Global Search Method for Locating General Slip Surface Using Monte Carlo Techniques

By Abdallah I. Husein Malkawi,[1] Waleed F. Hassan,[2] and Sarada K. Sarma[3]

Abstract: Searching for the critical slip surface and the lowest factor of safety in slope stability analysis can be achieved by means of optimization techniques. A new search procedure in generating kinematically admissible slip surfaces has been introduced in this paper. Such a procedure is based, mainly, on the Monte Carlo methods, where both the critical global slip surface as well as its associated factor of safety is determined. Several practical examples, of known minimum factor of safety and its associated slip surface, have been used to demonstrate the efficiency and capability of the proposed method. The method is intended to be robust and effective to solve problems that involve extremely complicated slope geometry. It is as powerful as any other powerful optimization methods.

INTRODUCTION

By the advent of computers, optimization-based techniques have become an effective means of searching for the critical slip surface in slope stability analysis. Examples of such methods that provide reasonable results can be found in Celestino and Duncan (1981), Baker (1980), Nguyen (1985), Li and White (1987), Arai and Tagyo (1985), and Chen and Shao (1988). However, practical feasibility and application of these methods are still of concern.

The use of optimization techniques in locating the critical slip surface and its associated factor of safety has been one of the most interesting topics for the researchers. Baker and Gaber (1977, 1978) used the calculus of variations to locate the critical slip surface and its associated factor of safety. However, Luceno and Castillo (1980) and Castillo and Luceno (1982) questioned the results of Baker and Gaber about the existence of such a minimum in their results. They concluded that their method has been incorrectly formulated. De Josselin De Jong (1980) argued about the correctness in using variational calculus. Celestino and Duncan (1981) and Li and White (1987) used alternating variable methods to locate the critical noncircular slip surface. Baker (1980) used dynamic programming to determine the critical slip surface associated with Spencer's method of slope stability analysis (Spencer 1967). Chen (1992) postulated that using a random trail search would lead to the global minimum factor of safety. Nguyen (1985) and De Natale (1991) used the simplex method, and Chen and Shao (1988) used simplex, steepest descent, and Davidson-Fletcher-Powell (DFP) methods in conjunction with a grid search solution. Duncan (1996) presented a state of the art comprehensive review of both limit equilibrium and finite-element analysis of slopes. Greco (1996) presented Monte Carlo based techniques of the random walk type to locate the critical slip surface. The trial solutions are randomly generated and then compared with the best solution for improvement. However, implementation of this method in an automatic search requires too many constraints. Recently, Husein Malkawi et al. (2001) developed an effective approach for locating the critical circular slip surfaces based on Monte Carlo techniques. The method proved to be accurate, efficient, and reliable; besides, it could be easily incorporated as a subroutine in any computer program for slope stability for the use as an engineering routine.

Monte Carlo based methods are simply structured, random searching and optimization techniques. In these methods, a large number of trial surfaces can be generated to ensure a minimum factor of safety. Through having the searching routine as a base, random jumping and random walk are the two main classes into which Monte Carlo techniques are divided. In random jumping, a large number of random trial surfaces are generated to locate the critical slip surface having the least safety factor, as defined by Siegal et al. (1981). Every trial solution is generated throughout those methods, without considering the previous trial. For this reason, this approach is recommended to be adopted only for the purpose of locating the initial estimate to be used by other methods of optimization. In random walk methods, iterative slip surfaces are constructively generated; that is, the ith trial is modified to get the $(i + 1)$th solution. As a result, a series of improved approximations to the minimum is obtained with less iterative effort (Cherubini and Greco 1987; Greco 1996, Husein Malkawi et al. 2001).

This paper presents a new search procedure based on the principle of the Monte Carlo method of the random walk type. The proposed method shows solutions of high quality, accuracy, and efficiency. The structure of the proposed method is easily programmed and can be implemented into an automatic searching technique to be used as a routine engineering application. Besides, the method produces accurate results of the safety factor and predicts the failure mechanisms; all slip surfaces generated are kinematically admissible.

IMPLEMENTATION OF SEARCH STRATEGY

The following proposed method could easily be integrated with limiting equilibrium-based methods such as the ordinary method of slices (Fellenius 1936), Bishop's modified method (1955), Janbu's method (1954, 1973), Spencer's method (1967, 1973), Morgenstern and Price's method (1965), and Sarma's method (1973, 1979). For this purpose, a software package called Stability Analysis of Slopes using Monte Carlo Techniques (SAS-MCT) was developed by Husein Malkawi and Hassan (2001), in which the limiting equilibrium-based methods (i.e., ordinary method of slice, Bishop's method, Janbu's method, Morgenstern and Price's method, and Spencer's method) were combined in the proposed method based on the principle of the Monte Carlo techniques. The program searches for the most critical circular and noncircular slip surface and respectively calculates its associated minimum factor of safety.

---

[1]PhD, Assoc. Prof. of Geotech. Engrg., Civ. Engrg. Dept., Jordan Univ. of Sci. and Technol., 221-10 Irbid, Jordan. E-mail: mhusein@just.edu.jo

[2]MS, Res. Asst., Civ. Engrg. Dept., Jordan Univ. of Sci. and Technol., 221-10 Irbid, Jordan.

[3]PhD, Reader in Engrg. Seismology, Civ. and Envir. Engrg., Imperial Coll. of Sci., Technol. and Medicine, Imperial College Rd., London SW7 2BU, U.K. E-mail: s.sarma@ic.ac.uk

Note. Discussion open until January 1, 2002. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on January 21, 2000; revised April 2, 2001. This paper is part of the ***Journal of Geotechnical and Geoenvironmental Engineering***, Vol. 127, No. 8, August, 2001. ©ASCE, ISSN 1090-0241/01/0008-0688–0698/$8.00 + $.50 per page. Paper No. 22258.

J. Geotech. Geoenviron. Eng., 2001, 127(8): 688-698

FORMULATION

The problem can be described in the *o*, *x*, *y*-plane (Fig. 1) by using the following functions:

- $Y = g(x)$ represents the topographic profile of the soil.
- $Y = L(x)$ represents the discontinuity surface in layered soil.
- $Y = r(x)$ represents the lower boundary (stiff or rock soil).
- $Y = s(x)$ represents the slip surface.
- $Y = w(x)$ represents the water table.

The development of every slip surface is represented by n vertices [$V_1, V_2, \ldots, V_n$ with coordinates $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$, respectively] and $n - 1$ segments. Each segment can be identified by two vertices. To make the slip surface kinematically admissible, these segments are assumed to be concave upward, which means that

$$\alpha_1 < \alpha_2 < \alpha_3 < \cdots < \alpha_i < \cdots < \alpha_{n-1} \tag{1}$$

where α_i = inclination of segment i.

Each specific slip surface can be expressed mathematically by a $2n$-dimensional array

$$S = [x_1, y_1, x_2, y_2, \ldots, x_n, y_n]^T \tag{2}$$

The objective function locating the critical slip surface, which has the minimum safety factor among all the available ones, can be stated

$$\min F(S) \tag{3}$$

The geometrical constraints that minimize $F(S)$ are crucial to ensure this target:

- The abscissas of all vertices should be enclosed within $x_{\min}$ and $x_{\max}$, or mathematically

$$x_{\min} \le x \le x_{\max} \tag{4}$$

  and subjected to the condition that

$$x_i < x_{i+1} \quad \text{for } i = 1, n - 1 \tag{5}$$

- The ordinates of these vertices should be confined between the topographic profile (upper boundary) and the bearing layer (lower boundary)

$$r(x) \le s(x) \le g(x) \tag{6}$$

$$y_i = g(x_i) \quad \text{for } i = 1 \text{ and } i = n \tag{7}$$

$$r(x_i) < s(x_i) < g(x_i) \quad \text{for } i = 2, n - 1 \tag{8}$$

  where n = number of vertices.

