

Validation Report for Slope Stability Analysis Program

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Contents

1 Introduction

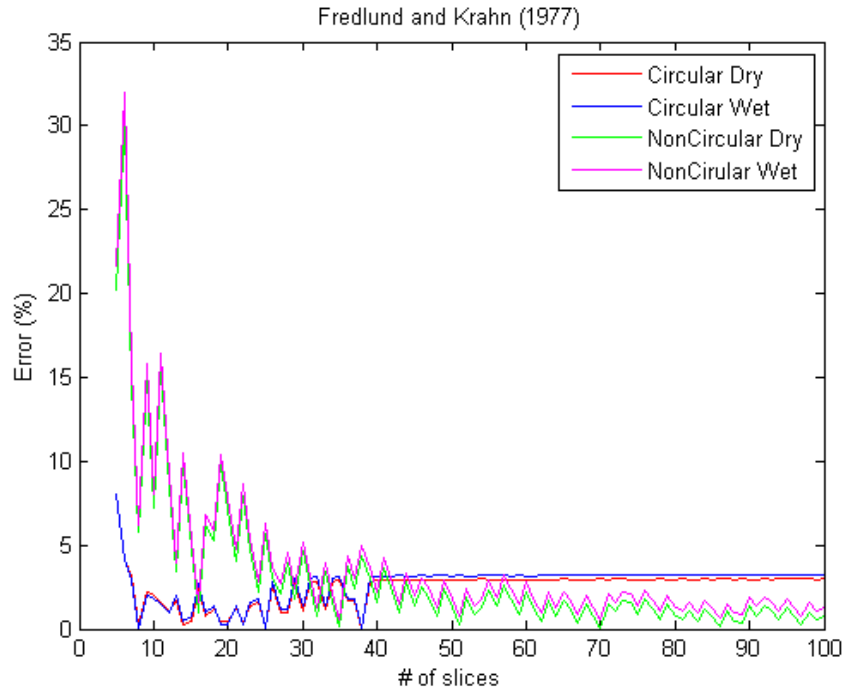
This document is a report on the results of testing a Slope Stability Analysis program for validity. The tests are introduced, the results are displayed, and the meaning of the results are analyzed. Possible further studies are suggested.

2 Morgenstern Price Slice Tester

Testing results obtained from the MorgPriceSolver.m program, a module in the SSA program. Factor of safety results from the program are compared to results from examples in slope stability papers, to judge the accuracy of the implemented algorithm. Due to the comparison analysis of the slip surfaces being imperfect definitive pass or fail assessments are not used, so results are measured on a relative and objective scale. The general rule of less relative error between results and comparisons suggests a more accurate solution can be used. For the same slip the result may be more accurate towards some comparisons more than others. In these cases ...?? . As a guideline relative error less than 10% could be considered acceptable, less than 5% good, and less than 1% excellent.

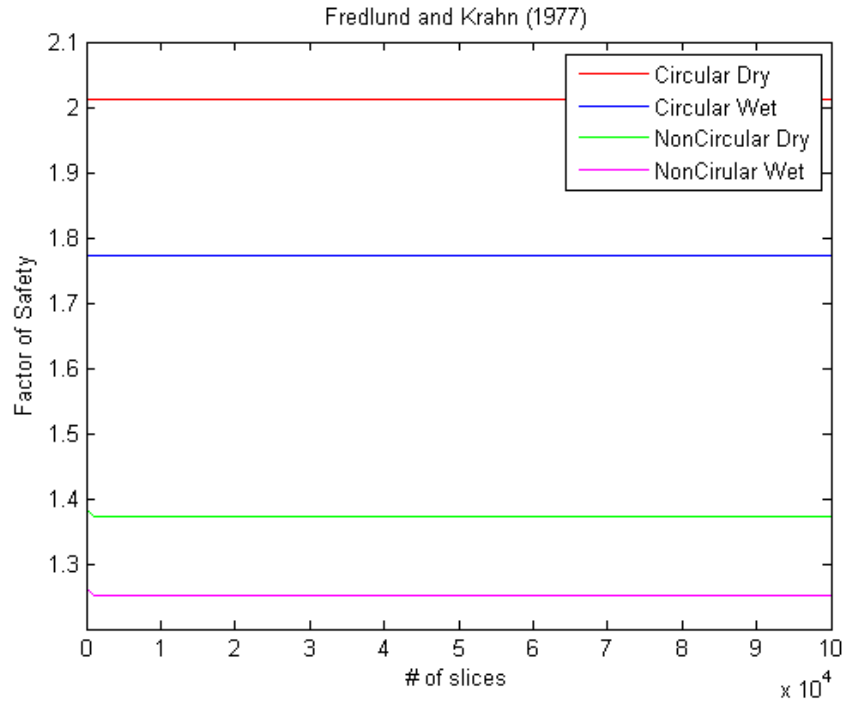
2.1 Fredlund and Krahn (1977)

Results compared to the Fredlund and Krahn (1977) paper. A graph of the relative error between the factor of safety calculated by the program with the factor of safety given in the paper, as a function of the number of analysis slices is used to analyze the results. For this paper a circular and non circular slip surface for the same slope are studied, under both dry and wet conditions.

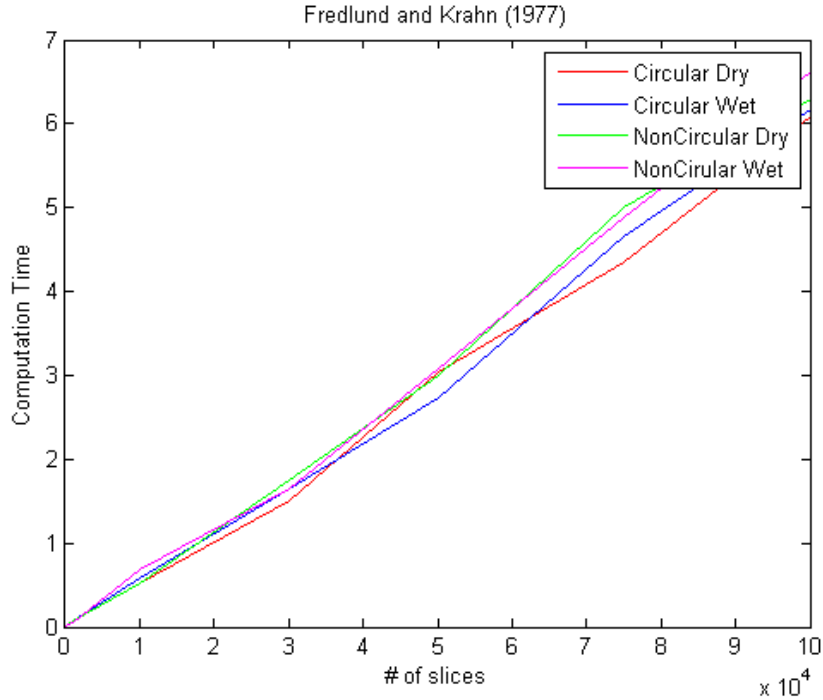


The figure shows very large error at low numbers of slices, up until approximately 20 slice analysis. Accuracy then begins to level off and stays consistent. The graph also shows that the dry analysis has less relative error than the wet analysis for both circular and non circular analysis. It can also be seen that the circular slip converges to less relative error than the non circular analysis. This suggests that the simpler dry and circular cases produce more accurate results, however further analysis of this type would be needed for a concrete conclusion.

Inspecting the computation of the factor of safety at extremely large numbers of slices shows no noticeable change in the returned value. Convergence to the same factor of safety occurs at 100 and 100000 slices, as seen on the following graph.

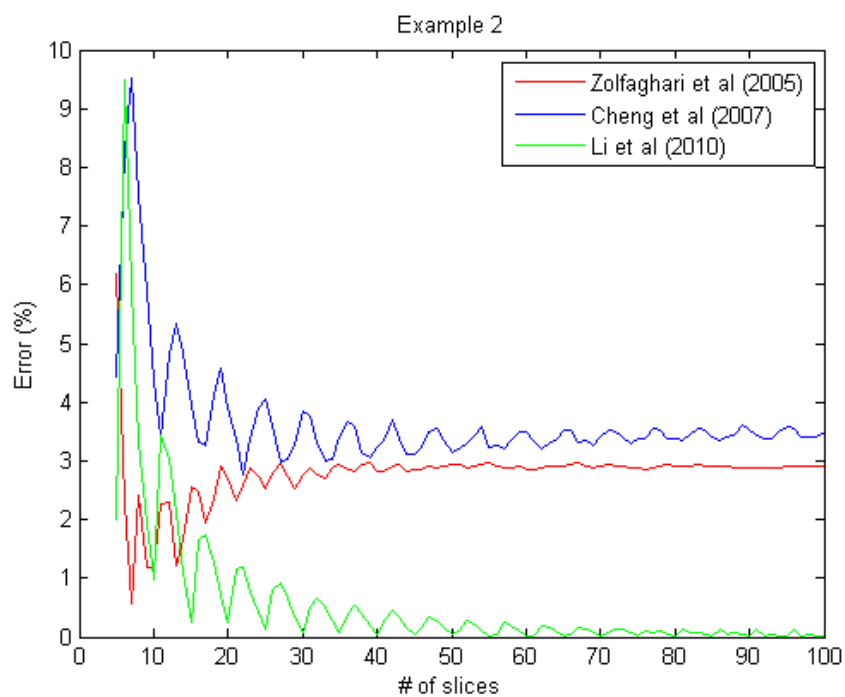
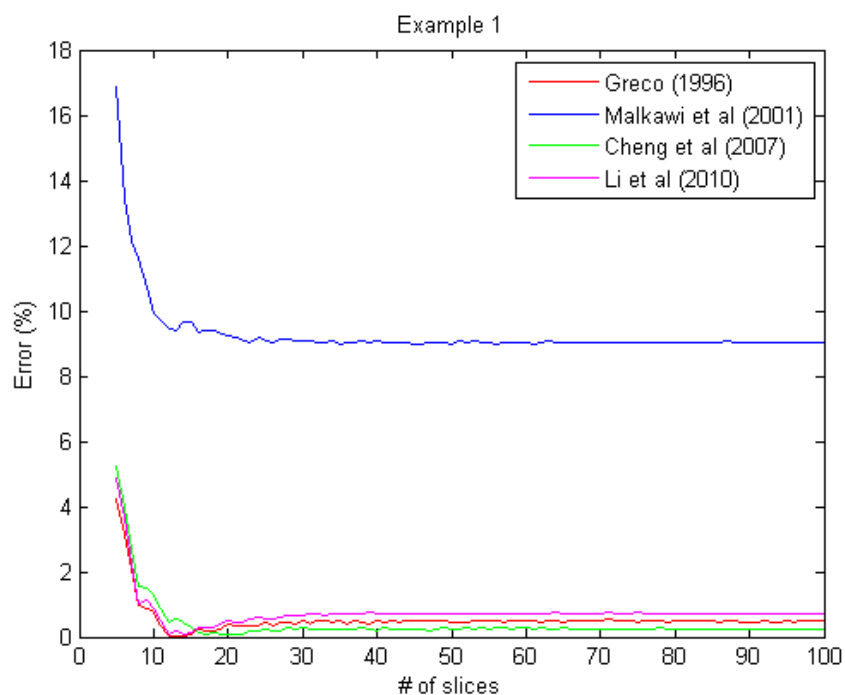


