Disjoint Union Types in P0

Project 9 / Group 8

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April 2021

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- An instance of a DUT may take on the form of **only one** of it's variants.
- They are often found in functional programming languages, where they are usually known as **Algebraic Data Types** (ADTs).

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a = Cons 1 (Cons 2 (Cons 3 Nil))
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- Instantiation in our implementation,
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a := Cons(1, Cons(2, Cons(3, Nil())))
b := Red()
```

How do we use DUTs?

The anatomy of a case statement.

- cases are the only way to access data inside of DUTs.

```
case <variable> of {
    [nil: <stmtSuite>]
    Kind A: <stmtSuite>
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    ...
    ...
    [default: <stmtSuite>]
    ... or ...
    [default nothing]
}
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    ... or ...
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```

Exhaust your cases!

If you create a non-exhaustive case statement, the compiler will warn you.

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Example: Maybe

```
type Maybe = Just(value: integer)
           | Nothing

procedure valOr(v: Maybe, n: integer) → (r: integer)
    case v of {
        Just:
            r := v.value
        default:
            r := n
    }

program Main
    var maybe: Maybe

    maybe <– Nothing()
    writeln(valOr(maybe, –1))

    maybe <– Just(1111)
    writeln(valOr(maybe, 0))
```

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```

Output

```
-1
1111
```

Example: Lists

```
type List = Cons(head: integer, tail: List)
          | Nil

procedure upToList(n: integer) → (l: List)
    if n < 1 then l := Nil() else l := Cons(n, upToList(n−1))

procedure consumeList(l: List)
    case l of {
        Cons: writeln(l.head); consumeList(l.tail)
        default nothing
    }

procedure sumList(l: List) → (n: integer)
    case l of {
        Cons: n := sumList(l.tail) + l.head
        default: n := 0
    }

program Main
    var myList: List
    myList := upToList(5)
    consumeList(myList)
    writeln(sumList(myList))
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program Main
    var myList: List
    myList := upToList(5)
    consumeList(myList)
    writeln(sumList(myList))
```

Output

```
5
4
3
2
1
15
```

Example: Strings

... lists in disguise?

```
type String = SCons(ch: integer, tail: String)
            | SNil

procedure printStr(s: String, ln: boolean)
    case s of {
        SCons: writeChar(s.ch); printStr(s.tail, ln)
        default: if ln then writeNewLine()
    }

// inclusively generating alphabets in a range
procedure genBetwn(start: integer, end: integer) -> (s: String)
    var ch: integer
    ch := end
    s := SNil()

    while start <= end do
        s, start, ch := SCons(ch, s), start + 1, ch - 1

program Main
    // print capital letters
    printStr(genBetwn('A', 'Z'), true)

    // print lowercase letters
    printStr(genBetwn('a', 'z'), true)

    // print numbers 0-9
    printStr(genBetwn('0', '9'), true)

    // print Greek letters
    printStr(genBetwn('α', 'ω'), true)
```

Note

We convert single-quoted characters into their UTF-8 integer representation when reading in P0 programs.

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    // print Greek letters
    printStr(genBetwn('α', 'ω'), true)
```

Output

ABCDEFGHIJKLMNOPQRSTUVWXYZ
abcdefghijklmnopqrstuvwxyz
0123456789
$\alpha\beta\gamma\delta\epsilon\zeta\eta\theta\iota\kappa\lambda\mu\nu\xi o\pi\rho\sigma\tau\upsilon\phi\chi\psi\omega$

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We convert single-quoted characters into their UTF-8 integer representation when reading in P0 programs.

Other Examples

```
type RainbowColour = Red | Orange | Yellow | Green | Blue | Indigo | Violet

type Either = Left(value: integer)
            | Right(value: boolean)

type Tree = Branch(left: Tree, right: Tree)
          | Leaf(value: integer)

type Expr = Add(left: Expr, right: Expr)
          | Sub(left: Expr, right: Expr)
          | Mul(left: Expr, right: Expr)
          | Div(num: Expr, den: Expr)
          | Pow(base: Expr, exponent: Expr)
          | Int(value: integer)

type StringIntMap = SIMCons(key: String, value: integer, tail: StringIntMap)
                  | SIMEmpty
```

Remark

Modelling is nice with disjoint union types!

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Focal Grammar Changes

- Disjoint union type declarations

```
type ::=
    ident ["(" typedIds ")"] {"|" ident ["(" typedIds ")"]}
    | ...
```

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```
type ::=
    ident ["(" typedIds ")"] {"|" ident ["(" typedIds ")"]}
    | ...
```

- case statements

```
statement ::= ... | "case" expression "of" "{" INDENT
                  ["nil" ":" statementSuite]
                  {ident ":" statementSuite}
                  ["default" (":" statementSuite | "nothing")]
                 DEDENT "}"
```

Supplementary Grammar and Procedure Changes

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- "$*$" as an alternative for "$\times$"

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- "$*$" as an alternative for "$\times$"
- Standard procedures
  - `write` - no longer prints a newline character
  - `writeln` - writes single integer to std. out. with a newline afterwards
  - `writeChar` - writes single integer converted into a utf-8 character to std. out.
  - `writeCharLn` - writes single integer converted into a utf-8 character to std. out. with a newline afterwards
  - `writeNewLine` - writes a newline character to std. out.

