Software Requirements Specification for BeamBending: examining a beam bending under load

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Contents

Revision History

Date	Version	Notes
Jan. 25, 2023	0.0.0	Template imported.
Jan. 25, 2023	0.1.0	Preliminary information added.
Jan. 25, 2023	0.1.1	Table of units added.
Jan. 25, 2023	0.1.2	Table of symbols added.
Jan. 25, 2023	0.1.3	Refining above tables.
Jan. 25, 2023	0.2.0	Refined introduction.
Jan. 25, 2023	0.3.0	Refining GSD, SSD, TMs, and IMs.
Feb. 4, 2023	0.4.0	Work on things other than the models.
Feb. 5, 2023	0.5.0	Refining models (again).
Feb. 6, 2023	0.6.0	Complete rough draft.
Feb. 6 - Mar. 15, 2023	0.6.0 - 0.9.0	Continue working on document, cleaning up misc. comments.
Mar. 16, 2023	1.0.0	Address Dr. Smith's feedback in full.
Apr. 24, 2023	2.0.0	Final submission.

1 Reference Material

This section records information for easy reference.

1.1 Table of Units

Throughout this document SI (Système International d'Unités) is employed as the unit system. In addition to the basic units, several derived units are used as described below. For each unit, the symbol is given followed by a description of the unit and the SI name.

symbol	**unit**	**SI**
m	length	metre
kg	mass	gram
Pa	pressure	pascal
rad	angle	radian
N	force	newton

Note that "unitless" quantities will be described as having the unit 1.

1.2 Table of Symbols

The table that follows summarizes the symbols used in this document along with their units. The choice of symbols was made to be consistent with the literature related to materials.

symbol	**unit**	**description**
L	m	abstract length of the beam
E	Pa	abstract Young's modulus (beam material modulus of elasticity)
I	m^4	abstract moment of second area
L_B	m	user-defined length of the beam
E_B	Pa	user-defined Young's modulus (beam material modulus of elasticity)
I_B	m^4	user-defined moment of second area of a cross-section of the beam
$w_B(x)$	$\mathrm{N\,m^{-1}}$	user-defined definition of the force of the distributed load applied at a specific point along the beam (directed downwards, towards the beam)
a_i	$\mathrm{N\,m^{-1}}$	coefficient of x^i from the polynomial defining $w_B(x)$.
x	m	distance of an arbitrary point along the beam, from the far left-side (at the pinned support)

$w(x)$	$\mathrm{N\,m^{-1}}$	hypothetical distributed force of the load applied at a specific point along the beam (directed downwards, towards the beam)
Pinned	m	1-dimensional position of the pinned support
Roller	m	1-dimensional position of the roller support
$y(x)$	m	deflection of a hypothetical loaded beam at a specific point along it (positive y values relate to amount deflected downwards)
$y_B(x)$	m	deflection of the loaded beam at a specific point along it (positive y values relate to amount deflected downwards)
a	1	point along the x-axis of a hypothetical graph
$f(a)$	1	point along the y-axis of a hypothetical graph, dependent on the value of a
$M(x)$	1	moment at a particular point along the beam
ρ	rad	ratio of curvature

1.3 Abbreviations and Acronyms

symbol	description
A	Assumption
BVP	Boundary Value Problem
DD	Data Definition
GD	General Definition
GS	Goal Statement
IM	Instance Model
LC	Likely Change
PS	Physical System Description
R	Requirement
SRS	Software Requirements Specification
BeamBending	Beam Bending Program
TM	Theoretical Model

1.4 Mathematical Notation

The common mathematical notation used in Canadian undergraduate-level mathematics will be used throughout this document.

2 Introduction

Beams safely distribute the stress of the load to its foundations. For example, beams are used in both residential and commercial applications. In residential homes, we expect floors (which might be supported by beams) to remain flat enough that inhabitants wouldn't care about minor deflections. However, in other applications of beams, such as the beds of industrial-strength mechanical tools, and bridges, we might expect beams to be relatively pliant under loading. For example, we might expect these industrial applications to have some vertical and horizontal deflection. *Simply supported* beams are one type of beam that are commonly found in bridges and beds of machine tools. It is important to understand how beams will react under load or else we risk damaging structures, floor bending (making inhabitants feel unsafe), or damaging loads. We may use software to analyze the reaction of beams under various loading scenarios. Henceforth, we will call some software artifact that conforms to these requirements as "BeamBending".