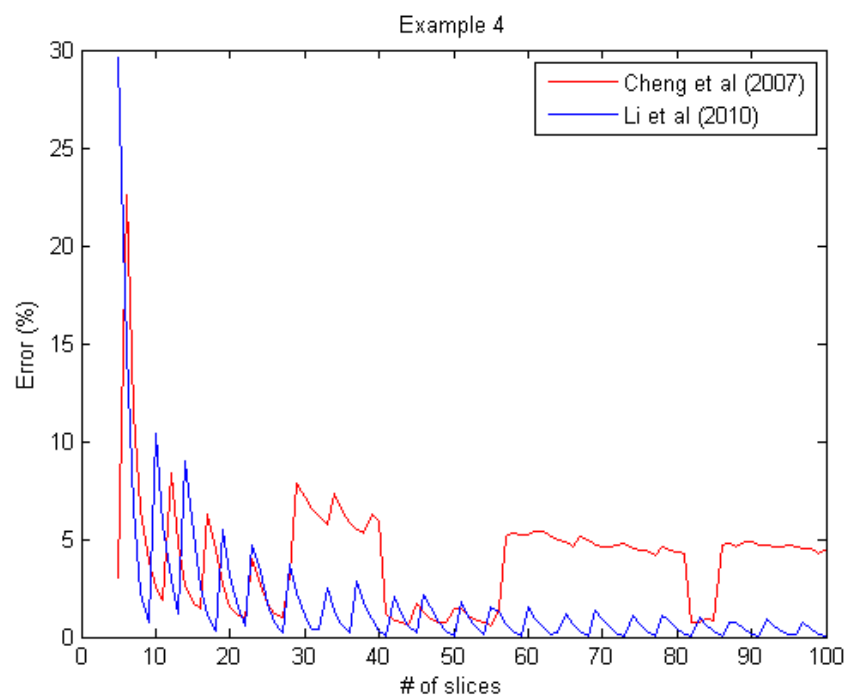
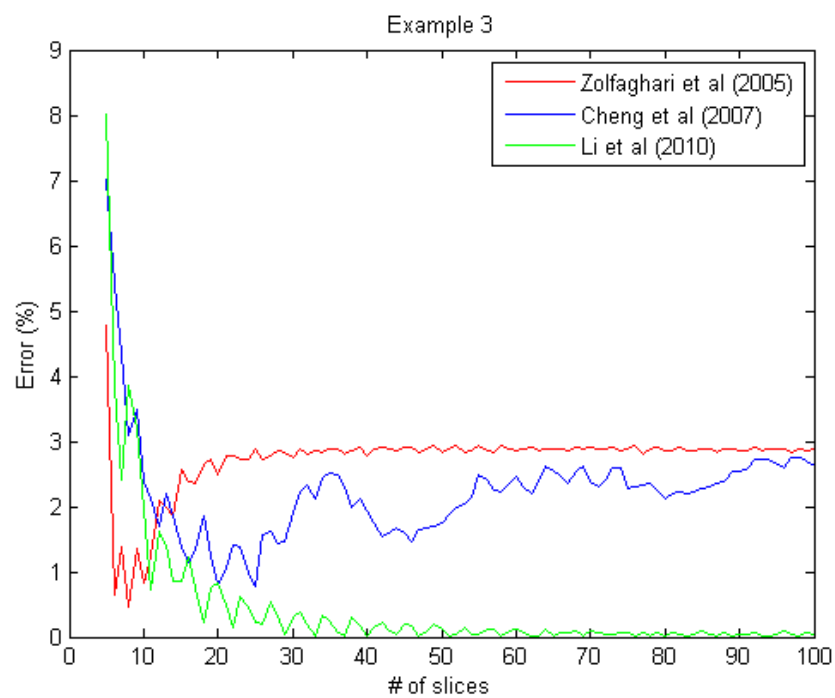
The next graph shows the change in computation time for calculation of the slope with different slice numbers. An approximately linear increase in computation time with number of analysis slices can be seen. As using a large number of slices sees no noticeable increase in the calculation of the Factor of Safety, the increased computation time makes using more slices than approximately 50 unnecessary. When compared to the computation time results the RFEM solver (??), it can be seen that the computation time for the morgenstern price algorithm is significantly less.

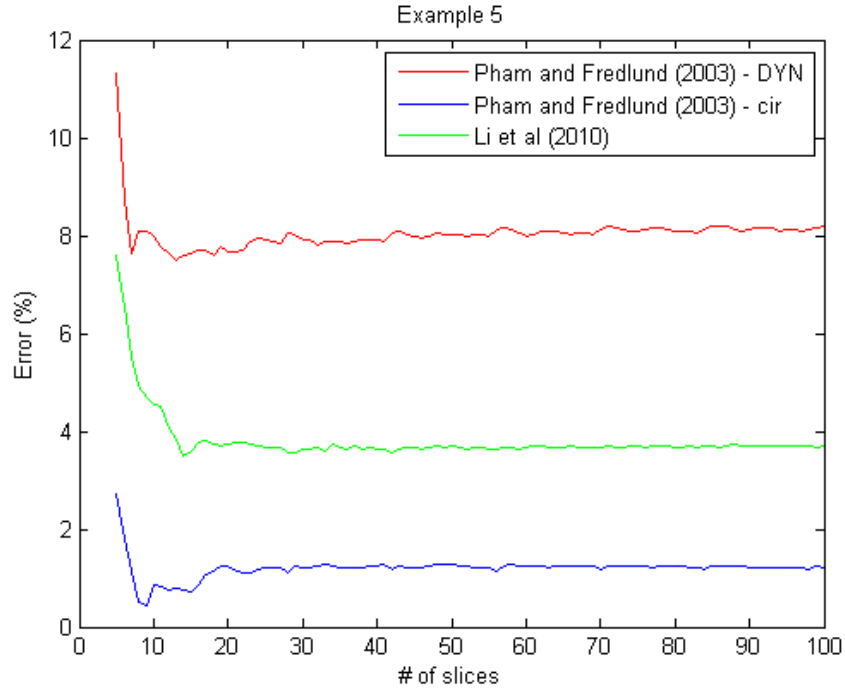


2.2 Examples 1 - 5

Results from the examples used in the papers: Greco (1996), Malkawi et al (2001) Zolfaghari et al (2005), Sarma and Tan (2006), Pham and Fredlund (2003), Cheng et al (2007), and Li et al (2010). Results followed the same general pattern as the previous test: Converging to a consistent factor of safety at approximately 25 slices. Relative error between results achieved and the results from the examples in the papers are all less than 10%, and for some lower than 1%. For examples with multiple comparisons the result usually converges very strongly to at least one of the comparisons. It should again be noted that the accuracy of results is relative only to the accuracy of the results from the comparison papers. However consistently high accuracy compared to results from different papers in many different examples, means a large true error would suggest a flaw in the slope stability analysis community as a whole. Not displayed here, but it was also seen that a large number of slices resulted in no noticeable change in the factor of safety calculation.

