Software Requirements Specification for Slope Stability Analysis

Henry Frankis and Brooks MacLachlan

October 3, 2018

1 Revision History

Date	Version	Notes
09/24/18	1.0	Removed RFEM
09/25/18	1.1	Traceability matrix work
09/26/18	1.2	Physical System Description expanded, Non-functional requirements itemized
10/01/18	1.3	Various improvements throughout
10/02/18	1.4	Initial revision of the solution characteristics specification
10/03/18	1.5	Completed revision of the solution characteristics specification and other sections

2 Reference Material

This section records information for easy reference.

2.1 Table of Units

The unit system used throughout is SI (Système International d'Unités). In addition to the basic units, several derived units are also used. For each unit, the table lists the symbol, a description and the SI name.

Symbol	**Unit**	**SI**
N	force	Newton
m	length	meter
$\mathrm{Pa} = \mathrm{N\,m^{-2}}$	pressure	Pascal
$^\circ$	angle	degree

2.2 Table of Symbols

The table that follows summarizes the symbols used in this document along with their units. Throughout the document, the subscript i implies that the value will be taken and analyzed at a slice or slice interface composing the total slip mass.

Symbol	**Unit**	**Description**
b	m	width of the base of a slice in the x direction
c'	Pa	effective cohesion
$C1_i$	Nm	interslice shear force expression used to calculate the numerator of the scaling factor
$C2_i$	Nm	interslice normal force expression used to calculate the denominator of the scaling factor
F	N	force
F_x	N	x-component of force
F_y	N	y-component of force
f		function describing variation of the interslice normal to shear force ratio; can be constant or a half-sine
FS		factor of safety
G	$\mathrm{N\,m^{-1}}$	interslice normal force
H	$\mathrm{N\,m^{-1}}$	interslice water force

h	m	height in the y-direction from the base of a slice to the slope surface, at the x-direction midpoint on the slice
i		index representing a single slice
K_c		horizontal seismic coefficient
M	N m	moment
N	$\mathrm{N\,m^{-1}}$	normal force
N'	$\mathrm{N\,m^{-1}}$	effective normal force
$N*$	$\mathrm{N\,m^{-1}}$	effective normal force without the influence of interslice forces
P	$\mathrm{N\,m^{-1}}$	resistive shear force
Q	$\mathrm{N\,m^{-1}}$	imposed surface load or external force
R	$\mathrm{N\,m^{-1}}$	resistive shear force without the influence of interslice forces
S	$\mathrm{N\,m^{-1}}$	mobilized shear force
s	Pa	mobilized shear stress
T	$\mathrm{N\,m^{-1}}$	mobilized shear force without the influence of interslice forces
U_b	$\mathrm{N\,m^{-1}}$	base hydrostatic force
U_t	$\mathrm{N\,m^{-1}}$	surface hydrostatic force
W	$\mathrm{N\,m^{-1}}$	self-weight
x	m	x-ordinate in the Cartesian coordinate system
x_{cs}	m	x-ordinate of a point on the critical slip surface
x_{slip}	m	x-ordinate of a point on a slip surface
x_{us}	m	x-ordinate of a point on the slope
X	$\mathrm{N\,m^{-1}}$	interslice shear force
y	m	y-ordinate in the Cartesian coordinate system
y_{cs}	m	y-ordinate of a point on the critical slip surface
y_{slip}	m	y-ordinate of a point on a slip surface
y_{us}	m	y-ordinate of a point on the slope
y_{wt}	m	y-ordinate of a point on the water table

z	m	height in the y-direction from the base of a slice to the center of the slice
z_w	(meter)	height in the y-direction from the base of a slice halfway to the water table
α	$^\circ$	angle between the base of a slice and the horizontal
β	$^\circ$	angle between the surface of a slice and the horizontal
γ	$\mathrm{N\,m^{-3}}$	soil dry unit weight
γ_{Sat}	$\mathrm{N\,m^{-3}}$	soil saturated unit weight
γ_w	$\mathrm{N\,m^{-3}}$	unit weight of water
ΔH	$\mathrm{N\,m^{-1}}$	difference between interslice water forces
λ		scaling factor for the interslice normal to shear force ratio
μ	Pa	pore pressure from water within the soil
σ	Pa	the total stress a soil mass needs to maintain itself as a rigid collection of particles
σ_N	Pa	normal stress
σ'	Pa	effective stress provided by the soil skeleton
τ	Pa	shear strength
Υ		generic minimization function or algorithm
φ'	$^\circ$	effective angle of friction
Φ		constant to convert resistive shear without the influence of interslice forces to resistive shear with the influence of interslice forces
Ψ		constant to convert mobile shear without the influence of interslice forces to mobile shear with the influence of interslice forces
ω	$^\circ$	angle between the imposed surface load acting into the surface and the vertical
ℓ_b	m	base length of a slice in the direction parallel to the slope of the base
ℓ_s	m	surface length of a slice in the direction parallel to the slope of the surface

2.3 Abbreviations and Acronyms

Symbol	Description
2D	Two-Dimensional
A	Assumption
DD	Data Definition
GD	General Definition
GS	Goal Statement
IM	Instance Model
LC	Likely Change
NFR	Non-Functional Requirement
PS	Physical System Description
R	Requirement
SRS	Software Requirements Specification
SSP	Slope Stability Analysis Program
T	Theoretical Model
TU	Typical Uncertainty
UC	Unlikely Change

Contents

1 Revision History **i**

2 Reference Material **ii**
2.1 Table of Units . . . ii
2.2 Table of Symbols . . . ii
2.3 Abbreviations and Acronyms . . . v

3 Introduction **1**
3.1 Purpose of Document . . . 1
3.2 Scope of Requirements . . . 1
3.3 Characteristics of Intended Reader . . . 1
3.4 Organization of Document . . . 1

4 General System Description **2**
4.1 System Context . . . 2
4.2 User Characteristics . . . 2
4.3 System Constraints . . . 3

5 Specific System Description **3**
5.1 Problem Description . . . 3
5.1.1 Terminology . . . 3
5.1.2 Physical System Description . . . 4
5.1.3 Goal statements . . . 4
5.2 Solution Characteristics Specification . . . 5
5.2.1 Assumptions . . . 6
5.2.2 Theoretical Models . . . 6
5.2.3 General Definitions . . . 8
5.2.4 Data Definition . . . 13
5.2.5 Instance Models . . . 22
5.2.6 Data Constraints . . . 26
5.2.7 Properties of a Correct Solution . . . 27

6 Requirements **28**
6.1 Functional Requirements . . . 28
6.2 Nonfunctional Requirements . . . 29

7 Likely Changes **29**

8 Unlikely Changes **29**

9 Traceability Matrices and Graphs **29**

10 Values of Auxiliary Constants **34**

3 Introduction

A slope of geological mass, composed of soil and rock and sometimes water, is subject to the influence of gravity on the mass. This can cause instability in the form of soil or rock movement. The effects of soil or rock movement can range from inconvenient to seriously hazardous, resulting in significant life and economic losses. Slope stability is of interest both when analyzing natural slopes, and when designing an excavated slope. Slope stability analysis is the assessment of the safety of a slope, identifying the surface most likely to experience slip and an index of its relative stability known as the factor of safety.

The following section provides an overview of the Software Requirements Specification (SRS) for a slope stability analysis problem. The developed program will be referred to as the Slope Stability Analysis Program (SSP). This section explains the purpose of this document, the scope of the system, the characteristics of the intended readers, and the organization of the document.

3.1 Purpose of Document

This document will be used as a starting point for subsequent development phases, including writing the design specification and the software verification and validation plan. The design document will show how the requirements are to be realized, including decisions on the numerical algorithms and programming environment. The verification and validation plan will show the steps that will be used to increase confidence in the software documentation and the implementation. Although the SRS fits in a series of documents that follow the so-called waterfall model, the actual development process is not constrained in any way. Even when the waterfall model is not followed, as Parnas and Clements point out [4], the most logical way to present the documentation is still to "fake" a rational design process.

3.2 Scope of Requirements

The scope of the requirements includes stability analysis of a 2-dimensional slope, composed of homogeneous soil layers. Factors which may change the slope properties over time will not be considered.

3.3 Characteristics of Intended Reader

Reviewers of this documentation should have an understanding of undergraduate Level 4 physics and should have completed a second year or higher level undergraduate course in solid mechanics. The users of SSP can have a lower level of expertise, as explained in Section 4.2.

3.4 Organization of Document

The organization of this document follows the template for an SRS for scientific computing software proposed by [2] and [5]. The presentation follows the standard pattern of presenting goals, theories, definitions, and assumptions. For readers that would like a more bottom up approach, they can start reading the instance models in Section 5.2.5 and trace back to find any additional information they require. The goal statements are refined to the theoretical models, and the theoretical models

(Section 5.2.2) to the instance models (Section 5.2.5). The instance models provide the set of algebraic equations that must be solved.