This document aims to develop a general scheme for understanding the reaction of a simply supported beam to imposed load under simplified conditions. This section aims to provide an overview of the Software Requirements Specification (SRS) for the BeamBending problem, discussing the scope and purpose of the work.

2.1 Purpose of Document

The purpose of this document is to provide the reader with a well-derived, verifiable explanation of a solution to the beam bending problem. This document outlines sufficient information for developers to create software tools that solve the BeamBending problem. The produced software artifact has an increased[1] degree of confidence in reliability and correctness by being traceable to and derived by these defined software requirements specifications. As such, a large focus on the development of this document is to have the domain knowledge captured and adequately codified such that the origins of fragments of code may be verified, up to development choices (e.g., numerical algorithm, language, tooling, etc.).

2.2 Scope of Requirements

The requirements analyze the problem (as defined in Section 4.1) and related solution (as per Section 4.2.6, most directly) under the assumption of the Euler-Bernoulli Beam Theory [1] in a two-dimensional space for a simply supported prismatic beam.

2.3 Characteristics of Intended Reader

Readers of this document should at least have an understanding of:

- at least a first-year university engineering mechanics concepts,

[1]Not that we will try to quantify it, however.

- first-year university linear algebra and calculus concepts, such as derivatives, integration, continuous functions, and vectors.

However, it would be an asset for users to have at least a second-year university understanding of physics and calculus concepts to confidently audit the document.

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2.4 Organization of Document

This document follows the SRS template as specified by Smith and Lai [2]. If you are already familiar with the SRS template, the author’s recommended, but not required, reading order is as follows:

- Problem Description,
- Goal Statements,
- Characteristics of Intended Reader,
- Scope of Requirements,
- General System Description,
- Assumptions,
- Specific System Description, and, finally,
- Instance Models to Theoretical Models.

The other material is referential and may be read as needed.

3 General System Description

This section provides general information about the system so that the next section will be easier to digest. It identifies the interfaces between the system and its environment, describes the user characteristics, and lists the system constraints.

3.1 System Context

Figure 1 represents an abstract view of the software. The rectangular node represents the BeamBending software itself, whilst circular nodes represent external entities interacting with the BeamBending software. The abstract view is that a user would provide information (e.g., beam and load specifications) to the software, which would use an external Boundary Value Problem (BVP) solver to approximate the deflection curve.

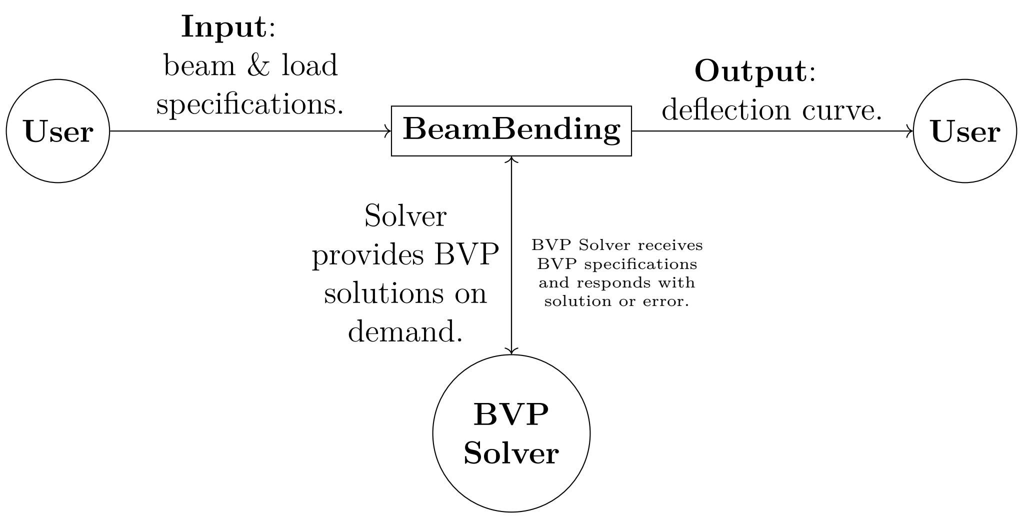

Figure 1: System Context

- User Responsibilities:
  - Providing material properties of the beam, taking required units, assumptions, and applicability of the beam into consideration.
  - Providing an explanation of the imposed load as a function of the distance from the leftward pinned support.
  - Interpret the output of the program taking this SRS document into consideration and use an audited and reliable copy of a software related to this SRS.
- BeamBending Responsibilities:

- Detect data type mismatches, such as a string of characters instead of a floating point number.
- Validating applicability of the input arguments to the proposed "solution" outlined in this document.
- Approximate the deflection of the beam under the imposed loading.

- External BVP Solver Responsibilities:
  - Attempt to solve 4^{th} order BVP problems on demand.

3.2 User Characteristics

The user of this software should have a working understanding of the deflection of beams, shear forces, bending moments, and related concepts. If applied in educational purposes, only an understanding of first-year university physics and calculus is required to generally understand the program and simulate a simply supported beam across various scenarios.

The author of this document accepts no liability and provides no warranty for any usage of the software. The software should be appropriately audited by appropriate authorities and conform to local law.

3.3 System Constraints

The final software should be built using Drasil to encode the problem and generate a problem using a BVP solver. There are no other system constraints. However, it should be noted that the usage of Drasil does impose the limitations of Drasil onto the produced software and SRS documents. One notable limitation is the inability to input general expressions into Drasil-generated programs. As such, the "user-defined" loading function, $w_B(x)$, will need to be hard-coded into both this SRS document and the software. However, changing the source for both is relatively straightforward and simple thanks to Drasil.

4 Specific System Description

This section first presents the problem description, which gives a high-level view of the problem to be solved. This is followed by the solution characteristics specification, which presents the assumptions, theories, definitions and finally the instance models.

4.1 Problem Description

BeamBending is intended to solve for the deflection experienced by a beam given a distributed loading function and properties of the beam.

4.1.1 Terminology and Definitions

This subsection provides a list of terms that are used in the subsequent sections and their meaning, with the purpose of reducing ambiguity and making it easier to correctly understand the requirements:

- beam: structural component intended to carry loads perpendicularly to their "long" axis,
- bending moment: the internal reaction of a structural element when an external force is imposed and bends it,
- deflection: the amount that a structural component is displaced due to deformation under load,
- load/loading: an applied force,
- modulus of elasticity: the measure of lengthwise stiffness of an element as a force is applied lengthwise,
- bending moment: a measure of how well a cross-section resists bending[3],
- pinned support: a structural support that allows for rotation but neither vertical nor horizontal movement,
- shear force: force applied perpendicularly to objects,
- simply supported beam: a beam that has a pinned support on one end and a roller support on the other,
- slender: has a length to height ratio greater than 10 to 1,
- roller support: a structural support that allows for rotation and horizontal movement, but not vertical movement, and
- Young's modulus: common synonym for modulus of elasticity.

4.1.2 Physical System Description

An overview of the physical system of BeamBending at rest is depicted in Figure 2.

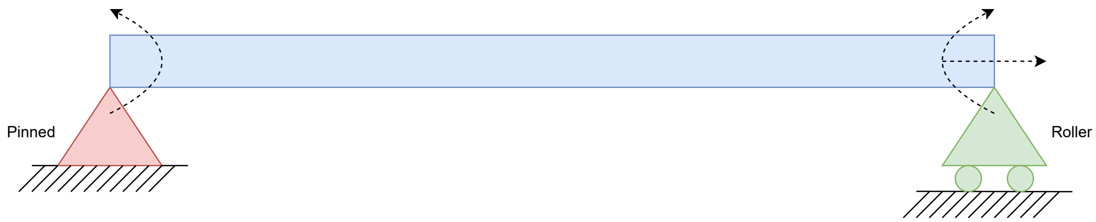

Figure 2: Diagram of a Simply Supported Beam

Figure 2 shows the physical system at rest. A slender beam is supported by a pinned support and a roller support (i.e., it is simply supported). For the sake of following convention, the pinned support is placed on the left side of the beam, while the roller support is on the right. Additionally, when thinking of distance along the beam, we think of it as a distance from the pinned support on the left-hand side. As shown in Figure 2, the pinned support allows for rotational movements about the support tip, while the roller support also allows for some horizontal movement along the roller. When the beam is loaded, the roller support allows for tensile/compressive changes in the beam (which would alter the length of the beam).

