

FUNCTIONS

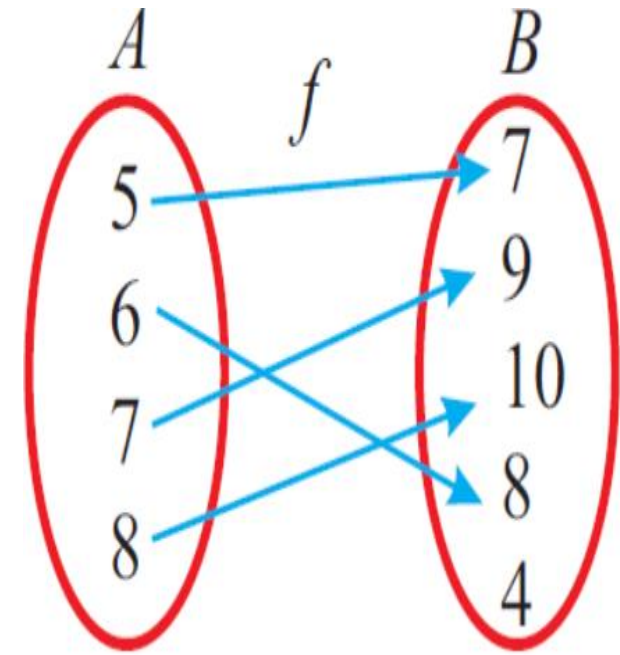
Nikhita Binu
19pw18

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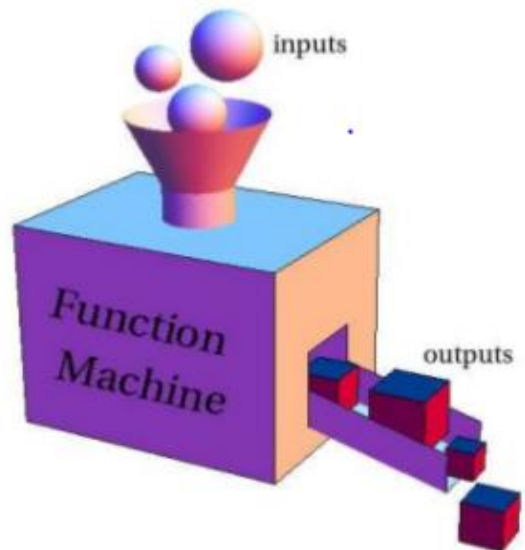
- › Functions
- › Functions vs. Relations
- › Standard functions
- › Vertical line test

Functions

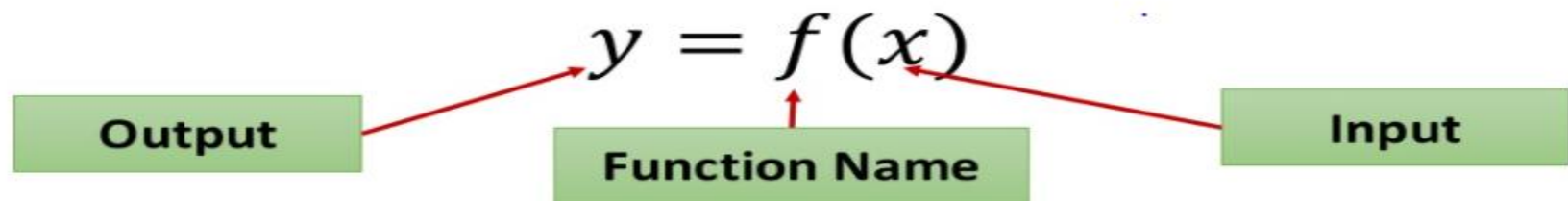
- Let A and B be two nonempty sets .A function f from A to B is an assignment of exactly one element of B to each element of A .
- A is the domain of the function ,while B is the range of the function.
- We write $f(x)=y$ if x is the unique element of B assigned by the function f to the element y of A .
- If f is a function from A to B , we write $f:A \rightarrow B$
 - If $f(a)=b$,we say that b is the image of a and a is the pre-image of b .
 - The range of $f:A \rightarrow B$ is the set of all images of elements of A .
 - We say that $f:A \rightarrow B$ maps A to B .



- › A function relates an input to an output.
- › A function is a rule which operates on an input and produces a single output from that input.

