Software Requirements Specification for SSP: Slope Stability Analysis Program

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1 Revision History

Date	Version	Notes
09/24/18	1.0	Removed RFEM
09/25/18	1.1	Traceability matrix work
09/26/18	1.2	Physical System Description expanded, Non-functional requirements itemized
10/01/18	1.3	Various improvements throughout
10/02/18	1.4	Initial revision of the solution characteristics specification
10/03/18	1.5	Completed revision of the solution characteristics specification and other sections
10/04/18	1.6	Minor fixes throughout
10/12/18	1.7	Minor fixes based on feedback
10/17/18	1.8	More fixes based on feedback

2 Reference Material

This section records information for easy reference.

2.1 Table of Units

The unit system used throughout is SI (Système International d'Unités). In addition to the basic units, several derived units are also used. For each unit, the table lists the symbol, a description and the SI name.

Symbol	**Unit**	**SI**
N	force	newton
m	length	meter
$\mathrm{Pa} = \mathrm{N\,m^{-2}}$	pressure	pascal
$^\circ$	angle	degree

2.2 Table of Symbols

The table that follows summarizes the symbols used in this document along with their units.

Symbol	**Unit**	**Description**
b	m	width of the base of a slice in the x direction
const_f		boolean decision on which form of f the user desires: constant if true, or half-sine if false
c'	Pa	effective cohesion
$C_{\text{num},i}$	N	expression used to calculate the numerator of the interslice normal to shear force proportionality constant
$C_{\text{den},i}$	N	expression used to calculate the denominator of the interslice normal to shear force proportionality constant
F_x	N	x-component of force
F_y	N	y-component of force
f		function describing variation of the interslice normal to shear force ratio; can be constant or a half-sine
FS		factor of safety
FS_{Min}		minimum factor of safety associated with the critical slip surface
G	$\mathrm{N\,m^{-1}}$	interslice normal force

H	$\mathrm{N\,m^{-1}}$	interslice water force
h	m	height in the y-direction from the base of a slice to the slope surface, at the x-direction midpoint on the slice
i		index representing a single slice
K_c		horizontal seismic coefficient
M	N m	moment
N	$\mathrm{N\,m^{-1}}$	normal force
N'	$\mathrm{N\,m^{-1}}$	effective normal force
N^*	$\mathrm{N\,m^{-1}}$	effective normal force without the influence of interslice forces
n		the total number of slices
P	$\mathrm{N\,m^{-1}}$	resistive shear force
Q	$\mathrm{N\,m^{-1}}$	imposed surface load or external force
R	$\mathrm{N\,m^{-1}}$	resistive shear force without the influence of interslice forces
S	$\mathrm{N\,m^{-1}}$	mobilized shear force
T	$\mathrm{N\,m^{-1}}$	mobilized shear force without the influence of interslice forces
U_b	$\mathrm{N\,m^{-1}}$	base hydrostatic force
U_t	$\mathrm{N\,m^{-1}}$	surface hydrostatic force
W	$\mathrm{N\,m^{-1}}$	self-weight
X	$\mathrm{N\,m^{-1}}$	interslice shear force
x	m	x-ordinate in the Cartesian coordinate system
x_{cs}	m	x-ordinate of a point on the critical slip surface
x_{slip}	m	x-ordinate of a point on a slip surface
x_{slip}^{maxEnd}	m	maximum potential x-ordinate of the ending point of a slip surface
$x_{slip}^{maxStart}$	m	maximum potential x-ordinate of the starting point of a slip surface
x_{slip}^{minEnd}	m	minimum potential x-ordinate of the ending point of a slip surface

$x_{slip}^{minStart}$	m	minimum potential x-ordinate of the starting point of a slip surface
x_{us}	m	x-ordinate of a point on the slope
x_{wt}	m	x-ordinate of a point on the water table
y	m	y-ordinate in the Cartesian coordinate system
y_{cs}	m	y-ordinate of a point on the critical slip surface
y_{slip}	m	y-ordinate of a point on a slip surface
y_{slip}^{max}	m	minimum potential y-ordinate of the of a point on a slip surface
y_{slip}^{min}	m	maximum potential y-ordinate of the of a point on a slip surface
y_{us}	m	y-ordinate of a point on the slope
y_{wt}	m	y-ordinate of a point on the water table
z	m	height in the y-direction from the base of a slice to the center of the slice
z_w	(m)	height in the y-direction from the base of a slice halfway to the water table
α	$^\circ$	angle between the base of a slice and the horizontal
β	$^\circ$	angle between the surface of a slice and the horizontal
γ	$\mathrm{N\,m^{-3}}$	soil dry unit weight
γ_{Sat}	$\mathrm{N\,m^{-3}}$	soil saturated unit weight
γ_w	$\mathrm{N\,m^{-3}}$	unit weight of water
λ		proportionality constant for the interslice normal to shear force ratio
μ	Pa	pore pressure from water within the soil
σ	Pa	the total stress a soil mass needs to maintain itself as a rigid collection of particles
σ_N	Pa	normal stress
σ'	Pa	effective stress provided by the soil skeleton
τ	Pa	shear strength

