MATH 273 – Lecture 13 (10/07/2014)

Average on Exam 1: ~55.5

Offer: * If you score 90+ in Exam 2, your Exam 2 score will replace your Exam 1 score.

* If you score 85-89 in Exam 2, the weights for Exams 1 and 2 will be 5% and 35%, respectively

* If you score 80-84 in Exam 2, the weights for Exams 1 and 2 will be 10% and 30%, respectively.

Offer also applies with Exam 1 and 2 swapped!

Need to show up for the Exams to apply!!

Hw 6 due this FRIDAY, Oct 10, by 4 PM in my mailbox (VSCI 130).

Linearization of $f(x,y)$

In 1D, for a function $f(x)$ that is differentiable at $x=a$,

$L(x) = f(a) + f'(a)(x-a)$ is the linearization (or linear approximation) of $f(x)$ at $x=a$.

$f'(a)dx$ is the differential of f at $x=a$

estimates the small change in f for a small change dx in x.

Extending to 2- and higher dimensions, at $P_0(x_0,y_0)$

$$L(x,y) = f(x_0,y_0) + \underbrace{\left.\frac{\partial f}{\partial x}\right|_{P_0}(x-x_0) + \left.\frac{\partial f}{\partial y}\right|_{P_0}(y-y_0)}$$ is

the **standard linear approximation** of f at P_0.

total differential of f at P_0

$$= f_x\big|_{P_0}dx + f_y\big|_{P_0}dy + \underbrace{f_z\,dz}_{\text{if we have 3 variables}}$$

The linearization is a good approximation to $f(x,y)$ only for small changes in x and y, i.e., when $x-x_0$ and $y-y_0$ (or dx, dy) are small.

13-3

<u>Prob 29</u> $f(x,y) = e^x \cos y$. Find standard linear approximation $L(x,y)$ of $f(x,y)$ at (a) $(0,0)$ and (b) $(0, \pi/2)$.

$$L(x,y)\Big|_{P_0} = f(x_0,y_0) + f_x\Big|_{P_0}(x-x_0) + f_y\Big|_{P_0}(y-y_0)$$

$$f_x = e^x \cos y \qquad f_y = -e^x \sin y$$

(a) $P_0\,(\underset{x_0,\,y_0}{0,0})$

$$f(x_0,y_0) = e^0 \cos 0 = 1$$

$$f_x = e^0 \cos 0 = 1$$

$$f_y = -e^0 \sin 0 = 0$$

$$L(x,y) = \underset{f(x_0,y_0)}{1} + \underset{f_x}{1}(x - \underset{x_0}{0}) + \overset{f_y}{0}(y - \overset{y_0}{0})$$

$$= 1 + x.$$

(b) $P_0\,(\underset{x_0,\,y_0}{0, \pi/2})$

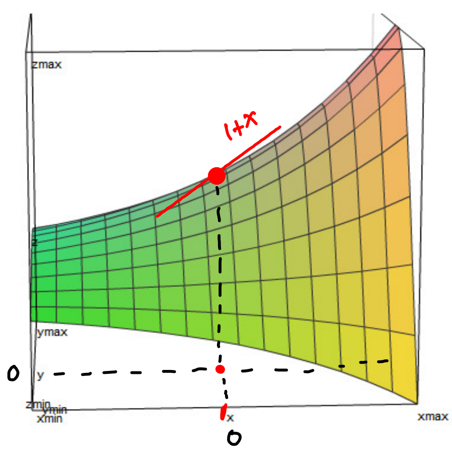
$$f(x_0,y_0) = e^0 \cos \pi/2 = 1 \times 0 = 0$$

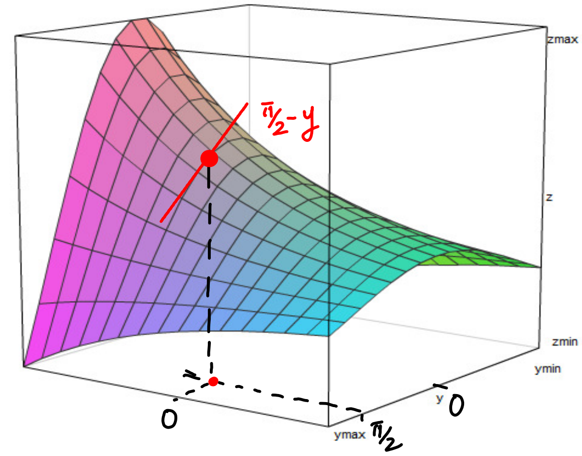
$$f_x = e^0 \cos \pi/2 = 0$$

$$f_y = -e^0 \sin \pi/2 = -1$$

$$L(x,y) = \underset{f(x_0,y_0)}{0} + \underset{f_x}{0}(x - \underset{x_0}{0}) + \underset{f_y}{(-1)}(y - \underset{y_0}{\pi/2})$$