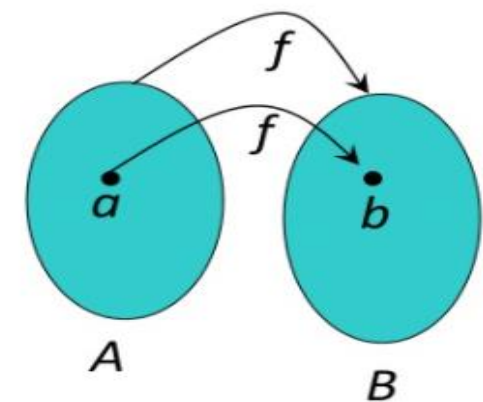


- › Function Notation

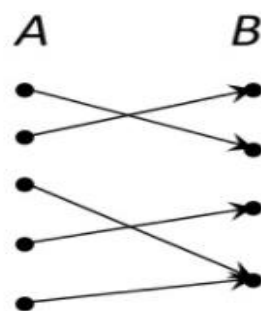


Representations of functions

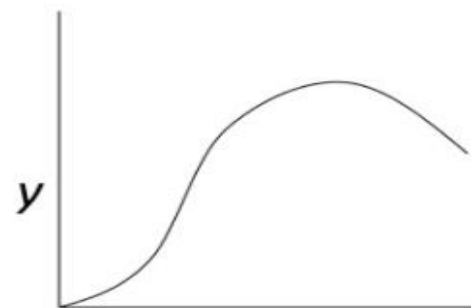
- Verbally
- Numerically, i.e. by a table
- Visually, i.e. by a graph
- Algebraically, i.e. by an explicit formula
- Functions can be represented graphically in several ways:



Like Venn diagrams



Graph



Plot

- EXAMPLE
- Function $f:P \rightarrow C$ with
- $P=\{\text{Linda, Max, Kathy, Peter}\}$
- $C=\{\text{Boston, New York, Hong Kong, Moscow}\}$

$f(\text{Linda})=\text{Moscow}$

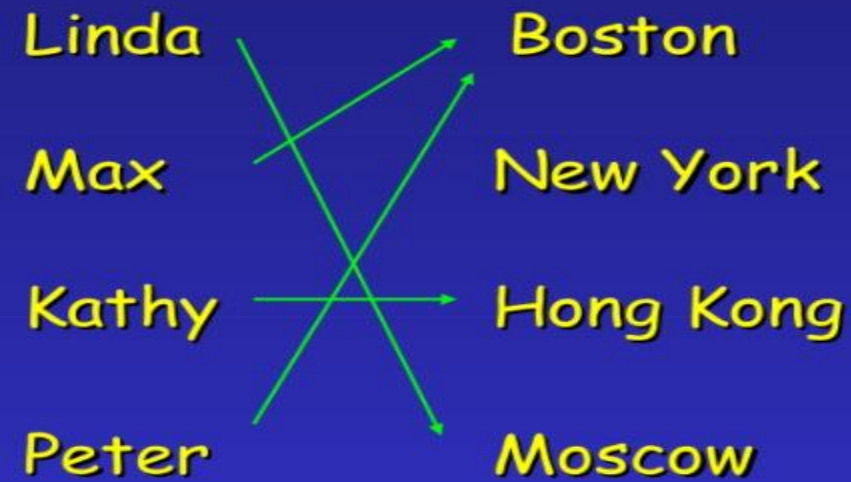
$f(\text{Max})=\text{Boston}$

$f(\text{Kathy})=\text{Hong Kong}$

$f(\text{Peter})=\text{New York}$

- Here, the range of f is C .

x	$f(x)$
Linda	Moscow
Max	Boston
Kathy	Hong Kong
Peter	New York



If we only regard a subset $S \subseteq A$, the set of all images of elements $s \in S$ is called the image of S .

We denote the image of S by $f(S) = \{f(s) \mid s \in S\}$

Example:

$f(\text{Linda}) = \text{Moscow}$

$f(\text{Max}) = \text{Boston}$

$f(\text{Kathy}) = \text{Hong Kong}$

$f(\text{Peter}) = \text{Boston}$

What is the image of $S = \{\text{Linda}, \text{Max}\}$?

$f(S) = \{\text{Moscow}, \text{Boston}\}$

What is the image of $S = \{\text{Max}, \text{Peter}\}$?

$f(S) = \{\text{Boston}\}$

FUNCTIONS vs. RELATIONS

- A “relation” is just a relationship between sets of information.
- In mathematics ,a **relation** is just a set (collection)of ordered pairs.
- A “function” is a well-behaved relation, that is,given a starting point we know exactly where to go.
- For a relation to be a function,there must be exactly one y value that corresponds to a given x value.
- Exampe:
- People and their heights.We can think of this relation as ordered pair.

(height,name)

OR

(name,height)

State the domain and range of the relation shown in the graph. Is the relation a function?