Υ		generic minimization function or algorithm
φ'	$^\circ$	effective angle of friction
Φ		first function used to convert shear without the influence of interslice forces to shear with the influence of interslice forces
Ψ		second function used to convert shear without the influence of interslice forces to shear with the influence of interslice forces
ω	$^\circ$	angle between the imposed surface load acting into the surface and the vertical
ℓ_b	m	base length of a slice in the direction parallel to the slope of the base
ℓ_s	m	surface length of a slice in the direction parallel to the slope of the surface

2.3 Abbreviations and Acronyms

Symbol	**Description**
2D	Two-Dimensional
A	Assumption
DD	Data Definition
GD	General Definition
GS	Goal Statement
IM	Instance Model
LC	Likely Change
NFR	Non-Functional Requirement
PS	Physical System Description
R	Requirement
SRS	Software Requirements Specification
SSP	Slope Stability Analysis Program
T	Theoretical Model
TU	Typical Uncertainty
UC	Unlikely Change

Contents

3 Introduction

A slope of geological mass, composed of soil and rock and sometimes water, is subject to the influence of gravity on the mass. This can cause instability in the form of soil or rock movement. The effects of soil or rock movement can range from inconvenient to seriously hazardous, resulting in significant life and economic losses. Slope stability is of interest both when analysing natural slopes, and when designing an excavated slope. Slope stability analysis is the assessment of the safety of a slope, identifying the surface most likely to experience slip and an index of its relative stability known as the factor of safety.

The following section provides an overview of the Software Requirements Specification (SRS) for a slope stability analysis problem. The developed program will be referred to as the Slope Stability Analysis Program (SSP). This section explains the purpose of this document, the scope of the system, the characteristics of the intended readers, and the organization of the document.

3.1 Purpose of Document

The primary purpose of this document is to record the requirements of SSP and the models that will be used to meet those requirements. Goals, assumptions, theoretical models, definitions, and other model derivation information are specified, allowing the reader to fully understand and verify the purpose and scientific basis of SSP. With the exception of system constraints in Section 4.3, this SRS will remain abstract, describing *what* problem is being solved, but not *how* to solve it.

This document will be used as a starting point for subsequent development phases, including writing the design specification and the software verification and validation plan. The design document will show how the requirements are to be realized, including decisions on the numerical algorithms and programming environment. The verification and validation plan will show the steps that will be used to increase confidence in the software documentation and the implementation. Although the SRS fits in a series of documents that follow the so-called waterfall model, the actual development process is not constrained in any way. Even when the waterfall model is not followed, as Parnas and Clements (February 1986) point out, the most logical way to present the documentation is still to “fake” a rational design process.

3.2 Scope of Requirements

The scope of the requirements includes stability analysis of a 2-dimensional slope, composed of homogeneous soil layers. The analysis will be at an instant in time; factors that may change the slope properties over time will not be considered.

3.3 Characteristics of Intended Reader

Reviewers of this documentation should have an understanding of undergraduate Level 4 physics and should have completed a second year or higher level undergraduate course in solid mechanics. The users of SSP can have a lower level of expertise, as explained in Section 4.2.

3.4 Organization of Document

The organization of this document follows the template for an SRS for scientific computing software proposed by Koothoor (2013) and Smith and Lai (2005). The presentation follows the standard pattern of presenting goals, theories, definitions, and assumptions. For readers that would like a more bottom up approach, they can start reading the instance models in Section 5.2.5 and trace back to find any additional information they require. The goal statements (Section 5.1.3) are refined to the theoretical models, and the theoretical models (Section 5.2.2) to the instance models (Section 5.2.5). The instance models provide the set of algebraic equations that must be solved.

4 General System Description

This section provides general information about the system. It identifies the interfaces between the system and its environment, describes the user characteristics, and lists the system constraints.

4.1 System Context

Figure 1 shows the system context. A circle represents an external entity outside the software. A rectangle represents the software system itself (SSP). Arrows are used to show the data flow between the system and its environment.

