Integer Optimization (Spring 2025): Project 1

- This project will be worth 35% of the total credit for projects.
- You must submit a concise report explaining your methodology (IP formulations, any other heuristics or algorithms used), and briefly analyze the solution(s) obtained. You must also submit your AMPL files.
- Your email submission should include all files in a **compressed (e.g., tar-gzip-ed or zip-ed) folder** that identifies you starting with **`FirstnameLastname`**, e.g., if you're Player Of Cards, **PlayerCards_Project1.zip**.
- Begin the **SUBJECT** of your email submission with the same `FirstnameLastname`, e.g., **"PlayerCards Project 1 submission"**.
- **This project is due by 11:59 pm on Tuesday, March 25**.

The Hiker's Tour Problem

We are given an directed graph $G = (N, A)$ with node set $N = \{1, \ldots, n\}$ and arc set $A = \{(i, j) \,|\, i, j \in N\}$. Each arc (i, j) has the associated cost c_{ij}. We want to find a single circuit, or tour, W, with the following properties.

- Some nodes may be skipped, i.e., every node need **not** be visited;
- each node may be visited more than once;
- no arc is used more than once;
- sub-tours are allowed, as long as they are connected (at nodes); and
- $\sum_{(i,j)\in W} c_{ij} \geq L$, a given lower bound.

The motivation comes from a so called *hiker's problem*. There is a network of trails that possibly meet at several trail junctions. The goal of the hiker is to start at one trail junction, hike at least a minimum number of miles (say L) of trails, and return to the starting point. The hiker need not visit every trail junction, but wants to start and finish at the same point without repeating any trails. As such, we denote this problem as the hiker's tour problem (HTP).

The goal of this project is to come up with IP-based methods to solve this problem. Naturally, IP formulations for TSP introduced in class, or described in other resources, can be used as guides. You will also solve an asymmetric instance of HTP for $n = 20$ using one of the formulations you propose. The cost matrix (htp20.txt) for this problem is available from the course web page. The c_{ij} values are in the range $[1, 20]$, with a value of 10000 indicating that the corresponding arc is not present in the network. You may have to modify the original network in order to implement a correct formulation (by adding extra nodes or arcs).

1. (40) Write two different IP formulations for the HTP—one based on, or similar to, the Miller-Tucker-Zemlin (MTZ) formulation, and the other based on the subtour formulation. Explain clearly any modification you may have to make to the original graph so that your formulations work. Your formulation must give as the solution the *entire* HTP tour W, and not a collection of disjoint subtours.

2. (65) Implement the IP model based on the MTZ formulation in AMPL using Gurobi as the MIP solver. Solve the $n = 20$ HTP instance htp20.txt for $L = 200, 500, 800, 1000$. Point out the number of nodes visited by the HTP tour for each L. You must also report the solution times and the number of branch-and-bound nodes taken by AMPL/Gurobi to solve the IP in each case.