The **relation** is:

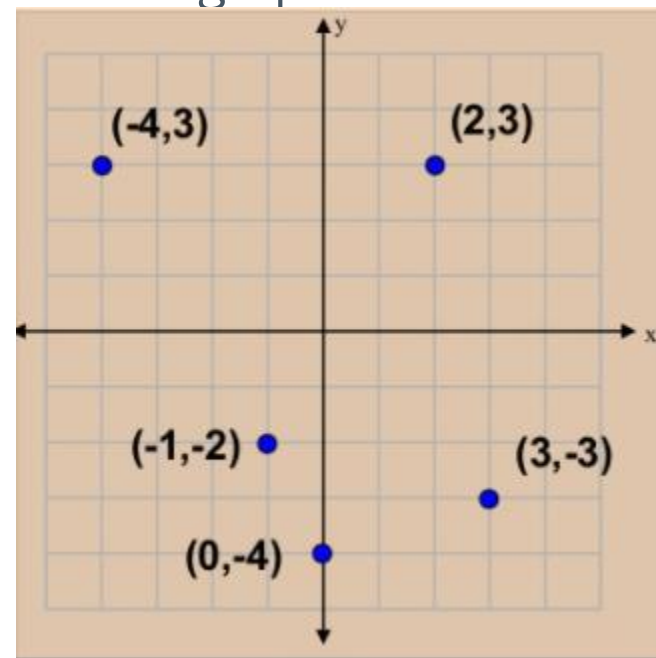
$$\{ (-4,3), (-1,-2), (0,-4), (2,3), (3,-3) \}$$

The **domain** is:

$$\{-4, -1, 0, 2, 3\}$$

The **range** is:

$$\{-4, -3, -2, 3\}$$



Each member of the domain is paired with exactly one member of the range ,so this **relation is a function**

CONCLUSION

- Not every relation is a function.
- Every function is a relation.

π

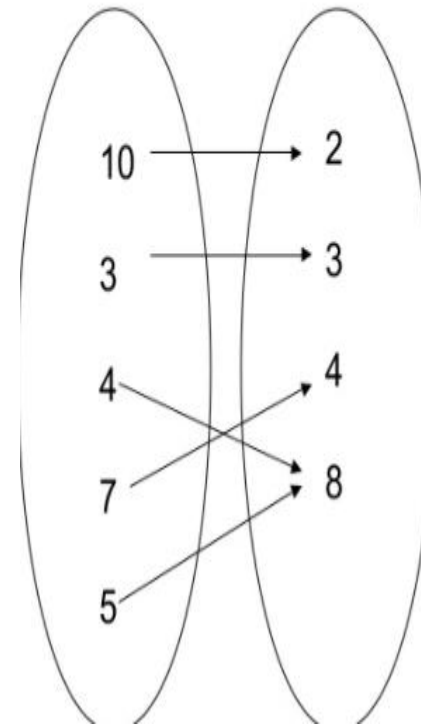
Function

(4,12)
(5,15)
(6,18)
(7,21)
(8,24)

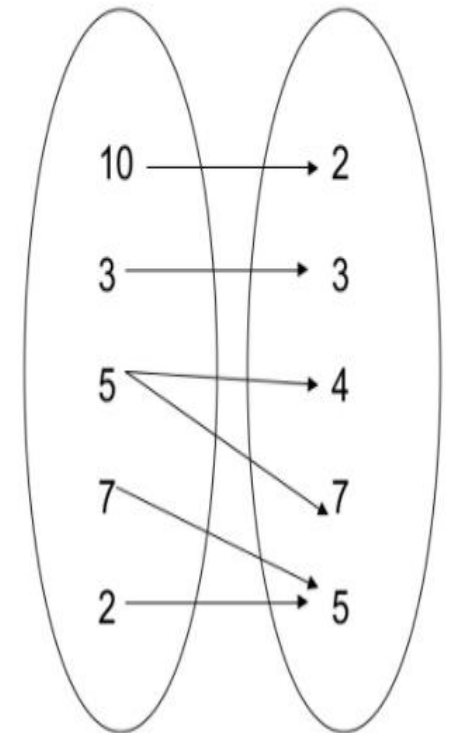
Not a Function

(4,12)
(4,15)
(5,18)
(5,21)
(6,24)