The physical system described in BeamBending examines Figure 2 under an inputted general loading function. As such, the "whole" system is as per Figure 3, containing:

PS1: a slender beam,

PS2: a pinned support,

PS3: a roller support, and

PS4: an applied distributed load.

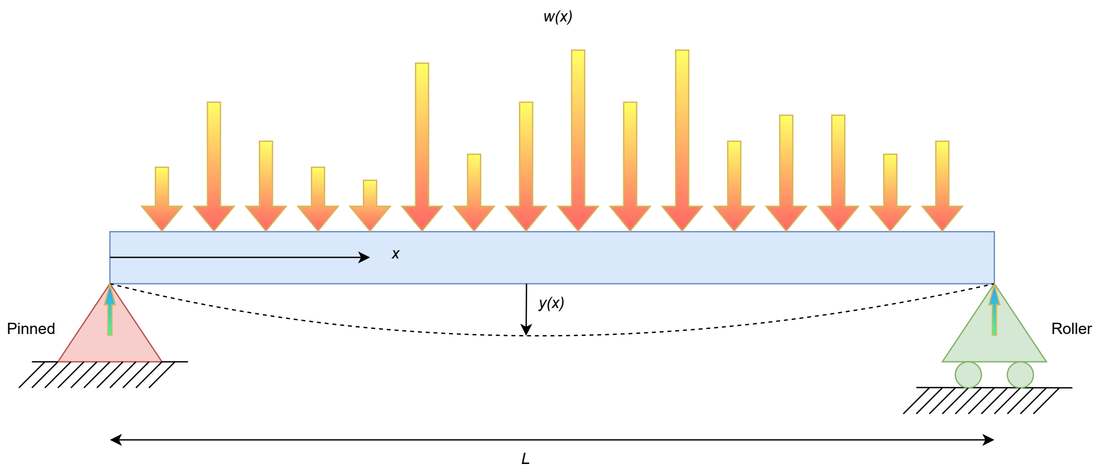

Figure 3: Diagram of a Simply Supported Beam Under Load

However, note that at the "boundaries" (the support tips), the beam may not have vertical deflection, and similarly has zero bending moment at the tips. Additionally, it should be noted that the beam is *statically determinant* [4].

4.1.3 Goal Statements

Given constant length, modulus of elasticity, and bending moment, and the applied distributed load on the beam as a function from the distance of the leftward pinned support, the goal statements are:

GS1: Calculate the deflection of the beam under load.

4.2 Solution Characteristics Specification

The instance models that govern BeamBending are presented in Subsection 4.2.6. The information to understand the meaning of the instance models and their derivation is also presented, so that the instance models can be verified.

4.2.1 Assumptions

This section simplifies the original problem and helps in developing the theoretical model by filling in the missing information for the physical system. The numbers given in the square brackets refer to the Theoretical Model (TM), General Definition (GD), Data Definition (DD), Instance Model (IM), or Likely Change (LC), in which the respective assumption is used.

A1: The "world" model is two-dimensional, observing the instant a load is applied on the beam.

A2: The beam is slender.

A3: The beam is prismatic (i.e., A4 & A5).

A4: The beam has a uniform cross-section.

A5: The beam is straight/flat within a reasonable tolerance.

A6: The beam has a constant bending moment.

A7: The beam experiences vertical linear elastic load.

A8: The beam's modulus of elasticity is constant along the beam.

A9: Only relatively small deflections will be examined (whereby the maximum deflection is at most `SLENDER` (10%) of the beam's length).

A10: The deflection will have locally small slopes across the beam.

A11: The beam is simply supported.

A12: The beam's loading is captured by a third-order polynomial of standard form, $w_B(x) = a_0 + a_1x + a_2x^2 + a_3x^3$.