SOLUTION OF PROBLEM

To start the search for the potential critical slip surface, the domain on the *o*, *x*, *y*-plane where the search is to be made must be specified. Then the end vertices V_1 and V_4 are randomly chosen (Fig. 2). Two straight lines are extended from these determined end vertices, the slope of these lines are similarly determined by generating a pair of random angles α_1 and α_3. The intersection of these two lines defines a new vertex $V(x, y)$; the coordinates of this vertex are determined mathematically. The third segment is found by similarly generating a new randomly determined angle α_2. The intersection of this segment with the other two lines at V_2 and V_3 defines the other two segments. This technique is suitable for generating the first random slip surface with four vertices and three segments (Fig. 2). Mathematically this is shown as follows, where $R_1, R_2, \ldots, R_7$ are uniformly distributed random numbers in the range (0, 1).

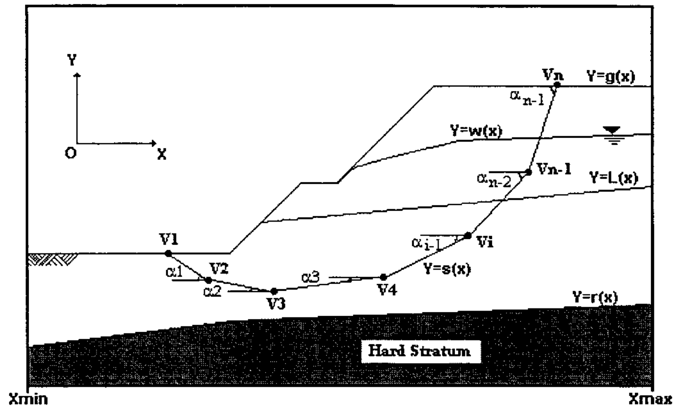

FIG. 1. General Cross Section of Slope

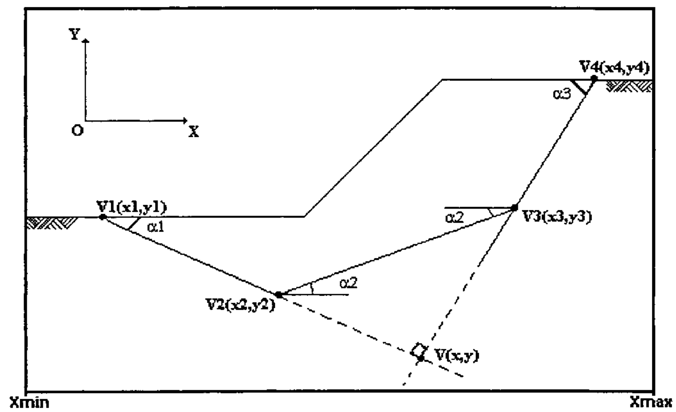

FIG. 2. Generation of First Slip Surface

The coordinates of both vertices V_1 and V_4 are determined according to the following equations.

$$x_1 = x_{\min} + R_1 \frac{(x_{\max} - x_{\min})}{2} \tag{9}$$

$$y_1 = g(x_1) \tag{10}$$

$$x_4 = x_{\max} - R_4 \frac{(x_{\max} - x_{\min})}{2} \tag{11}$$

$$y_4 = g(x_4) \tag{12}$$

Two lines are extended from vertices V_1 and V_4 with slopes randomly generated as given below

$$\alpha_1 = (R_2 - 3) \times \frac{\pi}{12} \tag{13}$$

$$\alpha_3 = \frac{\pi}{2} - R_3 \times \alpha_1 \tag{14}$$

where α_1 and α_3 are in radian scale and α_1 (at the toe) is negative while α_3 is positive (at the crest). Constants 3 and 12 in (13) are selected in such a way that α_1 is limited between 30° and 45°. This is made to avoid computational difficulties encountered in solving for the safety factor. Ching and Fredlund (1983) discussed such limitations on the α values on both the active and the passive zone to avoid any numerical difficulties and geometrical conditions imposed on the stability computations.

The two lines are intersecting each other at vertex $V(x, y)$. The angle of inclination of segment $\mathbf{V}_2\mathbf{V}_3$ α_2 is given by

$$\alpha_2 = R_5 \times \alpha_3 \tag{15}$$

J. Geotech. Geoenviron. Eng., 2001, 127(8): 688-698

The y-coordinate of vertex V_2 is randomly generated and the x-coordinate is mathematically calculated as follows:

$$y_2 = y + R_6 \times (y_1 - y) \tag{16}$$

$$x_2 = x_1 + (y_2 - y_1) \times \tan \alpha_1 \tag{17}$$

Similarly for vertex V_3, which is the intersection of segment two and three

$$x_3 = x_2 + R_7 \times (x_4 - x_2) \tag{18}$$

$$y_3 = y_2 + (x_3 - x_2) \times \tan \alpha_2 \tag{19}$$

SEARCH METHOD

To achieve a reduction in the safety factor, each segment of the specific slip surface has to rotate randomly around its vertices. Segment i will rotate around vertex V_{i+1} as follows:

$$\alpha_i^{j+1} = \alpha_i^j + |R \times \Omega_i^j| \times N_d \tag{20}$$

where R = uniformly distributed random number in the range (-0.5, $+0.5$); Ω_i^j = angle of rotation for segment i at stage j, where an initial value for Ω can be assumed as $\Omega = \alpha_i/(2m - 1)$ and m is the number of segments; and N_d = number that defines the direction of rotation: ($+1$) clockwise, (-1) counterclockwise.

The coordinates of vertex V_i are then calculated from the geometry, and the slip surface is tested according to constraints [(4)–(8)]. The safety factor for the new slip surface is then calculated. If this safety factor is less than the pervious minimum value, this trail will be considered successful and the angle of rotation is extended to

$$\Omega_i^{j+1} = \Omega_i^j \times \varepsilon_1 \tag{21}$$

Consequently, a new rotation trail is evaluated. If this trail is unsuccessful, the angle of rotation is reduced to

$$\Omega_i^{j+1} = \Omega_i^j \times \varepsilon_2 \tag{22}$$

where ε_1 and ε_2 = factors used to increase and decrease the angle of rotation. These factors must be >1.0 and <1.0, respectively. Their values should be chosen in a way so as not to cause the segment to vibrate around a constant angle and to reduce the computational time and produce aceptable results. In this study, values of 1.666 and 0.333 were recommended for ε_1 and ε_2, respectively.

The iterative rotational procedure for this segment around vertex V_{i+1} stops and starts with vertex V_i, when

$$\Omega_i^{j+1} = \Omega_i^1 \times \Psi \tag{23}$$

The same procedure can be repeated by rotating segment one about vertex one to get more reduction in the safety factor.

When the rotational procedure is completed with segment i, the treatment of a new segment starts with the same procedure. The whole procedure stops and the current slip surface is assumed as minimum when the following two criteria are achieved

$$\Omega_i^{j+1} \le \Omega_i^1 \times \Psi, \quad i = 1 \text{ to } m \tag{24}$$

$$|F(S^j) - F(S^{j+1})| < \Delta \tag{25}$$

where $\Psi = 0.01$; and Δ = tolerance difference between two consecutive safety factors.

Fig. 3 illustrates a step-by-step procedure for rotating a three-segment example to minimize the safety factor. Fig. 4 depicts the complete rotation of the three segments around their four vertices. Similarly, for m segments, the step-by-step procedure is repeated following the same steps presented in Fig. 3 [i.e., a total of 2(n) steps, where n is the number of vertices].

To minimize the factor of safety further, the number of segments is increased by one. This is achieved by locating a new point at the middle of the largest horizontal distance between any adjacent two vertices. Then a new rotational procedure similar to the step-by-step procedure shown in Fig. 3 starts, but this time with four segments and five vertices, and this continues further to obtain a reduction in the safety factor. The degree of accuracy of the shape of the critical slip surface increases as the number of segments increase. It is noticed that 10–12 segments are enough to approximate the actual shape of the critical slip surface. The parameter Δ in (25) varies from 0.001 with three segments and is reduced to 0.00001 for the 12 segments.

TEST PROBLEMS AND RESULTS

To verify and evaluate the applicability of the new proposed method, the following benchmark problems were selected from the literature.

