Transform Techinques

Transformation
Change from on form to another.

Generalform

$$T[f(t)] = \int_a^b K(s,t)\, f(t)\, dt = F(s).$$

$K(s,t) \downarrow$ Kernel.

Laplace Transform

Kernel $\rightarrow e^{-st}$.

List of Transforms.

- $\rightarrow$ $L\{e^{at}\} = 1/(s-a)$
- $\rightarrow$ $L\{e^{-at}\} = 1/(s+a)$
- $\rightarrow$ $L\{\cos at\} = \frac{s}{s^2+a^2}$
- $\rightarrow$ $L\{\sin at\} = \frac{a}{s^2+a^2}$
- $\rightarrow$ $L\{\sinh at\} = \frac{a}{s^2-h^2}$
- $\rightarrow$ $L\{\cosh at\} = \frac{s}{s^2-h^2}$

$L\{t^n\} = \frac{\Gamma(n+1)}{s^{n+1}}$

$L\{1\} = 1/s$

$L\{K\} = K/s$

→ $\Gamma(n+1) = n\Gamma(n)$

$= n!$ → n is +ve integer.

→ $\Gamma(1) = 1$

→ $\Gamma(1/2) = \sqrt{\pi}$

→ K/E1

Inverse Laplace Transforms list

→ $L^{-1}(1/s-a) = e^{at}$

→ $L^{-1}(1/s+a) = e^{-at}$ → $L^{-1}\left(\frac{\Gamma(n+1)}{s^{n+1}}\right) = t^n$

→ $L^{-1}(s/s^2+a^2) = \cos at$ → $L^{-1}[K/s] = K$

→ $L^{-1}(a/s^2+a^2) = \sin at$ → $L^{-1}(1/s^n) = \frac{t^{n-1}}{\Gamma(n)}$

→ $L^{-1}(s/s^2-a^2) = \cosh at$

→ $L^{-1}(a/s^2-a^2) = \sinh at$.

→ Laplace Transform does not exist for all function.

Exponential Order :- A function $f(t)$ is said to be of EO if there exists c and positive const t_0 and M such that $|f(t)| < Me^{ct}$ for all $t > t_0$ at which $f(t)$ is defined.

classmate

Piecewise cont. function

-> If there are Jump discontinuties then they have to be not infinity.

-> If a function is both PWC & EO then Laplace will 100% exists.

-> If anything is nothere the Transform may or maynot exist.

-> LT dosent exist for e^{t^2}, $\tan t$, $\log t$.

S- shifting Theorem

-> $L\{f(t)\} = F(s)$.

-> $L\{e^{at} f(t)\} = F(s-a)$

-> $L^{-1}\{F(s-a)\} = e^{at} f(t)$

-> $L^{-1}\{F(s-a)\} = e^{at} L^{-1}[F(s)]$

-> $L\{e^{-at} f(t)\} = F(s+a)$

-> $L^{-1}\{F(s+a)\} = e^{-at} L^{-1}[F(s)]$

Transform of Derivative

→ $L\{f'(t)\} = s\,L\{f(t)\} - f(0)$

→ $L\{f''(t)\} = s^2\,L\{f(t)\} - s\,f(0) - f'(0).$

Derivative of Transform

→ $L\{t\,f(t)\} = -F'(s)$

→ $L\{t^2 f(t)\} = F''(s)$

→ $L^{-1}[F'(s)] = -t\,L^{-1}[F(s)]$

Transform of Integral

$$L\left\{\int_0^t f(t)\,dt\right\} = \frac{F(s)}{s}$$

$$L^{-1}\left[\frac{F(s)}{s}\right] = \int_0^t f(t)\,dt$$

$$= \int_0^t L^{-1}[F(s)]\,dt$$

Integration of Transform

If $L\{f(t)\} = F(s)$ and $\lim_{t \to 0^+} \frac{f(t)}{t}$ exists then

$L\left\{\frac{f(t)}{t}\right\} = \int_s^{\infty} F(s)\, ds.$

$\Rightarrow \quad L\left\{\frac{f(t)}{t}\right\} = \int_s^{\infty} F(s)\, ds.$

Transform of special Functions

unit step function

$\rightarrow \quad L\{u(t-a)\} = \frac{e^{-as}}{s}$

1. $L\{f(t-a)\, u(t-a)\} = e^{-as} F(s)$
2. $L\{f(t-a)\, u(t-a)\} = e^{-as} L\{f(t)\}$