4 General System Description

This section provides general information about the system. It identifies the interfaces between the system and its environment, describes the user characteristics, and lists the system constraints.

4.1 System Context

Figure 1 shows the system context. A circle represents an external entity outside the software, the user in this case. A rectangle represents the software system itself (SSP). Arrows are used to show the data flow between the system and its environment.

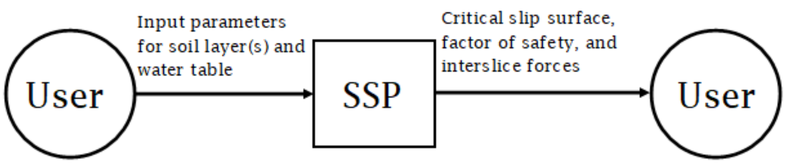

Figure 1: System Context

The interaction between the product and the user is through a user interface. The responsibilities of the user and the system are as follows:

- User Responsibilities:
  - Provide the input data related to the soil layer(s) and water table (if applicable), ensuring conformation to input data format required by SSP
  - Ensure that consistent units are used for input variables
  - Ensure required software assumptions (Section 5.2.1) are appropriate for any particular problem input to the software
- SSP Responsibilities:
  - Detect data type mismatch, such as a string of characters input instead of a floating point number
  - Verify that the inputs satisfy the required physical constraints
  - Identify the critical slip surface within the possible input range
  - Find the factor of safety for the slope
  - Find the interslice normal and shear forces along the critical slip surface

4.2 User Characteristics

The end user of SSP should have an understanding of undergraduate Level 1 Calculus and Physics, and be familiar with soil and material properties, specifically cohesion, effective angle of friction, and unit weight.

4.3 System Constraints

The Morgenstern-Price method, which involves dividing the slope into vertical slices, will be used to derive the equations for analysing the slope.

5 Specific System Description

This section first presents the problem description, which gives a high-level view of the problem to be solved. This is followed by the solution characteristics specification, which presents the assumptions, theories, definitions and finally the instance models that model the slope.

5.1 Problem Description

SSP is a computer program developed to evaluate the factors of safety for a slope's slip surfaces and identify the critical slip surface of the slope, as well as the interslice normal and shear forces along the critical slip surface. It is intended to be used as an educational tool for introducing slope stability issues, and to facilitate the analysis and design of a safe slope.

5.1.1 Terminology

This subsection provides a list of terms that are used in the subsequent sections and their meaning, with the purpose of reducing ambiguity and making it easier to correctly understand the requirements.

- *Factor of safety:* The global stability metric of a slip surface of a slope.
- *Slip surface:* A surface within a slope that has the potential to fail or displace due to load or other forces.
- *Critical slip surface:* Slip surface of the slope that has the lowest global factor of safety, and is therefore most likely to experience failure.
- *Water table:* The upper boundary of a saturated zone in the ground.
- *Stress:* Forces that are exerted between planes internal to a larger body subject to external loading.
- *Strain:* Stress forces that result in deformation of the body/plane.
- *Normal force:* A force applied perpendicular to the plane of the material.
- *Shear force:* A force applied parallel to the plane of the material.
- *Resistive shear force:* Shear force in the direction opposite of the direction of potential motion, thus hindering motion along the plane.
- *Mobile shear force:* Shear force in the direction of potential motion, thus encouraging motion along the plane.
- *Cohesion:* An attractive force between adjacent particles that holds the matter together.

- *Isotropic:* A condition where a the value of a property is independent of the direction in which it is measured.

- *Plane strain:* A condition where the resultant stresses in one of the directions of a 3-dimensional material can be approximated as 0. Results when the length of one dimension of the body dominates the others. Stresses in the direction of the dominant dimension can be approximated as 0.

5.1.2 Physical System Description

The physical system of SSP, as shown in Figure 2, includes the following elements:

PS1: A slope comprised of one or more layers of soil.

PS2: A water table within the soil layers, which may or may not exist.

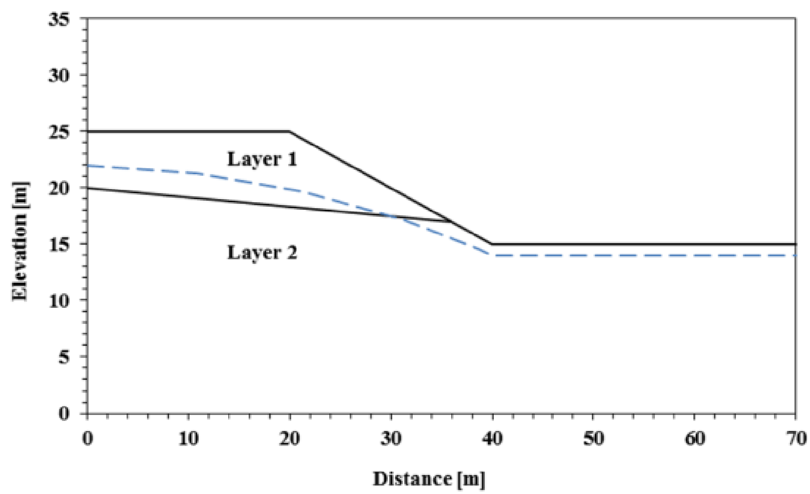

Figure 2: An example slope for analysis by SSP

Morgenstern-Price analysis of the slope involves representing the slope as a series of vertical slices. As shown in Figure 3, the index i is used to denote a value for a single slice, and an interslice value at a given index i refers to the value between slice i and adjacent slice i+*1*.

A free body diagram of the forces acting on a slice is displayed in Figure 4.

5.1.3 Goal statements

Given the geometry of the soil layers and water table composing the plane of a slope and the material properties of the layers, the goal statements are:

GS1: Evaluate the factors of safety for possible slip surfaces along the slope.

GS2: Identify the critical slip surface for the slope, with the lowest factor of safety.

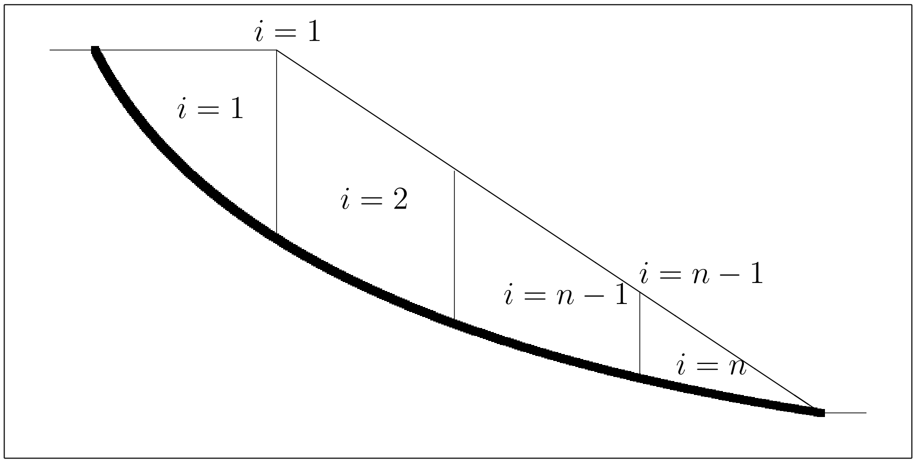

Figure 3: Index convention for slice and interslice values

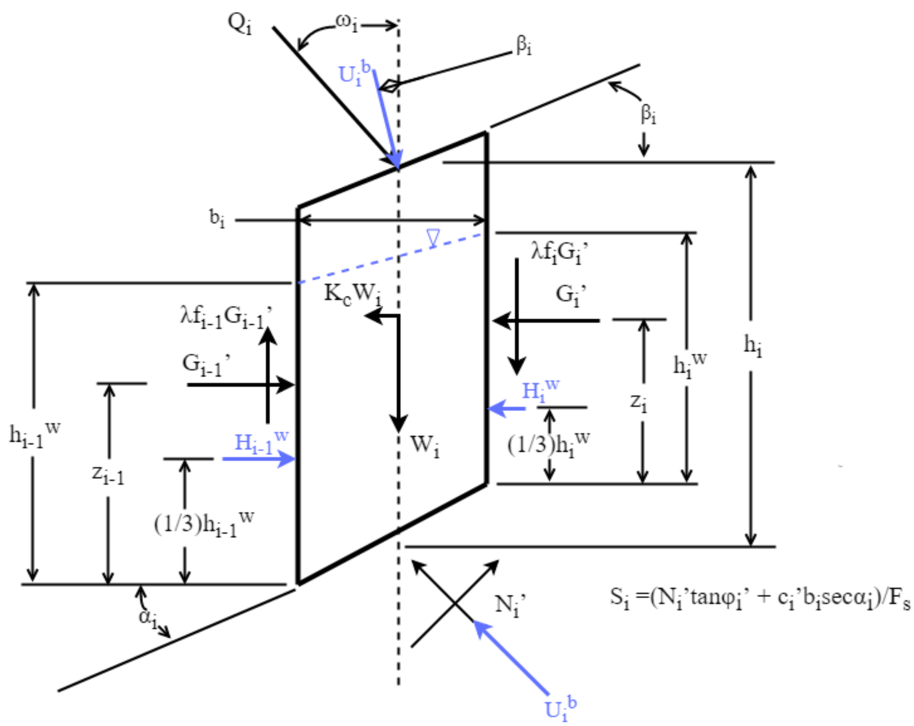

Figure 4: Free body diagram of forces acting on a slice

GS3: Determine the interslice normal force between each pair of vertical slices of the slope.

