

Algebraic Topology (Fall 2023): Homework 2

- You **must email your submission** as a **PDF file** to kbala@wsu.edu. You are welcome to write answers by hand, and scan or take photos of the writings. Put all the images on a PDF file, though.
 - Your file name should identify you **in the following manner**. For instance, if you are Scott Tenorman, you should name your submission ScottTenorman_Hw2.pdf (and **NOT** “Scott Tenorman*.pdf” or Scotttenorman_*.pdf or ...). **Please avoid white spaces in the file name :-).**
 - **Begin the SUBJECT of your email submission with the same FirstnameLastname, e.g., “ScottTenorman Hw2 submission”.**
 - The book of Munkres (Elements of Algebraic Topology) is denoted [M].
 - The total points (given in parentheses) add up to 110.
 - **This homework is due by 11:59 PM on Thursday, September 14.**
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1. (20) Given a simplicial complex K , show that each point $x \in |K|$, its underlying space, lies in the interior of exactly one simplex of K .
 2. (25) **[M] Prob 4(a), Page 7:** Let σ be the simplex spanned by $\{a_0, \dots, a_n\}$, τ its face spanned by $\{a_0, \dots, a_p\}$ for $p < n$, and ρ its face spanned by $\{a_{p+1}, \dots, a_n\}$. We call ρ the face of σ *opposite* τ . Show that σ is the union of all line segments joining points of τ to points of ρ , and that two such line segments intersect in at most one common end point.
 3. (20) **[M] Prob 5(b), Page 7:** Let U be a bounded open set in \mathbb{R}^d that is **star-convex** relative to 0 (origin), i.e., for every $x \in U$ the line segment from 0 to x lies in U . Show by example that \bar{U} (closure of U) need not be homeomorphic to \mathbb{B}^d , the d -dimensional unit ball.
 4. (25) Show that the vertices a_0, \dots, a_n of a simplicial complex span a simplex of K if and only if the intersection of their stars is nonempty.
 5. (20) **[M] Prob 2, Page 14:** Show that in general, the star and closed star are path connected, i.e., there exists a path between any two points in the set.