Function



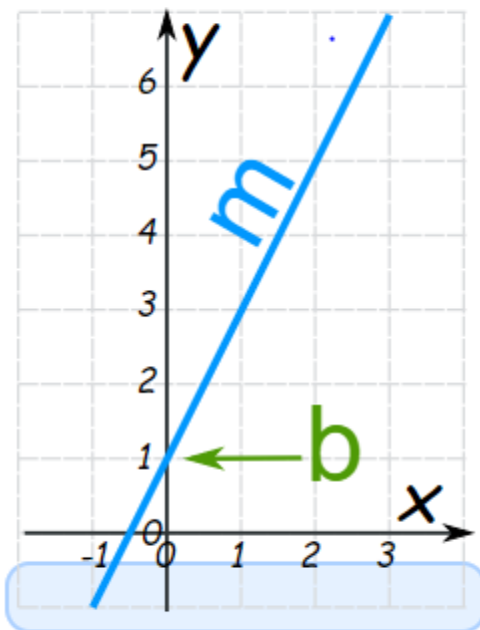
Not a Function



STANDARD FUNCTIONS

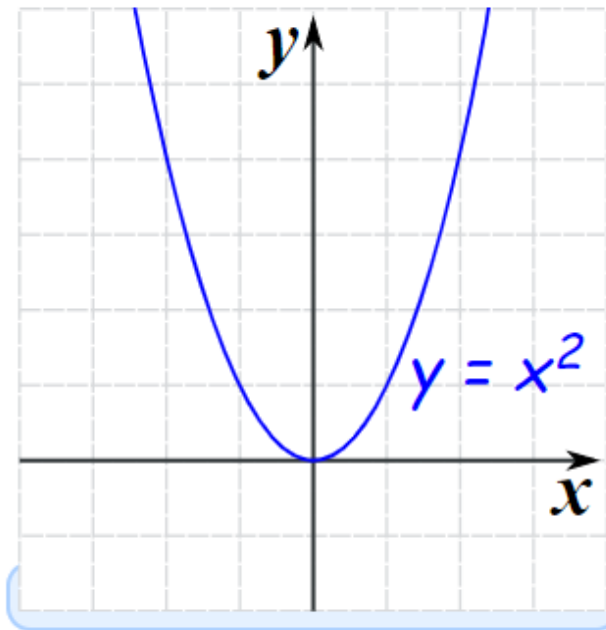
Here are the most commonly used functions and their graphs:

Linear Function :



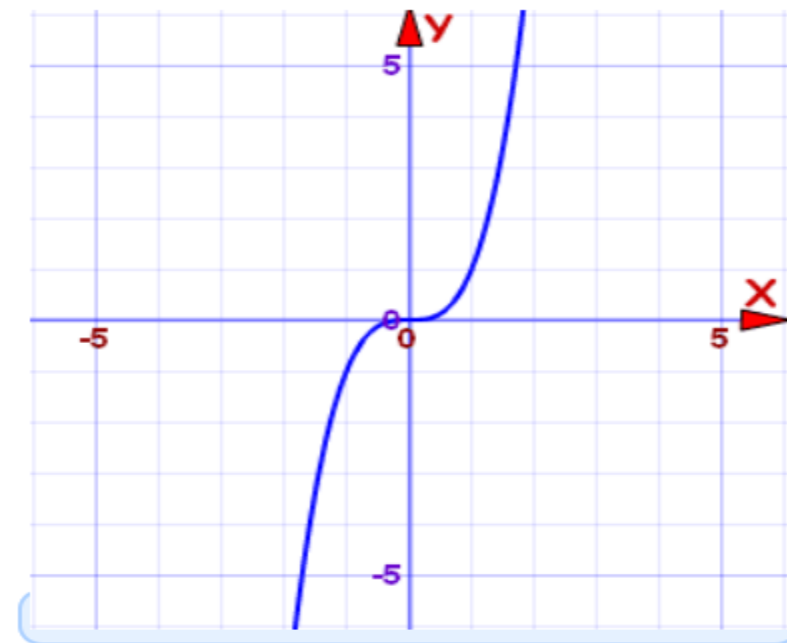
$$f(x) = mx + b$$

Square Function :



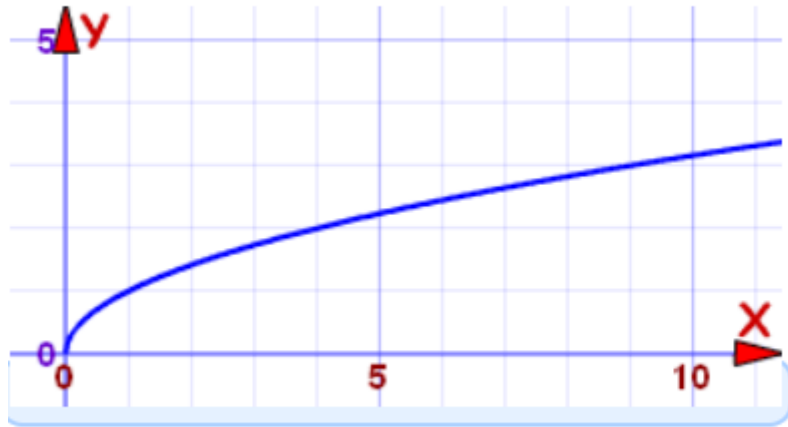
$$f(x) = x^2$$

Cube Function :