A13: The beam contains no point loads.

A14: The beam has no loading applied axially.

A15: The deflection function ($y_B(x)$) belongs to the class of functions that are continuously differentiable 4 times on $[0, L]$.

A16: The loading function ($w_B(x)$) belongs to the class of functions that are integrable 4 times on $[0, L]$.

4.2.2 Theoretical Models

This section focuses on the general equations and laws that BeamBending is based on.

RefName: TM1

Label: Curvature of a Plane

Equation:

$$\frac{1}{\rho} = \frac{\frac{d^2 f}{da^2}}{(1 + (\frac{df}{da})^2)^{\frac{3}{2}}}$$

Description: From elementary calculus, the curvature of a plane curve at a point $(a, f(a))$ of a curve may be expressed by the above equation [5], where:

- ρ is the radius of curvature (1),
- f is the vertical component of the point (1), and
- a is the horizontal component of the point (1).

Notes: None.

Source: [5]

Ref. By: TM3

Preconditions for TM1: None.

Derivation for TM1: Not Applicable

RefName: TM2

Label: Prismatic Beam Pure Bending into Arc

Equation:

$$\frac{1}{\rho} = \frac{M(x)}{EI}$$

Description: For a beam, the reciprocal of the radius of curvature is a ratio of the bending moment ($M(x)$) to the flexural rigidity (EI).

Notes: None.

Source: [5]

Ref. By: TM3

Preconditions for TM2: None.

Derivation for TM2: Not Applicable

RefName: TM3

Label: ODE Relationship of an Elastic Curve

Equation:

$$\frac{d^2y}{dx^2} = \frac{M(x)}{EI}$$

Description: For a beam, the second derivative of the deflection function (i.e., the shear force at a point along the beam) may be expressed as a ratio of the bending moment ($M(x)$) to the flexural rigidity (EI).

Notes: None.

Source: [5]

Ref. By: TM4

Preconditions for TM3: None.

Derivation for TM3: Starting with TM1:

$$\frac{1}{\rho} = \frac{\frac{d^2y}{dx^2}}{(1 + (\frac{dy}{dx})^2)^{\frac{3}{2}}}$$

We may recognize that in the context of an abstract prismatic beam, the slope $\frac{dy}{dx}$ is relatively "small" (A10). Similarly, its square is also small. Considering it negligible, we may drop it:

$$\frac{1}{\rho} = \frac{d^2y}{dx^2}$$

Substituting $\frac{1}{\rho}$ from TM2, we obtain:

$$\frac{d^2y}{dx^2} = \frac{M(x)}{EI}$$

RefName: TM4

Label: Euler-Bernoulli Beam Deflection Equation

Equation:

$$EI\frac{d^4y}{dx^4} = w(x)$$

Description: The above equation describes the relationship between a beam's deflection ($y(x)$) and the applied load ($w(x)$) for any point along the beam (x). Notably, for a simply supported beam (A11), the ends observe no deflection, thus:

- $y(0) = 0$, and
- $y(L) = 0$.

Additionally, as there is no resistance to rotation at the ends, the moments are zero at the ends (e.g., $M(0) = M(L) = 0$), and thus, as $M = \frac{d^2}{dx^2}y$:

- $\frac{d^2}{dx^2}y(0) = 0$, and
- $\frac{d^2}{dx^2}y(L) = 0$

Notes: None.

Source: [1, 5]

Ref. By: IM1

Preconditions for TM4: None.

Derivation for TM4: When a prismatic beam supports an applied loading, ($w(x)$), its elastic curve is governed by the fourth-order ODE found by integrating the equation found in TM3 twice [1].

4.2.3 General Definitions

There are no general definitions.

Detailed derivation of simplified rate of change of temperature

4.2.4 Data Definitions

Number	DD1
Label	**Applied Load**
Symbol	$w_B(x)$
SI Units	$\mathrm{N\,m^{-1}}$
Equation	$w_B(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$ $(\mathrm{N\,m^{-1}})$
Description	$w_B(x)$ is the function describing the load applied along the beam. x is a point along the beam (m). a_i is the coefficient of x^i from the polynomial $(\mathrm{N\,m^{-1}})$.
Ref. By	IM1

4.2.5 Data Types

The conventional data types used in undergraduate-level physics courses is sufficient. As such, the specifications define no extra data types.