Example 1

Example 1 is of a homogeneous slope (Fig. 5) with the following geotechnical properties: friction angle $\phi' = 10°$, cohesion $c' = 9.8$ kPa, unit weight $\gamma = 17.64$ kN/m^3, and pore pressures $r_u = 0.0$.

This example is found in the literature and solved by different researchers; Yamagami and Ueta (1988) used nonlinear programming methods in searching for the critical slip surfaces and used the Morgenstern and Price method with the assumption of $f(x) = 1$ to calculate the factor of safety. The minimization was achieved by employing several optimization methods: the DFP method as suggested by Davidon (1959) and modified by Fletcher and Powell (1963); Broyden-Fletcher-Goldfarb-Sganno (BFGS) method proposed by Broyden (1970), Fletcher (1970), Goldfarb (1970), and Shanno (1970); method of conjugate directions by Powell (1964); and simplex method proposed by Nelder and Mead (1965). Also the problem was analyzed by Greco's optimization methods of pattern search and Monte Carlo type (Greco 1996). In this study, the problem is analyzed and Table 1 summarizes the results obtained in comparison with those obtained by different researchers.

Example 2

Baker (1980) used a dynamic programming to locate the critical slip surface and its associated minimum factor of safety using Spencer's method of the following homogeneous slope (Fig. 6). The geotechnical properties are angle of friction $\phi' = 20°$, $c' = 28.4$ kPa, and unit weight $\gamma = 18.8$ kN/m^3. Similarly, Greco (1996) employed pattern-search and Monte Carlo methods to solve this example. In this study, the problem is analyzed and the results are presented in Table 2. The critical slip surface is close to that obtained by Greco (1996).

Example 3

This example is a case where a weak layer is sandwiched between two strong formations. The geotechnical properties for layers 1–3 are respectively angle of friction 12°, 5°, and 40°; cohesion 29.4, 9.8, and 294.0 kPa; and unit weight 18.82 kN/m^3 for all three layers.

Arai and Tagyo (1985) used Janbu's simplified method in combination with the conjugate gradient method Fletcher and Reeves (1964) tested in this example. Similarly Sridevi and Deep (1992) used random search technique RST-2 (Shanker and Mohan 1987) to locate the critical slip surface. This problem was examined by using the proposed method. Table 3 compares the results obtained from this study with the previ-

J. Geotech. Geoenviron. Eng., 2001, 127(8): 688-698

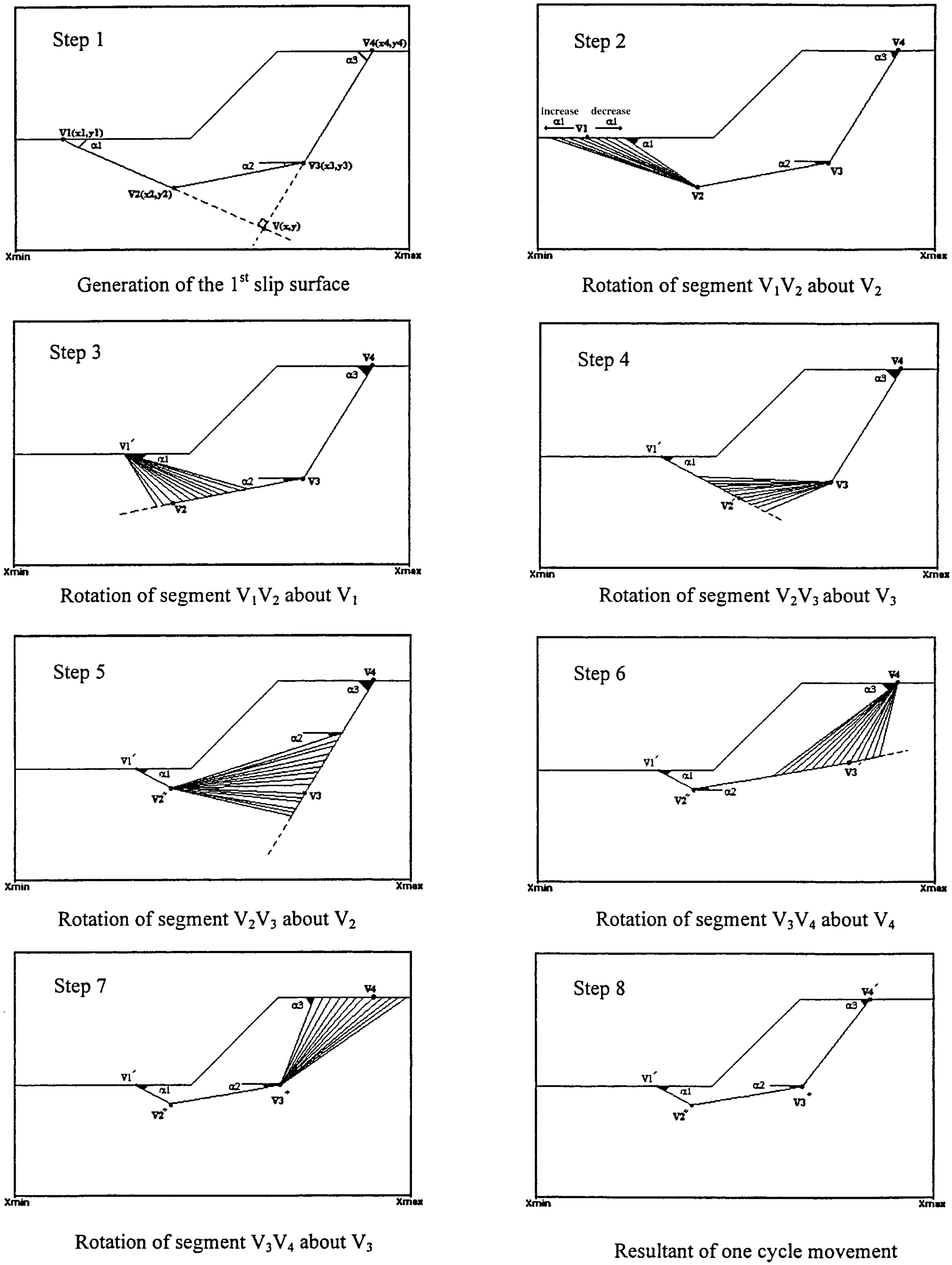

FIG. 3. Step-by-Step Search Procedure for Rotating Three-Segment Example to Minimize Factor of Safety