Example: case WebAssembly Generation

For example, the WebAssembly code on the right-hand side for the below case statement.

```
type Colour = R | G | Unknown

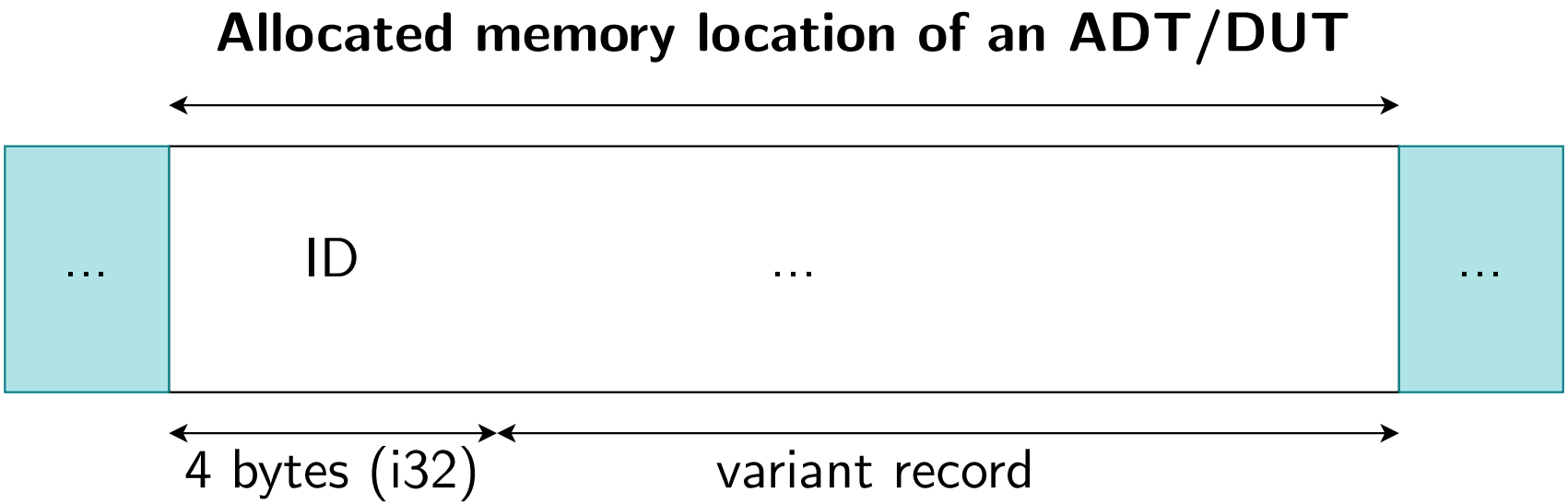
procedure printCol(col: Colour)
    case col of {
        nil: writeCharLn('?')
        R: writeCharLn('R')
        G: writeCharLn('G')
        default: writeCharLn('?')
    }
```

```
...
local.get $col
i32.load
i32.const 0          ;; check if nil
i32.eq
if                   ;; if it is nil
i32.const 63
call $writeCharLn    ;; print '?'
else                 ;; otherwise
local.get $col
i32.load
i32.const 1          ;; check if 'R'
i32.eq
if                   ;; if it is 'R'
i32.const 82
call $writeCharLn    ;; print 'R'
else                 ;; otherwise
local.get $col
i32.load
i32.const 2          ;; check if 'G'
i32.eq
if                   ;; if it is 'G'
i32.const 71
call $writeCharLn    ;; print 'G'
else
i32.const 63         ;; otherwise, default
call $writeCharLn    ;; print '?'
end
end
end
```