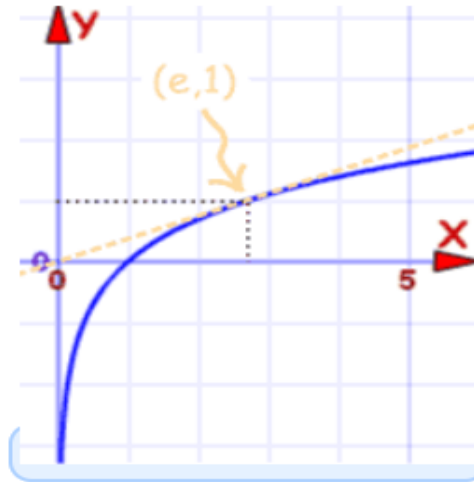
$$f(x) = x^3$$

Square Root Function :



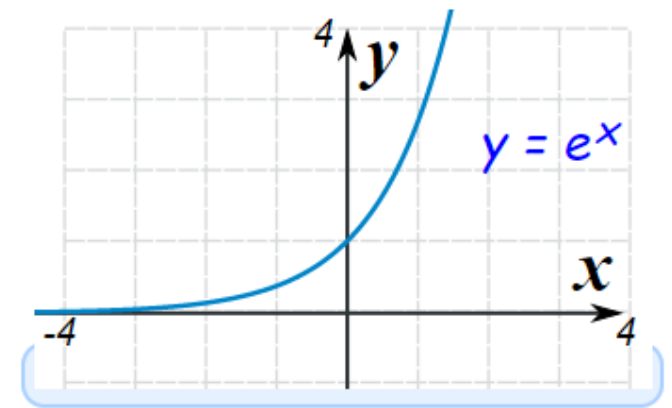
$$f(x) = \sqrt{x}$$

Logarithmic Function :



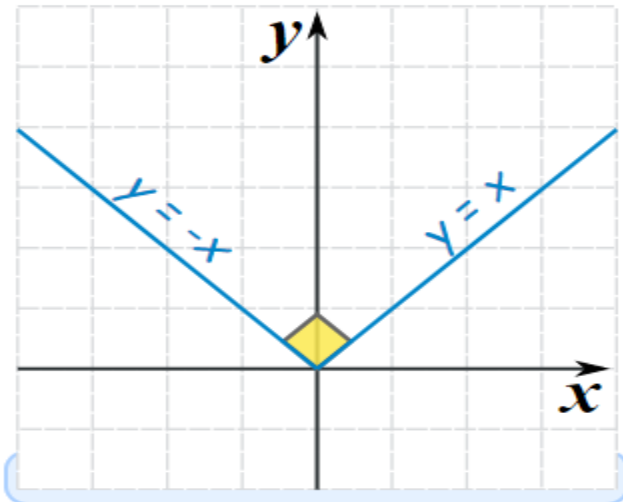
$$f(x) = \ln(x)$$

Exponential Function :



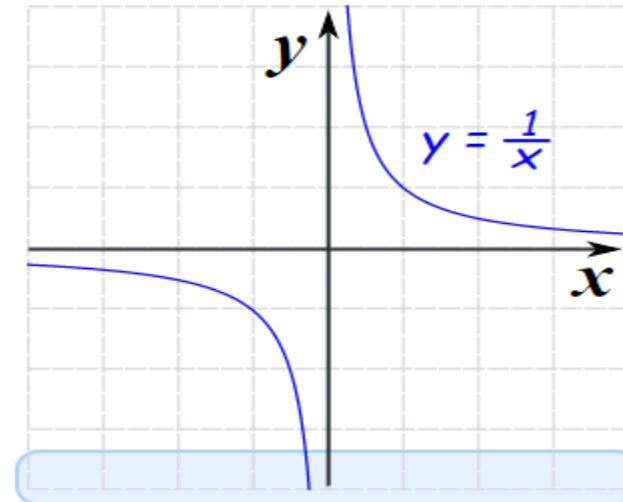
$$f(x) = e^x$$

Absolute Value Function :



