Experiment 1

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Introduction:

The first part of the assignment mainly deals with plotting certain periodic/non-periodic signals and finding their power spectral densities. The time domain signal has a time window chosen as per sampling rate and number of cycles to be taken. The power spectral density is calculated by averaging over the absolute square of discrete fourier transform value samples.The following shows the PSD relation for a real signal:

$$PSD = \frac{2}{N * fs}|X(k)|^2 \tag{1}$$

where, 1 <= k<=N/2-1 (integer k ϵ [0,N-1])
N is number of time, frequency samples
fs is sampling rate
$|X(k)|$ is the DFT at kth frequency bin
Since the signal is real, we take twice the absolute square of DFT for frequencies in between 0, fs/2 considering symmetric conjugate DFT. This formula in a sense approximates the energy in the kth frequency bin and takes an average over the frequency resolution - fs/N.

The time window size, time resolution/sampling period , frequency window size, frequency resolution are decided by the nyquist criterion and the DFT relations:

$$fs >= 2 * fb \tag{2}$$

$$Freq.\ Resolution = fs/N \tag{3}$$

$$Freq.\ window = fs/2 \tag{4}$$

$$Time\ window = N * Ts \tag{5}$$

$$Time\ resolution = Ts = 1/fs \tag{6}$$

As we can see all of them are related by N, Ts the total samples(decided by total cycles) and sampling period. The second part of this assignment deals with finding the 3dB bandwidth of the emission spectrum of an LED.

Voltage and PSD plots:

To find the PSD of various signals using fft, the following procedure is used:

```
ydft = fft(y);
N = length(y);
ydft = ydft(1:N/2+1); %[0 or dc  to pi freq.] or [0 to fs/2]
y_enrg = (abs(ydft).^2);
%so if we want the spectral energy in (-pi, pi],
%we need to twice the samples between 0,pi
%excluding those at 0, pi. (The signal is also real so symmetric conjugate
%FT).
y_enrg(2:end-1) = 2*y_enrg(2:end-1);
%N time samples, and fs occupies -pi to pi, hence the division to get PSD.
y_psd = (y_enrg/N)/fs;
```

As we can see, we take twice the value only for k= 1 to N/2-1 (k ϵ [0,N-1], since the values corresponding to 0 and fs/2 frequencies occur only once.

The Sinusoidal signal:

We can plot the sinusoid easily with the 'sin' math function and the number of time samples taken for 10 cycles is chosen as per the sampling frequency which is taken as 100KHz, sufficiently large enough to satisfy the nyquist criterion. Therefore the time resolution is chosen as 0.01ms. The time window is fixed at 10ms as per the given frequency of the sinusoid and the total number of cycles. The frequency resolution is also fixed given sampling rate and no. of time domain samples as per eqn.2 = 100Hz. Since the signal is real, we take a frequency window upto fs/2. Hence, the frequency window is 50KHz. The plots of signal and its PSD are shown in fig.1 and 2,

The Square wave:

The square wave is also easily plotted with the 'square' function (with argument of 50 for duty cycle). The sampling rate is chosen again as 100kHz hence giving a time resolution of 0.01ms. The time window is fixed as 10ms since the frequency given is 1kHz and no. of cycles is 10. The sampling rate is chosen as such because although the square wave has infinite number of odd harmonics, we can approximately take the one sided bandwidth of around 49kHz (as we see in the psd its contribution is around -70dB/Hz) and hence, the sampling frequency of 100khz > 2*49kHz satisfies the nyquist criterion. The frequency window decided by sampling rate is fs/2 = 50kHz