Memory Impact & Management

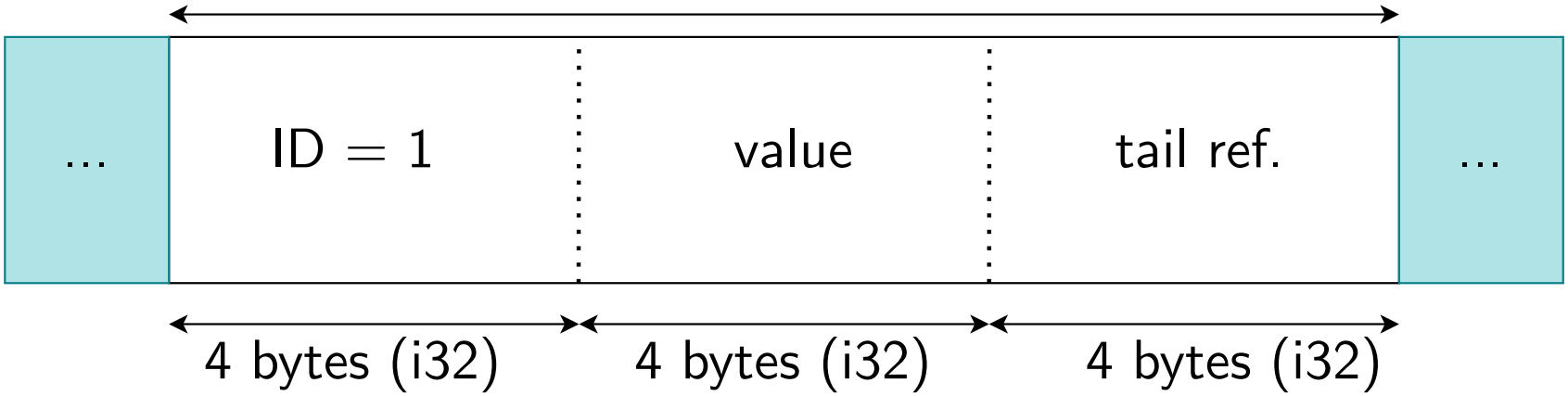
Each instance of a DUT is located on the heap, and instances of local/global DUTs are pointers to the locations of their corresponding DUT on the heap.

- Size of an allocation depends on the size of the variant being instantiated
- Offsets to accessing variables work similar to records, with a 4 byte offset for the variant id.

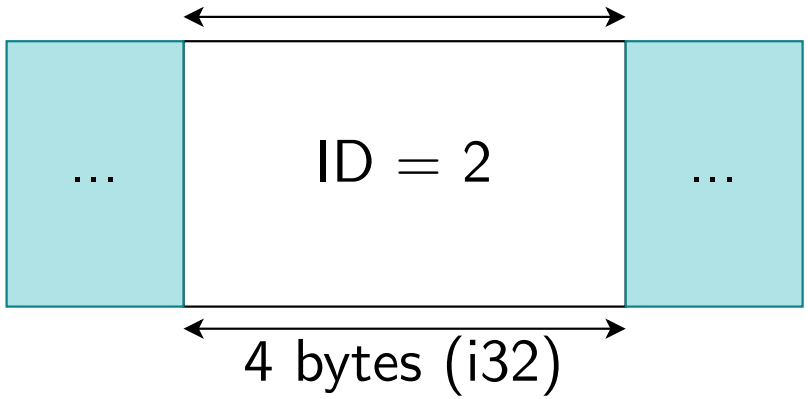


Example: Lists in Memory

Allocated memory location of a ‘Cons‘ (12 bytes)



Allocated memory location of a ‘Nil‘ (4 bytes)



Notable Design Decisions

- The first 4 bytes of a program are to be always initialized to 0 so that we can always have uninitialized DUT pointers pointing to it, then when this uninitialized DUT is read in, we will always see that the "instance" has $kindId = 0$, meaning it hasn't been instantiated.

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- When DUTs are read in, we create "helper functions" for each DUT variant. These "helper functions" are then used whenever we want to instantiate any particular DUT variant. DUT variant instantiation is hence secretly just a function call (to some function that creates a DUT on the heap and returns it's memory location).

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- Disjoint union type variants are mutable records. You may modify the values of a DUT variant only when caseing on it from within it's case.
- DUT variant kind identifiers are immutable!

Example of DUT instantiation helper

This function is used when wanting to instantiate a "Cons" variant (of a List).

```
(func $__mk_Cons (param $head i32) (param $tail i32) (result i32)
global.get $_memsize          ;; get known unused memory location
i32.const 1                   ;; get Cons's kind index
i32.store                     ;; store it
global.get $_memsize          ;; get known unused memory location
i32.const 4                   ;; get offset of the next type
i32.add                       ;; impose offset onto total memory size
local.get $head               ;; get param head
i32.store                     ;; store it in it's area
global.get $_memsize          ;; get known unused memory location
i32.const 8                   ;; get offset of the next type
i32.add                       ;; impose offset onto total memory size
local.get $tail               ;; get param tail
i32.store                     ;; store it in it's area
global.get $_memsize          ;; get global memory size
global.get $_memsize          ;; get global memory size (again)
i32.const 12                  ;; get size of kind (Cons)
i32.add                       ;; add to memory size
global.set $_memsize          ;; set memory size, leftover i32 on stack which is the
    returned pointer to the generated Cons
)
```

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- This causes issues for **pywasm**
  - Due to being interpreted in Python, a recursive call stack size limitation is imposed onto our programs.
- Thankfully, **wasmer** has no issues!
- In-browser WebAssembly execution also has no issues, but we don't ship a web browser with the compiler.

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  - Stronger syntactic sugar for String generation (e.g., “abcd...” for quickly instantiating large strings)
- Improved Memory Management
  - Memory freeing!
  - Memory reuse!
  - Allocation specialization for built-in DUTs!

References