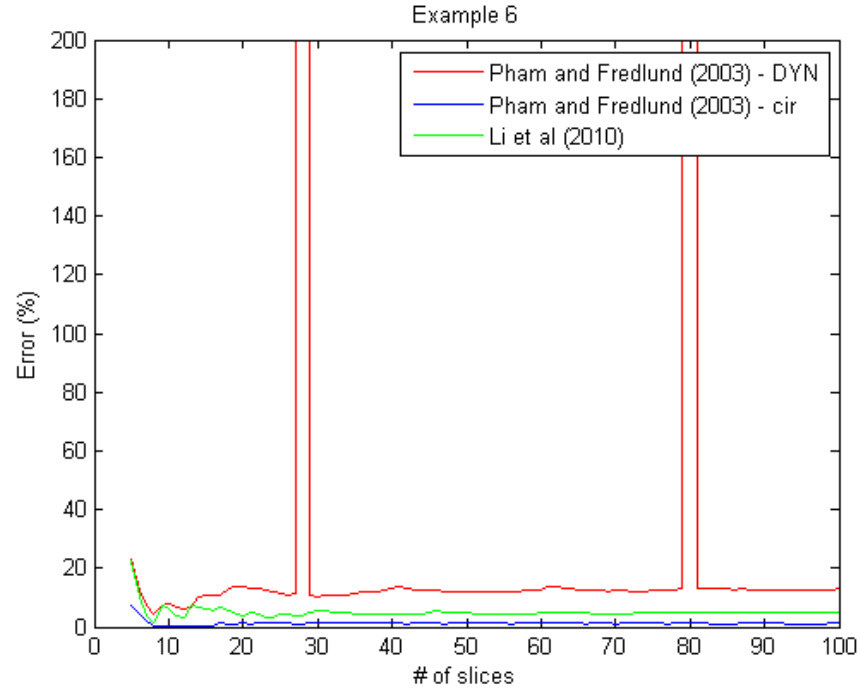




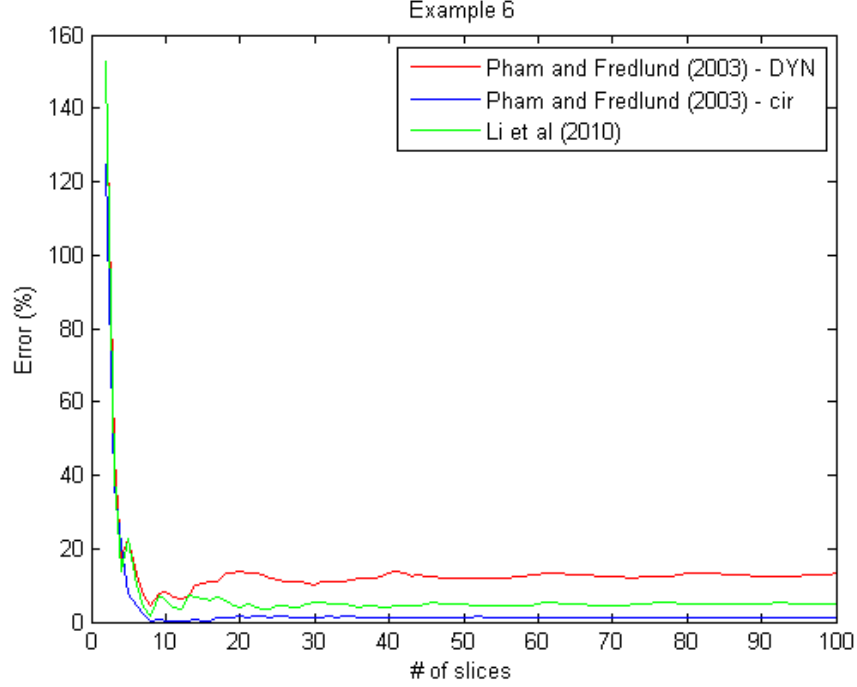


2.3 Example 6

Results from the example used in the papers: Pham and Fredlund (2003), Li et al (2010). Analysis of this example demonstrated non convergence of the Factor of Safety calculation for specific slice counts of the circular Pham and Fredlund analysis. Non convergence occurs when the algorithm calculates a very low factor of safety, or if the algorithm solution doesn't become consistent within the limited amount of iterations.



As a special case the limiting number of iterations allowed for the analysis was raised from 20 iterations to 30. The previously non converging results now converge to approximately 15 seen in the following figure. This is less accurate than the results seen previously, which were generally under 10 off between raising the iteration limit allowing better convergence, but decreasing the overall accuracy of the solver.

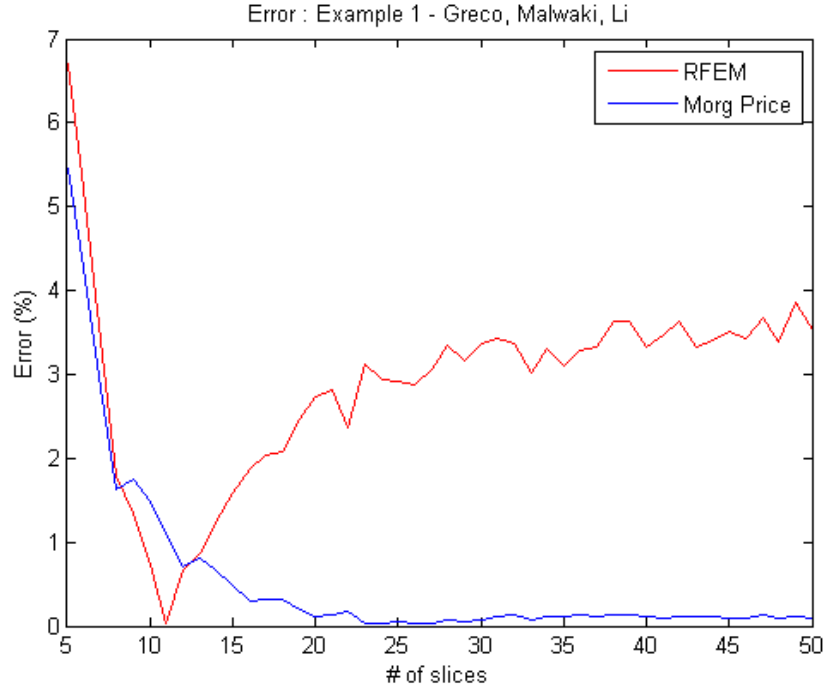


3 RFEM Slice Tester

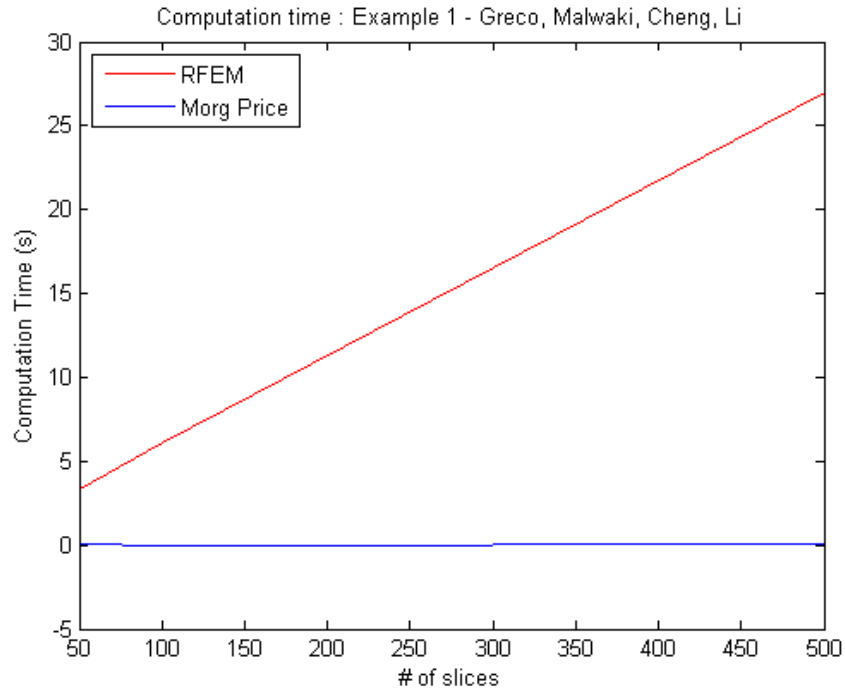
Testing results obtained from the RFEMSolver.m program, a module in the SSA program. Factor of safety results from the program are compared to results from examples in slope stability papers, to judge the accuracy of the implemented algorithm. As seen mentioned in the Morgenstern Price solver testing (section ??), due to the imperfect nature of the comparisons, results are judged on a relative basis. Example numbers refer to the same slope/slip problem as analyzed in the Morgenstern Price tests. The comparisons from scientific papers were performed using non RFEM solver algorithms, such as Morgenstern Price or Spencer's method. Therefore the relative accuracy of the implemented Morgenstern Price algorithm may appear higher, this does not also suggest the Morgenstern Price solver is truly more accurate however. Ideally and as is seen the RFEM solver should converge to a solution similar to the comparison slip, as two different methods calculating similar answers suggest accuracy of both methods.

3.1 Example 1

Results compared to those from Greco (1996), Malwaki et al (2001), Cheng et al(2007), and Li et al (2010). A graph of the relative error between the factor of safety calculated by the program with the factor of safety given in the papers is used to analyze the results. The results using the Morgenstern Price algorithm is also plotted. As the comparison was also performed using the Morgenstern Price algorithm this relative error is almost 0.

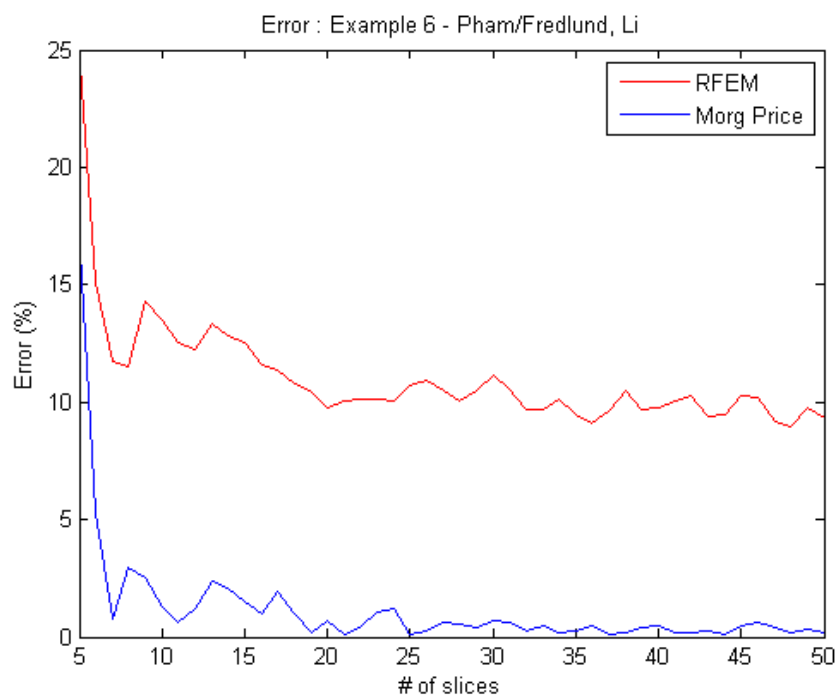
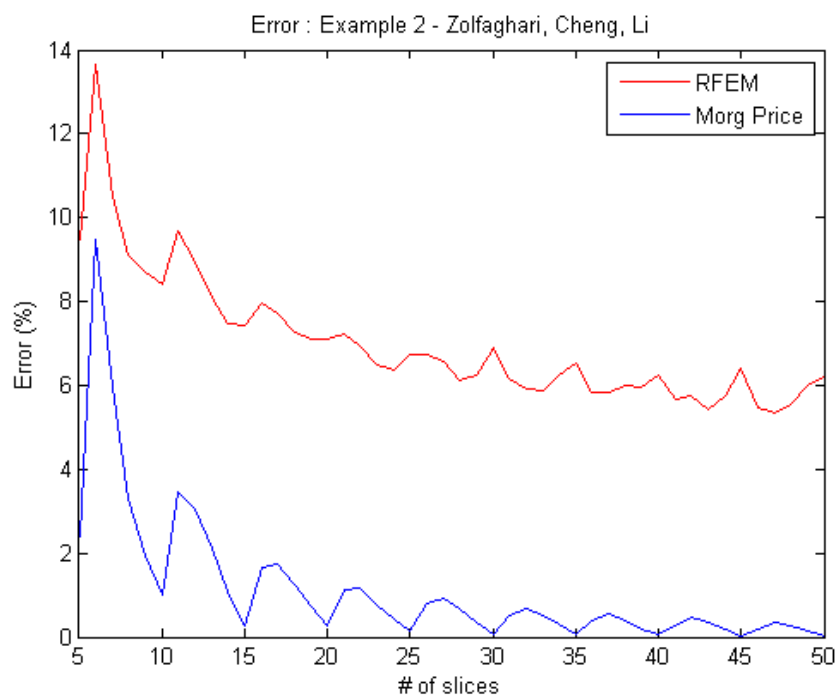


The figure shows convergence to a consistent relative error of less than 5% after approximately 25 slices for the RFEM solver. This is a positive result suggesting accuracy of both methods. However as seen in the following figure the RFEM algorithm requires significantly more computation time than the Morgenstern Price algorithm (section ??). The large difference in calculation time makes the Morgenstern Price solver much more efficient, especially when performing repeated analysis in the Genetic Algorithm search.