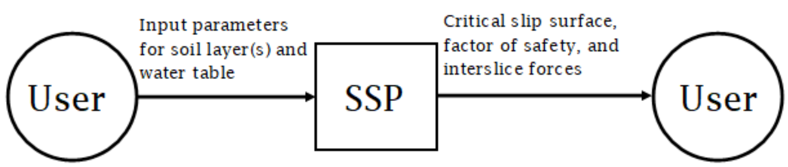

Figure 1: System Context

The responsibilities of the user and the system are as follows:

- User Responsibilities:
  - Provide the input data related to the soil layer(s) and water table (if applicable), ensuring conformation to input data format required by SSP
  - Ensure that consistent units are used for input variables
  - Ensure required software assumptions (Section 5.2.1) are appropriate for the problem to which the user is applying the software
- SSP Responsibilities:
  - Detect data type mismatch, such as a string of characters input instead of a floating point number
  - Verify that the inputs satisfy the required physical constraints and other data constraints (Section 5.2.6)
  - Identify the critical slip surface within the possible input range
  - Find the factor of safety for the slope
  - Find the interslice normal and shear forces along the critical slip surface

4.2 User Characteristics

The end user of SSP should have an understanding of undergraduate Level 1 Calculus and Physics, and be familiar with soil and material properties, specifically cohesion, effective angle of friction, and unit weight.

4.3 System Constraints

The Morgenstern-Price method, which involves dividing the slope into vertical slices, will be used to derive the equations for analysing the slope.

5 Specific System Description

This section first presents the problem description, which gives a high-level view of the problem to be solved. This is followed by the solution characteristics specification, which presents the assumptions, theories, definitions and finally the instance models.

5.1 Problem Description

SSP is a computer program developed to evaluate the factors of safety for a slope's slip surfaces and identify the critical slip surface of the slope, as well as the interslice normal and shear forces along the critical slip surface. It is intended to be used as an educational tool for introducing slope stability issues, and to facilitate the analysis and design of a safe slope.

5.1.1 Terminology and Definitions

This subsection provides a list of terms that are used in the subsequent sections and their meaning, with the purpose of reducing ambiguity and making it easier to correctly understand the requirements.

- *Factor of safety:* The global stability metric of a slip surface of a slope.
- *Slip surface:* A surface within a slope that has the potential to fail or displace due to load or other forces.
- *Critical slip surface:* Slip surface of the slope that has the lowest factor of safety, and is therefore most likely to experience failure.
- *Water table:* The upper boundary of a saturated zone in the ground.
- *Stress:* Force applied over an area.
- *Strain:* A measure of deformation of a body or plane under stress.
- *Normal force:* A force applied perpendicular to the plane of the material.
- *Shear force:* A force applied parallel to the plane of the material.
- *Resistive shear force:* Shear force in the direction opposite of the direction of potential motion, thus hindering motion along the plane.

- *Mobile shear force:* Shear force in the direction of potential motion, thus encouraging motion along the plane.
- *Cohesion:* An attractive force between adjacent particles that holds the matter together.
- *Isotropic:* A condition where the value of a property is independent of the direction in which it is measured.
- *Plane strain:* A condition where the resultant stresses in one of the directions of a 3-dimensional material can be approximated as zero. Results when the length of one dimension of the body dominates the others. Stresses in the direction of the dominant dimension can be approximated as zero.

5.1.2 Physical System Description

The physical system of SSP, as shown in Figure 2, includes the following elements:

PS1: A slope comprised of one or more layers of soil.

PS2: A water table within the soil layers, which may or may not exist.

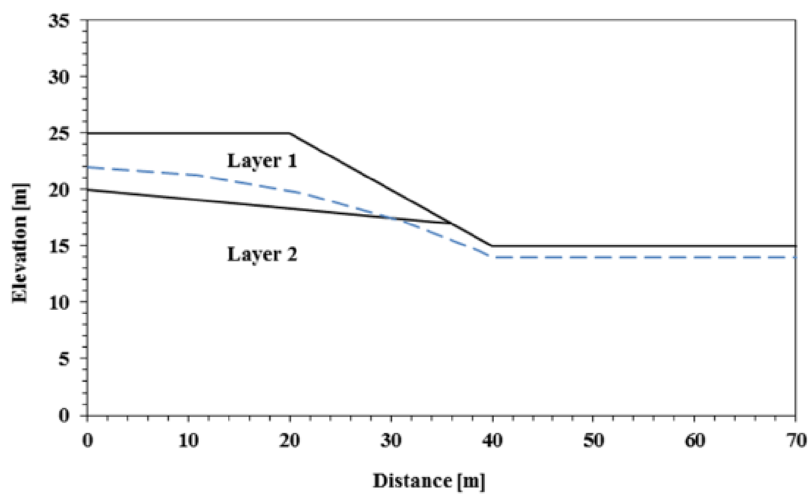

Figure 2: An example slope for analysis by SSP

Morgenstern-Price analysis of the slope involves representing the slope as a series of vertical slices. As shown in Figure 3, the index i is used to denote a value for a single slice, and an interslice value at a given index i refers to the value between slice i and adjacent slice i+*1*.

A free body diagram of the forces acting on a slice is displayed in Figure 4.