GS4: Determine the interslice shear force between each pair of vertical slices of the slope.

5.2 Solution Characteristics Specification

The instance models that govern SSP are presented in Section 5.2.5. The information to understand the meaning of the instance models and their derivation is also presented, so that the instance models can be verified.

5.2.1 Assumptions

This section simplifies the original problem and helps in developing the theoretical model by filling in the missing information for the physical system. The numbers given in the square brackets refer to the theoretical model [T], general definition [GD], data definition [DD], instance model [IM], or likely change [LC], in which the respective assumption is used.

A1: The slip surface is concave with respect to the slope surface. The (x, y) coordinates of a slip surface follow a concave up function. [IM4]

A2: The factor of safety is assumed to be constant across a whole slip surface. [GD4, IM1, IM3]

A3: The different layers of the soil are homogeneous, with consistent soil properties throughout. [GD3, GD4, DD10, DD11, LC1]

A4: The soil properties are independent of dry or saturated conditions, with the exception of unit weight. [GD3, GD4, DD10, DD11]

A5: Soil layers are treated as if they have isotropic properties. [GD3, GD4, DD10, DD11]

A6: Interslice normal and shear forces have a linear relationship, proportional to a constant (λ) and an interslice force function (f) depending on x position. [GD6, DD**??**]

A7: The slope and slip surface extends far into and out of the geometry (z coordinate). This implies plane strain conditions, making 2D analysis appropriate. [T2]

A8: The effective normal stress is large enough that the resistive shear to effective normal stress relationship can be approximated as a linear relationship. [T3]

A9: The surface and base of a slice are approximated as straight lines [DD1, DD2, DD3, DD4, DD5, DD6, DD7, DD8, DD9].

A10: There is no seismic force acting on the slope. [DD10, DD11, IM1, IM2, IM3]

A11: There is no imposed surface load, and therefore no external force, acting on the slope. [DD10, DD11, IM1, IM2, IM3]

5.2.2 Theoretical Models

This section focuses on the general equations and laws that SSP is based on.

Number	T1
Label	**Factor of Safety**
Equation	$\mathrm{FS} = \frac{P}{S}$
Description	FS is the factor of safety, or stability metric of the slope. S is the mobile shear force ($\mathrm{N\,m^{-1}}$). P is the resistive shear force ($\mathrm{N\,m^{-1}}$).
Source	[1]
Ref. By	IM1, GD4

Number	T2
Label	**Static Equilibrium**
Equation	$\sum F_{\mathrm{x}} = \sum F_{\mathrm{y}} = \sum M = 0$
Description	For a body in static equilibrium the net forces and net moments acting on the body will cancel out. This equation assumes a 2D space (A7). F_x is the x-component of the net force (N). F_y is the y-component of the net force (N). M is the net moment ($\mathrm{N\,m}$).
Source	[1]
Ref. By	GD1, GD2, GD7, IM2

Number	T3
Label	**Mohr-Coulomb Shear Strength**
Equation	$\tau = \sigma_N \cdot \tan(\varphi') + c'$
Description	The τ versus σ_N relationship is not truly linear, but assuming the effective normal force is strong enough, it can be approximated with a linear fit (A8), where the cohesion c' represents the τ intercept of the fitted line. τ is the shear strength (Pa). σ_N is the normal stress (Pa). φ is the effective angle of friction (°). c' is the effective cohesion (Pa).
Source	[1]
Ref. By	GD3, GD4, DD10, DD11

Number	T4
Label	**Effective Stress**
Equation	$\sigma' = \sigma - \mu$
Description	σ is the total stress a soil mass needs to maintain itself as a rigid collection of particles (Pa). σ' is the effective stress provided by the soil skeleton (Pa). μ is the pore pressure from water within the soil.
Source	[1]
Ref. By	GD5, DD10, IM1, IM2, IM3

5.2.3 General Definitions

This section collects the laws and equations that will be used to build the instance models.

Number	GD1
Label	**Normal Force Equilibrium**
SI Units	$\mathrm{N\,m^{-1}}$
Equation	$N_{\mathrm{i}} = [W_{\mathrm{i}} - X_{\text{i-1}} + X_{\mathrm{i}} + U_{\mathrm{t,i}} \cos(\beta_{\mathrm{i}}) + Q_{\mathrm{i}} \cos(\omega_{\mathrm{i}})] \cos(\alpha_{\mathrm{i}}) + [-K_{\mathrm{c}}\, W_{\mathrm{i}} - G_{\mathrm{i}} + G_{\text{i-1}} - H_{\mathrm{i}} + H_{\text{i-1}} + U_{\mathrm{t,i}} \sin(\beta_{\mathrm{i}}) + Q_{\mathrm{i}} \sin(\omega_{\mathrm{i}})] \sin(\alpha_{\mathrm{i}})$
Description	This equation satisfies T2 in the normal direction. Force equilibrium is derived from the free body diagram of Figure 4 in section 5.1.2. Index i refers to the values of the properties for slice/interslices following convention in Figure 3 in section 5.1.2. Force variable mathematical definitions can be found in DD1 to DD**??**. N is the normal force ($\mathrm{N\,m^{-1}}$). W is the weight ($\mathrm{N\,m^{-1}}$). X is the interslice shear force ($\mathrm{N\,m^{-1}}$). U_t is the surface hydrostatic force ($\mathrm{N\,m^{-1}}$). β is the angle between the surface of a slice and the horizontal (°). Q is the external force ($\mathrm{N\,m^{-1}}$). ω is the angle between the imposed surface load acting into the surface and the vertical (°). α is the angle between the base of a slice and the horizontal (°). K_c is the seismic coefficient. G is the interslice normal force ($\mathrm{N\,m^{-1}}$). H is the interslice water force ($\mathrm{N\,m^{-1}}$).
Source	[6]
Ref. By	DD10, DD11, IM3

Number	GD2
Label	**Shear Force Equilibrium**
SI Units	$\mathrm{N\,m^{-1}}$
Equation	$S_{\text{i}} = \begin{array}{l}[W_{\text{i}} - X_{\text{i-1}} + X_{\text{i}} + U_{\text{t,i}} \cos(\beta_{\text{i}}) + Q_{\text{i}} \cos(\omega_{\text{i}})] \sin(\alpha_{\text{i}}) \\ - [-K_{\text{c}}\, W_{\text{i}} - G_{\text{i}} + G_{\text{i-1}} - H_{\text{i}} + H_{\text{i-1}} + U_{\text{t,i}} \sin(\beta_{\text{i}}) + Q_{\text{i}} \cos(\omega_{\text{i}})] \cos(\alpha_{\text{i}})\end{array}$
Description	This equation satisfies T2 in the shear direction. Force equilibrium is derived from the free body diagram of Figure 4 in section 5.1.2. Index i refers to the values of the properties for slice/interslices following convention in Figure 3 in section 5.1.2. Force variable mathematical definitions can be found in DD1 to DD**??**. S is the mobile shear force ($\mathrm{N\,m^{-1}}$). W is the weight ($\mathrm{N\,m^{-1}}$). X is the interslice shear force ($\mathrm{N\,m^{-1}}$). U_t is the surface hydrostatic force ($\mathrm{N\,m^{-1}}$). β is the angle between the surface of a slice and the horizontal (°). Q is the external force ($\mathrm{N\,m^{-1}}$). ω is the angle between the imposed surface load acting into the surface and the vertical (°). α is the angle between the base of a slice and the horizontal (°). K_c is the seismic coefficient. G is the interslice normal force ($\mathrm{N\,m^{-1}}$). H is the interslice water force ($\mathrm{N\,m^{-1}}$).
Source	[6]
Ref. By	DD10, DD11,IM3

Number	GD3
Label	**Resistive Shear Force**
SI Units	$\mathrm{N\,m^{-1}}$
Equation	$P_{\mathrm{i}} = N'_{\mathrm{i}} \cdot \tan\left(\varphi'_{\mathrm{i}}\right) + c' \cdot \ell_{b,i}$
Description	The Mohr-Coulomb resistive shear strength from T3 implemented with forces. i is the index representing a single slice. P is the resistive shear force ($\mathrm{N\,m^{-1}}$). N' is the effective normal force ($\mathrm{N\,m^{-1}}$). φ' is the effective angle of friction ($^{\circ}$). c' is the effective cohesion (Pa). ℓ_b is the width of the base of a slice in the x direction (m).
Source	[6]
Ref. By	GD4, DD10, DD11