3.2 Examples 2 and 6

Results compared to those from Zolfaghari et al (2005), Cheng et al (2007), Li et al (2010) and Pham and Fredlund (2003). The graphs of these examples continue to show convergence to relative error of approximately 5% - 10% after approximately 25 steps. Again these are all positive results suggesting accuracy of both solvers.



4 Genetic Algorithm Tester

This script file tests the function GenAlg.m program, a module in the SSA program. The program will be tested for the relative error of the critical factor of safety for an example slip compared to the critical factor of safety from a paper analyzing the same example slip. Repeated analysis of an example slip will also be performed to analyze the consistency of the algorithm.

4.1 Example 1

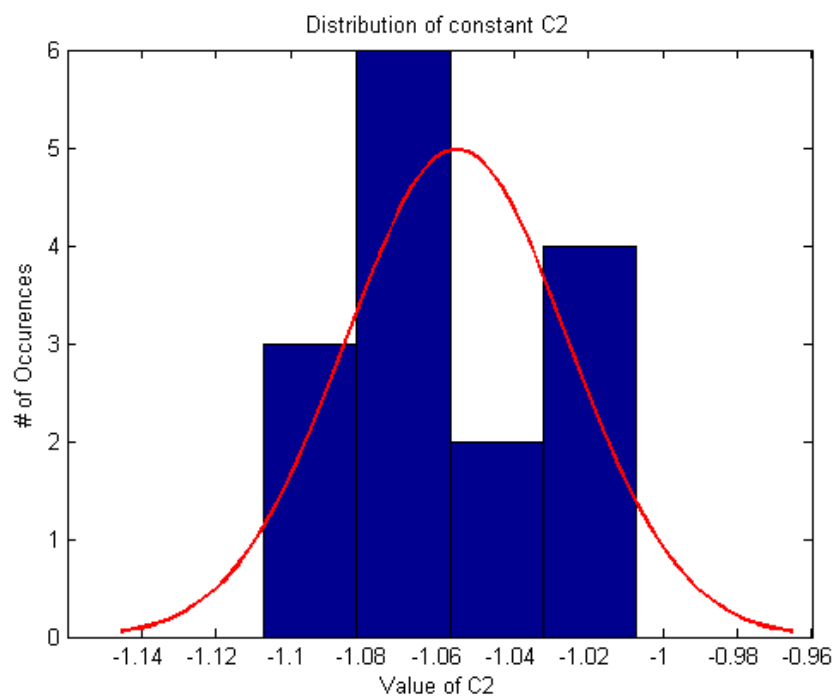
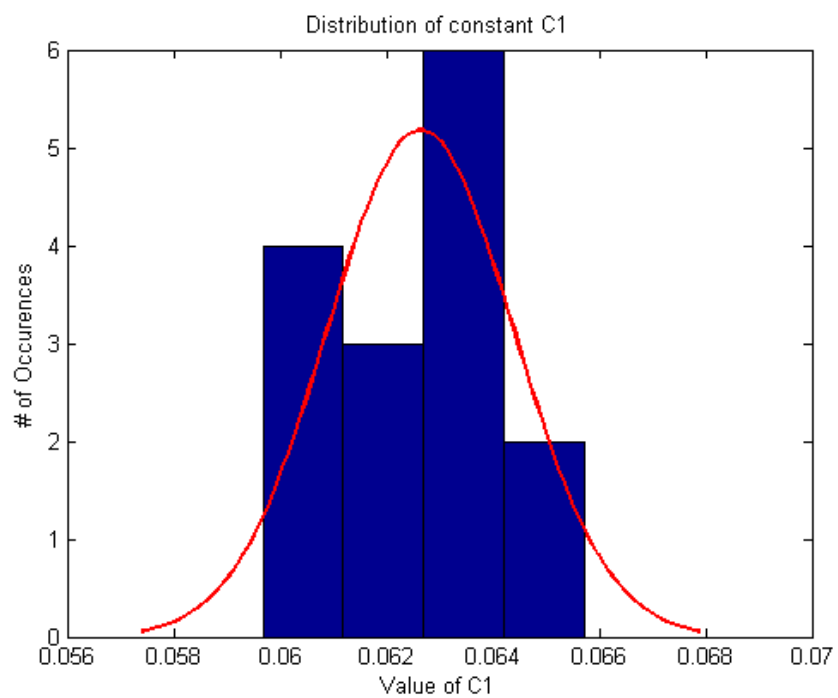
Comparing results for the example from Greco (1996), Malkawi et al (2001), Cheng et al (2007), Li et al (2010).

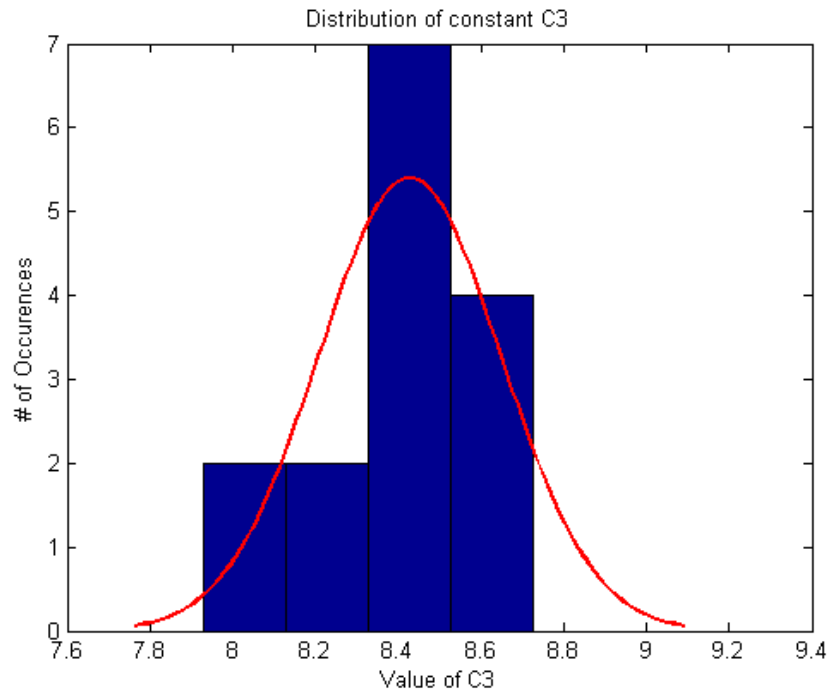
4.1.1 Consistency Testing

Firstly this example is used as a method of testing the consistency of the implemented genetic algorithm. Using the same input for the genetic algorithm to solve for the critical slip surface 15 times the physical and critical factor of safety range the algorithm generates as critical clip surfaces will be investigated. For each solution a second order polynomial is fit to the vertexes describing that solutions critical slip surface. The approximately quadratic shape of the slip surface makes a second order polynomial an appropriate fit. Solutions are of the form:

$$y(x) = C_1 \cdot x^2 + C_2 \cdot x + C_3$$

Where y is the vertical height of the slip surface at horizontal point x . The following histograms show the spread of the fitting constants C_1 , C_2 , and C_3 . If the algorithm is consistent the slip surfaces will follow similar shapes and the constants will have little spread.



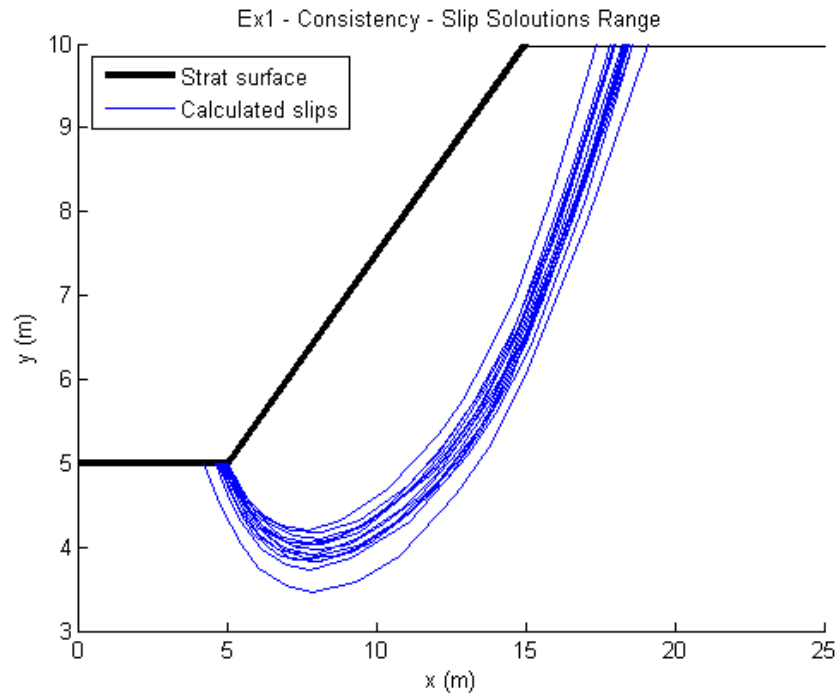


The results in the figures show that the normal distribution of results is approximately followed with few outlying data points, and that the spread of the constants for the different fits is small, differing over a small range. This suggests a consistent solution.

