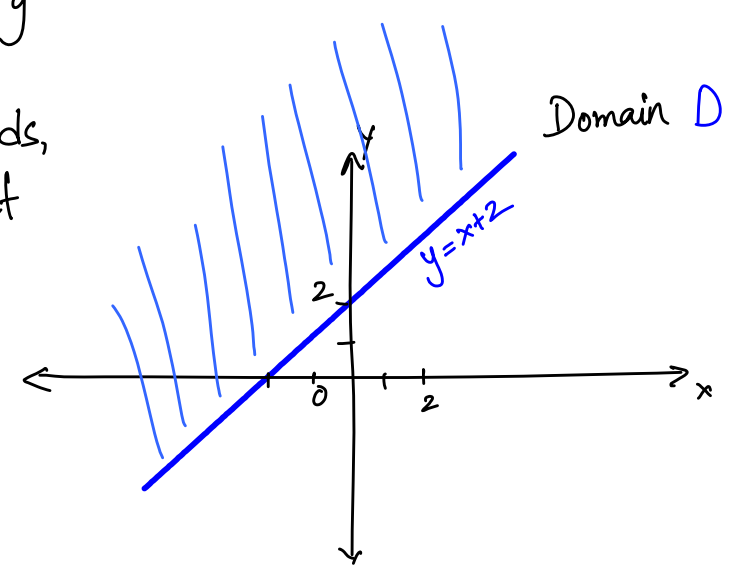
We need $y-x-2 \geq 0$, as the number inside the square root must be nonnegative. Indeed, the domain D of f here is the set of all pairs (x,y) of values in real 2D space that satisfy $y-x-2 \geq 0$.

Let's try to visualize D . For sets described by an inequality of this sort, we first plot the equation, and then pick the correct side of the line.

We first plot $y-x-2=0$, or $y=x+2$. This is the 45° line passing through $(0,2)$ and $(-2,0)$.

To pick the correct side, we try any point on the inequality to see if it holds. If it holds, that point is on the correct side. Since we could try any point, we try $(0,0)$ to keep things simple.

But, $0-0-2 \not\geq 0$. So $(0,0)$ is on the wrong side.



We now define properties of sets or regions that could describe the domain (and the function itself). In particular, we want to know if D is "open" or "closed", "bounded" (or finite) or "unbounded" etc.

Def

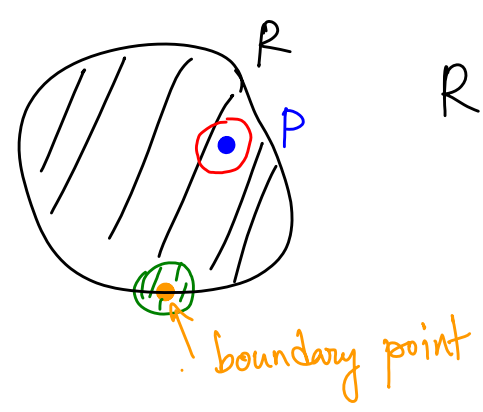
A point $P(x,y)$ in a set R in the plane is an **interior point** of R if P is the center of a disc of some positive radius lying entirely in R .

$P(x,y)$ is a **boundary point** of R if every disc centered at P has some points inside R and some outside. Notice that P itself need not be in R .

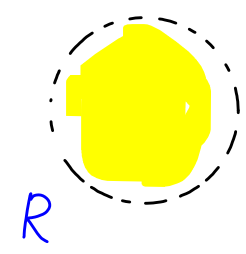
The set of all interior points : **interior** of R
 The set of all boundary points : **boundary** of R .

If R contains only interior points, it is **open**.

If R contains its boundary, it is **closed**.



R here is bounded.



R here is open, as it does not contain boundary points.