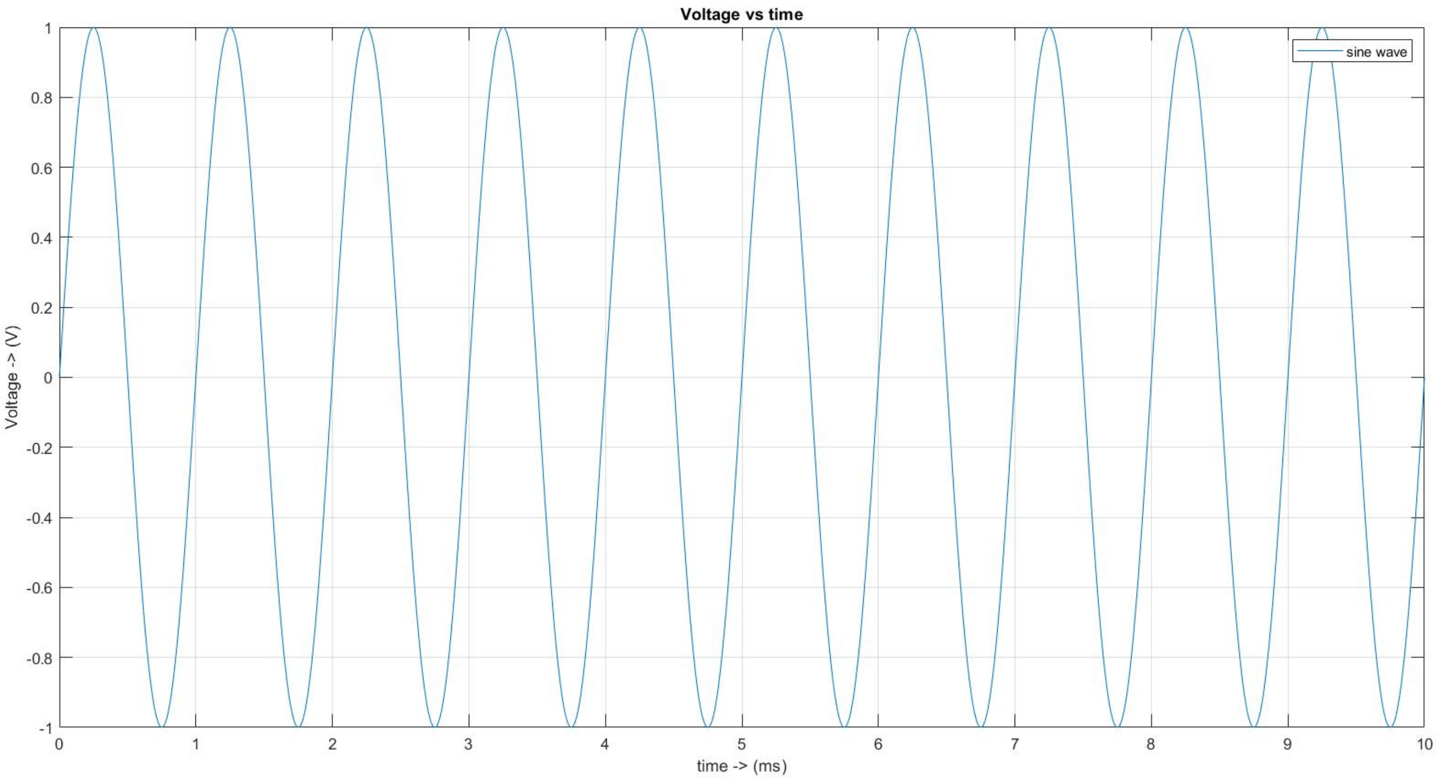

Figure 1: Voltage vs time for a sinusoid

for the real signal.The frequency resolution = fs/N is 100Hz. Fig. 3 and 4 show the voltage plots and PSD of the square wave of a duty cycle of 0.5.

PRBS pulse signals:

Generation of PRBS signals is done by the following code:

```
%generating prbs sequences of order 9.
%Hence, for an order n we put n in place of 9
y_prbs9= prbs(9,no_seq*9);
%replicating each value over the time window of the bit.
y_p9 = repmat(transpose(y_prbs9),1,N_cyc);
```

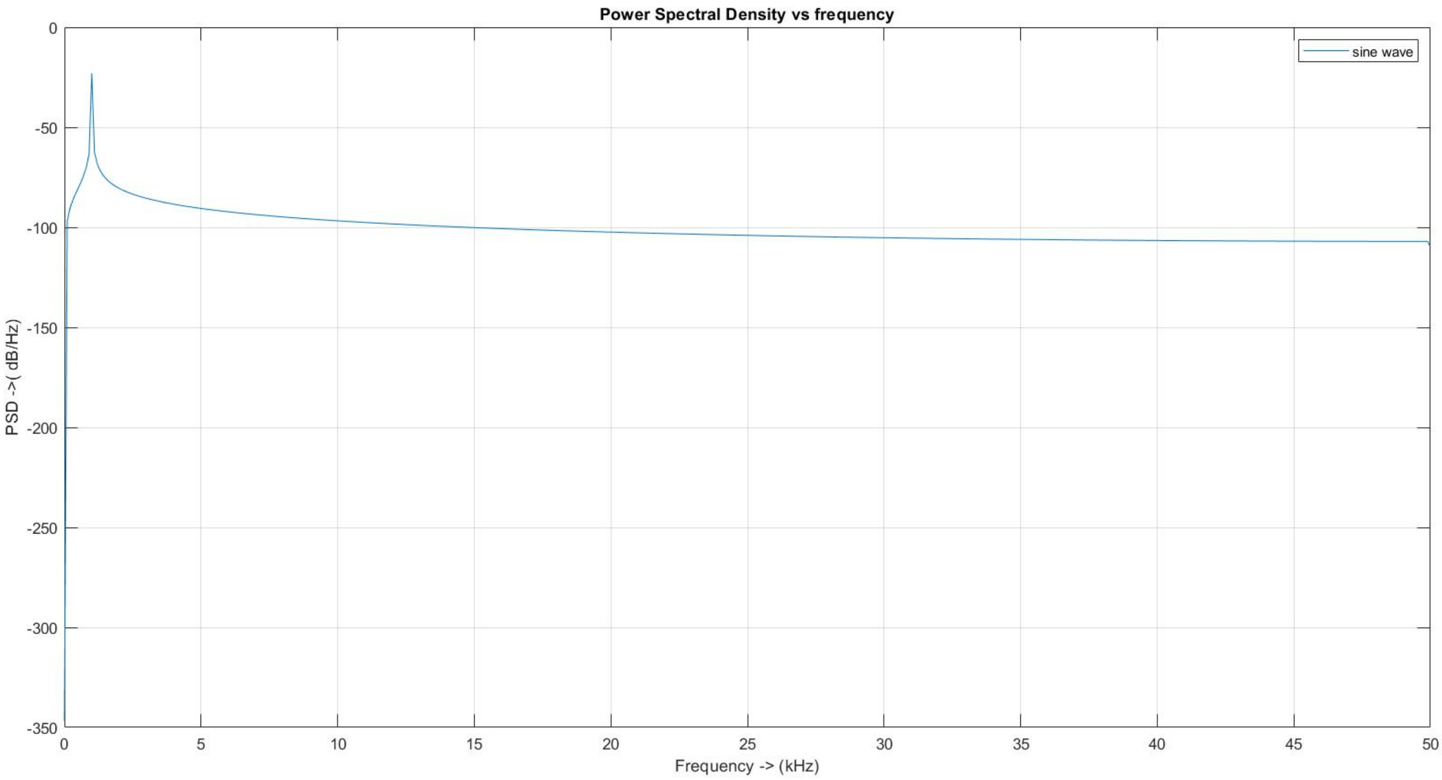

Figure 2: PSD of a sine wave derived by fft

```
%concatenating and generating the final time domain signal.
for i=1:no_seq*9
    c1 = [ c1 y_p9(i,:)];
end
```

The time window for each bit is fixed at 1ms as per the repetition rate(1kHz). So, the frequency spectrum of a fixed time window would be approximately like a sinc function (because of bit 1 giving a rectangular window). The first dip to zero for it would be 1/ T(time slot of the bit) = 1kHz. Hence, by nyquist we can choose a sampling frequency of >= 2kHz, but this is by approximating the signal to be bandlimited. Hence, we take a sufficiently large sampling frequency of 100kHz. Therefore ,the time resolution is 0.01ms.
10 PRBS sequences are taken for each case, hence the total time window is given by: 10*order*T(time slot of a bit). The time slot is given as 1ms.