C1 has a standard deviation of 0.00
C2 has a standard deviation of 0.03
C3 has a standard deviation of 0.22

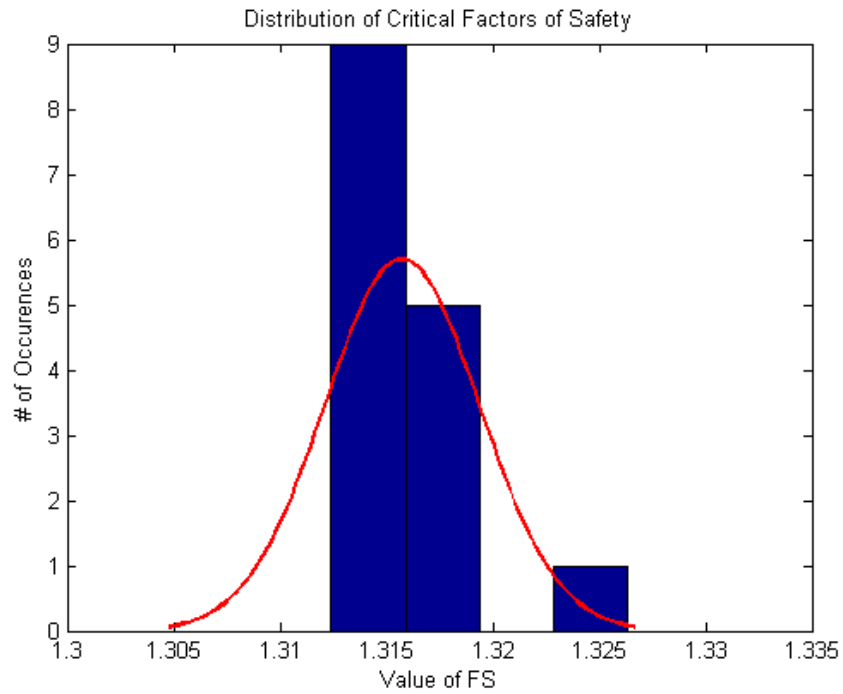
The next figure shows a plot of all the critical slip surfaces generated, supporting the previous findings by showing all the slip surfaces existing along a narrow band. The band width was measured at the entry and exit points of the slope for context.

The entry band width was : 0.837
The exit band width was : 1.692



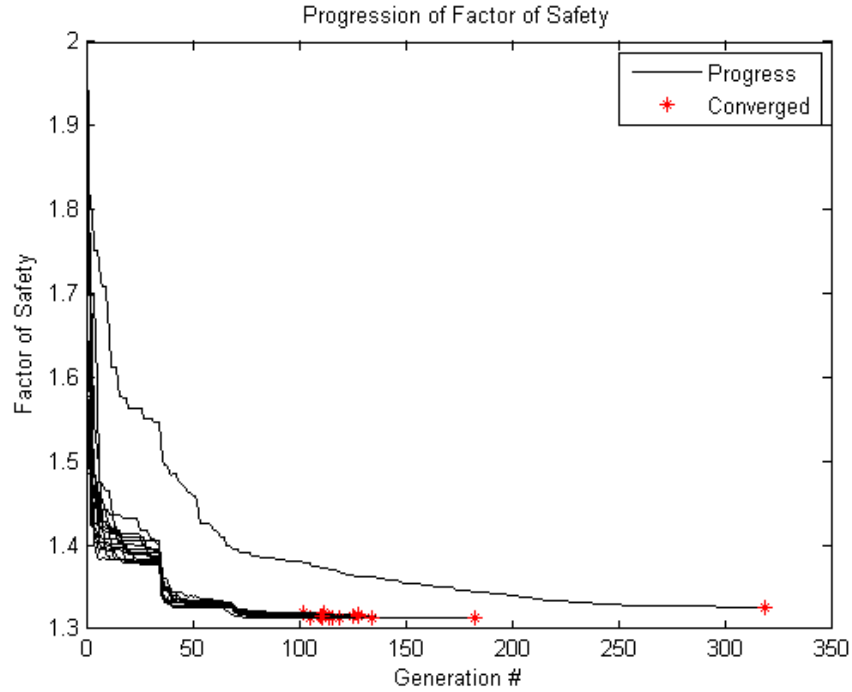
The previous results demonstrate that the algorithm generates spatially consistent solutions. The consistency of the Factor of Safety for the critical slips surface will now be investigated. The following histogram summarizes the distribution of calculated critical factors of safety. The figure shows a consistent factor of safety calculation, with results differing over a small range.

The average critical slip Factor of Safety
 is $FS=1.3157$, with a standard deviation
 of 0.0036 , and a minimum of $FS=1.3124$



The following figure shows the progression of the critical slip factor of safety over each generation of the genetic algorithm. The red dot represents the final generation. The figure generally shows convergence after approximately 130 generations. All solutions also seem to follow the same general path to convergence.

The average number of generations for convergence is 134
The maximum is 319, and the minimum is 102
The standard deviation of all trials is 55



The results seen in this section all suggest that the genetic algorithm can consistently converge to a common critical slip, with a common critical factor of safety.

4.1.2 Error Test

The Example 1 slope problem will now be measured for accuracy based on the difference between results generated by the algorithm, and results found scientific papers that analyzed the same example slope. A comparison between the slip surfaces and factors of safety is made. For this example the average of all generated slip surfaces was used.

The accuracy of the slip is shown visually with a plot, and also measured using a *distance error* metric. The metric slices the calculated slip surface, and comparison slip surface equally into 10 description vertexes. The average of distance between vertexes of the same indice, is then considered the *distance error*.

The accuracy of the factor of safety is measured based on relative error with the comparison factor of safety.

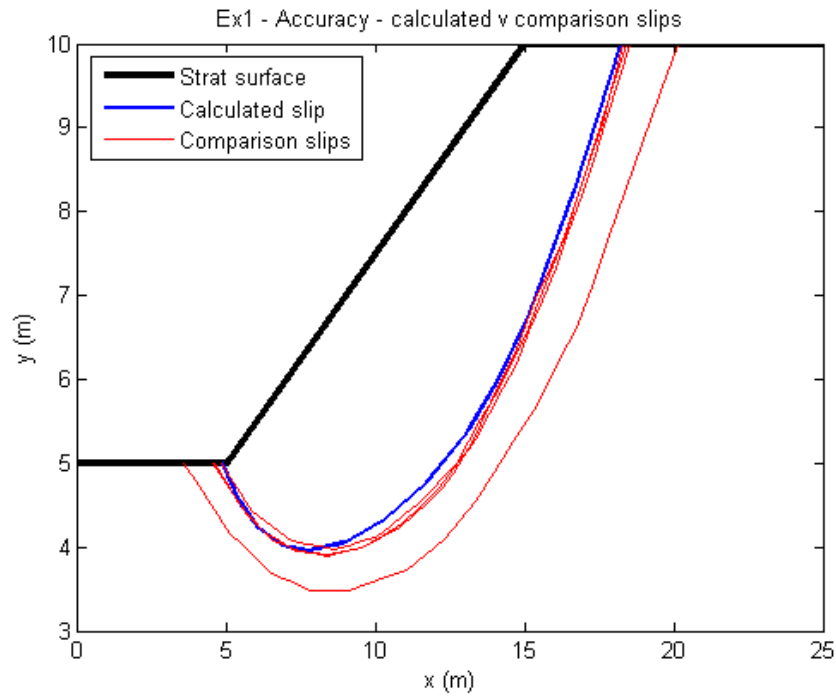
Example 1 - Slip

Author	entry x	exit x	distance error
Calculated	4.8225	18.2362	
Greco (1996)	4.8077	18.2911	2.3715

Malkawi et al (2001)	3.5400	20.1419	10.0779
Cheng et al (2007)	4.5258	18.3943	2.9385
Li et al (2010)	4.6000	18.5300	2.7468

Example 1 - Factor of Safety

Author	Fs	err (%)	time
Calculated	1.3157		18.9853
Greco (1996)	1.3270	0.8494	
Malkawi et al (2001)	1.2380	6.2786	
Cheng et al (2007)	1.3250	0.6997	
Li et al (2010)	1.3270	0.8494	



4.2 Examples 2-7

Examples 2-7 compare critical factor of safety results for the program to the results of many different papers. The tables show that results are very accurate compared to the results in the paper, with a relative factor of safety errors generally less than 5% for at least one of the comparison slips. The plots of the slip surfaces also show a critical slip that is reasonably similar to the comparison slips.

The only exception comes in Example 6. Approximately half the time a

critical factor of safety equal to the result in the paper will be calculated, while the other half a factor of safety of approximately 0.6 will be calculated. Further investigation of this is found in ??.

Example 2

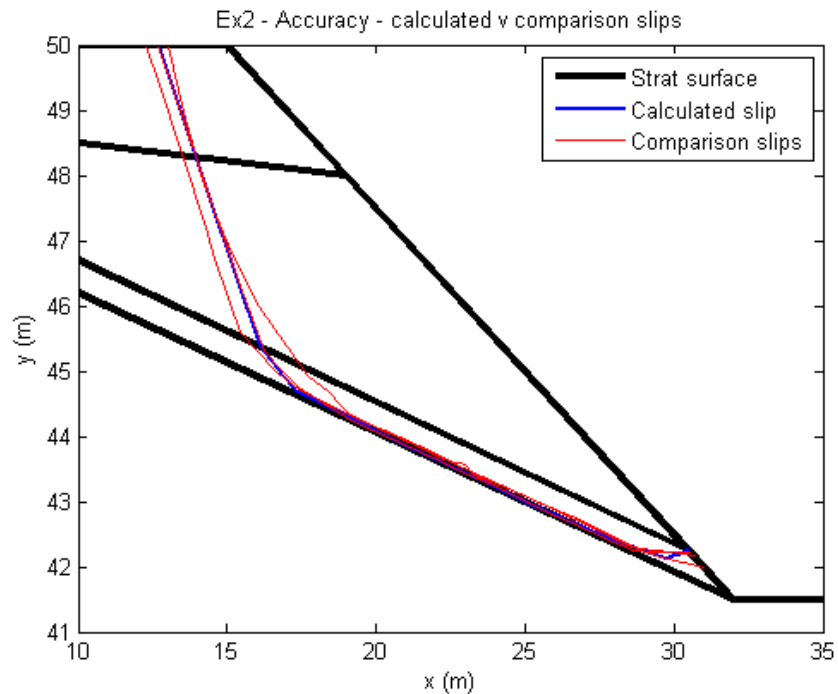
Papers: Zolfaghari et al (2005), Cheng et al (2007), Li et al (2010)