Number	GD4
Label	**Mobile Shear Force**
SI Units	$\mathrm{N\,m^{-1}}$
Equation	$S_{\mathrm{i}} = \frac{P_{\mathrm{i}}}{\mathrm{FS}} = \frac{N'_{\mathrm{i}} \cdot \tan\left(\varphi'_{\mathrm{i}}\right) + c' \cdot \ell_{b,i}}{\mathrm{FS}}$
Description	Mobile shear force as derived from the definition of the factor of safety in T1, and the definition of P in GD3. i is the index representing a single slice. S is the mobile shear force ($\mathrm{N\,m^{-1}}$). P is the resistive shear force ($\mathrm{N\,m^{-1}}$). N' is the effective normal force ($\mathrm{N\,m^{-1}}$). φ' is the effective angle of friction ($^{\circ}$). c' is the effective cohesion (Pa). ℓ_b is the width of the base of a slice in the x direction (m). FS is the factor of safety.
Source	[6]
Ref. By	DD10

Number	GD5
Label	**Effective Normal Force**
SI Units	$\mathrm{N\,m^{-1}}$
Equation	$N'_i = N_i - U_{b,i}$
Description	Effective normal force as derived from T4 and implemented with forces. i is the index representing a single slice. N' is the effective normal force ($\mathrm{N\,m^{-1}}$). N is the normal force ($\mathrm{N\,m^{-1}}$). U_b is the base hydrostatic force ($\mathrm{N\,m^{-1}}$).
Source	[6]
Ref. By	DD10

Number	GD6
Label	**Interslice Normal and Shear Force Proportionality**
Equation	$X = \lambda \cdot f \cdot G$
Description	Mathematical representation of the primary assumption for the Morgenstern-Price method (A6). X is the interslice shear force ($\mathrm{N\,m^{-1}}$). G is the interslice normal force ($\mathrm{N\,m^{-1}}$). λ is the proportionality constant. f is a function describing variation of the interslice normal to shear force ratio; can be constant or a half-sine.
Source	[6]
Ref. By	IM1, IM2, IM3

Number	GD7
Label	**Moment Equilibrium**
Equation	$0 = \begin{array}{l} -G_{\text{i}}\left[z_{\text{i}} + \frac{b_{\text{i}}}{2}\tan\left(\alpha_{\text{i}}\right)\right] + G_{\text{i-1}}\left[z_{\text{i-1}} - \frac{b_{\text{i}}}{2}\tan\left(\alpha_{\text{i}}\right)\right] - H_{\text{i}}\left[z_{\text{w,i}} + \frac{b_{\text{i}}}{2}\tan\left(\alpha_{\text{i}}\right)\right] \\ +H_{\text{i-1}}\left[z_{\text{w,i-1}} - \frac{b_{\text{i}}}{2}\tan\left(\alpha_{\text{i}}\right)\right] + \frac{b_{\text{i}}}{2}\left(X_{\text{i}} + X_{\text{i-1}}\right) - K_{\text{c}}W_{\text{i}}\frac{h_{\text{i}}}{2} + U_{\text{t,i}}\sin\left(\beta_{\text{i}}\right)h_{\text{i}} \\ +Q_{\text{i}}\sin\left(\omega_{\text{i}}\right)h_{\text{i}} \end{array}$
Description	This equation satisfies T2 for the net moment. Force equilibrium is derived from the free body diagram of Figure 4 in section 5.1.2. Index i refers to the values of the properties for slice/interslices following convention in Figure 3 in section 5.1.2. Force variable mathematical definitions can be found in DD1 to DD**??**. G is the interslice normal force ($\text{N}\,\text{m}^{-1}$). z is the height in the y-direction from the base of a slice to the center of the slice (m). b is the width of the base of a slice in the x direction (m). α is the angle between the base of a slice and the horizontal (°). H is the interslice water force ($\text{N}\,\text{m}^{-1}$). z_w is the height in the y-direction from the base of a slice halfway to the water table (m). X is the interslice shear force ($\text{N}\,\text{m}^{-1}$). K_c is the seismic coefficient. W is the weight ($\text{N}\,\text{m}^{-1}$). h is the height in the y-direction from the base of a slice to the slope surface, at the x-direction midpoint on the slice (m). U_t is the surface hydrostatic force ($\text{N}\,\text{m}^{-1}$). β is the angle between the surface of a slice and the horizontal (°). Q is the external force ($\text{N}\,\text{m}^{-1}$). ω is the angle between the imposed surface load acting into the surface and the vertical (°).
Source	[6]
Ref. By	IM2

5.2.4 Data Definition

This section collects and defines all the data needed to support the general definitions of 5.2.3 or build the instance models of 5.2.5. The dimension of each quantity is also given.

Number	DD1
Label	**Weight**
Symbol	W
SI Units	$\mathrm{N\,m^{-1}}$
Equation	$W_i = b_i \begin{cases} (y_{us,i} - y_{slip,i})\,\gamma_{Sat}, & y_{wt,i} \geq y_{us,i} \\ (y_{us,i} - y_{wt,i})\,\gamma + (y_{wt,i} - y_{slip,i})\,\gamma_{Sat}, & y_{us,i} > y_{wt,i} > y_{slip,i} \\ (y_{us,i} - y_{slip,i})\,\gamma, & y_{wt,i} \leq y_{slip,i} \end{cases}$
Description	i is the index representing a single slice. W is the weight ($\mathrm{N\,m^{-1}}$). b is the width of the base of a slice in the x direction (m). y_{us} is the y-ordinate of a point on the slope (m). y_{slip} is the y-ordinate of a point on a slip surface (m). γ_{Sat} is the soil saturated unit weight ($\mathrm{N\,m^{-3}}$). y_{wt} is the y-ordinate of a point on the water table (m). γ is the soil dry unit weight ($\mathrm{N\,m^{-3}}$).
Sources	[1]
Ref. By	GD1, GD2, GD7, DD10, DD11, IM1, IM2, IM3

Number	DD2
Label	**Base Water Force**
Symbol	U_b
SI Units	$\mathrm{N\,m^{-1}}$
Equation	$U_{b,i} = \ell_{b,i} \begin{cases} (y_{wt,i} - y_{slip,i})\,\gamma_w, & y_{wt,i} > y_{slip,i} \\ 0, & y_{wt,i} \leq y_{slip,i} \end{cases}$
Description	i is the index representing a single slice. U_b is the base hydrostatic force ($\mathrm{N\,m^{-1}}$). ℓ_b is the base length of a slice in the direction parallel to the slope of the base (m). y_{wt} is the y-ordinate of a point on the water table (m). y_{slip} is the y-ordinate of a point on a slip surface (m). γ_w is the unit weight of water ($\mathrm{N\,m^{-3}}$).
Sources	[1]
Ref. By	DD10, IM1, IM2, IM3

Number	DD3
Label	**Surface Hydrostatic Force**
Symbol	U_t
SI Units	$\mathrm{N\,m^{-1}}$
Equation	$U_{t,i} = \ell_{s,i} \begin{cases} (y_{wt,i} - y_{us,i})\,\gamma_w, & y_{wt,i} > y_{us,i} \\ 0, & y_{wt,i} \leq y_{us,i} \end{cases}$
Description	i is the index representing a single slice. U_t is the surface hydrostatic force ($\mathrm{N\,m^{-1}}$). ℓ_s is the surface length of a slice in the direction parallel to the slope of the surface (m). y_{wt} is the y-ordinate of a point on the water table (m). y_{us} is the y-ordinate of a point on the slope (m). γ_w is the unit weight of water ($\mathrm{N\,m^{-3}}$).
Sources	[1]
Ref. By	GD1, GD2, GD7, DD10, DD11, IM1, IM2, IM3

Number	DD4
Label	**Interslice Water Force**
Symbol	H
SI Units	$\mathrm{N\,m^{-1}}$
Equation	$H_i = \begin{cases} \frac{[y_{us,i} - y_{slip,i}]^2}{2}\gamma_{Sat} + [y_{wt,i} - y_{us,i}]^2\gamma_{Sat}, & y_{wt,i} \geq y_{us,i} \\ \frac{[y_{wt,i} - y_{slip,i}]^2}{2}\gamma_{Sat}, & y_{us,i} > y_{wt,i} > y_{slip,i} \\ 0, & y_{wt,i} \leq y_{slip,i} \end{cases}$
Description	i is the index representing a single slice. H is the interslice water force ($\mathrm{N\,m^{-1}}$). y_{us} is the y-ordinate of a point on the slope (m). y_{slip} is the y-ordinate of a point on a slip surface (m). γ_{Sat} is the soil saturated unit weight. ($\mathrm{N\,m^{-3}}$). y_{wt} is the y-ordinate of a point on the water table (m).
Sources	[1]
Ref. By	GD1, GD2, GD7, DD10, DD11, IM1, IM2, IM3