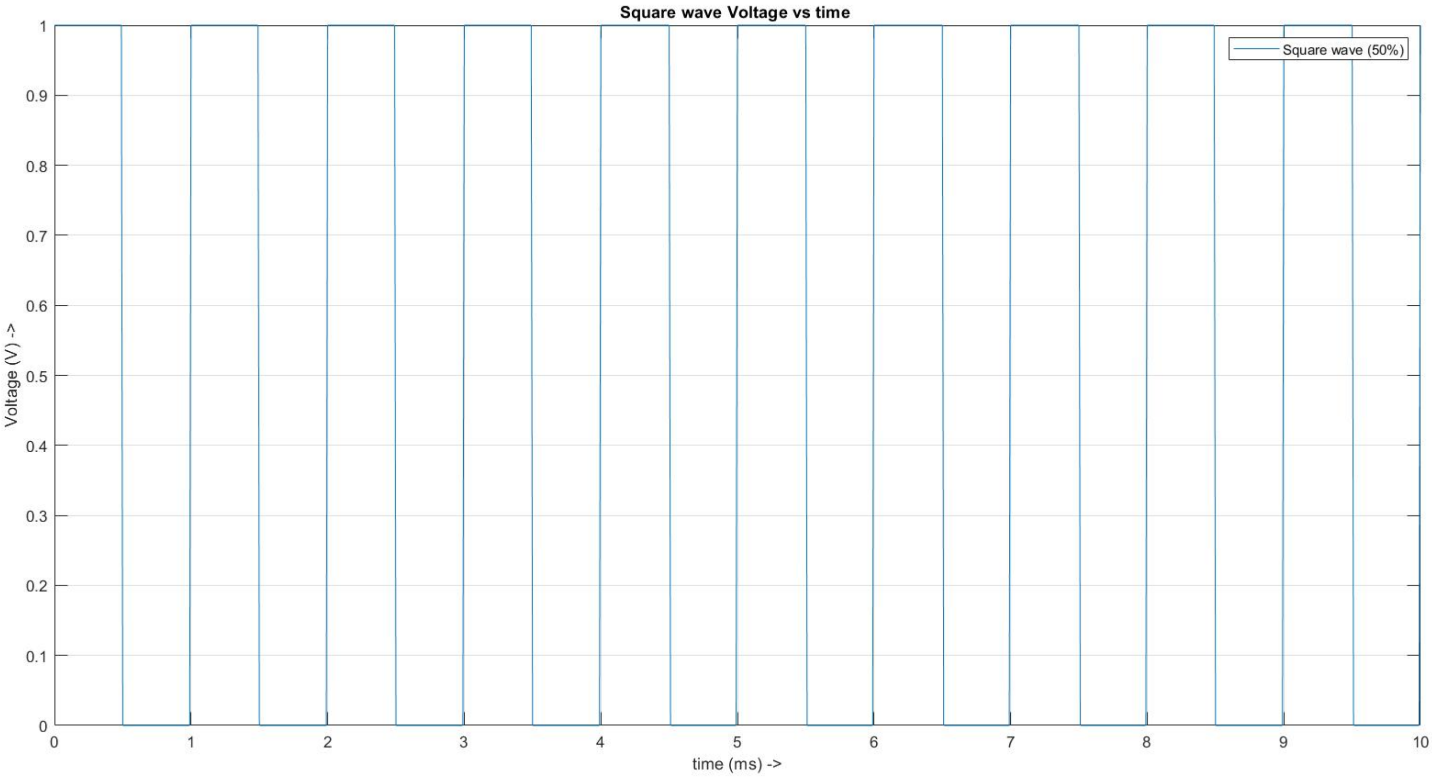

Figure 3: Voltage vs time for a square wave of duty cycle 0.5

Then, the time windows for the PRBS sequences are:
PRBS9: 90ms.
PRBS13: 130ms.
PRBS15: 150ms.

The frequency window is fs/2 = 50kHz decided by sampling rate. The frequency resolution is fs/N. N is given as: N = 10*order*(1/f)*fs, where 1/T(1 bit) = f. Hence, frequency resolution is: f/(10*order). 'f' is given as 1kHz. The frequency resolutions for the PRBS sequences are:

PRBS9: 11.111Hz
PRBS13: 7.692Hz.
PRBS15: 6.667Hz.

