Assignment - Adaptive signal Processing ①

Name - Balasubramaniam M C   Roll - EE18B155

Q.1 (a).

Prob III.14 :-

$x =$ non zero real valued

To show :- $\frac{d|x|}{dx} = \text{sign}(x)$ where $\text{sign}(x) \triangleq \begin{cases} +1 & x>0 \\ -1 & x<0 \end{cases}$

$|x| = +x$ if $x>0$ $\therefore \frac{d|x|}{dx}, x>0 = +1$

$|x| = -x$ if $x<0$

and $\frac{d|x|}{dx} = -1$ if $x<0$

$\rightarrow \therefore \frac{d|x|}{dx} = \text{sign}(x) \triangleq \begin{cases} +1 & x>0 \\ -1 & x<0 \end{cases}$ Verified

$\boxed{\text{defined} - \text{sign}(0) = 0}$

Now, x is complex $= x_r + j x_i$

l_1 norm of $x \triangleq |x_r| + |x_i| = |x|$

For any function $g(x_r, y_i) = u(x_r, y_i) + j(v(x_r, y_i))$

$$\boxed{(z = x_r + j y_i) \quad \frac{\partial g}{\partial z} = \frac{1}{2}\left(\frac{\partial g}{\partial x_r} - j\frac{\partial g}{\partial y_i}\right)} \rightarrow \text{def}^n \text{ of complex gradient}$$

(x - complex valued) $|x| = g(x)$.

So, for $\frac{\partial |x|}{\partial x} = \frac{1}{2}\left(\frac{\partial |x|}{\partial x_r} - j\frac{\partial |x|}{\partial x_i}\right)$

$$\frac{\partial|x|}{\partial x} = \frac{1}{2}\left(\frac{\partial|x_r|}{\partial x_r} - j\frac{\partial|x_i|}{\partial x_i}\right)$$

$$\rightarrow \boxed{\frac{\partial|x|}{\partial x} = \frac{1}{2}\left[\text{sign}(x_r) - j\,\text{sign}(x_i)\right]}$$

Prob. III.15 :-

(a). Minimizing E (l_1 norm of e) ie $E(|e|)$.

$$\boxed{e = d - Uw} \qquad \min_{w} E(|e|).$$

$$e = e_r + je_i \qquad \boxed{J(w) = E(|e|).}$$

To show $\nabla_w J(w) = -E\left(U\left[\text{sign}(e_r) - j\,\text{sign}(e_i)\right]\left(\frac{1}{2}\right)\right)$.

w is a col. vector $= \text{col}\{w_1, w_2 \cdots w_m\}$

$$\therefore \nabla_w J(w) = \left[\partial J/\partial w_1 \;\; \partial J/\partial w_2 \cdots \partial J/\partial w_m\right]$$

row vector.

$$\nabla_w J(w) = \nabla_w E(|e|) = \nabla_w E(|e_r| + |e_i|).$$

$$\nabla_w J(w) = \left[\frac{\partial E(|e_r|)}{\partial w_1} \; \frac{\partial E(|e_r|)}{\partial w_2} \cdots \frac{\partial E(|e_r|)}{\partial w_m}\right] + \left[\frac{\partial E(|e_i|)}{\partial w_1} \; \frac{\partial E(|e_i|)}{\partial w_2} \cdots \frac{\partial E(|e_i|)}{\partial w_m}\right]$$

$$\frac{\partial E(|e_r|)}{\partial w_k} = \frac{\partial E(|e_r|)}{\partial e_r} \cdot \frac{\partial e_r}{\partial w_k} \qquad \text{Ily } \frac{\partial E(|e_i|)}{\partial w_k} = \frac{\partial E(|e_i|)}{\partial e_i}\frac{\partial e_i}{\partial w_k}.$$

$$e = d - UW = e_r + je_i = d_r + jd_i - [U_1 \cdots U_m]\begin{bmatrix} w_1 \\ \vdots \\ w_m \end{bmatrix}$$

$$\begin{bmatrix}(U_{1r}+jU_{1i}) & (U_{2r}+jU_{2i}) & \cdots & (U_{mr}+jU_{mi})\end{bmatrix}\begin{bmatrix} w_{1r}+jw_{1i} \\ \vdots \\ w_{mr}+jw_{mi} \end{bmatrix}$$

$$= (U_{1r}w_{1r} - U_{1i}w_{1i}) + j(U_{1r}w_{1i} + U_{1i}w_{1r})$$
$$+ (U_{2r}w_{2r} - U_{2r}w_{2i}) + j(U_{2r}w_{2i} + U_{2i}w_{2r})$$
$$\vdots$$

mth term

$$\therefore e = d_r + jd_i - \sum_{k=1}^{m}(U_{kr}w_{kr} - U_{ki}w_{ki})$$
$$- j\sum_{k=1}^{m}(U_{kr}w_{ki} + U_{ki}w_{kr})$$

$$\frac{\partial e_r}{\partial w_k} = \frac{1}{2}\left(\frac{\partial e_r}{\partial w_{kr}} - j\frac{\partial e_r}{\partial w_{ki}}\right) = -\frac{1}{2}\left(U_{kr} - j(-U_{ki})\right)$$
$$= -\frac{1}{2}(U_{kr} + jU_{ki})$$
$$= -\frac{1}{2}U_k$$

$$\frac{\partial e_i}{\partial w_k} = \frac{1}{2}\left(\frac{\partial e_i}{\partial w_{kr}} - j\frac{\partial e_i}{\partial w_{ki}}\right) = -\frac{1}{2}\left(U_{ki} - j(U_{kr})\right)$$
$$= +j\frac{(U_k)}{2}$$