Number	DD5
Label	**Base Angle**
Symbol	α
SI Units	$^\circ$
Equation	$\alpha_{\mathrm{i}} = \arctan\left(\frac{y_{\mathrm{slip,i}} - y_{\mathrm{slip,i\text{-}1}}}{x_{\mathrm{slip,i}} - x_{\mathrm{slip,i\text{-}1}}}\right)$
Description	i is the index representing a single slice. α is the angle between the base of a slice and the horizontal ($^\circ$). y_{slip} is the y-ordinate of a point on a slip surface (m). x_{slip} is the x-ordinate of a point on a slip surface (m).
Sources	[1]
Ref. By	GD1, GD2, GD7, DD8, DD10, DD11, IM1, IM2, IM3

Number	DD6
Label	**Surface Angle**
Symbol	β
SI Units	$^\circ$
Equation	$\beta_{\text{i}} = \arctan\left(\frac{y_{\text{us,i}} - y_{\text{us,i-1}}}{x_{\text{us,i}} - x_{\text{us,i-1}}}\right)$
Description	i is the index representing a single slice. β is the angle between the surface of a slice and the horizontal ($^\circ$). y_{us} is the y-ordinate of a point on the slope (m). x_{us} is the x-ordinate of a point on the slope (m).
Sources	[1]
Ref. By	GD1, GD2, GD7, DD9, DD10, DD11, IM1, IM2, IM3

Number	DD7
Label	**Base x-Direction Width of a Slice**
Symbol	b
SI Units	m
Equation	$b_i = x_{slip,i} - x_{slip,i-1}$
Description	i is the index representing a single slice. b is the width of the base of a slice in the x direction (m). x_{slip} is the x-ordinate of a point on a slip surface (m).
Sources	[1]
Ref. By	GD7, DD1, DD8, DD9, DD10, DD11, IM1, IM2, IM3

Number	DD8
Label	**Total Base Length of a Slice**
Symbol	ℓ_b
SI Units	m
Equation	$\ell_{b,i} = b_i \sec(\alpha_i)$
Description	i is the index representing a single slice. ℓ_b is the base length of a slice in the direction parallel to the slope of the base (m). b is the width of the base of a slice in the x direction (m). α is the angle between the base of a slice and the horizontal (°).
Sources	[1]
Ref. By	GD3, GD4, DD2

Number	DD9
Label	**Total Surface Length of a Slice**
Symbol	ℓ_s
SI Units	m
Equation	$\ell_{s,i} = b_i \sec(\beta_i)$
Description	i is the index representing a single slice. ℓ_s is the surface length of a slice in the direction parallel to the slope of the surface (m). b is the width of the base of a slice in the x direction (m). β is the angle between the surface of a slice and the horizontal (°).
Sources	[1]
Ref. By	DD3

Number	DD10
Label	**Resistive Shear, Without Interslice Normal and Shear Forces**
Symbol	R
SI Units	$\mathrm{N\,m^{-1}}$
Equation	$R_i = \left(\begin{array}{l}[W_\mathrm{i} + U_\mathrm{t,i}\cos(\beta_\mathrm{i})]\cos(\alpha_\mathrm{i}) \\ +[-\Delta H_\mathrm{i} + U_\mathrm{t,i}\sin(\beta_\mathrm{i})]\sin(\alpha_\mathrm{i}) - U_\mathrm{b,i}\end{array}\right) \cdot \tan(\varphi') + c'_\mathrm{i} \cdot b_\mathrm{i} \cdot \sec(\alpha_\mathrm{i})$
Description	i is the index representing a single slice. R is the resistive shear force without the influence of interslice forces ($\mathrm{N\,m^{-1}}$). W is the weight ($\mathrm{N\,m^{-1}}$). U_t is the surface hydrostatic force ($\mathrm{N\,m^{-1}}$). β is the angle between the surface of a slice and the horizontal (°). α is the angle between the base of a slice and the horizontal (°). ΔH is the difference between interslice water forces ($\mathrm{N\,m^{-1}}$). U_b is the base hydrostatic force ($\mathrm{N\,m^{-1}}$). φ' is the effective angle of friction (°). c' is the effective cohesion (Pa). b is the width of the base of a slice in the x direction (m).
Sources	[6]
Ref. By	IM1, IM3

Resistive Shear, Without Interslice Normal and Shear Forces Derivation

The resistive shear force of a slice is defined as P_i in GD3, and depends upon the effective normal force. The definition for normal force in GD?? can be substituted into the definition for effective normal force in GD5 to obtain equation (1).

$$N'_\mathrm{i} = \begin{array}{l}[W_\mathrm{i} - X_\mathrm{i\text{-}1} + X_\mathrm{i} + U_\mathrm{t,i}\cos(\beta_\mathrm{i}) + Q_\mathrm{i}\cos(\omega_\mathrm{i})]\cos(\alpha_\mathrm{i}) \\ +[-K_\mathrm{c}\,W_\mathrm{i} - G_\mathrm{i} + G_\mathrm{i\text{-}1} - H_\mathrm{i} + H_\mathrm{i\text{-}1} + U_\mathrm{t,i}\sin(\beta_\mathrm{i}) + Q_\mathrm{i}\sin(\omega_\mathrm{i})]\sin(\alpha_\mathrm{i}) \\ -U_\mathrm{b,i}\end{array} \tag{1}$$

The values of the interslice forces G and X in the equation are unknown. Thus, a force equilibrium without the effect of interslice normal and shear forces is considered, from which a value for N^*_i can be obtained, as shown in equation (2).

$$N^*_\mathrm{i} = \begin{array}{l}[W_\mathrm{i} + U_\mathrm{t,i}\cos(\beta_\mathrm{i}) + Q_\mathrm{i}\cos(\omega_\mathrm{i})]\cos(\alpha_\mathrm{i}) \\ +[-K_\mathrm{c}\,W_\mathrm{i} - H_\mathrm{i} + H_\mathrm{i\text{-}1} + U_\mathrm{t,i}\sin(\beta_\mathrm{i}) + Q_\mathrm{i}\sin(\omega_\mathrm{i})]\sin(\alpha_\mathrm{i}) - U_\mathrm{b,i}\end{array} \tag{2}$$

Using N_{i}^{*}, a resistive shear force neglecting the influence of interslice normal and shear forces can be expressed in terms of only known values, as shown in equation (3).

$$R_{\text{i}} = N_{\text{i}}^{*} \tan(\varphi') + c_{\text{i}}' \cdot b_{\text{i}}' \sec(\alpha_t exti')$$

$$R_{\text{i}} = \begin{pmatrix} [W_{\text{i}} + U_{\text{t,i}} \cos(\beta_{\text{i}}) + Q_{\text{i}} \cos(\omega_{\text{i}})] \cos(\alpha_{\text{i}}) \\ + [-K_{\text{c}} W_{\text{i}} - \Delta H_{\text{i}} + U_{\text{t,i}} \sin(\beta_{\text{i}}) + Q_{\text{i}} \sin(\omega_{\text{i}})] \sin(\alpha_{\text{i}}) - U_{\text{b,i}} \end{pmatrix} \cdot \tan(\varphi') + c_{\text{i}}' \cdot b_{\text{i}} \cdot \sec(\alpha_{\text{i}}) \quad (3)$$

This can be further simplified by considering assumptions A10 and A11, which state that the seismic coefficient and the external force, respectively, are 0. Removing seismic and external forces yields equation (4).

$$R_{\text{i}} = \begin{pmatrix} [W_{\text{i}} + U_{\text{t,i}} \cos(\beta_{\text{i}})] \cos(\alpha_{\text{i}}) \\ + [-\Delta H_{\text{i}} + U_{\text{t,i}} \sin(\beta_{\text{i}})] \sin(\alpha_{\text{i}}) - U_{\text{b,i}} \end{pmatrix} \cdot \tan(\varphi') + c_{\text{i}}' \cdot b_{\text{i}} \cdot \sec(\alpha_{\text{i}}) \quad (4)$$

Number	DD11
Label	**Mobile Shear, Without Interslice Normal and Shear Forces**
Symbol	T
SI Units	$\text{N}\,\text{m}^{-1}$
Equation	$T_i = (W_i + U_{t,i} \cos(\beta_i)) \sin(\alpha_i) - (-\Delta H_i + U_{t,i} \sin(\beta_i)) \cos(\alpha_i)$
Description	i is the index representing a single slice. T is the mobilized shear force without the influence of interslice forces ($\text{N}\,\text{m}^{-1}$). W is the weight ($\text{N}\,\text{m}^{-1}$). U_t is the surface hydrostatic force ($\text{N}\,\text{m}^{-1}$). β is the angle between the surface of a slice and the horizontal (°). α is the angle between the base of a slice and the horizontal (°). ΔH is the difference between interslice water forces ($\text{N}\,\text{m}^{-1}$).
Sources	[6]
Ref. By	IM1