$$f(x) = |x|$$

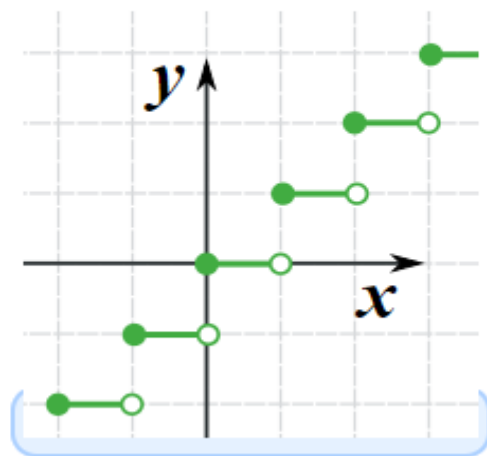
Reciprocal Function



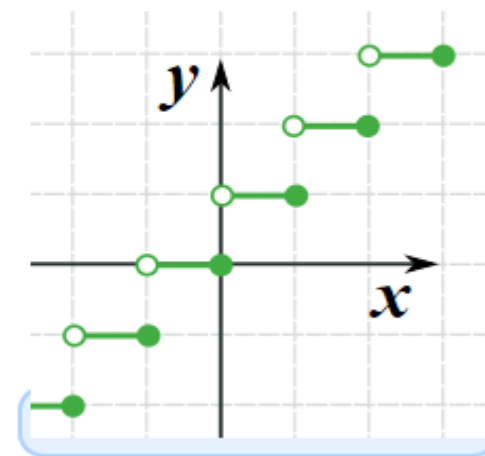
$$f(x) = 1/x$$

π

Floor and Ceiling Functions :

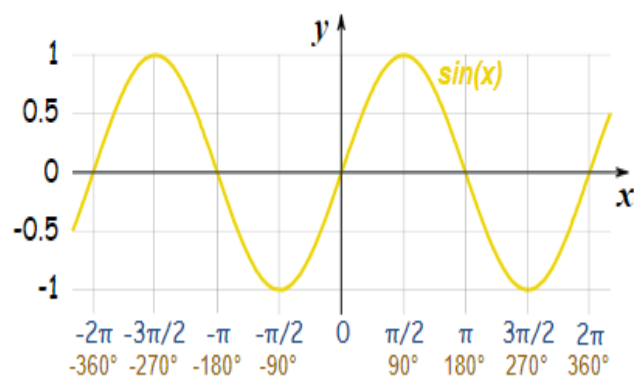


The Floor Function

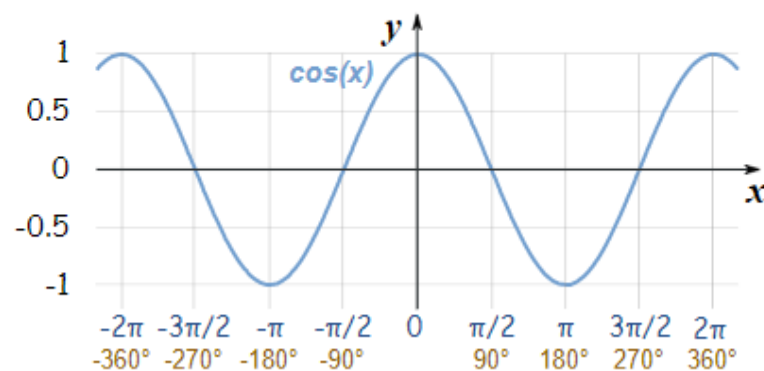


The Ceiling Function

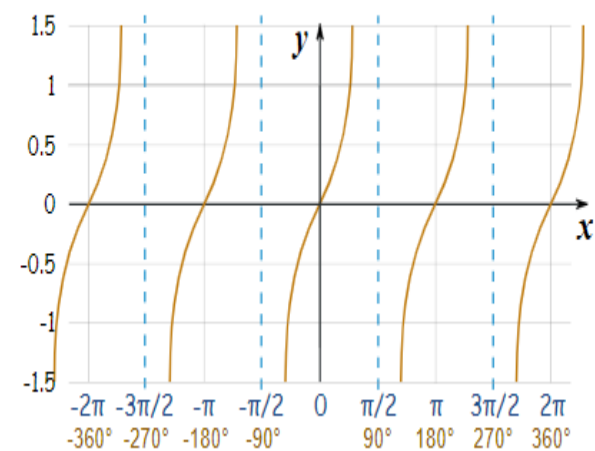
Sine Function :



Cosine Function :



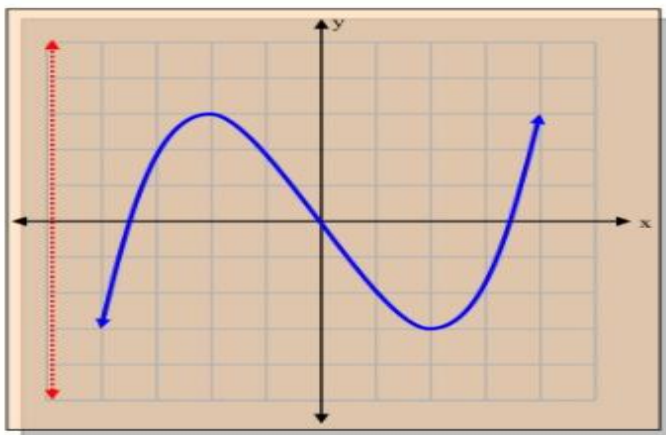
Tangent Function :