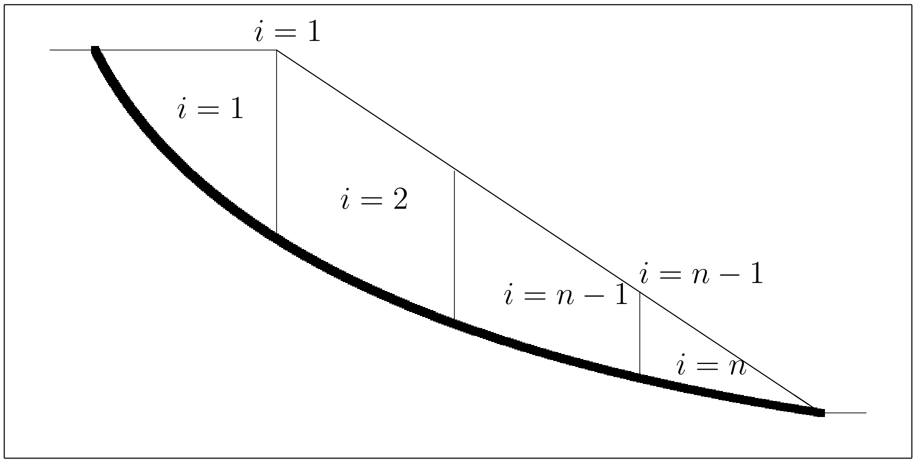

Figure 3: Index convention for slice and interslice values

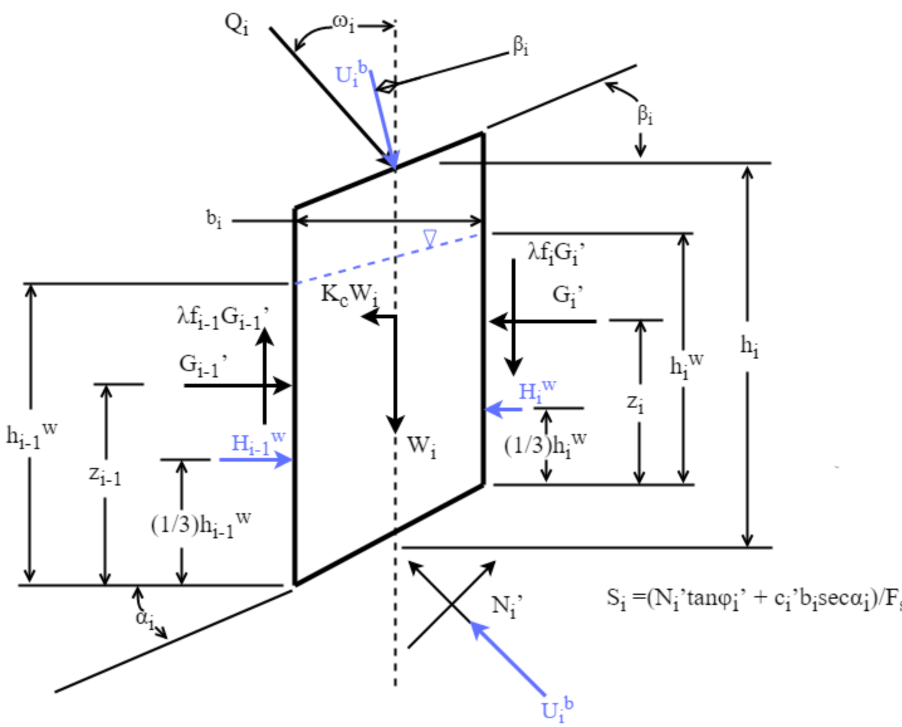

Figure 4: Free body diagram of forces acting on a slice

5.1.3 Goal statements

Given the geometry of the soil layers and water table composing the plane of a slope and the material properties of the layers, the goal statements are:

GS1: Evaluate the factors of safety for possible slip surfaces along the slope.

GS2: Identify the critical slip surface for the slope, with the lowest factor of safety.

GS3: Determine the interslice normal force between each pair of vertical slices of the slope.

GS4: Determine the interslice shear force between each pair of vertical slices of the slope.

5.2 Solution Characteristics Specification

The instance models that govern SSP are presented in Section 5.2.5. The information to understand the meaning of the instance models and their derivation is also presented, so that the instance models can be verified.

5.2.1 Assumptions

This section simplifies the original problem and helps in developing the theoretical models by filling in the missing information for the physical system. The numbers given in the square brackets refer to the theoretical model [T], general definition [GD], data definition [DD], instance model [IM], or likely change [LC], in which the respective assumption is used.