J. Geotech. Geoenviron. Eng., 2001, 127(8): 688-698

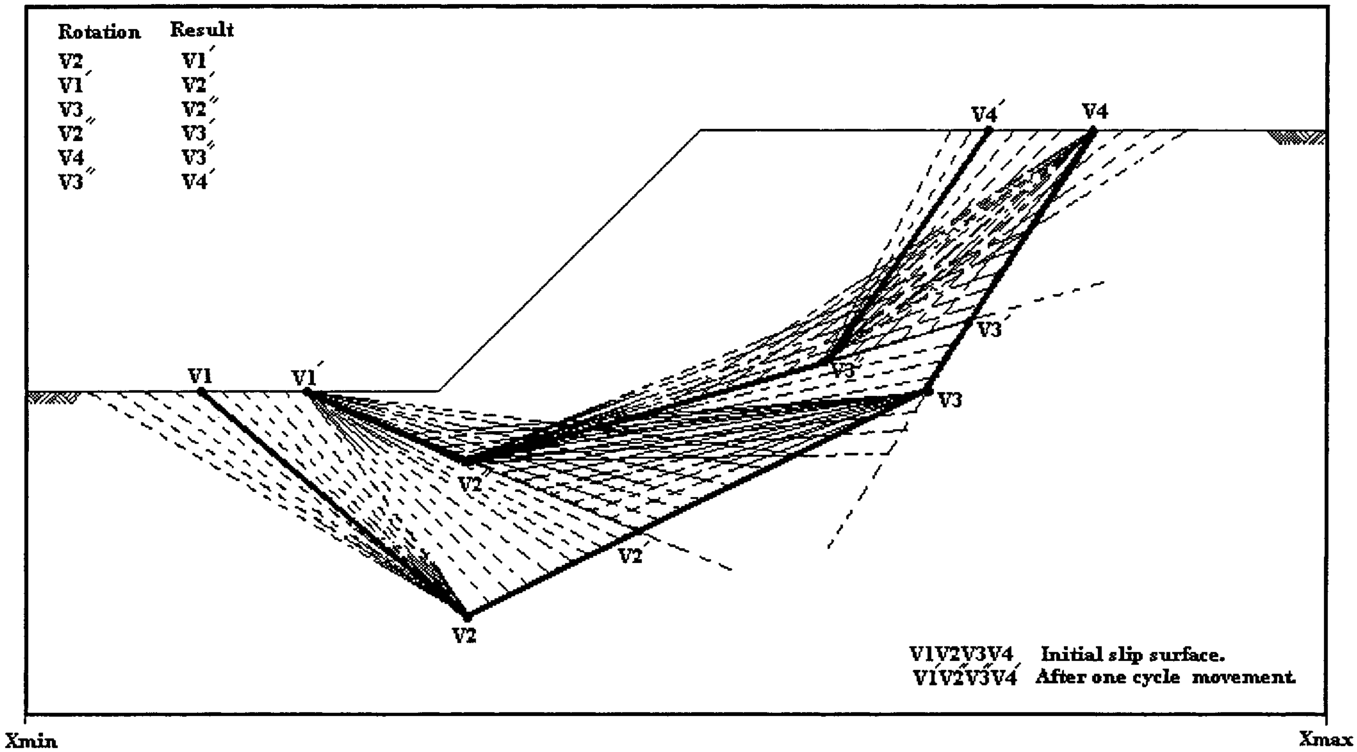

FIG. 4. Depicts Complete Rotation of Three-Segment Example around Their Four Vertices

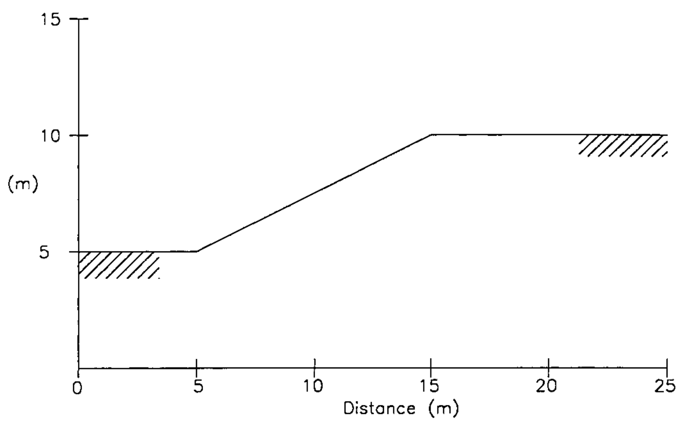

FIG. 5. Cross Section of Slope in Example 1

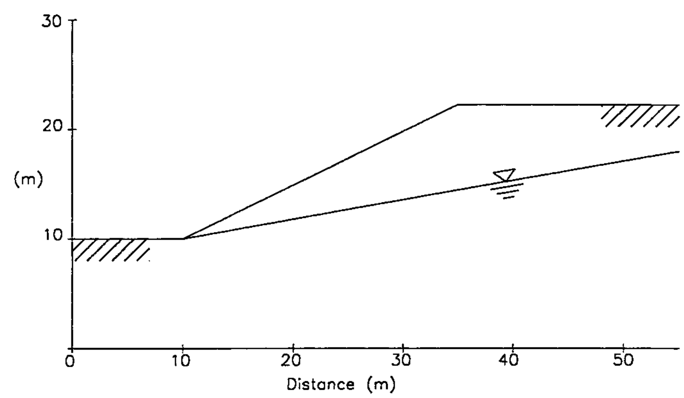

FIG. 6. Cross Section of Slope in Example 2

TABLE 1. Minimum Safety Factor Given by Minimization Procedures for Example 1

Method	Range of safety factor
(a) Yamagami and Ueta (1988)	
BFGS	1.338
DFP	1.338
Powell	1.338
Simplex	1.339–1.348
(b) Greco (1996)	
Pattern search	1.327–1.33
Monte Carlo	1.327–1.333
(c) This Study	
Monte Carlo (random walking)	1.238

TABLE 2. Minimum Safety Factor Given by Minimization Procedures for Example 2

Method	Range of safety factor
(a) Greco (1996)	
Pattern search	1.744–1.745
Monte Carlo	1.744–1.751
(b) Baker (1980)	
Dynamic programming	1.77
(c) This Study	
Monte Carlo (random walking)	1.502

J. Geotech. Geoenviron. Eng., 2001, 127(8): 688-698

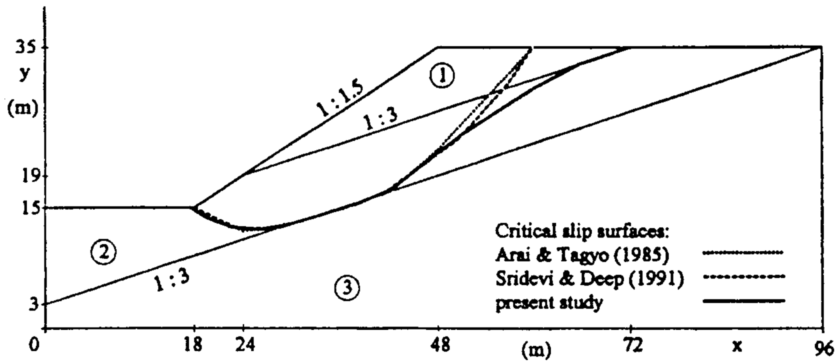

FIG. 7. Cross Section of Slope in Example 3 Showing Different Failure Surfaces [after Greco (1996)]

TABLE 3. Minimum Safety Factor Given by Minimization Procedures for Example 3

Method	Safety factor
(a) Fletcher and Reeves (1964)	
Conjugate gradient	0.405
(b) Sridevi and Deep (1992)	
RST-2	0.401
(c) Greco (1996)	
Pattern search	0.388
Monte Carlo	0.388
(d) This Study	
Monte Carlo (random walking)	0.401

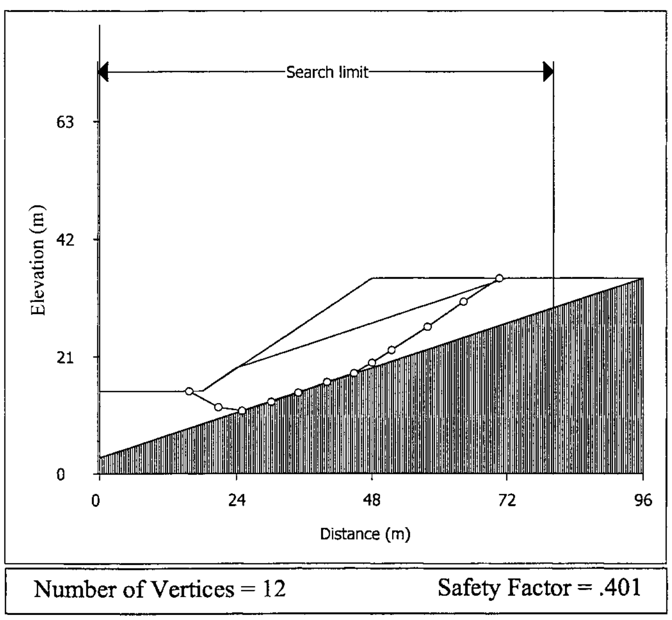

FIG. 8. Results Obtained from SAS-MCT Program for Example 3 Showing Critical Slip Surface