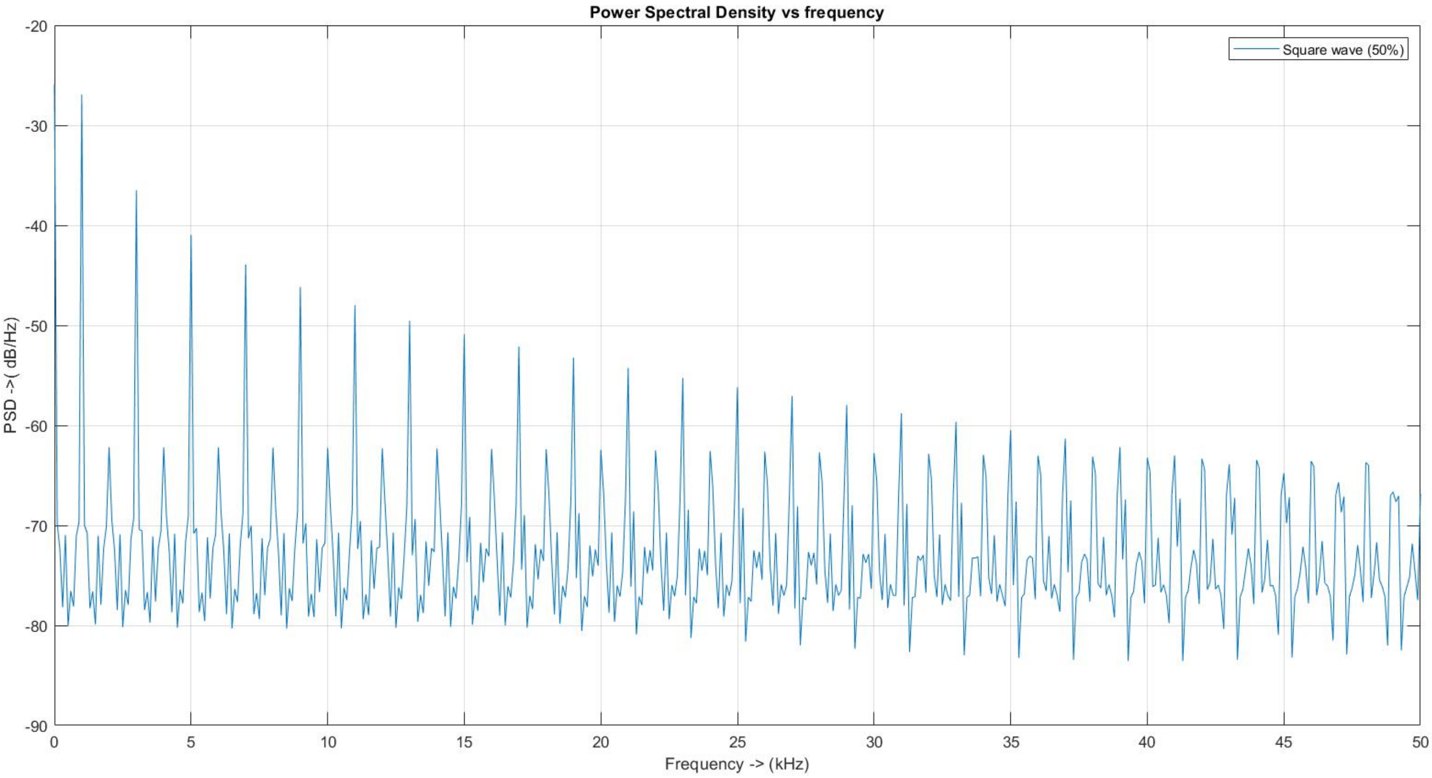

Figure 4: PSD of a square wave of duty cycle 0.5 derived by fft

The voltage and PSD plots are shown in fig. 5,6 , fig. 7,8 fig. 9,10 for PRBS 9, 13 and 15 respectively.

Emission Spectrum of InGaAsP LED:

The Rate of spontaneous emission for an LED is given by the following eqn.:

$$Rsp = R_0\sqrt{hf - Eg}\; e^{(-(hf-Eg)/kT)} \tag{7}$$

The Eg is calculated as per the given values of x and y and is found to be 1.0537 eV. We calculate the value at each frequency point and plot the output array with frequency:

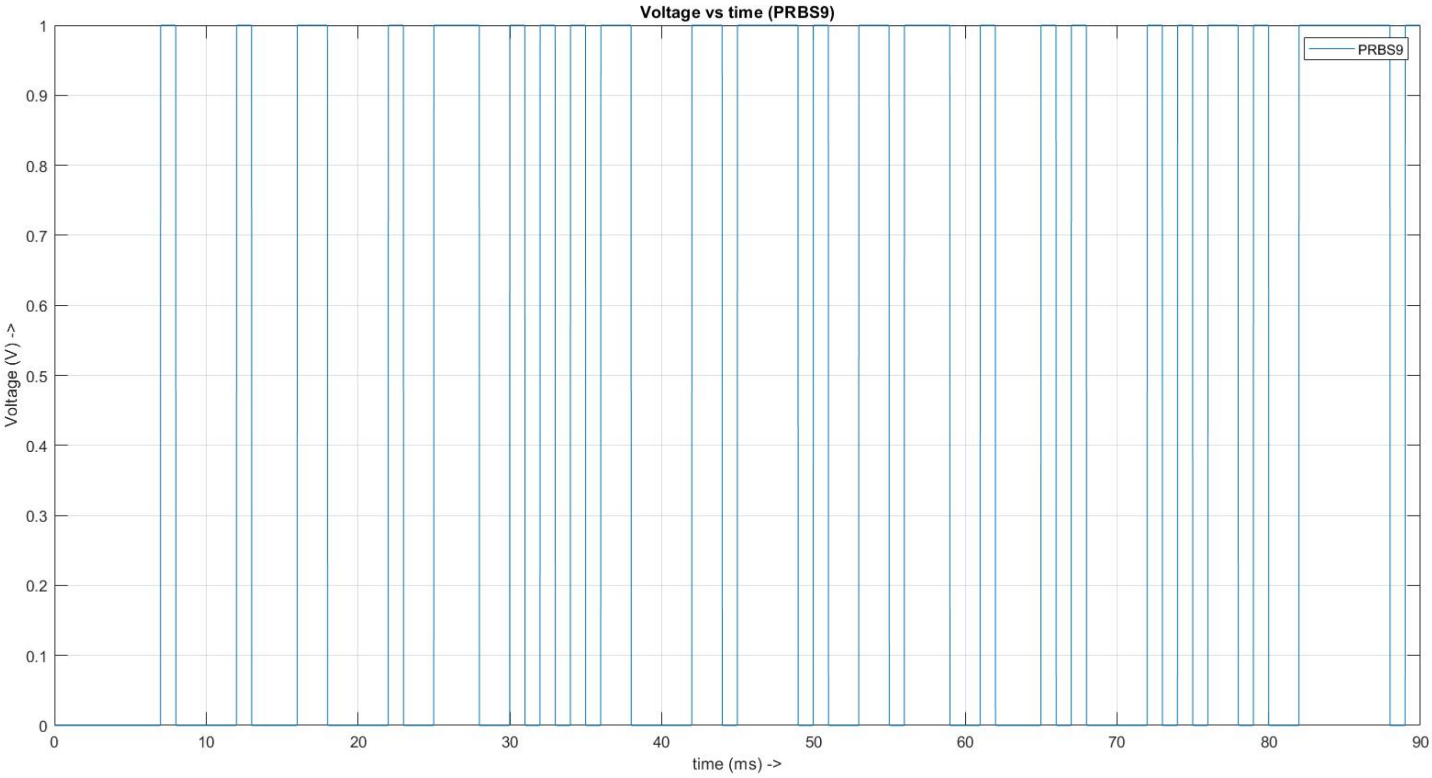

Figure 5: Voltage vs time for PRBS9 pulse signal

```
%calculating Rsp for each frequency.
for i= 1:length(f)
R(i) = ((h*f(i) - Eg)^0.5)* exp(- (h*f(i) - Eg)/(k*T));
end
```

