

Disjoint Union Types in P0

Project 9 / Group 8

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- 4 Future Work
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- An instance of a DUT may take on the form of **only one** of it’s variants.
- They are often found in functional programming languages, where they are usually known as **Algebraic Data Types** (ADTs).

What do they look like?

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- Instantiation in our implementation,
a := Cons(1, Cons(2, Cons(3, Nil())))
b := Red()

How do we use DUTs?

The anatomy of the case statement.

- cases are the only way to access data inside of DUTs.

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- default case allows you to perform either a statement suite or a no-op on all non-covered cases.

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  ...  
  [default:  <stmtSuite>]
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  ... or ...  
  [default nothing]  
}
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case <variable> of {  
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    ...  
    [default: <stmtSuite>  
    ... or ...  
    [default nothing]  
}
```

Cover your cases!

If you create a non-exhaustive case statement, the compiler will warn you.

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Example: Maybe

```
type Maybe = Just(value: integer)
           | Nothing

procedure valOr(v: Maybe, n: integer) → (r: integer)
case v of {
    Just:
        r := v.value
    default:
        r := n
}

program Main
var maybe: Maybe

maybe <- Nothing()
writeln(valOr(maybe, -1))

maybe <- Just(1111)
writeln(valOr(maybe, 0))
```

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program Main
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```

Output

-1

1111

Example: Integer Lists

```
type List = Cons(head: integer, tail: List)
           | Nil

procedure upToList(n: integer) → (l: List)
  if n < 1 then l := Nil() else l := Cons(n, upToList(n-1))

procedure printList(l: List)
  case l of {
    Cons: writeln(l.head); printList(l.tail)
    default nothing
  }

procedure sumList(l: List) → (n: integer)
  case l of {
    Cons: n := sumList(l.tail) + l.head
    default: n := 0
  }

program Main
  var myList: List
  myList := upToList(5)
  printList(myList)
  writeln(sumList(myList))
```


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```

Output

5
4
3
2
1
15

Example: Strings

... lists in disguise?

```
type String = SCons(ch: integer, tail: String)
             | SNil

procedure printStr(s: String, ln: boolean)
  case s of {
    SCons: writeChar(s.ch); printStr(s.tail, ln)
    default: if ln then writeNewLine()
  }

// inclusively generating alphabets in a range
procedure genBetwn(start: integer, end: integer) -> (s: String)
  var ch: integer
  ch := end
  s := SNil()

  while start <= end do
    s, start, ch := SCons(ch, s), start + 1, ch - 1

program Main
  // print capital letters
  printStr(genBetwn('A', 'Z'), true)

  // print lowercase letters
  printStr(genBetwn('a', 'z'), true)

  // print numbers 0-9
  printStr(genBetwn('0', '9'), true)

  // print Greek letters
  printStr(genBetwn('α', 'ω'), true)
```

Note

We convert single-quoted characters into their Unicode integer representation when reading in P0 programs.

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```

Output

```
ABCDEFGHIJKLMNOPQRSTUVWXYZ
abcdefghijklmnopqrstuvwxyz
0123456789
αβγδεζηθικλμνξοπρστυφχψω
```

Note

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Other Examples

```
type RainbowColour = Red | Orange | Yellow | Green | Blue | Indigo | Violet

type Maybe = Just(value: integer)
           | Nothing

type Either = Left(value: integer)
           | Right(value: boolean)

type Tree = Branch(left: Tree, right: Tree)
          | Leaf(value: integer)

type Expr = Add(left: Expr, right: Expr)
          | Sub(left: Expr, right: Expr)
          | Mul(left: Expr, right: Expr)
          | Div(num: Expr, den: Expr)
          | Pow(base: Expr, exponent: Expr)
          | Int(value: integer)

type StringIntMap = SIMCons(key: String, value: integer, tail: StringIntMap)
                  | SIMEmpty
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Remark

Modelling is nice with disjoint union types!

