NLMS Adaptive FIR Filter Design Method

Haichen Zhao[1,2], Shaolu Hu[1], Linhua Li[1], Xiaobo Wan[1]

[1] Micro-nanoelectronic and Software Key Laboratory of Guizhou Province, Guizhou University, Guiyang, China
[2] Shenzhen Key Laboratory of Information Science and Technology,
Graduate School at Shenzhen, Tsinghua University, Shenzhen, China

***Abstract*—This article presents a FIR filter design method which utilizes the NLMS (Normalized Least Mean Square) adaptive algorithm's system identification abilities. With such a method, any filters in hand including FIR, IIR or even analog ones can copy its responses to design the desired FIR filters. The FIR filter generated by such method can have the exact amplitude and phase responses with a target FIR filter which has a smaller or equal length, or obtain an amplitude response the same with an IIR or analog filter. In addition, it can design a FIR filter utilizing the structures' maximum, with a more excellent roll-off steepness and stopband attenuation than the traditionally designed FIR filter.**

Keywords—NLMS adaptive filter, FIR filter, IIR filter, system identification

I. INTRODUCTION

Traditionally there are some procedures to design a digital filter, and whenever the designed filter should be modified it needs to redesign the frequencies, calculate the parameters and then modify the filter. In some special practices, the redesign needed filters should change their filter types or even its lengths to match their new requirements. In a FPGA or ASIC design, the filter types and lengths are always fixed after the implementation; it can be a tough thing to change them.

Adaptive filters have the abilities of system identification, obtaining the unknown systems' linear characteristic. Such an unknown system can be a FIR filter, an IIR filter, or even, an analog one. In this way, it is possible to easily and fast adjust the filter by using a prototype in hand, or, a filter realized by a DSP to modify or set the filters in a FPGA/ASIC system.

Adaptive filters can use either FIR or IIR filter structures to implement [1~2], and use the adaptive algorithm to update their parameters. In practice, FIR structured adaptive filters can be easily achieved and with a nice performance thus being more widely used. Our research has used the FIR structured adaptive filter.

LMS (Least Mean Square) adaptive algorithm was mentioned by B. Widrow and Hoff in 1960 [3]. It is a converging algorithm based on the norm of MMSE (Minimum Mean Square Error) in Wiener filter theory and using a steepest descent method to achieve it. NLMS algorithm adds normalization to the LMS algorithm, its converging process behaves more stable than the counterpart.

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II. THE STRUCTURE OF NLMS ADAPTIVE FILTERS

NLMS Adaptive algorithm is always implemented with a direct-form structure [4]. The structure is displayed in Fig.1.

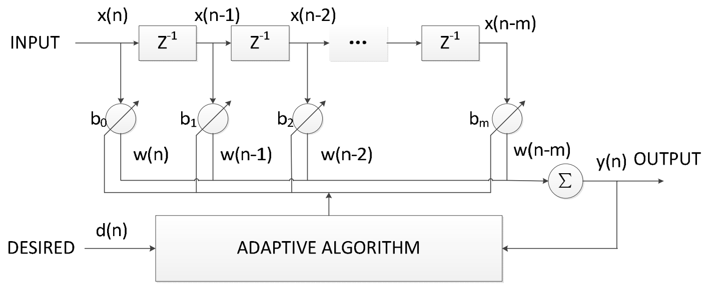

Fig. 1. NLMS adaptive filter with a direct-form structure.

The output of the NLMS adaptive filter is

$$y(n) = \sum_{k=0}^{N-1} b(k)x(n-k) = \mathbf{b}^T \mathbf{x} \tag{1}$$

y(n) and the desired signal d(n) make the error signal e(n)

$$\mathrm{e}(n) = d(n) - y(n) \tag{2}$$

The parameter vector $\mathbf{b}(n)$ is updated automatically by the feedbacks of error signal e(n)

$$\mathbf{b}(n) = \mathbf{b}(n-1) + f(\mathbf{x}(n), e(n), \mu) \tag{3}$$

$$f(\mathbf{x}(n), e(n), \mu) = \mu e(n) \frac{\mathbf{x}^*(n)}{\varepsilon + \mathbf{x}^H(n)\mathbf{x}(n)} \tag{4}$$

When get rid of the adaptive algorithm part, direct-form NLMS adaptive filter becomes the direct-form FIR filter as in Fig.2.