4.2.6 Instance Models

This section transforms the problem defined in Section 4.1 into one which is expressed in mathematical terms. It uses concrete symbols defined in Section 4.2.4 to replace the abstract symbols in the models identified in Sections 4.2.2 and 4.2.3. Notably, GS1 is solved by IM1.

Number	IM1
Label	**Beam Deflection**
Input	L_B, E_B, I_B, $w_B(x)$
Output	$y_B(x)$ as the solution to the BVP defined as: $E_B I_B \frac{d^4 y_B}{dx^4} = w_B(x)$ where $y_B(0) = 0$, $\frac{d^2 y_B}{dx^2}(0) = 0$, $y_B(L_B) = 0$, and $\frac{d^2 y_B}{dx^2}(L_B) = 0$
Description	The above output is the solution of the above defined BVP. L_B is the beam's length (m). E_B is the beam's modulus of elasticity (Pa). I_B is the beam's moment of second area (m^4). $w_B(x)$ is the function describing the load applied along the beam ($\mathrm{N\,m^{-1}}$). x is a point along the beam (m). $y_B(x)$ is a function describing the deflection along the beam (m).
Sources	—
Ref. By	—

Derivation of Beam Deflection:

This model is a direct instantiation of TM4 observed as a BVP for the simply supported beam in question as provided by user inputs (i.e., with L_B, E_B, I_B, w_B, y_B substituted in).

4.2.7 Input Data Constraints

Table 2 shows the data constraints on the input & output variables. The column for physical constraints gives the physical limitations on the range of values that can be taken by the variable. The column for software constraints restricts the range of inputs to reasonable values. The software constraints will be helpful in the design stage for picking suitable algorithms. The constraints are conservative, to give the user of the model the flexibility to experiment with unusual situations. The column of typical values is intended to provide a feel for a common scenario. The uncertainty column provides an estimate of the confidence with which the physical quantities can be measured. This information would be part of the input if one were performing an uncertainty quantification exercise.

The specification parameters in Table 2 are listed in Table 4.

Table 2: Input Variables

Var	Physical Constraints	Software Constraints	Typical Value	Uncertainty
L_B	$0 < L_B$	$0 < L_B < L_{\text{Max}}$	10 m	0%
E_B	$0 < E_B$	$0 < E_B < E_{\text{Max}}$	2E^{9} Pa	5%
I_B	$0 < I_B$	$0 < I_B < I_{\text{Max}}$	1.3E^{-5} m^4	5%
a_0	—	—	—	0%
a_1	—	—	—	0%
a_2	—	—	—	0%
a_3	—	—	—	0%

Table 4: Specification Parameter Values

Var	Value
L_{Max}	60 m
E_{Max}	2E^{12} Pa
I_{Max}	10 m^4

4.2.8 Properties of a Correct Solution

A correct solution must comply with the limitations as defined in Table 6.

Table 6: Output Variables

Var	Physical Constraints
$y_B(x)$	$y_B(0) = 0$ $y_B(L_B) = 0$ $\forall\ x \in [0, L_B]\ .\ \lvert y_B(x)\rvert \leq L_B\texttt{SLENDER}$ (by A9)

5 Requirements

This section provides the functional requirements, the business tasks that the software is expected to complete, and the nonfunctional requirements, the qualities that the software is expected to exhibit.

5.1 Functional Requirements

R1: The inputs must be provided by the end user according to Table 2.

R2: The inputs of BeamBending should be displayed to the user before any meaningful computation begins.

R3: The provided inputs must be validated against Table 2.

R4: The output variables (see Table 6) should be calculated following the instance models as per Section 4.2.6.

5.2 Nonfunctional Requirements

NFR1: **Accuracy** The accuracy of the computed solutions should meet the level needed for the usage domain (this is up to applicable regulatory bodies). To gain confidence that the software is accurate, BeamBending shall be evaluated by the steps outlined in the Verification and Validation Plan.