A1: The slip surface is concave with respect to the slope surface. The (x_{slip}, y_{slip}) coordinates of a slip surface follow a concave up function. [IM4]

A2: The factor of safety is assumed to be constant across a whole slip surface. [GD4, IM1, IM3]

A3: The different layers of the soil are homogeneous, with consistent soil properties throughout. [GD3, GD4, GD6, GD7, LC1]

A4: The soil properties are independent of dry or saturated conditions, with the exception of unit weight. [GD3, GD4, GD6, GD7]

A5: Soil layers are treated as if they have isotropic properties. [GD3, GD4, GD6, GD7]

A6: Interslice normal and shear forces have a proportional relationship, depending on a proportionality constant (λ) and an function (f) describing variation depending on x position. [GD8, IM1, IM2, IM3]

A7: The slope and slip surface extends far into and out of the geometry (z-coordinate). This implies plane strain conditions, making 2D analysis appropriate. [T2]

A8: The effective normal stress is large enough that the resistive shear to effective normal stress relationship can be approximated as a linear relationship. [T3, GD5]

A9: The surface and base of a slice are approximated as straight lines [DD2, DD3, DD5, DD6, DD8, DD9].

A10: The interslice forces at the 0th and nth interslice interfaces are zero. [IM1, IM2, IM3].

A11: There is no seismic force acting on the slope. [GD6, GD7, IM1, IM2, IM3, LC2]

A12: There is no imposed surface load, and therefore no external force, acting on the slope. [GD6, GD7, IM1, IM2, IM3, LC3]

5.2.2 Theoretical Models

This section focuses on the general equations and laws that SSP is based on.

Number	T1
Label	**Factor of Safety**
Equation	$\mathrm{FS} = \frac{P}{S}$
Description	FS is the factor of safety, or stability metric of the slope. S is the mobile shear force ($\mathrm{N\,m^{-1}}$). P is the resistive shear force ($\mathrm{N\,m^{-1}}$).
Source	Fredlund and J.Krahn (4 April 1977)
Ref. By	IM1, GD4

Number	T2
Label	**Static Equilibrium**
Equation	$\sum F_{\mathrm{x}} = \sum F_{\mathrm{y}} = \sum M = 0$
Description	For a body in static equilibrium the net forces and net moments acting on the body will cancel out. This equation assumes a 2D space (A7). F_x is the x-component of the net force (N). F_y is the y-component of the net force (N). M is the net moment ($\mathrm{N\,m}$).
Source	Fredlund and J.Krahn (4 April 1977)
Ref. By	GD1, GD2, GD9, IM2

Number	T3
Label	**Mohr-Coulomb Shear Strength**
Equation	$\tau = \sigma_N \cdot \tan(\varphi') + c'$
Description	The τ versus σ_N relationship is not truly linear, but assuming the effective normal force is strong enough, it can be approximated with a linear fit (A8), where the cohesion c' represents the τ intercept of the fitted line. τ is the shear strength (Pa). σ_N is the normal stress (Pa). φ is the effective angle of friction (°). c' is the effective cohesion (Pa).
Source	Fredlund and J.Krahn (4 April 1977)
Ref. By	GD3, GD4, GD6, GD7

Number	T4
Label	**Effective Stress**
Equation	$\sigma' = \sigma - \mu$
Description	σ is the total stress a soil mass needs to maintain itself as a rigid collection of particles (Pa). σ' is the effective stress provided by the soil skeleton (Pa). μ is the pore pressure from water within the soil (Pa).
Source	Fredlund and J.Krahn (4 April 1977)
Ref. By	GD5, GD6, IM1, IM2, IM3

5.2.3 General Definitions

This section collects the laws and equations that will be used to build the instance models.

Number	GD1
Label	**Normal Force Equilibrium**
SI Units	$\mathrm{N\,m^{-1}}$
Equation	$N_{\mathrm{i}} = [W_{\mathrm{i}} - X_{\text{i-1}} + X_{\mathrm{i}} + U_{\mathrm{t,i}} \cos(\beta_{\mathrm{i}}) + Q_{\mathrm{i}} \cos(\omega_{\mathrm{i}})] \cos(\alpha_{\mathrm{i}}) + [-K_{\mathrm{c}}\, W_{\mathrm{i}} - G_{\mathrm{i}} + G_{\text{i-1}} - H_{\mathrm{i}} + H_{\text{i-1}} + U_{\mathrm{t,i}} \sin(\beta_{\mathrm{i}}) + Q_{\mathrm{i}} \sin(\omega_{\mathrm{i}})] \sin(\alpha_{\mathrm{i}})$
Description	This equation satisfies T2 in the normal direction. Force equilibrium is derived from the free body diagram of Figure 4 in section 5.1.2. i is the index representing a single slice. N is the normal force ($\mathrm{N\,m^{-1}}$). W is the weight ($\mathrm{N\,m^{-1}}$). X is the interslice shear force ($\mathrm{N\,m^{-1}}$). U_t is the surface hydrostatic force ($\mathrm{N\,m^{-1}}$). β is the angle between the surface of a slice and the horizontal ($^\circ$). Q is the external force ($\mathrm{N\,m^{-1}}$). ω is the angle between the imposed surface load acting into the surface and the vertical ($^\circ$). α is the angle between the base of a slice and the horizontal ($^\circ$). K_c is the seismic coefficient. G is the interslice normal force ($\mathrm{N\,m^{-1}}$). H is the interslice water force ($\mathrm{N\,m^{-1}}$).
Source	Zhu et al. (19 February 2005)
Ref. By	GD6, GD7, IM3