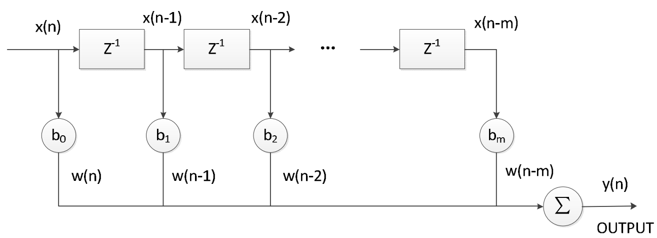

Fig. 2. Direct-form structure of FIR filter.

The output y(n) in the direct-form FIR filter structure is

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$$y(n) = \sum_{k=0}^{N-1} b(k)x(n-k) = \mathbf{b}^T \mathbf{x} \quad (5)$$

Comparing (1) and (5) we could easily find the filtering principles between the direct-form FIR filter and the direct-form NLMS filter are almost the same.

III. THE CONSTRUCTION OF RESEARCH SYSTEM

The direct-form NLMS filter is actually a variable parameters FIR filter with an adaptive algorithm. If it fixes the parameters when they are desired, the adaptive filter becomes the needed parameters fixed direct-form FIR filter.

The adaptive filter and the target filter should be paralleled when using the adaptive filter to model a target filter as in Fig.3. A signal generator is used to generate the drive signal x(n) which drives both input ports of target filter and adaptive filter. The target filter output d(n) and the adaptive filter output y(n) are to make the error signal (2). The adaptive process equals to make mean square error $\mathrm{e}^2(n)$ to its minimum to obtain the optimal wiener parameters. The responses of the adaptive filter become the optimal converged result when $\mathrm{e}^2(n)$ reaches its minimum.

When the adaptive filter is converging to a FIR or IIR filter, analyzed from frequency domain it is like the adaptive filter's responses of every frequency components converging to the target filter's one by one. Therefore, a drive signal which has non-zero PSD (power spectral density) across the whole frequency is needed to make the adaptive filter's responses converging to the target filter's on each corresponding frequency. As a white noise have the needed non-zero PSD on the whole frequency it is selected as the drive signal x(n).

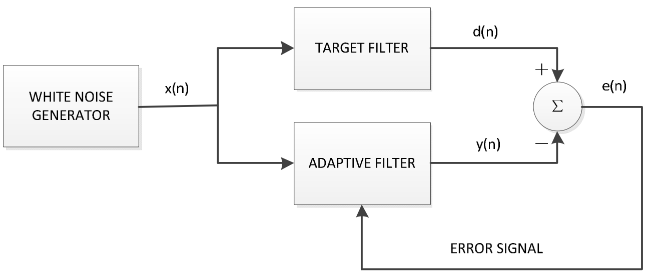

Fig. 3. Target filter modeled by the adaptive filter.

Based on the discussions above, a research system is constructed in MATLAB. The calculations in the system are double precision. The drive signal is a band limited noise with a sample rate at 8000Hz, and the height of its PSD is 1.

As white noise x(n) inputs to the target filter to get its output signal d(n), d(n) is used as the desired signal to the desired signal input port on the NLMS adaptive filter. The input port on the NLMS adaptive filter inputs the same source band limited white noise x(n). The NLMS adaptive algorithm is implemented with a filter length 99 and a step parameter fixed at $\mu = 0.01$ [6].

To assess the converging progress of the NLMS adaptive filter, the matrix squares to the parameters vector $\mathbf{b}$ of the adaptive filter is computed:

$$A = \mathbf{b}^H * \mathbf{b} \quad (6)$$

$\mathbf{b}^H$ is the Hermitian transpose and the parameters vector $\mathbf{b}$ is with a dimension N*1.