Example 2 - Slip

Author	entry x	exit x	distance error
Calculated	12.6982	30.5063	
Zolfaghari et al (2005)	13.0126	30.6803	0.1271
Cheng et al (2007)	12.2707	31.0044	0.1077
Li et al (2010)	12.6800	30.5300	0.0091

Example 2 - Factor of Safety

Author	Fs	err (%)	time
Calculated	1.1095		16.5205
Zolfaghari et al (2005)	1.2400	10.5203	
Cheng et al (2007)	1.1010	0.7765	
Li et al (2010)	1.1130	0.3101	



Example 3

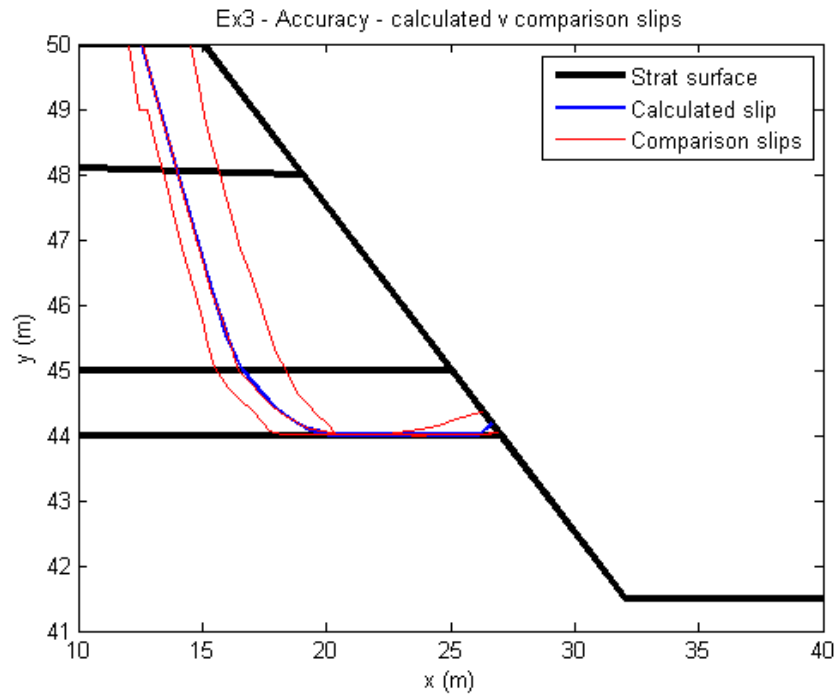
Papers: Zolfaghari et al (2005), Cheng et al (2007), Li et al (2010)

Example 3 - Slip

Author	entry x	exit x	distance error
Calculated	12.5196	26.6608	
Zolfaghari et al (2005)	14.4928	26.2681	0.3857
Cheng et al (2007)	12.0100	26.9014	0.1146
Li et al (2010)	12.5800	26.9200	0.0672

Example 3 - Factor of Safety

Author	Fs	err (%)	time
Calculated	1.3334		26.4734
Zolfaghari et al (2005)	1.4800	9.9039	
Cheng et al (2007)	1.3490	1.1548	
Li et al (2010)	1.3350	0.1182	



Example 4

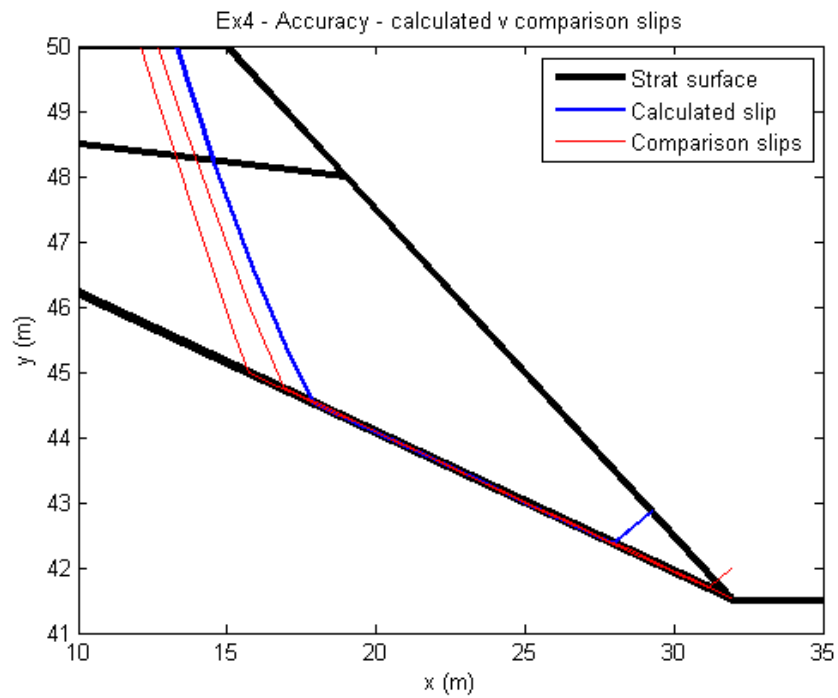
Papers: Cheng et al (2007), Li et al (2010)

Example 4 - Slip

Author	entry x	exit x	distance error
Calculated	13.2859	29.2819	
Cheng et al (2007)	12.0749	31.9000	0.4879
Li et al (2010)	12.6500	31.9100	0.4900

Example 4 - Factor of Safety

Author	Fs	err (%)	time
Calculated	1.3275		26.8166
Cheng et al (2007)	1.1840	12.1204	
Li et al (2010)	1.1970	10.9027	



Example 5

Papers: Pham and Fredlund (2003), Li et al (2010)

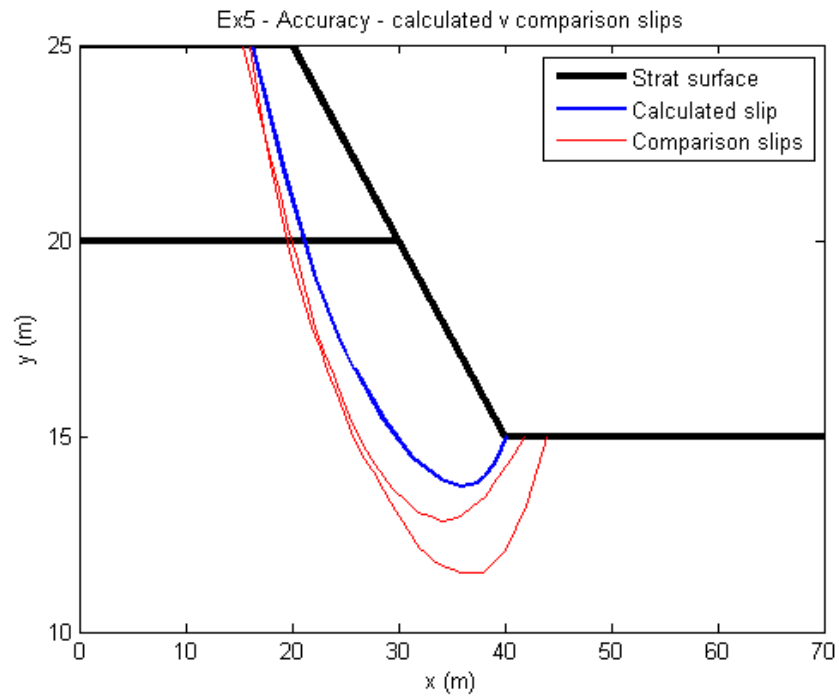
Example 5 - Slip

Author	entry x	exit x	distance err
Calculated	16.1920	40.1603	
Pham and Fredlund (2003)	15.9700	43.9000	1.7744
Li et al (2010)	15.4100	41.8900	0.8691

Example 5 - factor of Safety

Author	Fs	err (%)	time
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Calculated	1.4317	19.3441
Pham and Fredlund (2003)	1.4130	1.3233
Li et al (2010)	1.4080	1.6831



Example 6

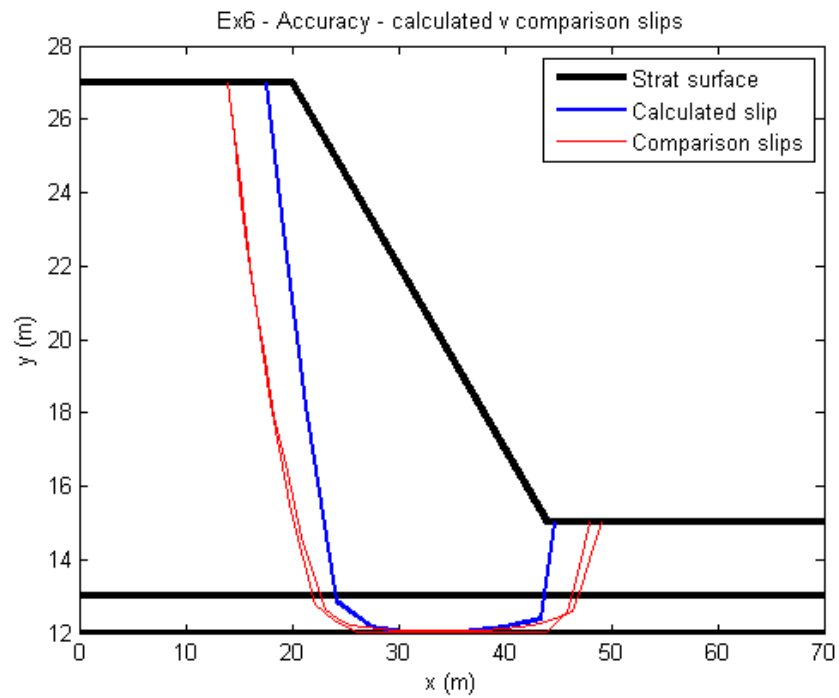
Papers: Pham and Fredlund (2003), Li et al (2010)

Example 6 - Slip

Author	entry x	exit x	distance error
Calculated	17.5144	44.7286	
Pham and Fredlund (2003)	14.0000	47.9868	1.1717
Li et al (2010)	13.9200	49.1400	1.3511

Example 6 - Factor of Safety

Author	Fs	err (%)	time
Calculated	0.5215		19.0789
Pham and Fredlund (2003)	1.0000	47.8494	
Li et al (2010)	1.0170	48.7212	