NFR2: **Usability** The program should allow for post-processing and visualization of the calculated data by having a standardized output (up to the design of the software). BeamBending is not intended to be used alone with manual analysis. The level of usability achieved by the software shall be described following the procedure given in Section X of the Verification and Validation Plan.

NFR3: **Maintainability** Through specialized tooling (Drasil), the effort needed to make any likely change should be relatively straightforward and already well-understood. Additionally, any other possible change should have a clear impact outlined by the specialized tools.

NFR4: **Portability** BeamBending should be usable on at least the 3 major home operating systems (Linux, macOS, and Windows).

6 Likely Changes

LC1: The loading function is currently pre-defined and should be changed.

LC2: The units of the variables may be changed if numbers have poor readability in certain units.

LC3: The beam's configuration is likely to change across different beam needs and positions.

7 Unlikely Changes

The focal problem of analyzing a beam bending under load is the only unlikely thing to change in this SRS. In any variants thereof, there should be some kind of reference to this problem.

8 Traceability Matrices

The purpose of the traceability matrices is to provide easy references on what has to be additionally modified if a certain component is changed. Every time a component is changed, the items in the column of that component that are marked with an "X" may have to be modified as well. Table 8 shows the dependencies of theoretical models, general definitions, data definitions, and instance models with each other. Table 9 shows the dependencies of instance models, requirements, and data constraints on each other. Table 10 shows the dependencies of theoretical models, general definitions, data definitions, instance models, and likely changes on the assumptions.

	TM1	TM2	TM3	TM4	DD1	IM1	GS1
TM1							
TM2							
TM3	X	X					
TM4			X				
DD1							
IM1				X	X		X
GS1							

Table 8: Traceability Matrix Showing the Connections Between Items of Different Sections

9 Development Plan

The software is to be constructed using Drasil. Additionally, Drasil should be used to re-construct this SRS document. Faking the "rational design process" in Drasil allows developers to scale products against changes in the background knowledge. Drasil has extensive

	IM1	4.2.7	R1	R2	R3	R4
IM1		X	X			
R1		X				
R2			X			
R3			X			
R4			X			

Table 9: Traceability Matrix Showing the Connections Between Requirements and Instance Models

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A15	A16
TM1	X	X		X							
TM2	X	X	X	X	X	X	X	X			
TM3	X	X	X	X	X	X	X	X			
TM4	X	X	X	X	X	X	X	X	X	X	X
DD1											
IM1	X	X	X	X	X	X	X	X	X	X	X
LC1										X	X
LC2											
LC3		X	X	X	X	X	X	X	X	X	X

Table 10: Traceability Matrix Showing the Connections Between Assumptions and Other Items

documentation on *how* to develop software using it and should be followed by any developer using this document.

10 Values of Auxiliary Constants

Table 11: Auxiliar Constants

Label	Value	Unit	Description
`SLENDER`	$\frac{1}{10}$	1	Maximum observable deflection allowed.

References

[1] Wikipedia contributors. *Euler-Bernoulli beam theory — Wikipedia, The Free Encyclopedia*. [Online; accessed 20-January-2023]. 2022. URL: https://en.wikipedia.org/w/index.php?oldid=1125198888 (cit. on pp. 1, 12).

[2] W. Spencer Smith and Lei Lai. "A New Requirements Template for Scientific Computing". In: *Proceedings of the First International Workshop on Situational Requirements Engineering Processes – Methods, Techniques and Tools to Support Situation-Specific Requirements Engineering Processes, SREP'05*. Ed. by J. Ralyté, P. Ȧgerfalk, and N. Kraiem. In conjunction with 13th IEEE International Requirements Engineering Conference. Paris, France, 2005, pp. 107–121 (cit. on p. 2).

[3] Spencer Smith. *Discussions between Dr. Spencer Smith and Jason Balaci*. 2023 (cit. on p. 5).

[4] Wikipedia contributors. *Statically indeterminate — Wikipedia, The Free Encyclopedia*. [Online; accessed 4-March-2023]. 2004. URL: https://en.wikipedia.org/wiki/Statically_indeterminate (cit. on p. 7).

[5] Ferdinand P. Beer and E. Russell Johnston Jr. *Mechanics of Materials*. McGraw-Hill Ryerson, 1981 (cit. on pp. 9–12).