When the converging is completed, (6) should almost be a constant. In this way it can assess whether the converging process of the NLMS adaptive filter is complete.

IV. PERFORMANCES WITH DIFFERENT TARGET FILTERS

Direct-form FIR filter structure can form most of the filters [7], including the usually used low pass filters, high pass filters, band pass filters, band stop filters, multiband filters and a lot other ones. The NLMS adaptive filters use a direct-form FIR structure processing their filtering; therefore any responses can be formed by the direct-form FIR structure may be possibly achieved by the NLMS adaptive filter.

Because the NLMS algorithm uses the steepest descent method converging to the optimal Wiener estimation, and the norm in Wiener filtering is the MMSE (Minimum Mean Square Error), which demand (7) to be at its minimum, The NLMS adaptive filter is able to explore the FIR structure's maximum abilities by such a converging.

$$\mathrm{E}[\mathrm{e}^2(n)] = E[d(n) - y(n)]^2 \quad (7)$$

A. A FIR filter target with a not larger length

With a proper step variable μ and an enough calculating precision, if the target filter is a FIR structure filter with a length not larger than the adaptive one, it can be modeled exactly the same responses of both amplitude and phase.

To test the efficiencies of the NLMS adaptive filter modeling a FIR filter whose length not larger than the adaptive one, a direct-form multiband equiripple filter [8~12] is chosen as the target filter, as the target's responses contain various components similar to different kinds of filters. Its length 99 is the same with the adaptive one, and the design parameters in detail are shown in Table I.

TABLE I. DESIGN PARAMETERS OF TARGET MULTIBAND EQUIRIPPLE FILTER

Parameters	Value
Response Type	Multiband
Design Method	Equiripple FIR
Filter Order	98 (Length=99)
Freq. vector (Normalized)	[0, 0.28, 0.3, 0.48, 0.5, 0.69, 0.7, 0.8, 0.81, 1]
Mag. Vector	[0, 0, 1, 1, 0, 0, 1, 1, 0, 0]
Weight vector	[1, 1, 1, 1, 1]

Shown in Fig.5, when matrix squares (6) are almost steady as a constant, it is indicating a completion of the converging. Comparing the responses between the converging result and the target, the amplitude and phase responses of both filters are drawn in the same figure of Fig.4.

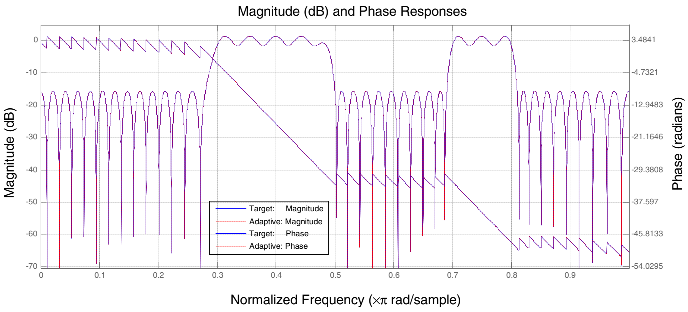

Fig. 4. Response comparisons between the converging result and the target.

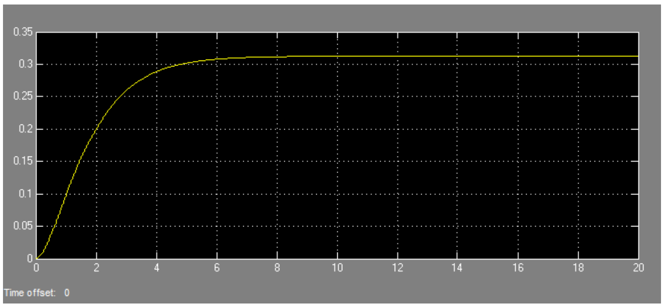

Fig. 5. Matrix squares (6) to the parameters vector $\mathbf{b}$ of the adaptive filter.

In Fig 4, both the amplitude and phase responses of both filters are almost the same. In Fig.5, it is clear matrix squares (6) hold a steady constant later.