Example 7

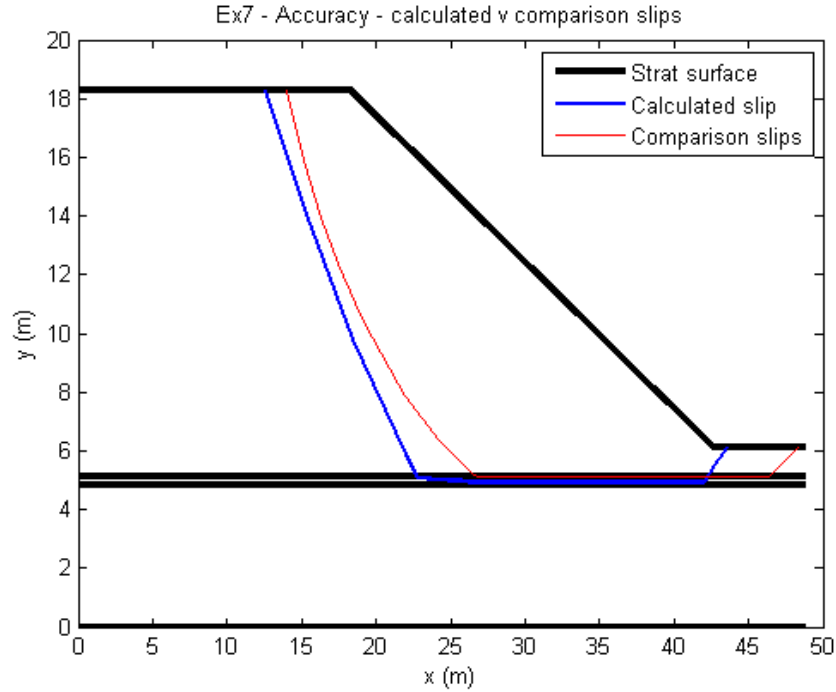
Papers: Fredlund and Krahn (1977)

Example 7 - Slip

Author	entry x	exit x	distance error
Calculated	12.5593	43.5584	
Fredlund and Krahn (1977)	13.9714	48.3809	1.6641

Example 7 - Factor of Safety

Author	Fs	err (%)	time
Calculated	1.1398		18.0649
Fredlund and Krahn (1977)	1.2450	8.4466	



5 Kinematic Admissibility Tester

Testing results obtained from the KinAdm.m program, a module in the Slope Stability Analysis program. The tester will test the algorithm to ensure it can correctly identify when a slip surface fails the 6 failure criterion.

5.1 Testing

The module will now be tested for the different failure criterion. A kinematically inadmissible test will return a value $pass=0$, and a failure code detailing the cause of the failure. All other values should return a value $pass=1$. All failure criteria will pass boundary cases. The failure criterion are tested individually with a simple slope surface, and slip surfaces designed to test the failure. Each failure test will give a slip designed to pass, a boundary case, and one designed to fail.

Failure (i)

The first criteria of a kinematically admissible surface is that the x-ordinates of the input slip surface vertexes do not decrease when reading the list of vertexes from beginning to end. (A) is a test with constantly increasing x-ordinates.

(B) is a test case with equivalent x-ordinates. (C) is a case with decreasing x-ordinates. Results follow:

```
Failure (i):
(A): pass=1
(B): pass=1
(C): pass=0 Failure Code1 - Non monotonic x
```

End Adjustments

The second criteria of a kinematically admissible surface is that the start and end vertexes of the slip surface match the y-ordinate of the uppermost stratigraphic layer at the specified x-ordinate. The module will not fail the slip surface, but will adjust the y value of the end vertexes. (A) is a test with vertexes above the uppermost stratigraphic layer. (B) is a test with vertexes on the uppermost stratigraphic layer. (C) is a test with vertexes below the uppermost stratigraphic layer. Vertex adjustment refers to the y values of the vertexes.

```
End Adjustments:
(A): pass=1, Start vertex adjustment,21->20 End vertex adjustment,13->12
(B): pass=1, Start vertex adjustment,20->20 End vertex adjustment,12->12
(C): pass=1, Start vertex adjustment,19->20 End vertex adjustment,11->12
```

Failure (ii)

The vertexes of the slip surface must be within the specified x-ordinate range of the uppermost stratigraphic layer. (A) is a test case with vertexes that stay within the uppermost stratigraphic layers range. (B) is a test with a vertex x-ordinate that goes below the minimum range of the uppermost stratigraphic layer. (C) is a test with a vertex x-ordinate that goes above the maximum range of the uppermost stratigraphic layer.

```
Failure (ii):
(A): pass=1
(B): pass=0 Failure Code2 - Vertex outside x range
(C): pass=0 Failure Code2 - Vertex outside x range
```

Failure (iii)

The non end vertexes of the slip surface must be below the uppermost stratigraphic layer. End vertexes will be moved onto the uppermost stratigraphic layer, and therefore don't interact with this case. This failure case checks only vertexes, line segments above the uppermost strat are checked in failure (iv). (A) is a test with vertexes below the uppermost stratigraphic layer. (B) is a test with vertexes on the uppermost stratigraphic layer. (C) is a test with the first interior vertex above the uppermost stratigraphic layer. (D) is a test with the last interior vertex above the uppermost stratigraphic layer of the slip surface.

Failure (iii):

(A): pass=1
(B): pass=1
(C): pass=0 Failure Code3 - Vertex above surface
(D): pass=0 Failure Code3 - Vertex above surface

Failure (iv)

Line segments between vertexes of the slip surface cannot go above the uppermost stratigraphic layer. (A) is a test with all line segments below the uppermost stratigraphic surface. (B) is a test case with a line segment on the uppermost stratigraphic layer. (C) is a test case with a line segment below the uppermost stratigraphic layer. the test is performed on the three interior line segments of the uppermost stratigraphic layer, going from slip entrance to exit as Strat Line Segments 1,2,3.

Failure (iv) Strat Line Segment 1:

(A): pass=1
(B): pass=1
(C): pass=0 Failure Code4 - Surface Intersection

Failure (iv) Strat Line Segment 2:

(A): pass=1
(B): pass=1
(C): pass=0 Failure Code4 - Surface Intersection

Failure (iv) Strat Line Segment 3:

(A): pass=1
(B): pass=1
(C): pass=0 Failure Code4 - Surface Intersection

Failure (v)

The slip surface must be concave upwards. The slope of line segments between vertexes of the slip surface must go from a large magnitude negative number towards a large magnitude positive number when connecting vertexes from slip entrance to exit. (A) test a case where slip surface slopes are increasing. (B) tests a case where slip surface slopes are constant. (C) tests a case where slip slopes experience a decrease.

Failure (v):

(A): pass=1
(B): pass=1
(C): pass=0 Failure Code5 - Concave Down, mcur=-1.02 mprv=-0.98

Failure (vi)

Slip surfaces cannot have angles less than 110 degrees (1.9199 rads) between adjacent line segments connecting vertexes. (A) tests a case with a greater than 110 degree slope. (B) tests a case with an exactly 110 degree slope. (C) tests a case with a less than 110 degree slope.

Failure (vi):

(A): pass=0 Failure Code6 - Sharp angle, Theta=1.9078

(B): pass=0 Failure Code6 - Sharp angle, Theta=1.9003

(C): pass=0 Failure Code6 - Sharp angle, Theta=1.8929

The results seen differ slightly from what's expected, with all cases failing reporting angles just below the 1.9199 rad cut off, despite case (A) being greater than this angle, and (B) being approximately equivalent, based on geometric analysis. The minor error in angle calculation likely comes from a π rounding error. Other than this small error, all other tests were successful.

6 Input Tester

Testing results obtained for the Input.m program a module in the Slope Stability Analysis program. Tested using the SSA_InputTester.m script. Tests used the script SSA_InputSpecial.m in place of Input.m, where SSA_InputSpecial is identical with the exception of predetermined command line inputs. The tester will test the algorithm to ensure it can correctly identify faulty input files, through various failure mechanisms.

Error Type	Case	Error Code
good file	/	
Dimensions - input file incorrectly identifies the number of stratigraphic layers describing the slope	understate	Input Error : Expected 1 and detected 2 stratigraphic layers
	overstate	Input Error : Expected 3 and detected 2 stratigraphic layers
Dimensions - input file incorrectly identifies the number of vertexes describing a soil layer	understate	Input Error : Expected 4 and detected 5 vertex sets describing stratigraphic layer 1
	overstate	Input Error : Expected 6 and detected 5 vertex sets describing stratigraphic layer 1

Dimensions - input file incorrectly gives the number of soil properties necessary to describe a layer	missing properties	Input Error : Stratigraphic soil properties not fully defined
	extra properties	Input Error : An extra soil data input has been given
Analysis - input files given soil motion direction does not match the geometry of the slope	left to right	Input Error : Detected soil motion direction (left to right), does not match given soil motion
	right to left	Input Error : Detected soil motion direction (right to left), does not match given soil motion
Analysis - input files given vertices of a soil layer or piezometric surface not ordered in terms of increasing x-ordinates.	soil layer	Input Error : Given x-ordinates describing layer 2 are not in an increasing order
	piezometric surface	Input Error : Given initial x-ordinate of 0.0, and final x-ordinate of 75.0, of layer 2, do not match those given in the first layer with an initial x of 0.0 and final x of 70.0
Analysis - input file gives end vertexes of a soil layer that does not match the end x-ordinate range of the uppermost stratigraphic layer.	end vertice - high	Input Error : Given initial x-ordinate of 0.0, and final x-ordinate of 75.0, of layer 2, do not match those given in the first layer with an initial x of 0.0 and final x of 70.0
	end vertice - low	Input Error : Given initial x-ordinate of 0.0, and final x-ordinate of 60.0, of layer 2, do not match those given in the first layer with an initial x of 0.0 and final x of 70.0

	start vertice - high	Input Error : Given initial x-ordinate of 5.0, and final x-ordinate of 70.0, of layer 2, do not match those given in the first layer with an initial x of 0.0 and final x of 70.0
	start vertice - low	Input Error : Given initial x-ordinate of -5.0, and final x-ordinate of 70.0, of layer 2, do not match those given in the first layer with an initial x of 0.0 and final x of 70.0
Analysis - input file gives end vertexes of a piezometric surface that does not match the end x-ordinate range of the uppermost stratigraphic layer.	end vertice - high	Input Error : Given initial x-ordinate of 0.0, and final x-ordinate of 75.0, of the piezometric surface, do not match those given in the first layer with an initial x of 0.0 and final x of 70.0
	end vertice - low	Input Error : Given initial x-ordinate of 0.0, and final x-ordinate of 65.0, of the piezometric surface, do not match those given in the first layer with an initial x of 0.0 and final x of 70.0
	start vertice - high	Input Error : Given initial x-ordinate of 10.0, and final x-ordinate of 70.0, of the piezometric surface, do not match those given in the first layer with an initial x of 0.0 and final x of 70.0
	start vertice - low	Input Error : Given initial x-ordinate of -10.0, and final x-ordinate of 70.0, of the piezometric surface, do not match those given in the first layer with an initial x of 0.0 and final x of 70.0