Mobile Shear, Without Interslice Normal and Shear Forces Derivation

The mobile shear force acting on a slice is defined as S_{i} from the force equilibrium in GD2, also shown in equation (5).

$$S_{\text{i}} = \begin{array}{l} [W_{\text{i}} - X_{\text{i-1}} + X_{\text{i}} + U_{\text{t,i}} \cos(\beta_{\text{i}}) + Q_{\text{i}} \cos(\omega_{\text{i}})] \sin(\alpha_{\text{i}}) \\ - [-K_{\text{c}}\, W_{\text{i}} - G_{\text{i}} + G_{\text{i-1}} - H_{\text{i}} + H_{\text{i-1}} + U_{\text{t,i}} \sin(\beta_{\text{i}}) + Q_{\text{i}} \cos(\omega_{\text{i}})] \cos(\alpha_{\text{i}}) \end{array} \quad (5)$$

The values of the interslice forces G and X in the equation are unknown. Thus, a force equilibrium without the effect of interslice normal and shear forces is considered, from which a value for T_i can be obtained, as shown in equation (6).

$$T_{\mathrm{i}} = \begin{aligned}&[W_{\mathrm{i}} + U_{\mathrm{t,i}}\cos(\beta_{\mathrm{i}}) + Q_{\mathrm{i}}\cos(\omega_{\mathrm{i}})]\sin(\alpha_{\mathrm{i}})\\ &-[-K_{\mathrm{c}}W_{\mathrm{i}} - \Delta H_{\mathrm{i}} + U_{\mathrm{t,i}}\sin(\beta_{\mathrm{i}}) + Q_{\mathrm{i}}\sin(\omega_{\mathrm{i}})]\cos(\alpha_{\mathrm{i}})\end{aligned} \tag{6}$$

The value of T_{i} is now defined completely in terms of known values. This can be further simplified by considering assumptions A10 and A11, which state that the seismic coefficient and the external force, respectively, are 0. Removing seismic and external forces yields equation (7).

$$T_{\mathrm{i}} = [W_{\mathrm{i}} + U_{\mathrm{t,i}}\cos(\beta_{\mathrm{i}})]\sin(\alpha_{\mathrm{i}}) - [-\Delta H_{\mathrm{i}} + U_{\mathrm{t,i}}\sin(\beta_{\mathrm{i}})]\cos(\alpha_{\mathrm{i}}) \tag{7}$$

5.2.5 Instance Models

This section transforms the problem defined in the Section 5.1 into one which is expressed in mathematical terms. It uses concrete symbols defined in Section 5.2.4 to replace the abstract symbols in the models identified in the Sections 5.2.2 and 5.2.3.

The goals GS1, GS3, and GS4 are met by the simultaneous solution of IM1, IM2, and IM3. The goal GS2 is met by IM4.

The Morgenstern-Price Method is a vertical slice, limit equilibrium slope stability analysis method. Analysis is performed by breaking the assumed slip surface into a series of vertical slices of mass. Static equilibrium analysis is performed, using two force equations and one moment equation as in T2. The problem is statically indeterminate with only these 3 equations and one constitutive equation (the Mohr Coulomb shear strength of T3), so the assumption A6 and corresponding equation GD6 are used. The force equilibrium equations can be modified to be expressed only in terms of known physical values, as done in DD10 and DD11.

Number	IM1
Label	**Factor of Safety**
Input	Ψ_{v} , Φ_{v} , T_{v} , R_{v}
Output	$FS = \dfrac{\sum_{v=1}^{n-1}\left[R_v \prod_{c=i}^{n-1} \frac{\Psi_u}{\Phi_u}\right] + R_n}{\sum_{v=1}^{n-1}\left[T_v \prod_{c=i}^{n-1} \frac{\Psi_u}{\Phi_u}\right] + T_n}$
Description	Equation for the Factor of Safety, the ratio between resistive and mobile shear the slip surface. The sum of values from each slice is taken to find the total resistive and mobile shear for the slip surface. The constants Φ and Ψ convert the resistive and mobile shear without the influence of interslice forces, to a calculation considering the interslice forces.
Sources	[6]
Ref. By	IM2, IM3

Factor of Safety Derivation

Using equation (17) from section 5.2.5, rearranging, and applying the boundary condition that E_0 and E_{n} are equal to 0 an equation for the factor of safety is found as equation (8), also seen in IM1.

$$\text{FS} = \frac{\sum_{v=1}^{n-1}\left[R_v \prod_{c=v}^{n-1} \frac{\Psi_u}{\Phi_u}\right] + R_n}{\sum_{v=1}^{n-1}\left[T_v \prod_{c=v}^{n-1} \frac{\Psi_u}{\Phi_u}\right] + T_n} \tag{8}$$

The constants Ψ and Φ described in equations 16 and 15 are functions of the unknowns: the interslice normal/shear force ratio λ (IM2) and the Factor of Safety FS (IM1).

Number	IM2
Label	**Normal/Shear Force Ratio**
Input	b_{v} , E_{v} , H_{v} , α_{v} , h_{v} , W_{v} , $U_{\text{t,v}}$, β_{v} , f_{v} , K_{c}
Output	$C1_{\text{i}} = \begin{cases} b_1\left[E_1 + H_1\right]\tan\left(\alpha_1\right) & \text{i} = 1 \\ b_{\text{i}}\left[\left(E_{\text{i}} + E_{\text{i-1}}\right) + \left(H_{\text{i}} + H_{\text{i-1}}\right)\right]\tan\left(\alpha_{\text{i}}\right) \\ +h_{\text{i}}\left(K_{\text{c}}\,W_{\text{i}} - 2\,U_{\text{t,i}}\sin\left(\beta_{\text{i}}\right) - 2\,Q_{\text{i}}\cos\left(\omega_{\text{i}}\right)\right) & 2 \leq \text{i} \leq \text{n-1} \\ b_{\text{n}}\left[E_{\text{n-1}} + H_{\text{n-1}}\right]\tan\left(\alpha_{\text{n-1}}\right) & \text{i} = \text{n} \end{cases}$ $C2_{\text{i}} = \begin{cases} b_1 E_1 f_1 & \text{i} = 1 \\ b_{\text{i}}\left(f_{\text{i}} E_{\text{i}} + f_{\text{i-1}} E_{\text{i-1}}\right) & 2 \leq \text{i} \leq \text{n-1} \\ b_{\text{n}} E_{\text{n-1}} f_{\text{n-1}} & \text{v} = \text{n} \end{cases}$ $\lambda = \frac{\sum_{i=1}^{n} C1_{\text{i}}}{\sum_{i=1}^{n} C2_{\text{i}}}$
Description	λ is the magnitude ratio between shear and normal forces at the interslice interfaces as the assumption of the Morgenstern-Price method in GD6. The inclination function f determines the relative magnitude ratio between the different interslices, while λ determines the magnitude. λ uses the sum of interslice normal and shear forces taken from each interslice.
Sources	[6]
Ref. By	IM1, IM3

Normal/Shear Force Ratio Derivation

The last static equation of T2 the moment equilibrium of GD7 about the midpoint of the base is taken, with the assumption of GD6. Results in equation (9).

$$0 = \begin{array}{l} -G_{\text{i}}\left[z_{\text{i}} - \frac{b_{\text{i}}}{2}\tan\left(\alpha_{\text{i}}\right)\right] + G_{\text{i-1}}\left[z_{\text{i-1}} + \frac{b_{\text{i}}}{2}\tan\left(\alpha_{\text{i}}\right)\right] - H_{\text{i}}\left[z_{\text{w,i}} - \frac{b_{\text{i}}}{2}\tan\left(\alpha_{\text{i}}\right)\right] \\ +H_{\text{i-1}}\left[z_{\text{w,i-1}} + \frac{b_{\text{i}}}{2}\tan\left(\alpha_{\text{i}}\right)\right] - \lambda\frac{b_{\text{i}}}{2}\left(G_{\text{i}} f_{\text{i}} + G_{\text{i-1}} f_{\text{i-1}}\right) + K_{\text{c}} W_{\text{i}}\frac{h_{\text{i}}}{2} - U_{\text{t,i}}\sin\left(\beta_{\text{i}}\right)h_{\text{i}} - Q_{\text{i}}\sin\left(\omega_{\text{i}}\right)h_{\text{i}} \end{array} \tag{9}$$

Rearranging the equation in terms of λ leads to equation (10).