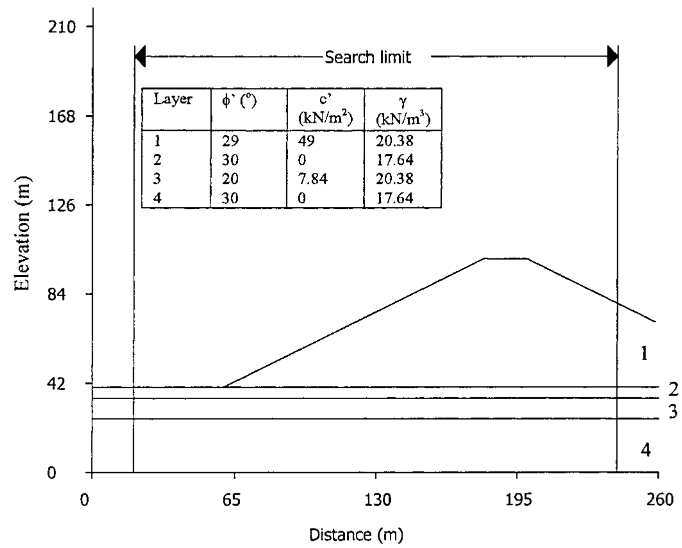

FIG. 9. Cross Section and Geotechnical Properties of Example 4

TABLE 4. Minimum Safety Factor Given by Minimization Procedures for Example 4

Method	Range of safety factor
(a) Yamagami and Ueta (1988)	
BFGS	1.423
DFP	1.453
Powell	1.402
Simplex	1.405
(b) Greco (1996)	
Pattern search	1.400
Monte Carlo	1.401
(c) This Study	
Monte Carlo (random walking)	1.33

ously reported results. Fig. 7 shows the critical slip surfaces obtained by different researchers, whereas Fig. 8 presents the results obtained in this study. It is observed that the critical slip surface is different from those obtained by Arai and Tagyo (1985) and by Sridevi and Deep (1991), where their presented slip surface is passing through the strong layer, but it is close to the slip surface obtained by Greco (1996). The problem that is observed in the Greco slip surface is that, once the slip surface approaches the ground, it turns back toward the crest (convex shape)—a situation that is overcome by the proposed method.

Example 4

A multilayer slope is presented by Yamagami and Ueta (1988). A cross section and soil properties are presented in Fig. 9. Yamagami and Ueta (1988) solved this example using limiting equilibrium [i.e., Morgenstern and Price with the assumption $f(x) = 1$] and employing different minimization procedures. Similarly, Greco (1996) solved this example using

J. Geotech. Geoenviron. Eng., 2001, 127(8): 688-698

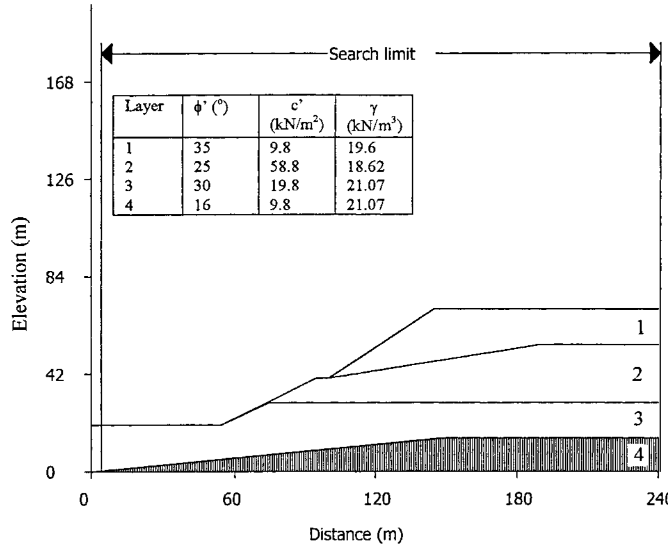

FIG. 10. Cross Section and Geotechnical Properties of Example 5

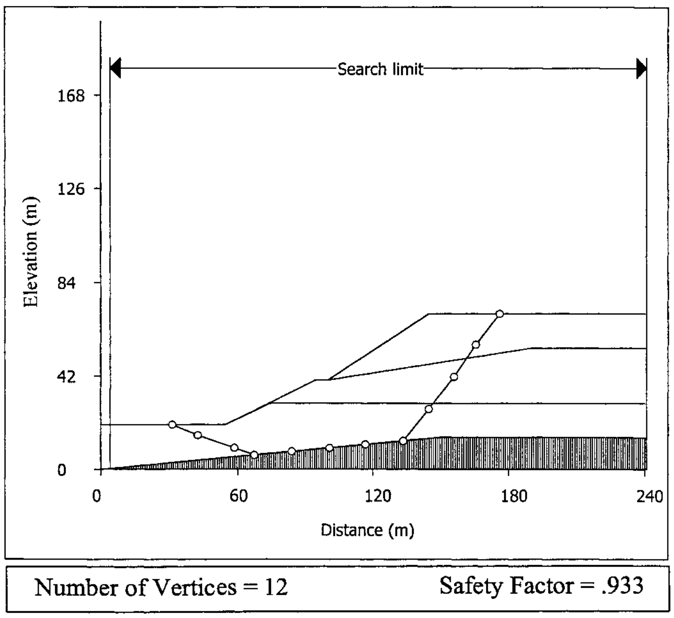

FIG. 11. Results Obtained from SAS-MCT program for Example 5 Showing Critical Slip Surface

TABLE 5. Minimum Safety Factor Given by Minimization Procedures for Example 5

Method	Range of safety factor
(a) Chen and Shao (1988)	
Original DFP	1.011–1.035
Modified DFP	1.009–1.025
Steepest descent	1.025
Simplex	1.025
(b) Greco (1996)	
Pattern search	0.973–1.033
Monte Carlo	0.973–0.974
(c) This Study	
Monte Carlo (random walking)	0.933

Spencer's method and employing pattern-search and Monte Carlo techniques. In this study, the slope was analyzed using Spencer's method and employing the proposed minimization method. Table 4 presents the results of both Yamagami and Ueta (1988) and Greco (1996) compared with this study. The critical slip surfaces reported by Yamagami and Ueta and Greco are in good agreement with the one obtained in this study.

Example 5

In this paper, Chen and Shao (1988), using the original method of DFP and its modified version, the simplex and steepest descent method, analyzed a layered slope where a weak thin layer exists (Fig. 10). The factor of safety was determined using Spencer's method. Similarly, Greco (1996) analyzed the same slope and found that the factor of safety falls in the range of 1.386–0.973. However, the used method of analysis was not reported. The same problem was analyzed using the SAS-MCT program. The determined factor of safety using Spencer's method was equal to 0.933. Table 5 presents the obtained results compared with those obtained by different researchers. Fig. 11 depicts the results obtained from the SAS-MCT program.

Example 6

The Tianshenqiao landslide is located on the right bank of the Nanpanjiang River in China. The collapsed mass com-

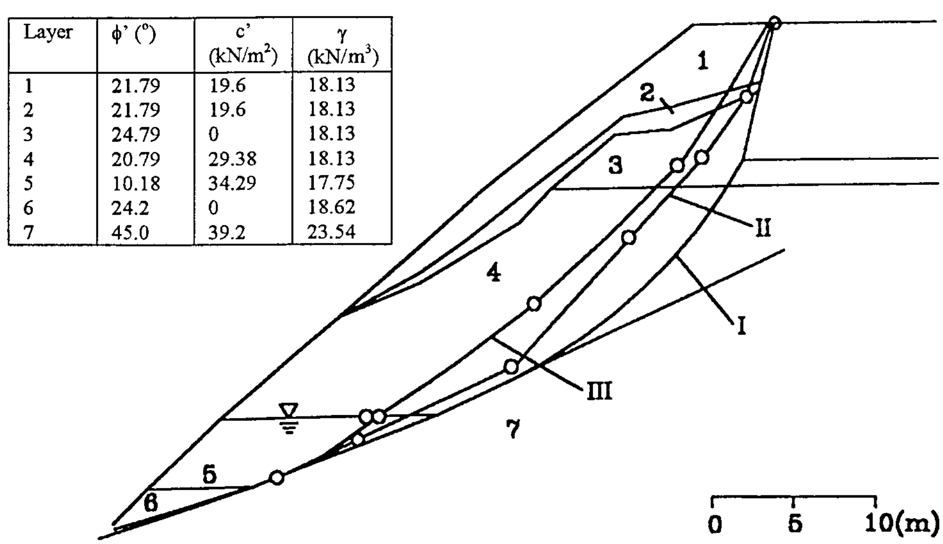

FIG. 12. Reevaluations of Tianshenqiao Landslide: Failure Surface I, Actual; II, that Obtained by Spencer's Method (F_m = 0.863); III, that Obtained by EMU (F_m = 0.882) [After Donald and Chen (1997)]