3. $L\{f(t)\, u(t-a)\} = e^{-as} L\{f(t+a)\}$

3. $L^{-1}[e^{-as} F(s)] = u(t-a) \{L^{-1}[F(s)]\}_{t \to t-a}$

~~4. $L^{-1}[e^{-as}$~~

Unit Impulse (Dirac delta) function.

-> $L\{\delta(t-a)\} = e^{-as}$.

Convolution Theorem

If $f(t)$ & $g(t)$ are two of function

$$f(t) * g(t) = \int_0^t f(u)\, g(t-u)\, du.$$

-> Convolution is Commutative.
-> distributive and associative.

-> $L\{f(t) * g(t)\} = L\{f(t)\}\, L\{g(t)\}$

-> $L^{-1}\{F(s)\, G(s)\} = L^{-1}[F(s)] * L^{-1}[G(s)]$.

Periodic function L.T

$$L\{f(t)\} = \frac{1}{1-e^{-Ps}} \int_0^P e^{-st} f(t)\, dt.$$

p is the period.

Fourier Transform

e^{-ist} is the kernel.

$$F\{f(t)\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-ist} f(t)\, dt.$$

$$F^{-1}[F(s)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{ist} F(s)\, dt.$$

Fourier Existence Condn

If $f(t)$ is absolutely integrable and piece wise cont. then Fourier Transform $f(t)$ exists and it is equal to $F(s)$

if $\int_{a} |f(t)|\, dt < \infty$ then its absolutely integrable.

$\frac{odd}{odd} = even$ $\quad o \times o = e$

$\frac{e}{e} = e$ $\quad e \times e = e.$

$o \times e = o.$

$\rightarrow \int_0^{\infty} \frac{\sin s}{s} = \pi/2.$

$\rightarrow F\{u(t-a)\} = \frac{1}{\sqrt{2\pi}} \frac{e^{-ias}}{is}$

$\rightarrow F\{g(t-a)\} = \cancel{\frac{1}{\sqrt{2\pi}}} / \cancel{\int_a^{a+h}} \; e^{-ias}$

$\rightarrow \int_0^{\infty} \frac{\sin x - x\cos x}{x^3} \cos x/_2 \, dx = \frac{3\pi}{16}.$

Self reciprocal

$F\{f(t)\} = f(s).$

$\rightarrow F\{e^{-a^2x^2}\} = \frac{e^{-s^2/4a^2}}{\sqrt{2}a}$ (root only for 2)

$\rightarrow F\{e^{-1/2 x^2}\} = e^{-s^2/2}.$

Fourier Sine

$F_s\{f(t)\} = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(t) \sin st \, dt.$

Inverse

$F_s^{-1}\{F_s(s)\} = \sqrt{\frac{2}{\pi}} \int_0^{\infty} F_s(s) \sin st \, ds$

Fourier Cosine

$$F_c\{f(t)\} = \sqrt{\frac{2}{\pi}} \int_0^\infty f(t) \cos st \, dt.$$
$$= F_c(s).$$

Inverse.

$$F_c^{-1}\{F_c(s)\} = \sqrt{\frac{2}{\pi}} \int_0^\infty F_c(s) \cos st \, ds.$$