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Focal Grammar Changes

- Disjoint union type declarations

```
type ::=  
  ident ["(" typedIds ")"] {"|" ident ["(" typedIds ")"]}  
  | ...
```

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type ::=  
  ident ["(" typedIds ")"] {"|" ident ["(" typedIds ")"]}  
  | ...
```

- case statements

```
statement ::= ... | "case" ident "of" "{" INDENT  
  ["nil" ":" statementSuite]  
  {ident ":" statementSuite}  
  ["default" (":" statementSuite | "nothing")]  
  DEDENT "}"
```


Supplementary Grammar and Procedure Changes

- Single-value returning procedures may be used “in-place” in expressions (e.g., “`m := 1 + pow(2, 3)`”)

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- “`< -`” and “`- >`” as alternatives for “`←`” and “`→`”, respectively

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- `< -` and `- >` as alternatives for `←` and `→`, respectively
- `>=` and `<=` as alternatives for `≥` and `≤`, respectively
- `*` as an alternative for `×`
- Standard procedures
 - `write` - no longer prints a newline character
 - `writeln` - writes single integer to std. out. with a newline afterwards
 - `writeChar` - writes single integer converted into a Unicode character to std. out.
 - `writeCharLn` - writes single integer converted into a Unicode character to std. out. with a newline afterwards
 - `writeNewLine` - writes a newline character to std. out.

Example: case WebAssembly Generation

The WebAssembly code on the right-hand side is generated for the below case statement.

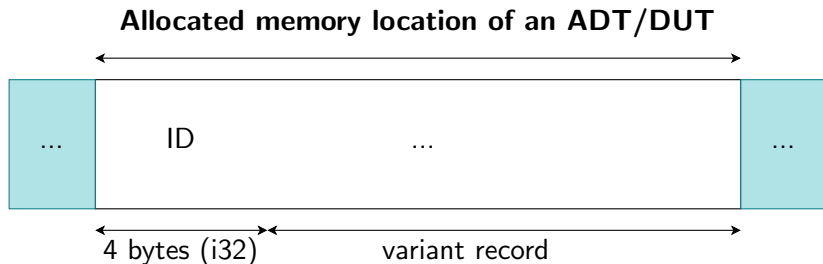
```
type Colour = R | G | Unknown

procedure printCol(col: Colour)
  case col of {
    nil: writeCharLn('?')
    R: writeCharLn('R')
    G: writeCharLn('G')
    default: writeCharLn('?')
  }
```

```
...
local.get $col
i32.load
i32.const 0                                ;; check if nil/0
i32.eq                                     ;; if it is nil
if
  i32.const 63
  call $writeCharLn                        ;; print '?'
else                                       ;; otherwise
  local.get $col
  i32.load
  i32.const 1                             ;; check if 'R'
  i32.eq
  if                                       ;; if it is 'R'
    i32.const 82
    call $writeCharLn                    ;; print 'R'
  else                                    ;; otherwise
    local.get $col
    i32.load
    i32.const 2                           ;; check if 'G'
    i32.eq
    if                                    ;; if it is 'G' kind
      i32.const 71
      call $writeCharLn                  ;; print 'G'
    else
      i32.const 63                       ;; otherwise, default
      call $writeCharLn                  ;; print '?'
    end
  end
end
end
```

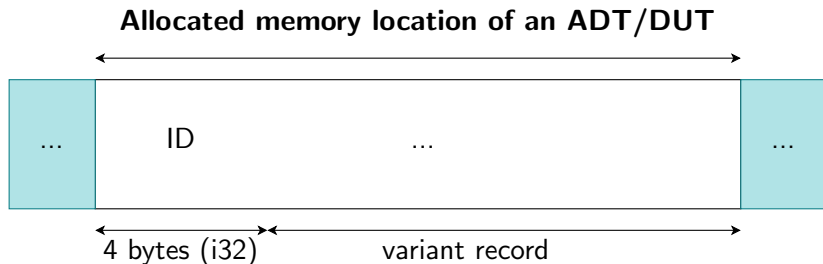
Memory Impact & Management

- Each instance of a DUT is located on the heap, and variables are just pointers to locations on the heap.



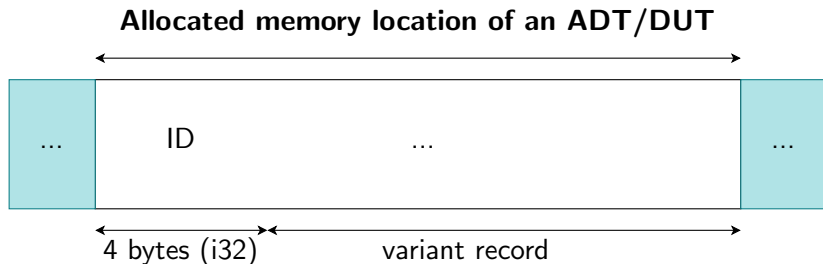
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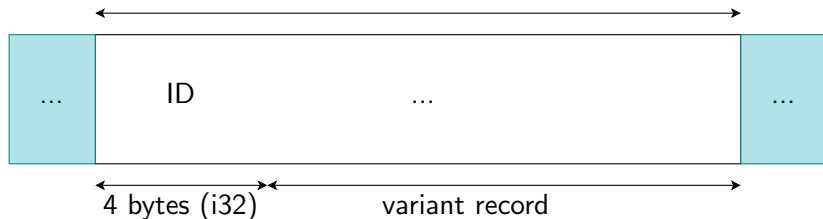
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Memory Impact & Management

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- Size of an allocation depends on the size of the variant being instantiated.
- Offsets to accessing variables works the same as records, with a 4 byte offset for the variant id.
- Memory usage can be fairly inefficient for certain constructions (e.g., Strings, Lists, etc).

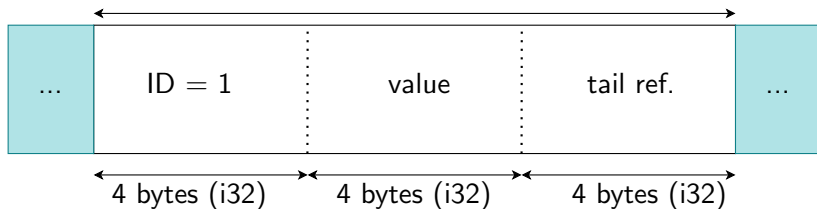
Allocated memory location of an ADT/DUT



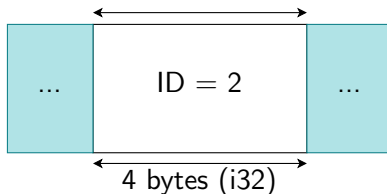
Example: Integer Lists in Memory

Strings are very similar

Allocated memory location of a 'Cons' (12 bytes)



Allocated memory location of a 'Nil' (4 bytes)



Notable Design Decisions

- The first 4 bytes of a program are always left as 0 so that we can always have uninitialized DUT pointers pointing to it naturally. When this uninitialized DUT is cased on, we will always see that the “instance” has $id = 0$, meaning it’s “nil”/not instantiated.

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- The data within the records are mutable, but you can only access them through caseing.
- When parsing DUTs, we create “instantiation helper functions” for each DUT variant.
 - “DUT variant instantiation” is secretly rewritten into procedure calls.
 - This simplifies parsing, and trims down generated code size.

Example of DUT instantiation helper

This function is used when wanting to instantiate a “Cons” variant (of a List).