J. Geotech. Geoenviron. Eng., 2001, 127(8): 688-698

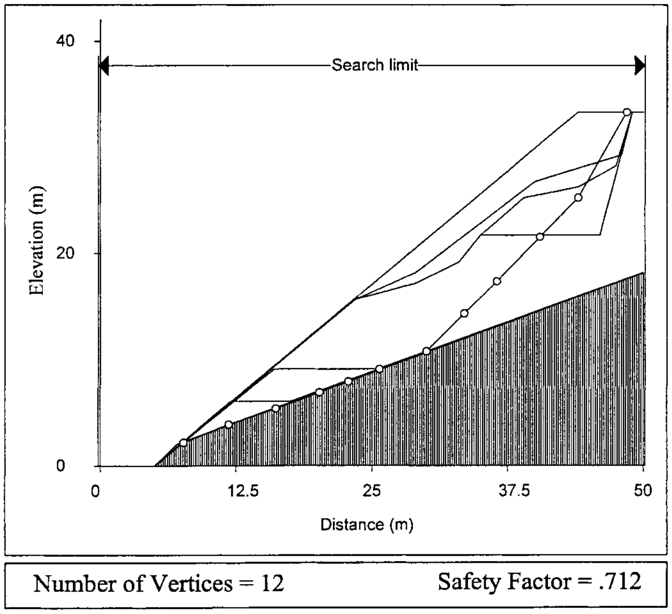

FIG. 13. Results Obtained from SAS-MCT Program for Example 6 Showing Critical Slip Surface

TABLE 6. Minimum Safety Factor Given by Minimization Procedures for Example 6

Method	Range of safety factor
(a) Chen and Shao (1988)	
Simplex	0.8631
(b) Donald and Chen (1997)	
EMU	0.882
(c) This Study	
Monte Carlo (random walking)	0.712

prised an estimated total volume of 7,115 m^3 (Chen and Shao 1988). This slide was analyzed by Chen and Shao (1988); the factor of safety obtained using Spencer's method is *SF* = 0.863. Similarly, the slope was reevaluated and the factor of safety obtained using the computer program Energy Method Upper bound (EMU) is equal to *SF* = 0.882 (Donald and Chen 1997) (Fig. 12).

This slide was analyzed using the SAS-MCT program, where the factor of safety obtained is *SF* = 0.712. The obtained critical slip surface is closer to the actual slip surface than that obtained by Chen and Shao (1988) and Donald and Chen (1997). Fig. 13 displays the results and presents the critical slip surface. Table 6 shows a comparison between those results obtained by different researchers and by this study. Fig. 14 displays the convergence rates obtained by the proposed method in terms of the number of tested slip surfaces with the safety factor.

ACCEPTABLE SOLUTION

Using limiting equilibrium methods to analyze slopes raises several computational difficulties in solving the factor of safety. Whitman and Bailey (1967) discussed some of these difficulties that were encountered during the use of computers for slope stability analysis. To be specific, they stated that the simplified Bishop's method could lead to trouble when the term ($\cos\alpha + \sin\alpha \tan\phi/F$) becomes <0.2 in any

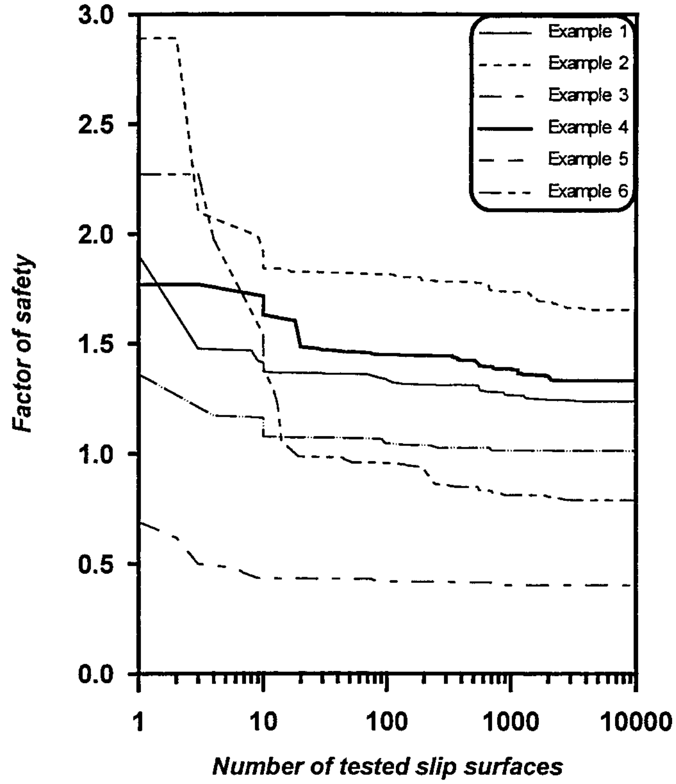

FIG. 14. Convergence Rates for Six Examples in Terms of Number of Tested Slip Surfaces and Safety Factor

slice; therefore, it should be used cautiously. Similarly, to reduce the error of estimating the factor of safety, Ting (1983) recommended, by using the simplified Bishop's method, that the area of each individual slice should be accurately determined and the base angle be obtained from the straight line at the slice bottom. Sarma (1987) rejected any slip surface when the value ($-\tan\alpha \tan\phi$) is greater than the unity for any slice. Morrison (1988) suggested changing the shear strength parameters used in the analysis for specific situations. Chowdhury and Zhang (1990) showed that to avoid false convergence, the initial safety factor F_0 must be assumed ($1 + \beta_1$), where $\beta_1 = -\tan\alpha \tan\phi$. Therefore, no slip surface should be rejected.

In the proposed method, no slip surface was rejected and the initial safety factor F_0, which is used herein, is the conventional safety factor calculated by the ordinary method of slice multiplied by 1.2 (Bromhead 1992). Both criteria were used (i.e., $1 + \beta_1$ and the constant value of 1.2) and achieved convergence without the need to reject any specific slip surface. Baker (1980) concluded that, in order that the Spencer method converges and yields reasonable results, the magnitude of the coefficient $P_{\alpha i} = \cos(\alpha_i - \delta)[1 + \tan\phi_i \cdot \tan(\alpha_i - \delta)/F]$ should be greater than or equal to 0.3–0.4. Fig. 15 shows the variation of the factor of safety and the coefficient $P_{\alpha i}$ with the number of slices for Example 4. Ching and Fredlund (1983) concluded that convergence problems might be encountered in the stability calculation as a result of using an inappropriate interslice force function. Sharma and Moudud (1992), Bhattacharya and Basudhar (1997), and Sarma (1979) discussed the validity of Spencer's method of analysis. They stated that a solution is acceptable if the associated line of thrust lies entirely within the sliding mass. Besides, the associated resultant interslice forces should also be positive; no tension forces are encountered between the slices and direction of forces are kinematically admissible. All of this is to guar-