→ $F_s(e^{-ax}) = \sqrt{\frac{2}{\pi}} \frac{s}{a^2+s^2}$

→ $F_c(e^{-ax}) = \sqrt{\frac{2}{\pi}} \frac{a}{s^2+a^2}.$

→ If $f(x)$ is even funct then $F_c\{f(x)\}$
$= F\{f(x)\}.$

→ $F_c\{x^{n-1}\} = \sqrt{\frac{2}{\pi}} \frac{\cos n\pi/2}{s^n}$

→ $F_s\{x^{n-1}\} = \sqrt{\frac{2}{\pi}} \frac{\sin n\pi/2}{s^n}.$

→ $F\left\{\frac{1}{\sqrt{|x|}}\right\} = F_c\left\{\frac{1}{\sqrt{|x|}}\right\} = \frac{1}{\sqrt{s}}$

$\rightarrow$ $F_c\left\{\frac{1}{x^2+1}\right\} = \sqrt{\frac{\pi}{2}}\, e^{-x}$

Properties

1. Linear property.

$$\mathcal{F}\{c_1 f(t) + c_2 g(t)\} = c_1 \mathcal{F}\{f(t)\} + c_2 \mathcal{F}\{g(t)\}$$

2. Change of Scale property

$$\mathcal{F}\{f(at)\} = \frac{1}{|a|} F(s/a).$$

3. Shifting prop (s-shifting).

$$F\{e^{-iat} f(t)\} = F(s+a)$$

4. Shifting prop (t-shifting).

$$\mathcal{F}\{f(t-a)\} = e^{-ias} F(s).$$

5. Modulation property

$$\mathcal{F}\{f(t)\cos at\} = \frac{1}{2}[F(s-a) + F(s+a)].$$

$$F_c\{f(t)\cos at\} = \frac{1}{2}[F_c(s+a) + F_c(s-a)]$$

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$F_c\{f(t)\sin at\} = \frac{1}{2}[F_s(s+a) - F_s(s-a)]$

$F_s\{f(t)\cos at\} = \frac{1}{2}[F_s(s+a) + F_s(s-a)]$

$F_s\{f(t)\sin at\} = \frac{1}{2}[F_c(s+a) - F_c(s-a)]$.

Transform of derivative

$F\{f'(t)\} = is\, f(s)$.

Derivative of Transform

If $F\{f(t)\} = F(s)$ then

$F\{t f(t)\} = i \frac{d}{ds} F(s)$.

Faultang theorem

The $F\{f(t) * g(t)\} = F(s)\, G(s)$

Parsevals identity or Energy Theorem

If $F\{f(t)\} = F(s) = \int_{-\infty}^{\infty} |f(t)|^2 dt = \int_{-\infty}^{\infty} |F(s)|^2 ds$

→ i) $\int_0^{\infty} f(t) g(t)\, dt = \int_0^{\infty} F_c(s)\, G_c(s)\, ds$

$= \int_0^{\infty} F_s(s)\, G_s(s)\, ds$

ii) $\int_0^{\infty} |f(t)|^2 = \int_0^{\infty} |F_s(s)|^2 = \int_0^{\infty} |F_c(s)|^2 ds.$

→ $F\left\{f(t)\left(1 + \cos\left(\frac{\pi t}{a}\right)\right)\right\}$ where

$f(t) = \begin{cases} 1 & |t| < a \\ 0 & \text{otherwise.} \end{cases}$

is $\cancel{\frac{2}{\sqrt{2\pi}}}\ \cancel{\frac{\sin as}{s}}$ $\frac{2}{\sqrt{2\pi}} \frac{\sin as}{s} \frac{\pi^2/a^2}{\pi^2/a^2 - s^2}.$

→ $F[x^2 e^{-ax} U(x)] = \frac{2}{\sqrt{2\pi}\,(is+a)^3}$

$\rightarrow$ $F_s\{x f(x)\} = -\frac{d}{ds} F_c\{f(x)\}$

$\rightarrow$ $F_c\{x f(x)\} = \frac{d}{ds} F_s\{f x\}$