The frequencies taken are from Eg/h to Eg/h +7*kT/h, since we know the Rsp is 0 for frequencies lower than the lower limit and the rate drops significantly for frequencies higher than the upper limit of this range. The frequency resolution is of the order of 10^-4 THz (by taking 10^5 frequency points) for sufficient accuracy. Additionally, we normalize it with the maximum value. The plot is as shown in fig. 11. We can find the 3dB bandwidth roughly from measuring frequencies at which Rsp(normalized) = 0.5 as shown in the plot or we can find it accurately by the following code:

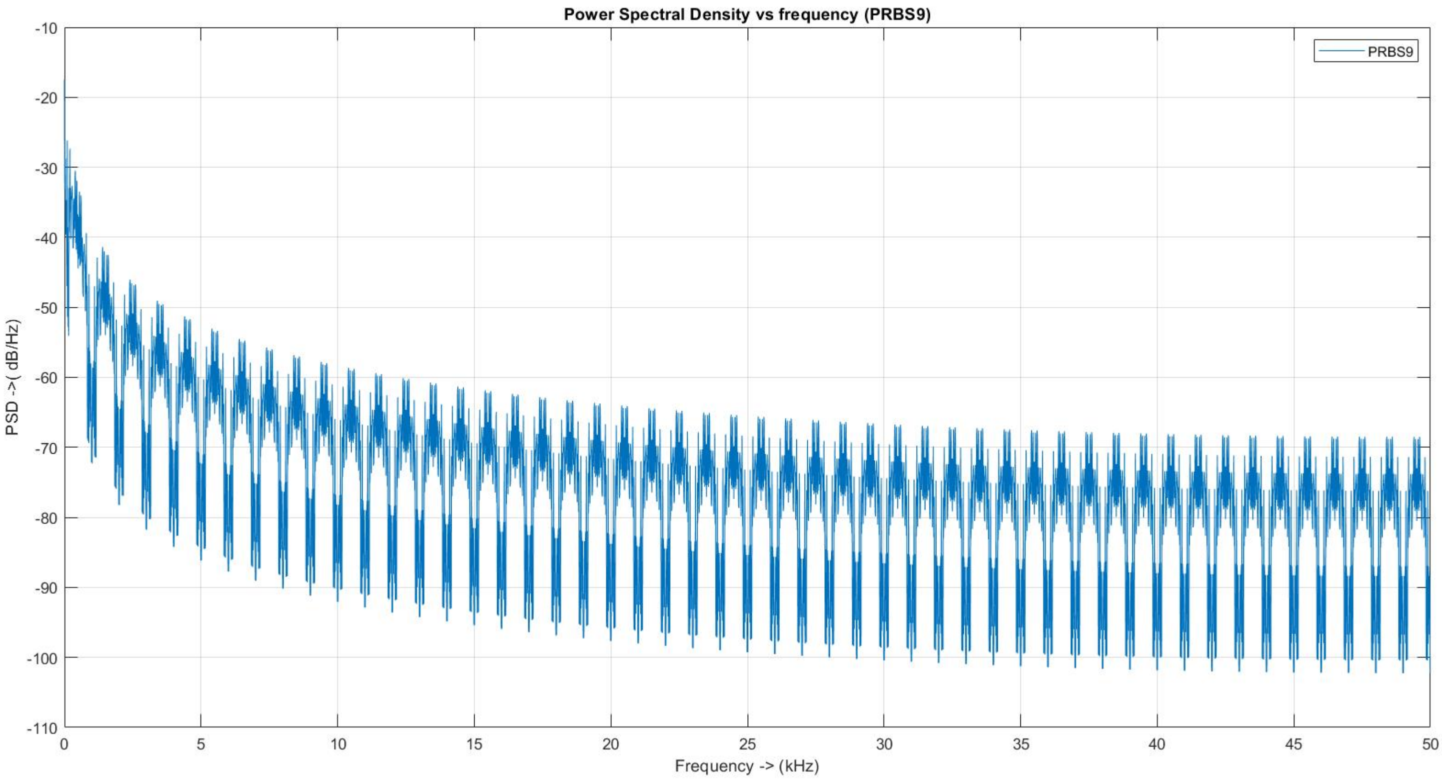

Figure 6: Power spectral density via FFT for a PRBS9 signal with 10 sequences.

```
for i = 1:length(f)

    if( (abs(R(i)-0.5)<=1e-3) && (f(i) < (Eg +k*T/2)/h))
       f1 =f(i);
    end
    if( (abs(R(i)-0.5)<=1e-3) && (f(i) > (Eg +k*T/2)/h))
        f2 = f(i);
    end
end
#1e-3 is the error limit between Rsp and 0.5
#f1 and f2 are the lower and higher frequencies corresponding to the bandwidth.
```

Computationally, the bandwidth is found to be 11.239 THz which is close to the actual spectral width of the led (11.25THz).