Whatever direct-form FIR target filters in the series of tests if the lengths of the targets are not larger than the adaptive one, the NLMS adaptive algorithm can always perform the same thing, that is, both the amplitude and phase responses of both filters can be almost the same.

B. A low order IIR filter target

Because an IIR filter can achieve a similar amplitude response with a much lower order than its FIR counterpart, and when the NLMS adaptive filter achieves something beyond the edge its structure or length can provide, some instability shall occur (which shall discuss later), a proper (small) length IIR filter should be chosen to test.

To test the efficiencies of the NLMS adaptive filter modeling a IIR filter with a smaller length, a 10th order Butterworth bandpass IIR filter is chosen as the target filter, the length of the NLMS adaptive filter is 99. The design parameters of the target filter in details are shown in Table II.

TABLE II. DESIGN PARAMETERS OF THE BUTTERWORTH BANDPASS IIR FILTER

Parameters	Value
Response Type	Bandpass
Design Method	Butterworth IIR
Filter Order	10
Fc1 (Normalized)	0.3
Fc2 (Normalized)	0.7

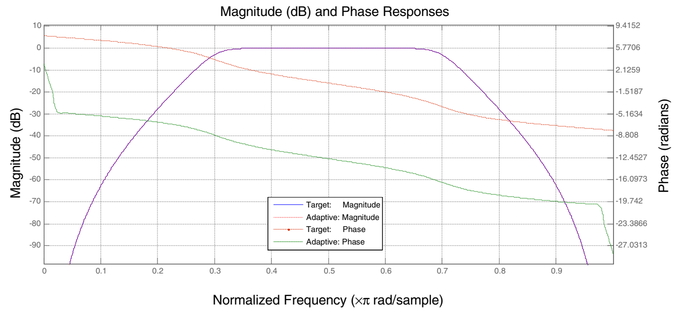

Fig. 6. Response comparisons between the converging result and the target.

Shown in Fig.6, both amplitude responses are almost the same though the phase responses are various.

There exist huge differences between the structures of FIR and IIR filters, as the transfer function of IIR filter is

$$H(z) = \sum_{k=0}^{M} b_k z^{-k} \,/\, \sum_{l=1}^{N} a_l z^{-l} \tag{8}$$

While the transfer function of FIR filter is

$$H(z) = \sum_{k=0}^{M} b_k z^{-k} \tag{9}$$

The obvious differences in the structures lead to an impossible task for a FIR structure to match totally exact responses both in amplitude and phase an IIR structure can have. When the seemingly same amplitude responses in Fig.6 enlarged as in Fig.7, some slight differences on some remote parts (under 100 dB) can be found.

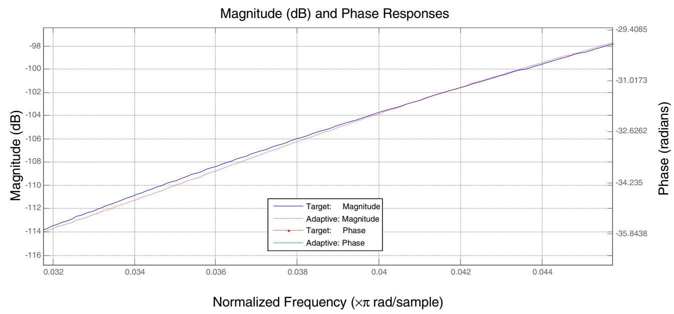

Fig. 7. Slight differences on some remote parts (under 100 dB).

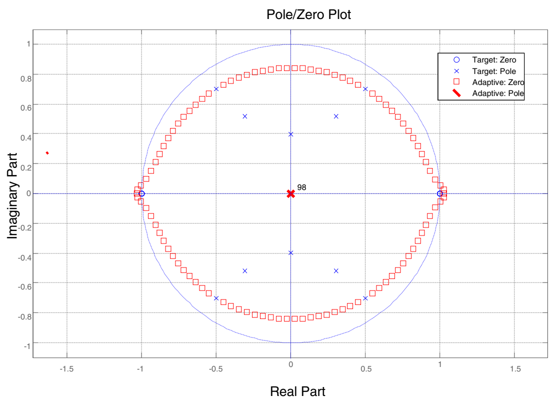

Fig. 8. The poles and zeros of the converging result and its IIR prototype.