J. Geotech. Geoenviron. Eng., 2001, 127(8): 688-698

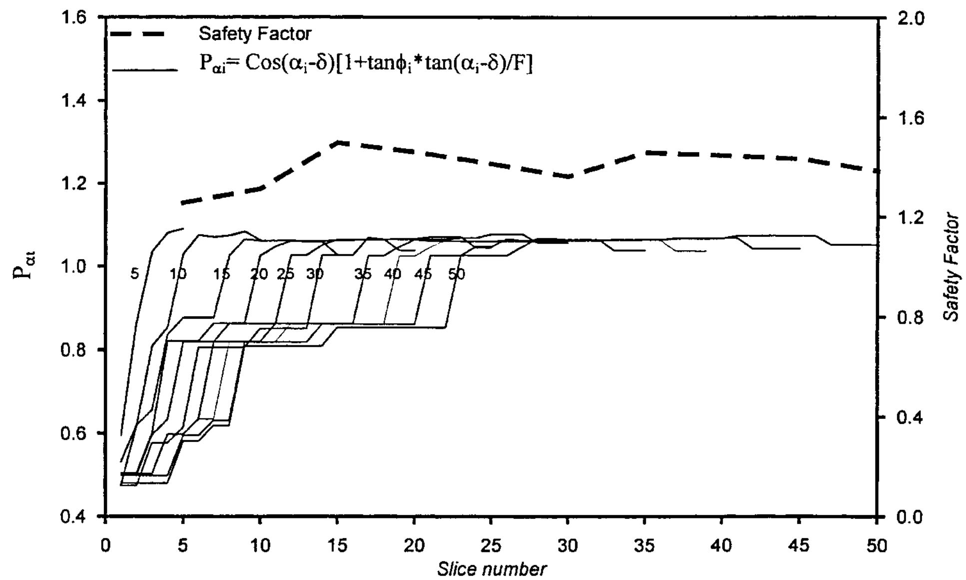

FIG. 15. Variation of Factor of Safety and $P_{\alpha i}$ Coefficient with Number of Slice

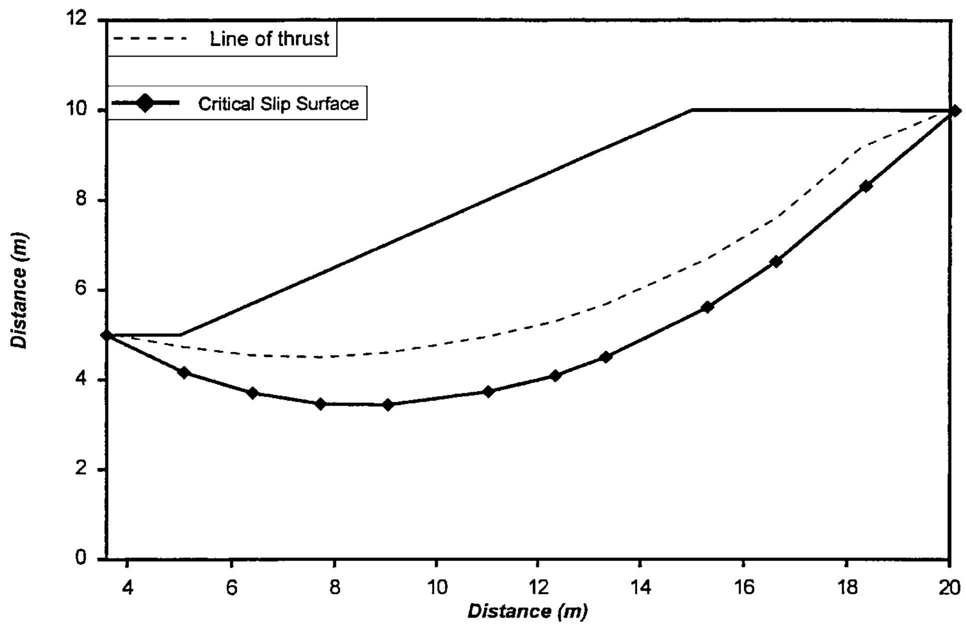

FIG. 16. Slope Profile and Location of Thrust Line for Example 1

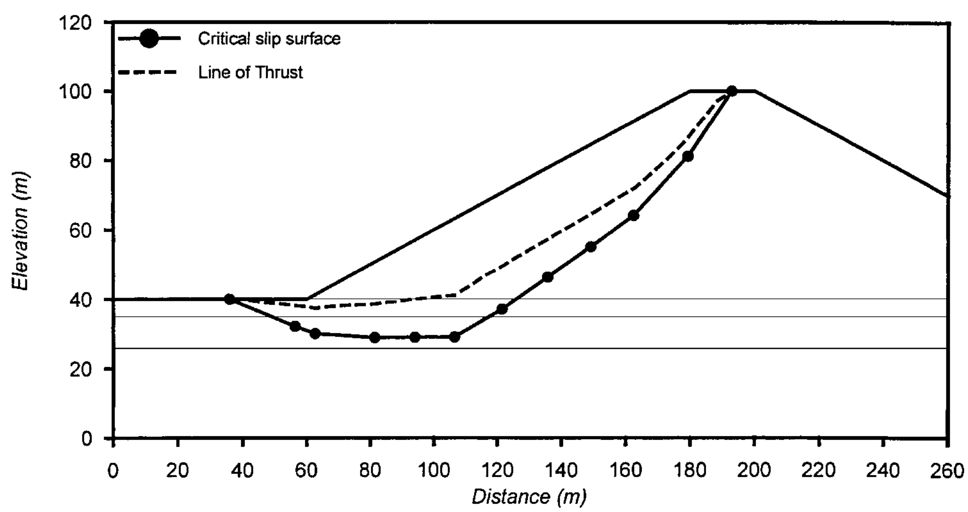

FIG. 17. Slope Profile, Critical Failure Surface, and Location of Thrust Line for Example

J. Geotech. Geoenviron. Eng., 2001, 127(8): 688-698

antee a geometrically reasonable and physically acceptable critical failure surface.

In all the slip surfaces obtained in the selected six examples, the admissibility and acceptability criteria were not violated. For a homogenous slope, Fig. 16 demonstrated this. It is clearly shown that the line of thrust lies entirely within the middle third of the sliding mass, thus ensuring the existence of compressive interslice forces only. Similarly, for the nonhomogeneous slope, Fig. 17 presents the line of thrust, critical slip surface, and slope profile of the fourth example.

RESULTS AND CONCLUSIONS

This paper introduces a new search procedure based, mainly, on the principle of Monte Carlo methods of the random walk type. The method shows accurate, efficient, and high quality solutions. The results are compared with the best nonlinear and dynamic programming methods. The quality of the results is demonstrated by six examples in the paper. The structure of the proposed method is easily programmed and can be implemented into an automatic searching technique in order to be used as a subroutine in any engineering slope stability software. The SAS-MCT program, incorporating the proposed method in association with various methods of limiting equilibrium, was developed. Besides, the method yields accurate results of the safety factor and prediction of failure mechanisms. It can handle any slope geometry, soil layering, and external concentrated or line loads. All generated slip surfaces are kinematically admissible and physically acceptable.

ACKNOWLEDGMENT

The writers are grateful to the three anonymous referees for critically reviewing the manuscript and improving the clarity of this paper as well.

REFERENCES

Arai, K., and Tagyo, K. (1985). ''Determination of non-circular slip surface giving the minimum factor of safety in slope stability analysis.'' *Soils and Found.*, Tokyo, 25(1), 43–51.

Baker, R. (1980). ''Determination of the critical slip surface in slope stability computations.'' *Int. J. Numer. and Analytical Methods in Geomech.*, 4, 333–359.

Baker, R., and Gaber, M. (1977). ''Variational approach to slope stability.'' *Proc., 9th Int. Conf. on Soil Mech. and Found. Engrg.*, Vol. 2, Japan Society for Soil Mechanics and Foundation Engineering, Tokyo, 9–12.

Baker, R., and Gaber, M. (1978). ''Theoretical analysis of the stability of slopes.'' *Géotechnique*, London, 28(4), 395–411.

Bhattacharya, G., and Basudhar, P. K. (1997). ''Convergence of slope stability computations using GPS.'' *Indian Geotech. J.*, 27(4).

Bishop, A. W. (1955). ''The use of slip circle in the stability analysis of slopes.'' *Géotechnique*, London, 5, 7–17.

Bromhead, E. N. (1992). *The stability of slopes*, 2nd Ed., Chapman & Hall, London.

Broyden, C. G. (1970). ''The convergence of a class of double-rank minimization algorithms.'' *J. Inst. Math. Applications*, 6, 76–90, 222–231.