```
(func $_mk.Cons (param $head i32) (param $tail i32) (result i32)
global.get $_memsize           ;; get known unused memory location
i32.const 1                     ;; get Cons's kind index
i32.store                       ;; store it
global.get $_memsize           ;; get known unused memory location
i32.const 4                     ;; get offset of the next type
i32.add                         ;; impose offset onto total memory size
local.get $head                ;; get param head
i32.store                       ;; store it in it's area
global.get $_memsize           ;; get known unused memory location
i32.const 8                     ;; get offset of the next type
i32.add                         ;; impose offset onto total memory size
local.get $tail                ;; get param tail
i32.store                       ;; store it in it's area
global.get $_memsize           ;; get global memory size
global.get $_memsize           ;; get global memory size (again)
i32.const 12                   ;; get size of kind (Cons)
i32.add                         ;; add to memory size
global.set $_memsize           ;; set memory size, leftover i32 on stack which is the
    returned pointer to the generated Cons
)
```

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- This causes issues for **pywasm**
 - Due to being interpreted in Python, a recursive call stack size limitation is imposed onto our programs.
- Thankfully, **wasmer** has no issues!
- In-browser WebAssembly execution also has no issues, but we don't ship a web browser with the compiler.

Challenges

- pywasm

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- pywasm
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Challenges

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- Recursive disjoint union types (self-references)

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- Recursive disjoint union types (self-references)
 - Assume type existence while parsing
 - Instances are references to other memory locations
- Making the grammar reasonably “natural” feeling in P0

- core components \approx 330 lines of code

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- supplementary components \approx 450 lines of code

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- tests and documentation \approx 1400 lines of code

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- \sim 230 commits
- \sim 18hrs for stable product
- \sim 30hrs for testing, debugging, documentation, and presentation slides

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Future Work

- Type variables!
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 - More code reuse!

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- Carette, Jacques, Oleg Kiselyov, and Chung-chieh Shan. “Finally tagless, partially evaluated: Tagless staged interpreters for simpler typed languages.” *Journal of Functional Programming* 19.5 (2009): 509.