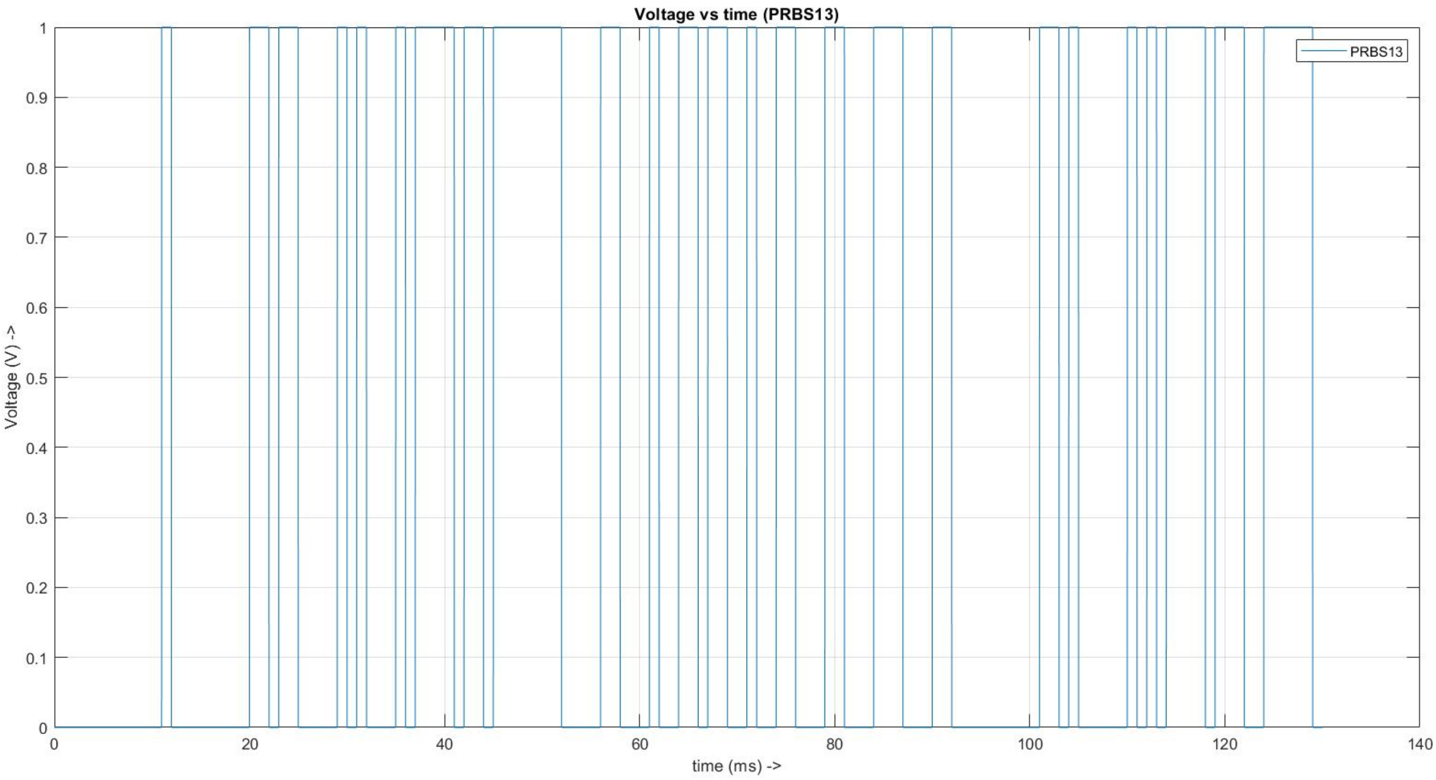

Figure 7: Voltage vs time for PRBS13 pulse signal

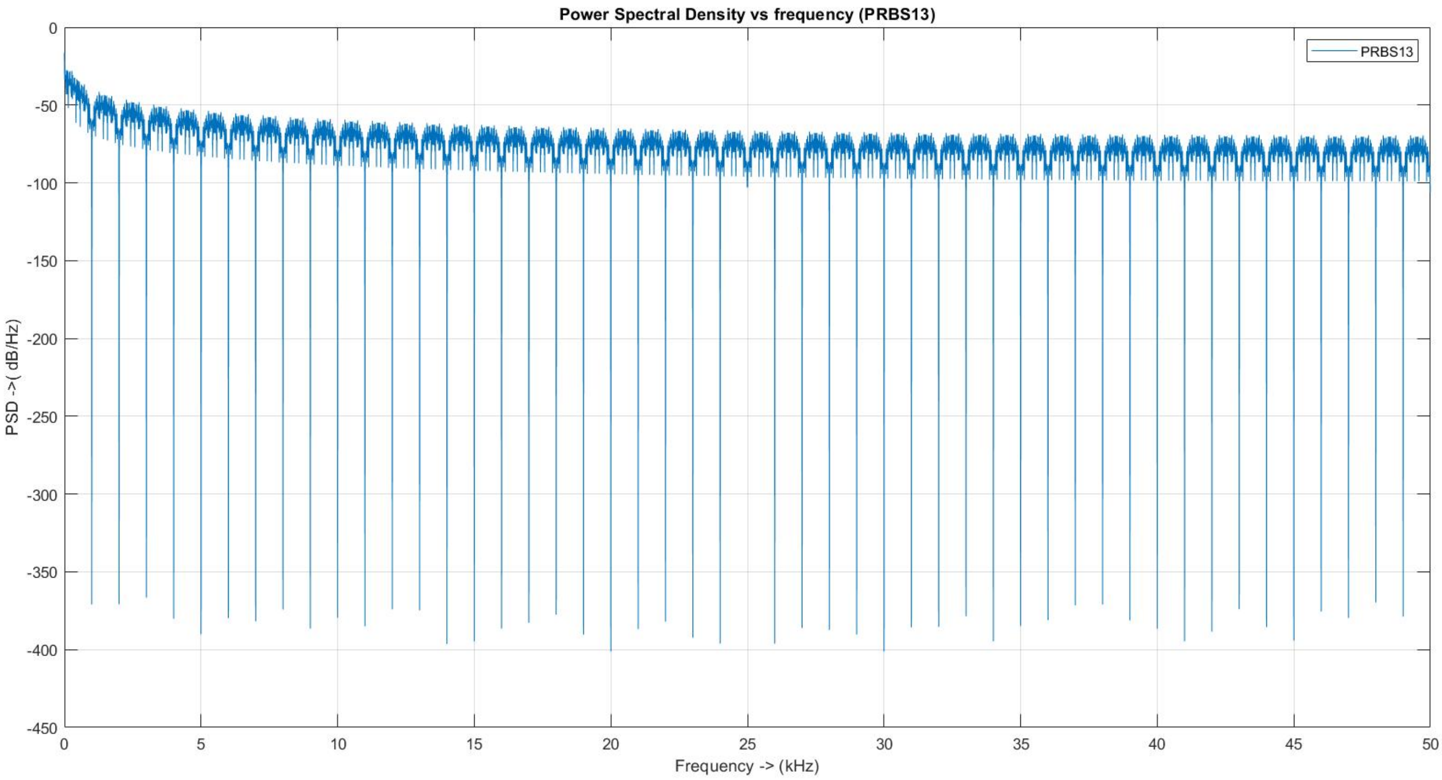

Figure 8: Power spectral density via FFT for a PRBS13 signal with 10 sequences.