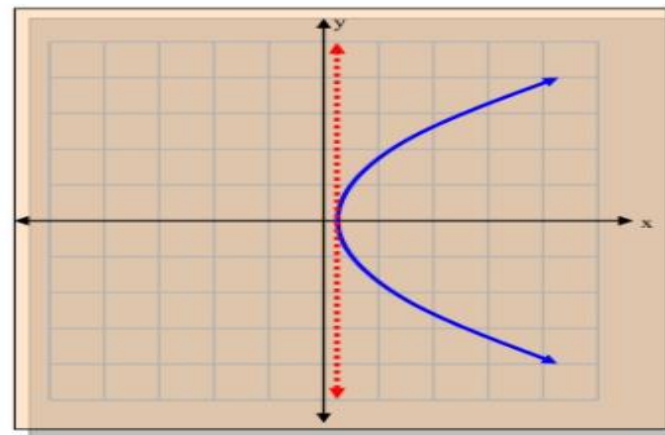
VERTICAL LINE TEST

- The vertical line test is used to determine whether a relation is a function or not.
- A relation is not a function if atleast one domain element x is paired with more than one value y .
- These two points(or a set of points) are aligned vertically in the xy -plane and a vertical line is drawn through one point also intersects the other point.
- Thus,if a vertical line drawn through a graph of a relation intersects the graph in more than one point,the relation cannot be a function.
- That is any vertical line drawn through the graph of a relation intersects the relation in more than one point,then the relation doesnot define y as function of x .

If no vertical line intersects a graph in more than one point, the graph represents a function.



If some vertical line intercepts a graph in two or more points, the graph does not represent a function.

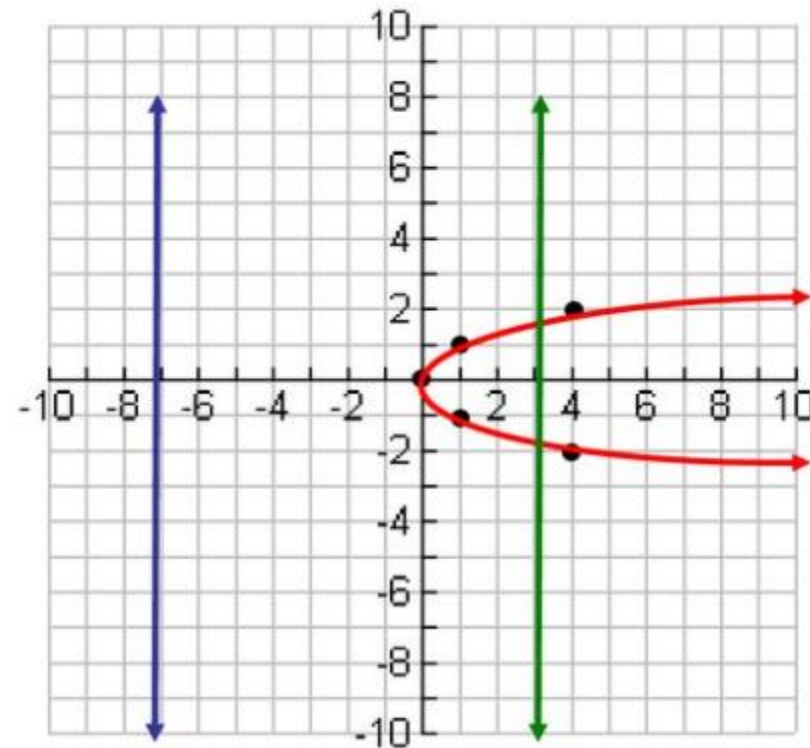


EXAMPLE:

