Calculus III (Math 273, Section 2) – Fall 2014
Practice Exam 1

- There are **eight** problems and **six** pages in this exam.
- Show all work, and provide appropriate **justifications** where required.
- Calculators, cell phones, laptops, or any other electronic devices are **not** allowed.
- Good luck!

---

1. **(14)** Find the domain and range of the function $f(x, y)$ given below, and identify its level curves. Sketch one typical level curve. Is the domain open or closed? Is the domain bounded?

$$f(x, y) = \sqrt{y - x^2}.$$

2. **(12)** Find the first partial derivatives with respect to each variable of the functions in each case.

   (a) $f(x, y) = \dfrac{x + y}{xy - 1}$. Simplify your answers.

   (b) $f(x, y, z) = \ln(2x + 3y - 5z)$.

3. **(12)** Find all second order partial derivatives of the function $g(x, y)$ given below.

$$g(x, y) = y \sin x - e^y$$

4. **(12)** Find $\dfrac{\partial w}{\partial s}$ when $r = \pi, s = 0$, if $w = \sin(2x - y),\ x = r + \sin s,\ y = rs$.

5. **(12)** Find dy/dx at the point $P(0, 1)$ when the following equation defines y implicitly as a function of x.

$$1 - x - y^2 - \sin xy = 0.$$

6. **(14)** Find the derivative of the function $f(x, y, z) = xyz$ in the direction of the velocity vector of the helix $\mathbf{r}(t) = (\cos 3t)\mathbf{i} + (\sin 3t)\mathbf{j} + 3t\mathbf{k}$ at $t = \pi/3$. Recall that the velocity vector of the curve $\mathbf{r}(t)$ is $d\mathbf{r}/dt$.

7. **(12)** What is largest value that the directional derivative of $f(x, y, z) = 1/xyz$ can have at the point $(1, 1, 1)$?

8. **(12)** Decide whether each of the following statements is *True* or *False*. **Justify** your answer.

   (a) If the domain of a function $f(x, y)$ is closed, then it cannot be unbounded.

   (b) When we have a dependent variable that depends on three intermediate variables and two independent variables, we draw three branch diagrams, one for each intermediate variable.

   (c) At the point (x_0, y_0), the vector ∇f is normal to the curve $f(x, y) = f(x_0, y_0)$.

   (d) the directional derivative of f in any direction $\mathbf{u}$ *different* from that of ∇f is strictly smaller than the derivative in the direction of ∇f.