Castillo, E., and Luceno, A. (1982). ''A critical analysis of some variational methods in slope stability analysis.'' *Int. J. Numer. and Analytical Methods in Geomech.*, 6, 195–209.

Celestino, T. B., and Duncan, J. M. (1981). ''Simplified search for noncircular slip surface.'' *Proc., 10th Int. Conf. SMFE*, 391–394.

Chen, Z.-Y. (1992). ''Random trials used in determining global minimum factors of safety of slopes.'' *Can. Geotech. J.*, Ottawa, 29, 225–233.

Chen, Z.-Y., and Shao, C.-M. (1988). ''Evaluation of minimum factor of safety in slope stability analysis.'' *Can. Geotech. J.*, Ottawa, 25, 735–748.

Cherubini, C., and Greco, V. R. (1987). ''A probabilistic method for locating the critical slip surface in slope stability analysis.'' *Proc., 5th ICSAP*, Vol. 2, 1182–1187.

Ching, R. K. H., and Fredlund, D. G. (1983). ''Some difficulties associated with the limit equilibrium method of slices.'' *Can. Geotech. J.*, Ottawa, 20, 661–672.

Chowdhury, R. N., and Zhang, S. (1990). ''Convergence aspect of limit equilibrium methods for slopes.'' *Can. Geotech. J.*, Ottawa, 27, 145–151.

Davidon, W. C. (1959). ''Variable metric method of minimization.'' *Rep. ANL-5990*, Argonne National Laboratory, Argonne, Ill.

De Josselin De Jong, G. (1980). ''Application of the calculus of variation to the vertical cut off in cohesive frictionless soil.'' *Géotechnique*, London, 30(1), 1–16.

De Natale, J. S. (1991). ''Rapid identification of critical slip surface: Structure.'' *J. Geotech. Engrg.*, ASCE, 117(10), 1568–1589.

Donald, I. B., and Chen, Z.-Y. (1997). ''Slope stability analysis by the upper bound approach: Fundamentals and methods.'' *Can. Geotech. J.*, Ottawa, 34, 853–862.

Duncan, J. M. (1996). ''State of the art: Limit equilibrium and finite element analysis of slopes.'' *J. Geotech. Engrg.*, ASCE, 122(7), 577–596.

Fellenius, W. (1936). ''Calculation of the stability of earth dams.'' *Trans. 2nd Congr. on Large Dams*, Vol. 4, 445–459.

Fletcher, R. (1970). ''A new approach to variable metric algorithms.'' *Comp. J.*, 13, 317–322.

Fletcher, R., and Powell, M. J. D. (1963). ''A rapidly convergent descent method for minimization.'' *Comp. J.*, 6(2), 163–168.

Fletcher, R., and Reeves, C. M. (1964). ''Function minimization by conjugate gradient.'' *Comp. J.*, 7, 149–154.

Goldfarb, D. (1970). ''A family of variable metric methods derived by variational means.'' *Math. Computation*, 24, 23–26.

Greco, V. R. (1996). ''Efficient Monte Carlo technique for locating critical slip surface.'' *J. Geotech. Engrg.*, ASCE, 122(7), 517–525.

Husein Malkawi, A. I., and Hassan, W. F. (2001). ''Stability analysis of slope using Monte Carlo Technique.'' *SAS-MCT software, User's manual, version 3.0. for Windows 95 and NT*, Geotech. Group, Civ. Engrg. Dept., Jordan University of Science and Technology, Irbid, Jordan.

Husein Malkawi, A. I., Hassan, W. F., and Sarma, S. K. (2001). ''An efficient search method for finding critical circular slip surface using Monte Carlo techniques.'' *Can. Geotech. J.*, Ottawa.

Janbu, N. (1954). ''Application of composite slip surface for stability analysis.'' *Proc., Eur. Conf. on Stability of Earth Slopes*, 43–49.

Janbu, N. (1973). ''Slope stability computations.'' *Embankment dam engineering, Casagrande memorial volume*, E. Hirschfield and S. Poulos, eds., Wiley, New York, 47–86.

Li, K. S., and White, W. (1987). ''Rapid evaluation of the critical slip surface in slope stability problems.'' *Int. J. Numer. and Analytical Methods in Geomech.*, 11, 449–473.

Luceno, A., and Castillo, E. (1980). ''Evaluation of variational methods in slope analysis.'' *Proc., 1st Int. Symp. on Landslides, Meerut: Sarita Prakashan*, Vol. 1, 255–258.

Morgenstern, N. R., and Price, V. E. (1965). ''The analysis of stability of general slip surface.'' *Géotechnique*, London, 15, 79–93.

Morrison, I. M. (1988). ''Discussion of 'A note on the stability analysis of slopes,' by S. K. Sarma.'' *Géotechnique*, London, 38, 157–159.

Nelder, J. A., and Mead, R. (1965). ''A simplex method for function minimization.'' *Comp. J.*, 7, 308–313.

Nguyen, V. U. (1985). ''Determination of critical slope failure surfaces.'' *J. Geotech. Engrg.*, ASCE, 111(2), 238–250.

Sarma, S. K. (1973). ''Stability analysis of embankments and slopes.'' *Géotechnique*, London, 23(3), 423–433.

Sarma, S. K. (1979). ''Stability analysis of embankments and slopes.'' *J. Geotech. Engrg. Div.*, ASCE, 105(12), 1511–1524.

Sarma, S. K. (1987). ''A note on the stability analysis of slopes.'' *Géotechnique*, London, 37, 107–111.

Shanker, K., and Mohan, C. (1987). ''A random technique for the global minima of constrained nonlinear optimization problems.'' *Proc., Int. Conf. on Optimization Techniques and Their Applications*, 905.

Shanno, D. F. (1970). ''Conditioning of quasi-Newton methods for function minimization.'' *Math. Computation*, 24, 647–657.

Sharma, S., and Moudud, A. (1992). ''Interactive slope analysis using Spencer's method.'' *Stability and performance of slopes and embankments—II A 25-Years Perspective, Vol. 1*, Geotech. Spec. Publ. No. 31, ASCE, New York.

Siegel, R. A., Kovacs, W. D., and Lovell, C. W. (1981). ''Random surface generation in stability analysis.'' *J. Geotech. Engrg. Div.*, ASCE, 107(7), 996–1002.

Spencer, E. E. (1967). ''A method of the analysis of the stability of embankments assuming parallel inter-slice forces.'' *Géotechnique*, London, 17, 11–26.

Spencer, E. E. (1973). ''The thrust line criterion in embankment stability analysis.'' *Géotechnique*, London, 23, 85–100.

Sridevi, B., and Deep, K. (1991). ''Application of global optimization technique to slope stability analysis.'' *Proc., 6th Int. Symp. on Landslides*, 573–578.

J. Geotech. Geoenviron. Eng., 2001, 127(8): 688-698

Ting, J. M. (1983). ‘‘Geometric concerns in slope stability analysis.’’ *J. Geotech. Engrg.*, ASCE, 109(11), 1487–1491.
Whitman, R. V., and Bailey, W. A. (1967). ‘‘Use of computers for slope stability analysis.’’ *J. Soil Mech. and Found. Engrg. Div.*, ASCE, 93(4), 475–498.
Yamagami, T., and Ueta, Y. (1988). ‘‘Search for noncircular slip surfaces by the Morgenstern-Price method.’’ *Proc., 6th Int. Conf. Numer. Methods in Geomechanics*, 1219–1223.

NOTATION

The following symbols are used in this paper:

c = cohesion of soil mass;
$f(x)$ = arbitrary mathematical expression;
m = number of segments;
N_d = number defines direction of rotation: (+1) clockwise, (−1) counterclockwise;
n = number of vertices;
R = uniformly distributed random number;
SF = factor of safety;
α_i = inclination of segment i;
γ = soil unit weight;
Δ = tolerance difference between two consecutive safety factors;
ε = factor used to increase and decrease angle of rotation;
ϕ = angle of internal friction;
Ψ = tolerance for angle of rotation; and
Ω_i^j = angle of rotation for segment i at stage j.

J. Geotech. Geoenviron. Eng., 2001, 127(8): 688-698