(redundant)

$$\therefore \frac{\partial E(|e_r|)}{\partial (w_k)} = \frac{\partial E(|e_r|)}{\partial e_r}\left(-\frac{1}{2}\times U_k\right); \quad \frac{\partial E(|e_i|)}{\partial (w_k)} = \frac{\partial E(|e_i|)}{\partial e_i}\left(+\frac{jU_k}{2}\right)$$

$$\therefore \frac{\partial E(|e_r|)}{\partial w_k} = \cancel{E[\text{sign}(e_r)]} \neq E\left(\frac{\partial |e_r|}{\partial e_r} \cdot \frac{\partial e_r}{\partial w_k}\right)$$

[This is because mathematically $\frac{\partial E(|e_r|)}{\partial e_i} \cancel{\neq E[\text{sign}\, e_r]}$ is not directly computable, as it involves ~~functi~~ (U_k and w_k) (Also $\frac{\partial |e_i|}{\partial e_i}$ is not exactly continuous).

and $\frac{\partial E(|e_r|)}{\partial w_k} = E\left(\frac{\partial |e_r|}{\partial e_r} \frac{\partial e_r}{\partial w_k}\right)$

$$= E\left(\text{sign}(e_r) \cdot \frac{\partial e_r}{\partial w_k}\right)]$$

$$\therefore \frac{\partial E(|e_r|)}{\partial w_k} = E\left(\text{sign}(e_r) \times \left(\frac{-U_k}{2}\right)\right)$$

$$\text{lly} \quad \frac{\partial E(|e_i|)}{\partial w_i} = E\left(\text{sign}(e_i) \cdot \left(\frac{+jU_k}{2}\right)\right)$$

$$\therefore \nabla_w J(w) = \left[-E\left(\text{sign}(e_r)\frac{U_1}{2}\right) - E\left(\text{sign}(e_r)\frac{U_2}{2}\right) \cdots\right] + \left[E\left(\text{sign}(e_i)\left(\frac{+jU_1}{2}\right)\right) \; E\left(\text{sign}(e_i)\left(\frac{+jU_2}{2}\right)\right) \cdots\right]$$

$$\therefore \to \boxed{\nabla_w J(w) = -E\left[U_*\left(\text{sign}(e_r) - j\,\text{sign}(e_i)\right)\right]\left(\tfrac{1}{2}\right)}$$

(b). To show a steepest descent method can be obtained via

$$w_i = w_{i-1} + \frac{\mu}{2} E\, U^* \{\text{sign}[e_r(i)] + j\,\text{sign}[e_i(i)]\}.$$

Normally for steepest descent

$w_i = w_{i-1} + \mu p$ where $p = -A \nabla_w J(w)^H$.

If we choose $A = I$ then it would be the basic form of (5) steepest descent where

$$w_i = w_{i-1} + \mu(-\nabla_w^H J(w)).$$

$$\therefore w_i = w_{i-1} + \mu E\left[u_i^H \times (\text{sign}(e_r) + j\,\text{sign}(e_{imag}))\right](1/2)$$

$\Rightarrow$

$$\boxed{w_i = w_{i-1} + \frac{\mu}{2} E\left[u_i^* \times (\text{sign}(e_r) + j\,\text{sign}(e_{imag}))\right]}$$

Hence, Verified.

Prob III.29 (Sign-error LMS).

From before, $w_i = w_{i-1} + \frac{\mu}{2} \underbrace{E\left[u_i^* \times (\text{sgn}(e_r) + j\,\text{sgn}(e_{imag}))\right]}$

have this statistic, but we dont normally so we can do instantaneous approximation where we assume $E(AB) = ab$ where a, b are the instantaneous values of A, B

$\|^{ly}$ in this situation $E(u_i^* \times (\text{sgn}(e_r) + j\,\text{sgn}(e_{imag}))$

$\simeq u_i^* \times (\text{sgn}(e_r) + j\,\text{sgn}(e_{imag}))$

$(\text{csgn}(x) = \text{sgn}(x_r) + j\,\text{sgn}(x_{imag}))$

$\therefore w_i = w_{i-1} + \frac{\mu}{2}(u_i^* \times (\text{sgn}(e_r) + j\,\text{sgn}(e_{imag}))$

$(\mu' = \mu/2)$ $\Rightarrow w_i = w_{i-1} + \mu' u_i^* \times \text{csgn}(e_i)$

$(e_i = d_i - u_i w_{i-1})$ $\rightarrow$ $\boxed{\therefore w_i = w_{i-1} + \mu u_i^* \times \text{csgn}(d(i) - u_i w_{i-1})}$

Hence, proved.