$$\lambda = \frac{\begin{array}{c} -G_{\text{i}}\left[z_{\text{i}} - \frac{b_{\text{i}}}{2}\tan\left(\alpha_{\text{i}}\right)\right] + G_{\text{i-1}}\left[z_{\text{i-1}} + \frac{b_{\text{i}}}{2}\tan\left(\alpha_{\text{i}}\right)\right] - H_{\text{i}}\left[z_{\text{i}} - \frac{b_{\text{i}}}{2}\tan\left(\alpha_{\text{i}}\right)\right] \\ +H_{\text{i-1}}\left[z_{\text{i-1}} + \frac{b_{\text{i}}}{2}\tan\left(\alpha_{\text{i}}\right)\right] + K_{\text{c}} W_{\text{i}}\frac{h_{\text{i}}}{2} - U_{\text{t,i}}\sin\left(\beta_{\text{i}}\right)h_{\text{i}} - Q_{\text{i}}\sin\left(\omega_{\text{i}}\right)h_{\text{i}} \end{array}}{\frac{b_{\text{i}}}{2}\left[G_{\text{i}} f_{\text{i}} + G_{\text{i-1}} f_{\text{i-1}}\right]} \tag{10}$$

Taking a summation of each slice, and considering the boundary conditions that G_0 and G_{n} are equal to zero, a general equation for the constant λ is developed in equation (11), also found in IM2.

$$\lambda = \frac{\sum_{i=1}^{n} b_{\text{i}} \left[(G_{\text{i}} + G_{\text{i-1}}) + (H_{\text{i}} + H_{\text{i-1}}) \right] \tan(\alpha_{\text{i}}) + h_{\text{i}} \ \left[K_{\text{c}} \, W_{\text{i}} - 2 \, U_{\text{t,i}} \sin(\beta_{\text{i}}) - 2 \, Q_{\text{i}} \sin(\omega_{\text{i}}) \right]}{\sum_{i=1}^{n} b_{\text{i}} \left[f_i G_{\text{i}} + f_{\text{i-1}} G_{\text{i-1}} \right]} \tag{11}$$

Equation (11) for λ, is a function of the unknown interslice normal force G (IM3).

Number	IM3
Label	**Interslice Forces**
Input	FS, T_{i}, R_{i}, Ψ, Φ
Output	$G_{\text{i}} = \begin{cases} \frac{(\text{FS}) T_1 - R_1}{\Phi_{\text{i}}} & \text{i=1} \\ \frac{\Psi_{\text{i-1}} \cdot G_{\text{i-1}} + (\text{FS}) \cdot T_{\text{i}} - R_{\text{i}}}{\Phi_{\text{i}}} & 2 \leq \text{i} \leq \text{n-1} \\ 0 & \text{i=0} \vee \text{i=n} \end{cases}$
Description	The value of the interslice normal force G_{i} at interface i. The net force the weight of the slices adjacent to interface i exert horizontally on each other.
Sources	[6]
Ref. By	IM1, IM2

Interslice Force Derivation

Taking the perpendicular force equilibrium of GD1 with the effective stress definition from T4 that $N_{\text{i}} = N'_{\text{i}} - U_{\text{b,i}}$, and the assumption of GD6 the equilibrium equation can be rewritten as equation (12).

$$N'_{\text{i}} = \begin{aligned} &\left[W_{\text{i}} - \lambda \cdot f_{\text{i-1}} \cdot G_{\text{i-1}} + \lambda \cdot f_{\text{i}} \cdot G_{\text{i}} + U_{\text{t,i}} \cos(\beta_{\text{i}}) + Q_{\text{i}} \cos(\omega_{\text{i}}) \right] \cos(\alpha_{\text{i}}) \\ &+ \left[-K_{\text{c}} W_{\text{i}} - G_{\text{i}} + G_{\text{i-1}} - H_{\text{i}} + H_{\text{i-1}} + U_{\text{t,i}} \sin(\beta_{\text{i}}) + Q_{\text{i}} \sin(\omega_{\text{i}}) \right] \sin(\alpha_{\text{i}}) - U_{\text{b,i}} \end{aligned} \tag{12}$$

Taking the base shear force equilibrium of GD2 with the definition of mobilized shear from GD4 and the assumption of GD6, the equilibrium equation can be rewritten as equation (13).

$$\frac{N_{\text{i}} \tan(\varphi'_{i}) + c'_{\text{i}} \cdot b'_{\text{i}} \cdot \sec(\alpha_{\text{i}})}{\text{FS}} = \begin{aligned} &\left[W_{\text{i}} - \lambda \cdot f_{\text{i-1}} \cdot G_{\text{i-1}} + \lambda \cdot f_{\text{i}} \cdot G_{\text{i}} + U_{\text{t,i}} \cos(\beta_{\text{i}}) + Q_{\text{i}} \cos(\omega_{\text{i}}) \right] \sin(\alpha_{\text{i}}) \\ &- \left[-K_{\text{c}} W_{\text{i}} - G_{\text{i}} + G_{\text{i-1}} - H_{\text{i}} + H_{\text{i-1}} + U_{\text{t,i}} \cdot \sin(\beta_{\text{i}}) + Q_{\text{i}} \sin(\omega_{\text{i}}) \right] \cos(\alpha_{\text{i}}) \end{aligned} \tag{13}$$

Substituting the equation for N'_{i} from equation (12) into equation (13) and rearranging results in equation (14)

$$G_{\text{i}}\begin{bmatrix}[\lambda\cdot f_{\text{i}}\cos(\alpha_{\text{i}})-\sin(\alpha_{\text{i}})]\tan(\varphi'_{i})\\-[\lambda\cdot f_{\text{i}}\sin(\alpha_{\text{i}})+\cos(\alpha_{\text{i}})]\,(\text{FS})\end{bmatrix}=G_{\text{i-1}}\begin{bmatrix}[\lambda\cdot f_{\text{i-1}}\cos(\alpha_{\text{i}})-\sin(\alpha_{\text{i}})]\tan(\varphi'_{i})\\-[\lambda\cdot f_{\text{i-1}}\sin(\alpha_{\text{i}})+\cos(\alpha_{\text{i}})]\,(\text{FS})\end{bmatrix}+(\text{FS})\cdot T_{\text{i}}-R_{\text{i}} \tag{14}$$

Where R and T are the resistive and mobile shear of the slice, without the influence of interslice forces G and X, as defined in DD10 and DD11. Making use of the constants ϕ and Ψ with full equations found below in equations (15) and (16) respectively, then equation (14) can be simplified to equation (17), also seen in IM3.

$$\Phi_{\text{i}}=[\lambda\cdot f_{\text{i}}\cos(\alpha_{\text{i}})-\sin(\alpha_{\text{i}})]\,[\tan(\varphi'_{\text{i}})]-[\lambda\cdot f_{\text{i}}\sin(\alpha_{\text{i}})+\cos(\alpha_{\text{i}})]\,(\text{FS}) \tag{15}$$

$$\Psi_{\text{i}}=[\lambda\cdot f_{\text{i}}\cos(\alpha_{\text{i+1}})-\sin(\alpha_{\text{i+1}})]\,[\tan(\varphi')]-[\lambda\cdot f_{\text{i}}\sin(\alpha_{\text{i+1}})+\cos(\alpha_{\text{i+1}})]\,(\text{FS}) \tag{16}$$

$$G_{\text{i}}=\frac{\Psi_{\text{i-1}}\,G_{\text{i-1}}+(FS)\ T_{\text{i}}-R_{\text{i}}}{\Phi_{\text{i}}} \tag{17}$$

The constants Ψ and Φ in equation (17) for are functions of the unknowns: the interslice normal/shear force ratio λ (IM2), and the Factor of Safety FS (IM1).

Number	IM4
Label	**Critical Slip Identification**
Input	The geometry of the water table, the geometry of the layers composing the plane of a slope, and the material properties of the layers.
Output	$\text{FS}_{\text{Min}}=\Upsilon(\{x_{\text{cs}},y_{\text{cs}}\},\text{Input})$
Description	Given the necessary slope inputs, a minimization algorithm or function Υ, will identify the critical slip surface of the slope, with the critical slip coordinates $\{x_{\text{cs}},y_{\text{cs}}\}$ and the minimum factor of safety FS_{Min} that results.
Sources	[3]

[Should this IM exist? It doesn't arise from any T or A —BM]

5.2.6 Data Constraints

Table 1 and 2 shows the data constraints on the input and output variables, respectively. The column for physical constraints gives the physical limitations on the range of values that can be taken by the variable. The uncertainty column provides an estimate of the confidence with which the physical quantities can be measured. This information would be part of the input if one were performing an uncertainty quantification exercise. The constraints are conservative, to give the user of the model the flexibility to experiment with unusual situations. The column of typical values is intended to provide a feel for a common scenario.