Number	GD2
Label	**Base Shear Force Equilibrium**
SI Units	$\mathrm{N\,m^{-1}}$
Equation	$S_{\mathrm{i}} = \begin{array}{l}[W_{\mathrm{i}} - X_{\mathrm{i\text{-}1}} + X_{\mathrm{i}} + U_{\mathrm{t,i}}\cos(\beta_{\mathrm{i}}) + Q_{\mathrm{i}}\cos(\omega_{\mathrm{i}})]\sin(\alpha_{\mathrm{i}}) \\ -[-K_{\mathrm{c}}\,W_{\mathrm{i}} - G_{\mathrm{i}} + G_{\mathrm{i\text{-}1}} - H_{\mathrm{i}} + H_{\mathrm{i\text{-}1}} + U_{\mathrm{t,i}}\sin(\beta_{\mathrm{i}}) + Q_{\mathrm{i}}\cos(\omega_{\mathrm{i}})]\cos(\alpha_{\mathrm{i}})\end{array}$
Description	This equation satisfies T2 in the shear direction. Force equilibrium is derived from the free body diagram of Figure 4 in section 5.1.2. i is the index representing a single slice. S is the mobile shear force ($\mathrm{N\,m^{-1}}$). W is the weight ($\mathrm{N\,m^{-1}}$). X is the interslice shear force ($\mathrm{N\,m^{-1}}$). U_t is the surface hydrostatic force ($\mathrm{N\,m^{-1}}$). β is the angle between the surface of a slice and the horizontal (°). Q is the external force ($\mathrm{N\,m^{-1}}$). ω is the angle between the imposed surface load acting into the surface and the vertical (°). α is the angle between the base of a slice and the horizontal (°). K_c is the seismic coefficient. G is the interslice normal force ($\mathrm{N\,m^{-1}}$). H is the interslice water force ($\mathrm{N\,m^{-1}}$).
Source	Zhu et al. (19 February 2005)
Ref. By	GD6, GD7,IM3

Number	GD3
Label	**Resistive Shear Force**
SI Units	$\mathrm{N\,m^{-1}}$
Equation	$P_{\mathrm{i}} = N'_{\mathrm{i}} \cdot \tan\left(\varphi'_{\mathrm{i}}\right) + c' \cdot \ell_{b,i}$
Description	The Mohr-Coulomb resistive shear strength from T3 implemented with forces. i is the index representing a single slice. P is the resistive shear force ($\mathrm{N\,m^{-1}}$). N' is the effective normal force ($\mathrm{N\,m^{-1}}$). φ' is the effective angle of friction ($^{\circ}$). c' is the effective cohesion (Pa). ℓ_b is the width of the base of a slice in the x direction (m).
Source	Zhu et al. (19 February 2005)
Ref. By	GD4, GD6, GD7

Number	GD4
Label	**Mobile Shear Force**
SI Units	$\mathrm{N\,m^{-1}}$
Equation	$S_{\mathrm{i}} = \frac{P_{\mathrm{i}}}{\mathrm{FS}} = \frac{N'_{\mathrm{i}} \cdot \tan\left(\varphi'_{\mathrm{i}}\right) + c' \cdot \ell_{b,i}}{\mathrm{FS}}$
Description	Mobile shear force as derived from the definition of the factor of safety in T1, and the definition of P in GD3. i is the index representing a single slice. S is the mobile shear force ($\mathrm{N\,m^{-1}}$). P is the resistive shear force ($\mathrm{N\,m^{-1}}$). N' is the effective normal force ($\mathrm{N\,m^{-1}}$). φ' is the effective angle of friction ($^{\circ}$). c' is the effective cohesion (Pa). ℓ_b is the width of the base of a slice in the x direction (m). FS is the factor of safety.
Source	Zhu et al. (19 February 2005)
Ref. By	GD6, IM3

Number	GD5
Label	**Effective Normal Force**
SI Units	$\mathrm{N\,m^{-1}}$
Equation	$N'_i = N_i - U_{b,i}$
Description	Effective normal force as derived from T4 and implemented with forces. i is the index representing a single slice. N' is the effective normal force ($\mathrm{N\,m^{-1}}$). N is the normal force ($\mathrm{N\,m^{-1}}$). U_b is the base hydrostatic force ($\mathrm{N\,m^{-1}}$).
Source	Zhu et al. (19 February 2005)
Ref. By	GD6