Because the direct-form FIR filter transfer function doesn't have a denominator variable, the poles of the converging results are always centered at the centre of the circle, as Fig.8 shows, it is stable.

C. A FIR filter target with a larger length

The NLMS adaptive filter converges to a larger length FIR filter contains two situations. The first situation is the responses of the target larger length FIR filter are within the description by the minor length structure. For example the result of a 150th order adaptive filter converging to a 90th order FIR filter, is again being converged by a 99th order adaptive. If the responses of the target larger length FIR filter exceed the describing abilities by the minor length structure, which is the second situation, the adaptive filter shall never completely converge to. Such a case is similar to the former test when IIR filter as the target.

If the responses of the target larger length FIR filter exceed but not far from the describing abilities by the minor length, the adaptive filter can form a FIR filter have a better amplitude response than any traditionally designed same length FIR filter can ever achieve. To get a more obvious result, the following test use a NLMS adaptive filter with a length of 19, and the target FIR filter is with the length of 21 (see Table III) and for better comparison, a comparing FIR filter is designed with the same design parameters but a same length with the adaptive one of 19 (see Table IV).

TABLE III. DESIGN PARAMETERS OF THE TARGET FIR FILTER

Parameters	Value
Response Type	Lowpass
Design Method	Window: Hann
Filter Length	21
Fc (Normalized)	0.5

TABLE IV. DESIGN PARAMETERS OF THE COMPARING FIR FILTER

Parameters	Value
Response Type	Lowpass
Design Method	Window: Hann
Filter Length	19
Fc (Normalized)	0.5

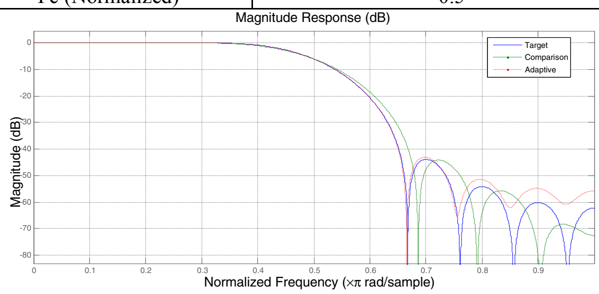

Fig. 9. Comparisons among converging result, target and comparing filters in amplitude responses.

Shown in Fig.9, the converging result has a similar roll-off steepness and stopband attenuation with the larger length FIR filter which are better than the comparing same length FIR filter. But one disadvantage such ways may result is the missing of the FIR filters' linear-phase feature, which is shown in Fig.10.

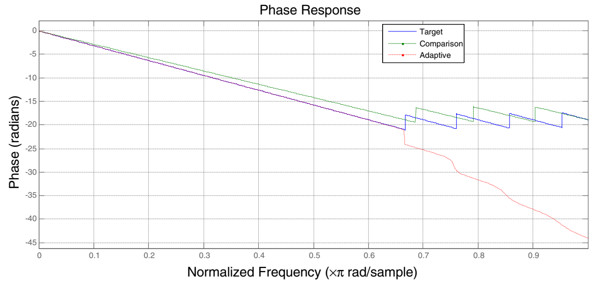

Fig. 10. Comparisons among converging result, target and comparing filters in phase responses.

D. A target far exceeding the edge

When the responses of the target filter far exceed the describing abilities by the minor length structure, it would lead to instability. A series of results indicate that the result responses trend to converge the bigger values in the amplitude response primarily, then the outline curve is following the bigger values stretching down. When reaching a point the adaptive structure cannot converge to, it starts to diffuse.

The following test uses a length 99 NLMS adaptive filter and a 50th order IIR filter (see Table V).

TABLE V. DESIGN PARAMETERS OF THE TARGET IIR FILTER.