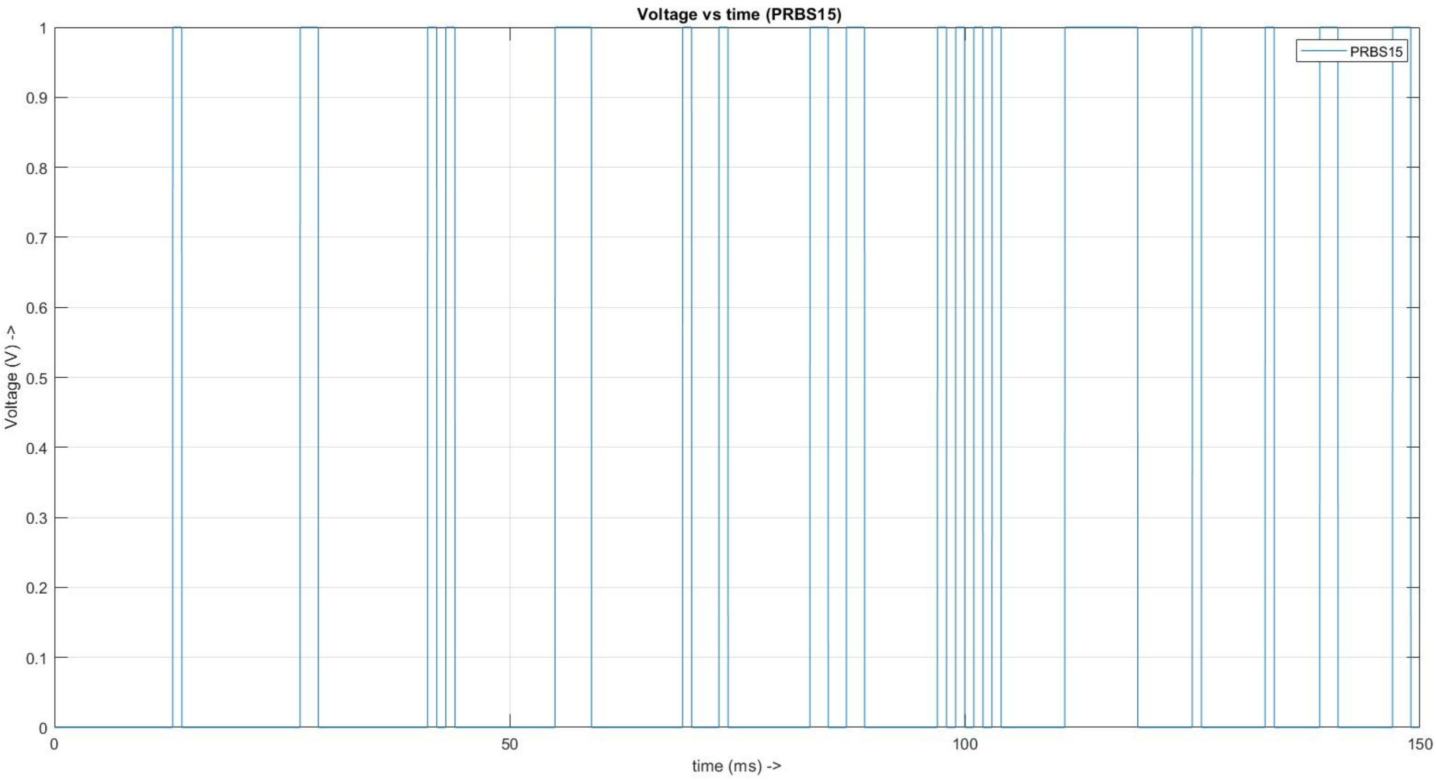

Figure 9: Voltage vs time for PRBS15 pulse signal

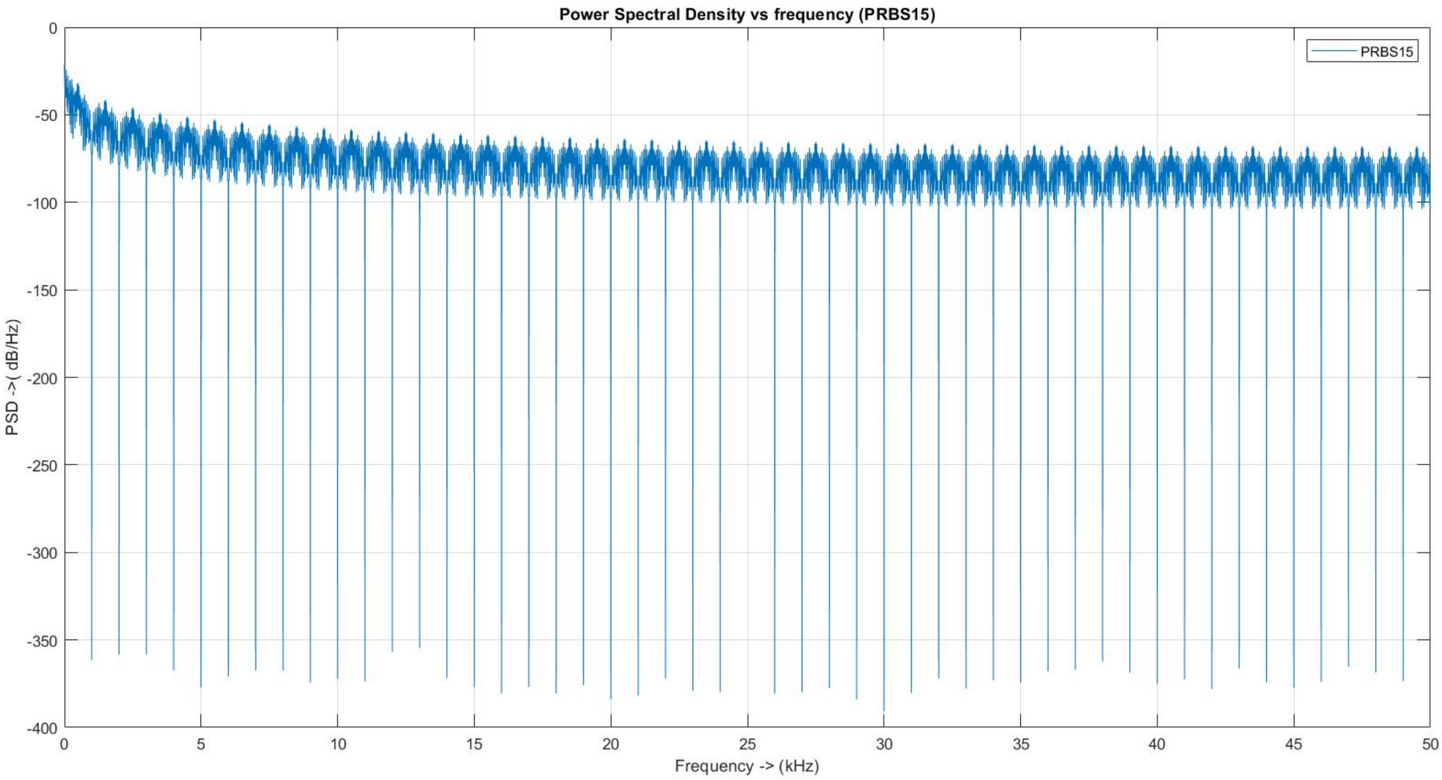

Figure 10: Power spectral density via FFT for a