Table 1: Input Variables

Var	Physical Constraints	Typical Value	Uncertainty
(x, y) of water table vertices'	Consecutive vertices have increasing x values. All layers start and end vertices' go to the same x values.	N/A	10%
(x, y) of slip vertices'	Consecutive vertices have increasing x values. All layers start and end vertices' go to the same x values.	N/A	10%
(x, y) of slope vertices' (*)	Consecutive vertices have increasing x values. All layers start and end vertices' go to the same x values.	N/A	10%
c (*)	$c > 0$	10	10%
v (*)	$0 < v < 1$	0.4	10%
φ' (*)	$0 < \varphi < 90$	25	10%
γ (*)	$\gamma > 0$	20	10%
γ_{Sat} (*)	$\gamma_{\text{Sat}} > 0$	20	10%
γ_{w}	$\gamma_{\text{w}} > 0$	9.8	10%

(*) Input coordinates needed for each layer.

Table 2: Output Variables

Var	Physical Constraints
FS	$FS > 0$
(x, y) Slip vertices'	Vertices are monotonic

5.2.7 Properties of a Correct Solution

A correct solution must exhibit

[What to put here? Only numerical output are coordinates of critical slip surface, factor of safety, and interslice forces, so showing that the outputs follow some physical law seems difficult. Static equilibrium? —BM]

[Or, could this be something along the lines of "Slope of critical slip surface must be monotonically increasing?" —BM]

6 Requirements

This section provides the functional requirements, the business tasks that the software is expected to complete, and the nonfunctional requirements, the qualities that the software is expected to exhibit.

6.1 Functional Requirements

R1: Read the input file, shown in the table below, and store the data. [A**??**, A3]

symbol	**unit**	**description**
(x, y)	m	x and y coordinates for vertices of the slope layers, and for the water table if one exists. Assumed straight line fits between vertices.
c'	kPa	Cohesion for each slope layer.
φ'	°	Effective angle of friction for each slope layer.
γ	kN m^{-3}	Unit weight of dry soil / ground layer for each slope layer.
γ_{Sat}	kN m^{-3}	Unit weight of saturated soil / ground layer for each slope layer.
γ_{w}	kN m^{-3}	Unit weight of water.

R2: Verify that the input data lies within physical constraints as shown in Table 1.

R3: Generate potential critical slip surfaces for the input slope.

R4: Calculate the factors of safety for each of the potential critical slip surfaces.

R5: Compare the factor of safety for each potential critical slip surface to determine the minimum factor of safety, corresponding to the critical slip surface.

[Would R3-R5 be better as a single requirement "Calculate the minimum factor of safety corresponding to the critical slip surface" —BM]

R6: Verify that the factor of safety and critical slip surface satisfy the physical constraints shown in Table 2.

R7: Display the critical slip surface of the 2D slope graphically.

R8: Display the value of the factor of safety for the critical slip surface.

R9: Calculate and graphically display the interslice normal forces.

R10: Calculate and graphically display the interslice shear forces.

6.2 Nonfunctional Requirements

SSP is intended to be an educational tool, therefore accuracy and performance speed are secondary program priorities. Instead, the following non-functional requirements are prioritized:

NFR1: Correctness, achieved if the outputs of the code have the properties described in 5.2.7.

NFR2: Understandability, achieved if the code is modularized with complete module guide and module interface specification.

NFR3: Reusability, achieved if the code is modularized.

NFR4: Maintainability, achieved if the traceability between requirements, assumptions, theoretical models, general definitions, data definitions, instance models, likely changes, and modules is completely recorded in traceability matrices in the SRS and module guide.

[Can a non-functional requirement refer to the software documentation? —BM]

7 Likely Changes

LC1: The system currently assumes the different layers of the soil are homogeneous. In the future, implementation can be added for inconsistent soil properties throughout.

8 Unlikely Changes

If changes were to be made with regard to the following, a different algorithm would be needed.

UC1: Changes related to A6 are not possible due to the dependency of the calculations on the linear relationship between interslice normal and shear forces.

UC2: A7 allows for 2D analysis with these models only because stress along z-direction is zero. These models do not take into account stress in the z-direction, and therefore cannot be used without manipulation to attempt 3-dimensional analysis.

[This section is not on the template —BM]

9 Traceability Matrices and Graphs

The purpose of the traceability matrices is to provide easy references on what has to be additionally modified if a certain component is changed. Every time a component is changed, the items in the column of that component that are marked with an "X" may have to be modified as well. Tables 6 and 7 show the dependencies of theoretical models, general definitions, data definitions,

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11
T1											
T2											
T3											
T4											
GD1											
GD2											
GD3											
GD4											
GD6											
GD7											
DD1											
DD2											
DD3											
DD4											
DD5											
DD6											
DD7											
DD8											
DD9											
DD??											
DD??											
DD??											
DD10											
DD11											
IM1											
IM2											
IM3											
IM4											
LC1											

Table 4: Traceability Matrix Showing the Connections Between Assumptions and Other Items

and instance models with each other. Table 5 shows the dependencies of instance models, requirements, and data constraints on each other. Table 4 shows the dependencies of theoretical models, general definitions, data definitions, instance models, and likely changes on the assumptions.

The purpose of the traceability graphs is also to provide easy references on what has to be additionally modified if a certain component is changed. The arrows in the graphs represent dependencies. The component at the tail of an arrow is depended on by the component at the head of that arrow. Therefore, if a component is changed, the components that it points to should

	IM1	IM2	IM3	IM4	5.2.6
R1					
R2					
R3					
R4					
R5					
R6					
R7					
R8					
R9					
R10					

Table 5: Traceability Matrix Showing the Connections Between Requirements and Instance Models

	T1	T2	T3	T4	GD1	GD2	GD3	GD4	GD6	GD7
T1										
T2										
T3										
T4										
GD1		X								
GD2		X								
GD3			X	X						
GD4	X		X	X			X			
GD6										
GD7		X								
DD1										
DD2										
DD3										
DD4										
DD5										
DD6										
DD7										
DD8										
DD9										
DD??										
DD??										
DD??										
DD10			X	X	X	X	X	X	X	
DD11			X	X	X	X	X	X	X	
IM1	X								X	
IM2		X							X	X
IM3				X	X	X			X	
IM4										

Table 6: Traceability Matrix Showing the Connections Between Items of Different Sections With Theory Models and General Definitions

	DD1	DD2	DD3	DD4	DD5	DD6	DD7	DD8	DD9	DD??	DD??	DD??	DD10	DD11	IM1	IM2	IM3	IM4
T1																		
T2																		
T3																		
T4																		
GD1																		
GD2																		
GD3																		
GD4																		
GD6																		
GD7																		
DD1																		
DD2																		
DD3																		
DD4																		
DD5																		
DD6																		
DD7																		
DD8																		
DD9																		
DD??																		
DD??																		
DD??																		
DD10	X	X	X	X	X	X	X	X	X	X	X	X						
DD11	X	X	X	X	X	X	X	X	X	X	X	X						
IM1	X	X	X	X	X	X	X	X	X	X	X	X	X	X		X	X	
IM2	X	X	X	X	X	X	X	X	X	X	X	X			X		X	
IM3	X	X	X	X	X	X	X	X	X	X	X	X			X	X		
IM4																		

Table 7: Traceability Matrix Showing the Connections Between Items of Different Sections with Data Definitions and Instance Models

also be changed. Figure **??** shows the dependencies of theoretical models, general definitions, data definitions, instance models, likely changes, and assumptions on each other. Figure **??** shows the dependencies of instance models, requirements, and data constraints on each other.

10 Values of Auxiliary Constants

There are no auxiliary constants.

[This section is not on the template, unless it's equivalent to symbolic constants in the appendix? —BM]

References

[1] D.G. Fredlund and J.Krahn. Comparison of slope stability methods of analysis. *Can. Geotech. J.*, (14):429–439, 4 April 1977.

[2] Nirmitha Koothoor. A document drive approach to certifying scientific computing software. Master's thesis, McMaster University, Hamilton, Ontario, Canada, 2013.

[3] Yu-Chao Li, Yun-Min Chen, Tony L.T Zhan, Dao-Sheng Ling, and Peter John Cleall. An efficient approach for locating the critical slip surface in slope stability analyses using a real-coded genetic algorithm. *Can. Geotech. J.*, (47):806–820, 25 June 2010.

[4] David L. Parnas and P.C. Clements. A rational design process: How and why to fake it. *IEEE Transactions on Software Engineering*, 12(2):251–257, February 1986.

[5] W. Spencer Smith and Lei Lai. A new requirements template for scientific computing. In J. Ralyté, P. Ȧgerfalk, and N. Kraiem, editors, *Proceedings of the First International Workshop on Situational Requirements Engineering Processes – Methods, Techniques and Tools to Support Situation-Specific Requirements Engineering Processes, SREP'05*, pages 107–121, Paris, France, 2005. In conjunction with 13th IEEE International Requirements Engineering Conference.

[6] D.Y. Zhu, C.F. Lee, Q.H. Qian, and G.R. Chen. A concise algorithm for computing the factor of safety using the morgenstern–price method. *Can. Geotech. J.*, (42):272–278, 19 February 2005.