Parameters	Value
Response Type	Bandpass
Design Method	Butterworth IIR
Filter Order	50
Fc1 (Normalized)	0.3
Fc2 (Normalized)	0.7

Shown in Fig.11, when the adaptive filter reaches the describing edge, it starts to diffuse. If the target filter's roll-off steepness is too tough, along with a low order adaptive filter, the adaptive filter may even impossibly form a stable response.

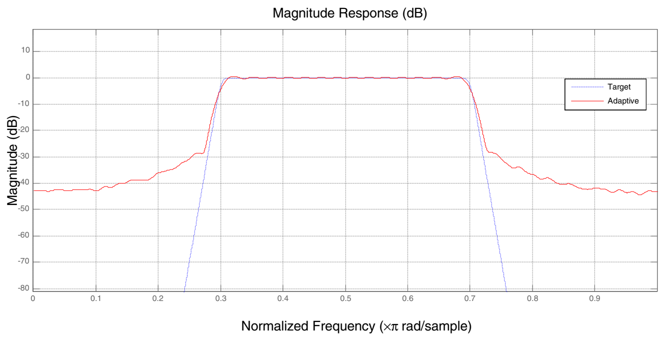

Fig. 11. Comparisons between converging result and the target IIR filter in amplitude responses.

E. An analog passive filter target

The responses of an analog filter are similar to the IIR filter which is designed by using the former as its prototype. Similarly, the prototype of a Butterworth IIR filter is the corresponding Butterworth analog filter. To model the analog ones an A/D converter should be set between the output port on the analog filter and the desired signal input port on the NLMS adaptive filter.

The drive signal in this test is still a band limited white noise with a sample rate 8 kHz, the height of its PSD is 1. SIMULINK is used to simulate such a test. The SIMULINK automatically holds the values during the intervals between each sample of the band limited white noise.

The "Analog Filter Design" block in the Simulink Library is used to design the analog filter [13], and a "Rate Transition" block is used to sample the output of the analog filter with a sample rate 8 kHz. The output port on the "Rate Transition" block links to the desired signal input port on the length 99 NLMS adaptive filter. The design parameters of the analog filter are shown in Table VI.

TABLE VI. DESIGN PARAMETERS OF THE ANALOG FILTER.

Parameters	Value
Response Type	Lowpass
Design Method	Butterworth
Filter Order	8
Passband Edge Frequency (rad/s)	12560(=2*3.14*2000)

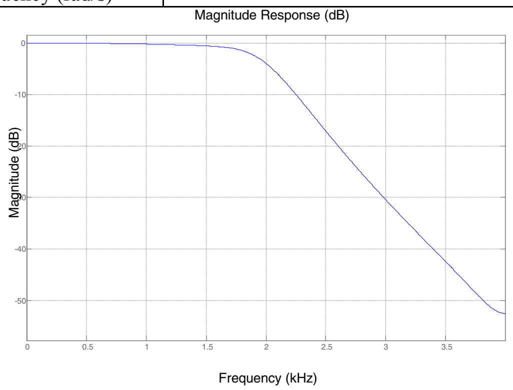

Fig. 12. The amplitude response of the converging result.

Shown in Fig.12, the amplitude response of the NLMS adaptive filter converging result is according with the analog filter desired amplitude response. The test proves that using the NLMS adaptive FIR design method the conversion from an analog filter to a FIR one is possible.

V. CONCLUSION

With the NLMS adaptive algorithm, NLMS adaptive filters are able to model any filters including the analog ones whose responses are within the description by the adaptive filters' length and their FIR structure.

When the target filter is a FIR one with a smaller or equal length than the NLMS adaptive filter, along with a proper step variable μ and an enough calculating precision, the adaptive filter can be converged to have both exact amplitude and phase responses of the target filter. When the target filter is a low order IIR filter, the adaptive filter can still model the amplitude response perfectly, but considering the huge differences between the FIR and the IIR filter structures, the phase response are reasonably various. When the target filter has responses exceeding the describing abilities by the adaptive filter's length and its FIR structure, the adaptive filter can still converge to some degree, but without stability.

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