$$= \pi/2 - y$$





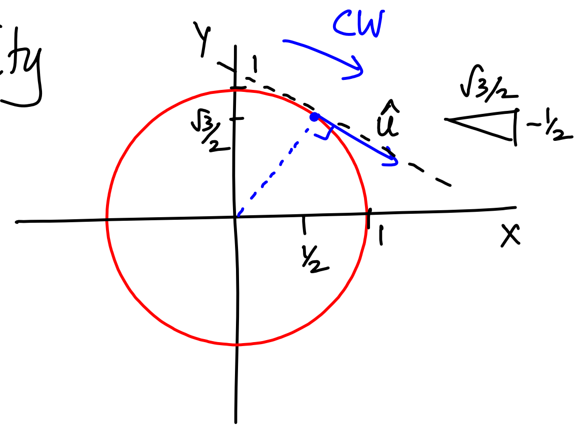
Prob 23 Temperature $T(x,y) = x \sin 2y$. A particle is moving clockwise along unit circle (centered at origin) at speed 2 m/s.

(a) How fast is T changing at $P\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ per meter?

(b) How fast is T changing with time at P?

$\hat{u}$: unit vector along the velocity direction

$$\hat{u} = \frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}$$



(a) $$\nabla T = \frac{\partial T}{\partial x}\hat{i} + \frac{\partial T}{\partial y}\hat{j}$$

$$= \sin 2y\,\hat{i} + 2x\cos 2y\,\hat{j}$$

$$(\nabla T)_P = \sin\sqrt{3}\,\hat{i} + 2\left(\tfrac{1}{2}\right)\cos\sqrt{3}\,\hat{j} = \sin\sqrt{3}\,\hat{i} + \cos\sqrt{3}\,\hat{j}$$

$P\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$$(D_{\hat{u}}T)_P = \nabla T_P \cdot \hat{u} = \left(\sin\sqrt{3}\,\hat{i} + \cos\sqrt{3}\,\hat{j}\right)\cdot\left(\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}\right)$$

$$= \frac{\sqrt{3}}{2}\sin\sqrt{3} - \frac{1}{2}\cos\sqrt{3} = 0.935\ {}^\circ C/m$$

(b) $$\frac{dT}{dt} = \nabla T \cdot \left(\frac{d\bar{r}}{dt}\right) \longrightarrow \text{velocity vector } \bar{v}$$

$$\frac{d\bar{r}}{dt} = |\bar{v}|\cdot\hat{u} = 2\left(\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}\right)$$

$$= \left(\sin\sqrt{3}\,\hat{i} + \cos\sqrt{3}\,\hat{j}\right)\cdot 2\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\hat{j}\right) = 2 \times 0.935 = 1.87\ {}^\circ C/s.$$

31. Wind chill factor Wind chill, a measure of the apparent temperature felt on exposed skin, is a function of air temperature and wind speed. The precise formula, updated by the National Weather Service in 2001 and based on modern heat transfer theory, a human face model, and skin tissue resistance, is

$$W = W(v, T) = 35.74 + 0.6215\,T - 35.75\,v^{0.16} + 0.4275\,T \cdot v^{0.16},$$

where T is air temperature in °F and v is wind speed in mph. A partial wind chill chart is given.

v (mph) \ T(°F)	**30**	**25**	**20**	**15**	**10**	**5**	**0**	**−5**	**−10**
5	25	19	13	7	1	−5	−11	−16	−22
10	21	15	9	3	−4	−10	−16	−22	−28
15	19	13	6	0	−7	−13	−19	−26	−32
20	17	11	4	−2	−9	−15	−22	−29	−35
25	16	9	3	−4	−11	−17	−24	−31	−37
30	15	8	1	−5	−12	−19	−26	−33	−39
35	14	7	0	−7	−14	−21	−27	−34	−41

a. Use the table to find $W(20, 25)$, $W(30, -10)$, and $W(15, 15)$.

b. Use the formula to find $W(10, -40)$, $W(50, -40)$, and $W(60, 30)$.

c. Find the linearization $L(v, T)$ of the function $W(v, T)$ at the point $(25, 5)$.

d. Use $L(v, T)$ in part (c) to estimate the following wind chill values.

i) $W(24, 6)$ **ii)** $W(27, 2)$

iii) $W(5, -10)$ (Explain why this value is much different from the value found in the table.)

at $P_0(v_0, T_0)$

$$L(v,T) = W(v_0, T_0) + \left.\frac{\partial W}{\partial v}\right|_{P_0}(v - v_0) + \left.\frac{\partial W}{\partial T}\right|_{P_0}(T - T_0)$$

At $P_0(\underset{v_0}{25}, \underset{T_0}{5})$

$$\frac{\partial W}{\partial v} = -35.75 \times 0.16\,v^{(0.16-1)} + (0.4275)(T)\,0.16\,v^{(0.16-1)}$$

$$\frac{\partial W}{\partial T} = 0.6215 + 0.4275\,v^{0.16}$$

We will finish this problem in the next lecture...