Number	GD6
Label	**Resistive Shear, Without Interslice Normal and Shear Forces**
Equation	$R_i = \begin{pmatrix} [W_{\text{i}} + U_{\text{t,i}} \cos(\beta_{\text{i}})] \cos(\alpha_{\text{i}}) \\ + [-H_{\text{i}} + H_{\text{i-1}} + U_{\text{t,i}} \sin(\beta_{\text{i}})] \sin(\alpha_{\text{i}}) - U_{\text{b,i}} \end{pmatrix} \cdot \tan(\varphi') + c'_{\text{i}} \cdot b_{\text{i}} \cdot \sec(\alpha_{\text{i}})$
Description	i is the index representing a single slice. R is the resistive shear force without the influence of interslice forces ($\text{N}\,\text{m}^{-1}$). W is the weight ($\text{N}\,\text{m}^{-1}$). U_t is the surface hydrostatic force ($\text{N}\,\text{m}^{-1}$). β is the angle between the surface of a slice and the horizontal (°). α is the angle between the base of a slice and the horizontal (°). H is the interslice water force ($\text{N}\,\text{m}^{-1}$). U_b is the base hydrostatic force ($\text{N}\,\text{m}^{-1}$). φ' is the effective angle of friction (°). c' is the effective cohesion (Pa). b is the width of the base of a slice in the x direction (m). This equation for R arises as part of the derivation for IM1, so that derivation should be consulted for information relating to the derivation of R.
Sources	Zhu et al. (19 February 2005), Karchewski et al. (2012)
Ref. By	IM1, IM3

Number	GD7
Label	**Mobile Shear, Without Interslice Normal and Shear Forces**
Equation	$T_i = (W_i + U_{t,i} \cos(\beta_i)) \sin(\alpha_i) - (-H_i + H_{i-1} + U_{t,i} \sin(\beta_i)) \cos(\alpha_i)$
Description	i is the index representing a single slice. T is the mobilized shear force without the influence of interslice forces ($\mathrm{N\,m^{-1}}$). W is the weight ($\mathrm{N\,m^{-1}}$). U_t is the surface hydrostatic force ($\mathrm{N\,m^{-1}}$). β is the angle between the surface of a slice and the horizontal (°). α is the angle between the base of a slice and the horizontal (°). H is the interslice water force ($\mathrm{N\,m^{-1}}$). This equation for T arises as part of the derivation for IM1, so that derivation should be consulted for information relating to the derivation of T.
Sources	Zhu et al. (19 February 2005), Karchewski et al. (2012)
Ref. By	IM1, IM3

Number	GD8
Label	**Interslice Normal and Shear Force Proportionality**
Equation	$X = \lambda \cdot f \cdot G$
Description	Mathematical representation of the primary assumption for the Morgenstern-Price method (A6). X is the interslice shear force ($\mathrm{N\,m^{-1}}$). G is the interslice normal force ($\mathrm{N\,m^{-1}}$). λ is the proportionality constant. f is a function describing variation of the interslice normal to shear force ratio; can be constant or a half-sine.
Source	Zhu et al. (19 February 2005)
Ref. By	IM1, IM2, IM3

Number	GD9
Label	**Moment Equilibrium**
Equation	$0 = \begin{array}{l} -G_{\text{i}}\left[z_{\text{i}} + \frac{b_{\text{i}}}{2}\tan\left(\alpha_{\text{i}}\right)\right] + G_{\text{i-1}}\left[z_{\text{i-1}} - \frac{b_{\text{i}}}{2}\tan\left(\alpha_{\text{i}}\right)\right] - H_{\text{i}}\left[z_{\text{w,i}} + \frac{b_{\text{i}}}{2}\tan\left(\alpha_{\text{i}}\right)\right] \\ +H_{\text{i-1}}\left[z_{\text{w,i-1}} - \frac{b_{\text{i}}}{2}\tan\left(\alpha_{\text{i}}\right)\right] + \frac{b_{\text{i}}}{2}\left(X_{\text{i}} + X_{\text{i-1}}\right) - K_{\text{c}}W_{\text{i}}\frac{h_{\text{i}}}{2} + U_{\text{t,i}}\sin\left(\beta_{\text{i}}\right)h_{\text{i}} \\ +Q_{\text{i}}\sin\left(\omega_{\text{i}}\right)h_{\text{i}} \end{array}$
Description	This equation satisfies T2 for the net moment. Force equilibrium is derived from the free body diagram of Figure 4 in section 5.1.2. i is the index representing a single slice. G is the interslice normal force ($\mathrm{N\,m^{-1}}$). z is the height in the y-direction from the base of a slice to the center of the slice (m). b is the width of the base of a slice in the x direction (m). α is the angle between the base of a slice and the horizontal (°). H is the interslice water force ($\mathrm{N\,m^{-1}}$). z_w is the height in the y-direction from the base of a slice halfway to the water table (m). X is the interslice shear force ($\mathrm{N\,m^{-1}}$). K_c is the seismic coefficient. W is the weight ($\mathrm{N\,m^{-1}}$). h is the height in the y-direction from the base of a slice to the slope surface, at the x-direction midpoint on the slice (m). U_t is the surface hydrostatic force ($\mathrm{N\,m^{-1}}$). β is the angle between the surface of a slice and the horizontal (°). Q is the external force ($\mathrm{N\,m^{-1}}$). ω is the angle between the imposed surface load acting into the surface and the vertical (°).
Source	Zhu et al. (19 February 2005)
Ref. By	IM2