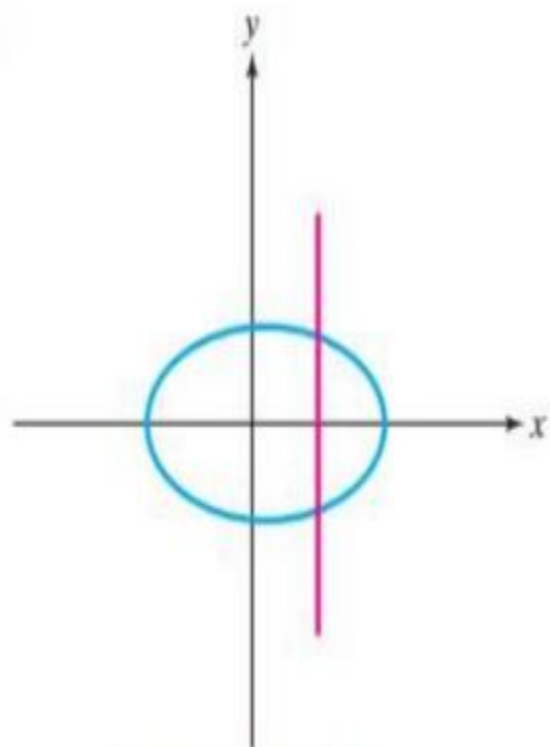
$$x = y^2$$

- We create a table of values
- Graph the points and draw the line or the curve if not linear.
- Run the vertical line across the graph and see if it intersects two places at the same time.
- The relation **fails** the vertical line test as it “intersects at two places” and thus **is not a function**.

X	Y
4	-2
1	-1
0	0
1	1
4	2



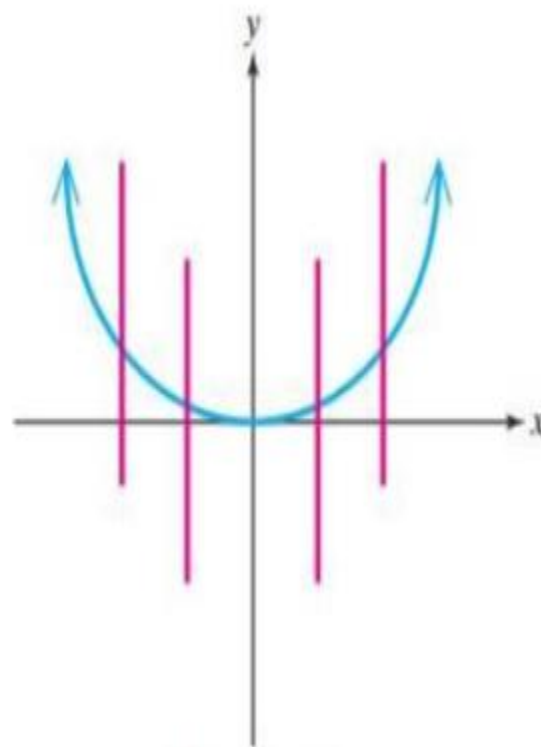
a.



Not a Function

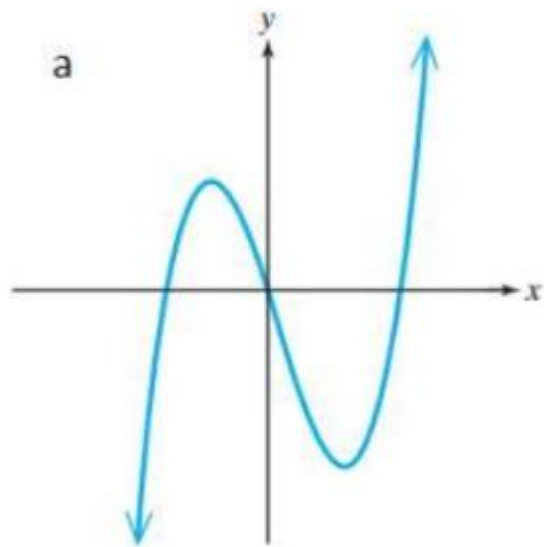
A vertical line intersects
in more than one point.

b.

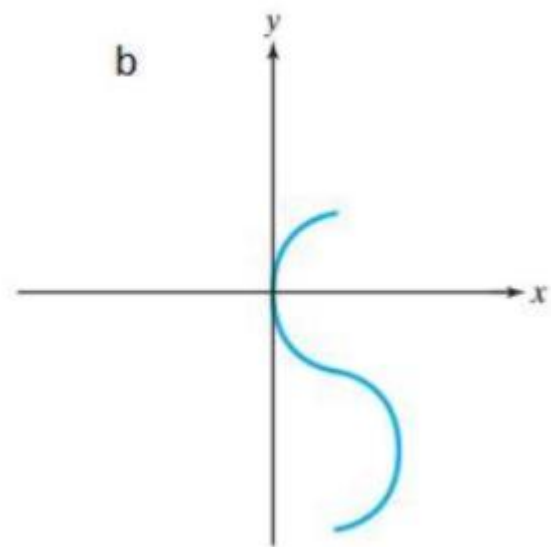


Function

No vertical line intersects
in more than one point.



Yes



No

THANK
YOU