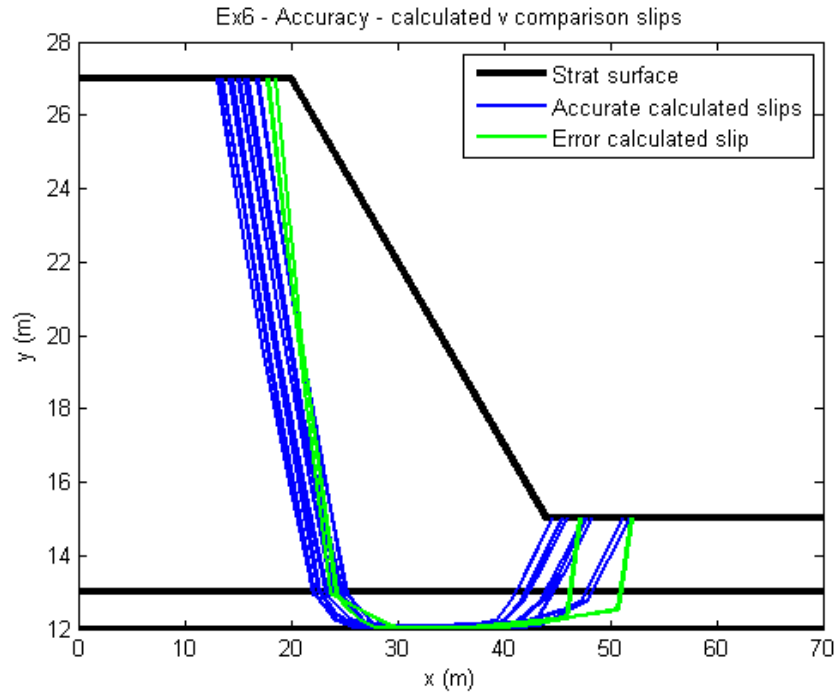
Constraints - The effective angle of friction given for all soil layers must be between 0 and 90 degrees.	> 90	Input Error : Effective angle of friction of layer 1 does not meet physical constraints, must be between 0 and 90 degrees, given 100.0
	< 90	Input Error : Effective angle of friction of layer 1 does not meet physical constraints, must be between 0 and 90 degrees, given -10.0
Constraints - The cohesion given for all soil layers must be greater than 0.	< 0	Input Error : cohesion of layer 1 does not meet physical constraints, must be greater than 0, given -5.0
Constraints - The soil weight given for all soil layers must be greater than 0.	< 0	Input Error : Soil weight of layer 1 does not meet physical constraints, must be greater than 0, given -15.0
Constraints - The saturated soil weight given for all soil layers must be greater than 0.	< 0	Input Error : Saturated soil weight of layer 1 does not meet physical constraints, must be greater than 0, given -15.0
Constraints - The poisson's ratio given for all soil layers must be between 0 and 90 degrees.	> 1	Input Error : Given initial x-ordinate of 0.0, and final x-ordinate of 75.0, of layer 2, do not match those given in the first layer with an initial x of 0.0 and final x of 70.0
	< 0	Input Error : Poissons ratio of layer 1 does not meet physical constraints, must be greater than 0 and less than 1, given -0.4

7 Example 6 - Further Study

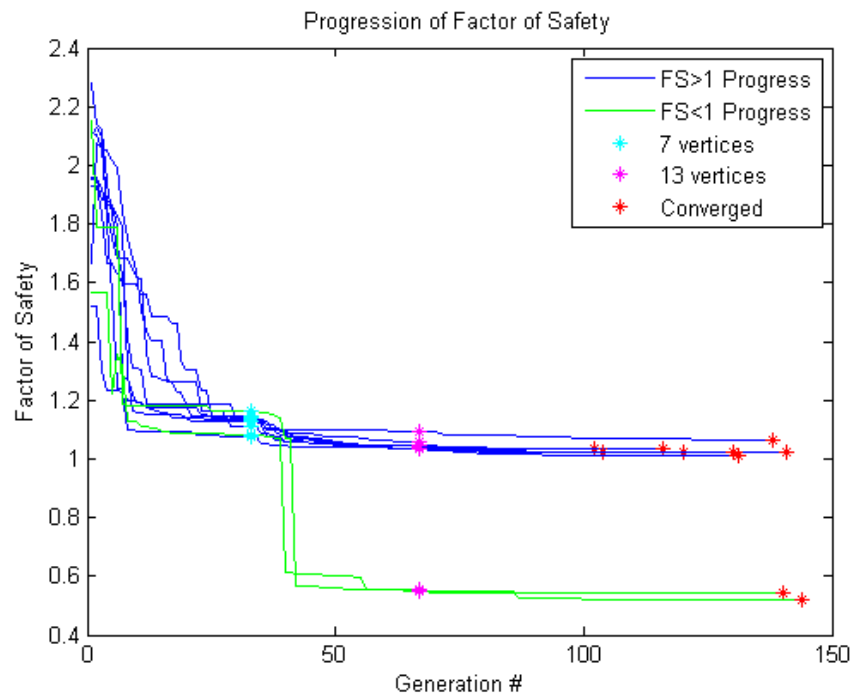
This script file tests the slope problem from example 6, which has been seen to create spurious results for the GenAlg module. The slope problem will be given a consistency test to investigate the performance issues.

7.1 Testing

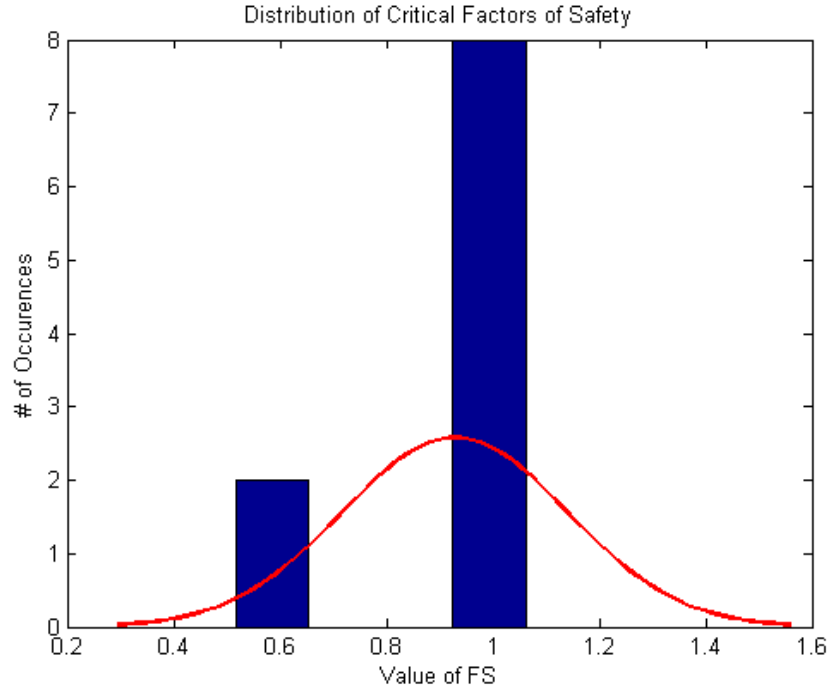
The genetic algorithm search is performed 10 times. The results of each critical slip are plotted in the following figure. The figure shows little difference between the slips that converge to factor of safety of 1 and those that converge below, other than the sharp incline the low factor of safety slopes show at the slip exit.



The next figure shows the progression of the factors of safety through the generations for each search. The results tend to not show a single specific path the slips with factors of safety less than 1 take towards convergence.



The next figure of the distribution of factors of safety clearly shows the bimodal distribution of critical factors of safety generated by the search.



The genetic algorithm operates by using the Morgenstern Price algorithm to calculate factors of safety. The factor of safety calculated for the critical slip by the Morgenstern Price solver is compared to the Rigid finite element algorithm. Results found in the following table.

Type	Morgenstern Price	RFEM	relative error
Accurate	1.0217	1.1683	14.3456
Accurate	1.0345	1.2078	16.7444
Accurate	1.0617	1.1922	12.2930
Accurate	1.0205	1.1641	14.0712
Accurate	1.0353	1.1929	15.2284
Accurate	1.0209	1.1672	14.3306
Accurate	1.0097	1.1520	14.0884
Accurate	1.0248	1.1572	12.9240
Error	0.5428	1.3583	150.2475
Error	0.5173	1.3861	167.9533

These results show disagreement between the Morgenstern Price and RFEM algorithms, specifically towards the slip surfaces that produce factors of safety less than 1 in the Morgenstern Price algorithm. This could suggest that slip surfaces in with the sharp angles shape seen are difficult for the Morgenstern Price algorithm to calculate accurately.

Next the sharp angle seen produced by the failure cases is studied by measuring the angle of the final rise of the slip surfaces. Results in the table below show that the $FS < 1$ cases have a significantly sharper exit angle. This continues to suggest that the shape of the failure slips may be the performance issues.

Type	exit angle	Kin Pass	Failure Code
Accurate	2.5570	1	
Accurate	2.4633	1	
Accurate	2.3903	1	
Accurate	2.6349	1	
Accurate	2.4936	1	
Accurate	2.5700	1	
Accurate	2.5328	1	
Accurate	2.4775	1	
Error	2.1022	1	
Error	2.0672	1	

The average rise angle of the slopes with factors of safety greater than 1 is 2.515 rads, while the average rise angle of slopes with factors of safety less than 1 is 2.085 rads.

Using a special test case, where the minimum allowable angle was raised (to 2.3 rads, 130 deg) the occurrence rate of low factor of safety results produced by the genetic algorithm is seen to drop. This could be a case that the shape of the failure surface is the performance issue, but may also simply be masking a different performance issue. This change produced no noticeable affect on the results of the other genetic algorithm calculations performed in ???. A case could be made for raising the minimum allowable angle, but a wider range of test cases would have to be studied before making this decision.