PRBS15 signal with 10 sequences.

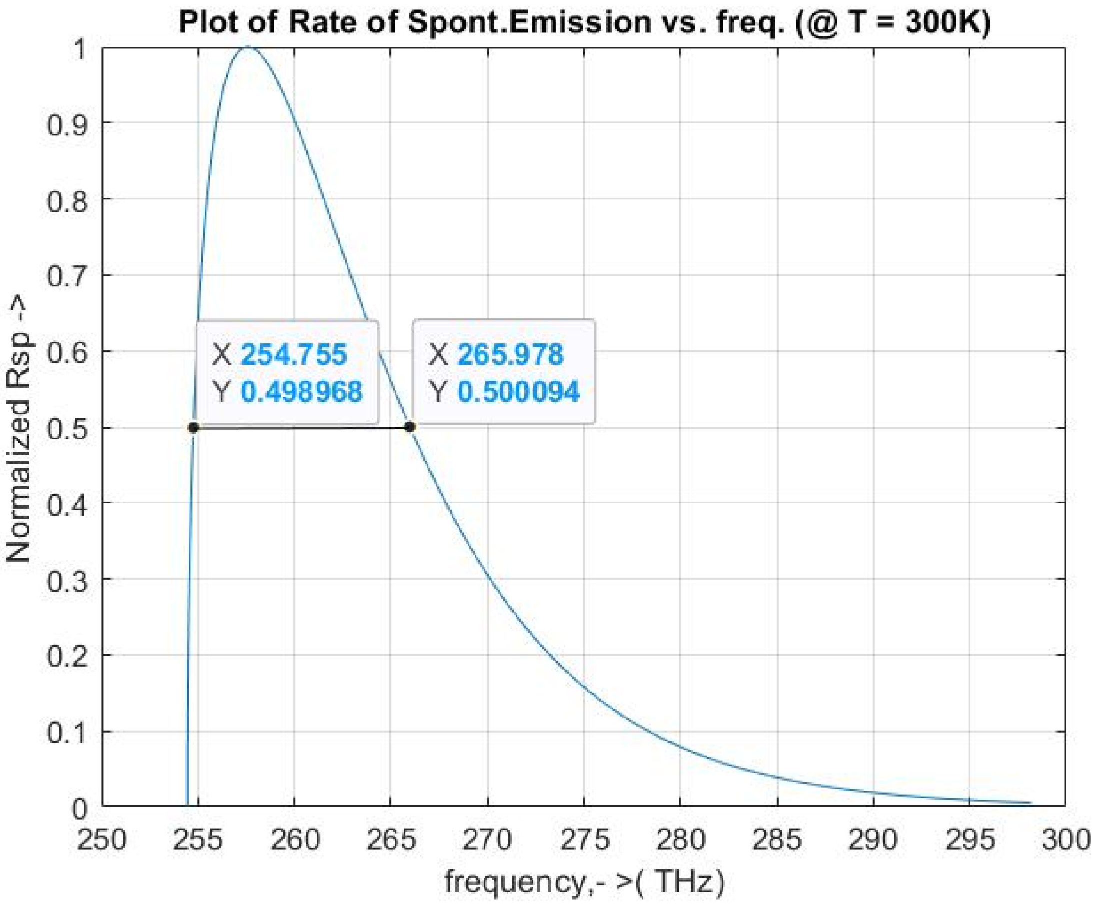

Figure 11: Rate of Spontaneous emission for the InGaAsP led at 300K with a spectral width of about 11.25 THz.