Karatsuba

$$T(n) = 3T(n/2) + n$$

$$T(n) = O(n)^{1.59}$$

Strassen's

$$T(n) = 7T(n/2) + n^2$$

$$T(n) = O(N)^{2.8}$$

Binary Search

$$T(n) = T(n/2) * C$$

$$T(n) = O(\log n)$$

B+ tree

$O(\log n)$

for every oper.

Merge Sort

Av, W, B $\rightarrow$ $O(n \log n)$

$T(n) = 2T(n/2) + n$

Quick sort

WC $T(n) = T(n-1) + n$

$O(n^2)$

BC

$T(n) = 2T(n/2) + n$

$O(n \log n)$

Splay Trees

Amortized
Analysis
Refer
Else where

$O(\log n)$ All ops
(Average time)

M-way trees

$$\boxed{\begin{aligned} h_{min} &= \log_m(n+1) \\ h_{max} &= n \end{aligned}}$$

$n \rightarrow$ no. of elements
$h \rightarrow$ height
$m \rightarrow$ order

$$\boxed{\begin{aligned} n_{min} &= h \\ n_{max} &= m^h - 1 \end{aligned}}$$

complexity $\rightarrow O(h)$

ensure ~~h_{min}~~ $h = h_{min}$

B-trees

~~Comp. $\rightarrow O(h)$~~ $\rightarrow O(\log n)$ for every op.

$$\boxed{\begin{aligned} h_{min} &= \lceil \log_m(n+1) \rceil - 1 \\ h_{max} &= \log_{\lceil m/2 \rceil}\left(\frac{n+1}{2}\right) \end{aligned}}$$

$n \rightarrow$ no. of nodes
$m \rightarrow$ max no. of children node can have (order)

AVL Trees

$O(\log n)$ for every operation

$n(h) = n(h-1) + n(h-2) + 1$

$\boxed{n(h) \geq (1.62)^h \text{ or } (2)^h} \rightarrow (2)$

$n(h) \rightarrow$ min. no of nodes with height h

Fibonacci relation

Roughly

$(1.44)^* \log_2 N$

~~Golden ratio~~

Rotation takes $O(1)$

height of a node $O(1)$

i.e height(node)

(2) $n \geq N(h)$ Always

$n \geq \phi^h$ $\quad \phi = 1.62$

taking log

$\log_\phi n \geq h \log_\phi(\phi)$

$\log_\phi n \geq h$

so $\boxed{h \leq \log_\phi n}$ $\rightarrow O(\log n)$