5.2.4 Data Definition

This section collects and defines all the data needed to support the general definitions of 5.2.3 or build the instance models of 5.2.5. The dimension of each quantity is also given.

Number	DD1
Label	**Weight**
Symbol	W
SI Units	$\mathrm{N\,m^{-1}}$
Equation	$W_i = b_i \begin{cases} (y_{us,i} - y_{slip,i})\,\gamma_{Sat}, & y_{wt,i} \geq y_{us,i} \\ (y_{us,i} - y_{wt,i})\,\gamma + (y_{wt,i} - y_{slip,i})\,\gamma_{Sat}, & y_{us,i} > y_{wt,i} > y_{slip,i} \\ (y_{us,i} - y_{slip,i})\,\gamma, & y_{wt,i} \leq y_{slip,i} \end{cases}$
Description	i is the index representing a single slice. W is the weight ($\mathrm{N\,m^{-1}}$). b is the width of the base of a slice in the x direction (m). y_{us} is the y-ordinate of a point on the slope (m). y_{slip} is the y-ordinate of a point on a slip surface (m). γ_{Sat} is the soil saturated unit weight ($\mathrm{N\,m^{-3}}$). y_{wt} is the y-ordinate of a point on the water table (m). γ is the soil dry unit weight ($\mathrm{N\,m^{-3}}$).
Sources	Fredlund and J.Krahn (4 April 1977)
Ref. By	GD1, GD2, GD9, GD6, GD7, IM1, IM2, IM3

Number	DD2
Label	**Base Water Force**
Symbol	U_b
SI Units	$\mathrm{N\,m^{-1}}$
Equation	$U_{b,i} = \ell_{b,i} \begin{cases} \left(y_{wt,i} - y_{slip,i}\right) \gamma_w, & y_{wt,i} > y_{slip,i} \\ 0, & y_{wt,i} \leq y_{slip,i} \end{cases}$
Description	i is the index representing a single slice. U_b is the base hydrostatic force ($\mathrm{N\,m^{-1}}$). ℓ_b is the base length of a slice in the direction parallel to the slope of the base (m). y_{wt} is the y-ordinate of a point on the water table (m). y_{slip} is the y-ordinate of a point on a slip surface (m). γ_w is the unit weight of water ($\mathrm{N\,m^{-3}}$).
Sources	Fredlund and J.Krahn (4 April 1977)
Ref. By	GD6, IM1, IM2, IM3

Number	DD3
Label	**Surface Hydrostatic Force**
Symbol	U_t
SI Units	$\mathrm{N\,m^{-1}}$
Equation	$U_{t,i} = \ell_{s,i} \begin{cases} (y_{wt,i} - y_{us,i})\,\gamma_w, & y_{wt,i} > y_{us,i} \\ 0, & y_{wt,i} \leq y_{us,i} \end{cases}$
Description	i is the index representing a single slice. U_t is the surface hydrostatic force ($\mathrm{N\,m^{-1}}$). ℓ_s is the surface length of a slice in the direction parallel to the slope of the surface (m). y_{wt} is the y-ordinate of a point on the water table (m). y_{us} is the y-ordinate of a point on the slope (m). γ_w is the unit weight of water ($\mathrm{N\,m^{-3}}$).
Sources	Fredlund and J.Krahn (4 April 1977)
Ref. By	GD1, GD2, GD9, GD6, GD7, IM1, IM2, IM3

Number	DD4
Label	**Interslice Water Force**
Symbol	H
SI Units	$\mathrm{N\,m^{-1}}$
Equation	$H_i = \begin{cases} \frac{\left[y_{us,i}-y_{slip,i}\right]^2}{2}\gamma_{\mathrm{w}} + \left[y_{wt,i}-y_{us,i}\right]^2\gamma_{\mathrm{w}}, & y_{wt,i} \geq y_{us,i} \\ \frac{\left[y_{wt,i}-y_{slip,i}\right]^2}{2}\gamma_{\mathrm{w}}, & y_{us,i} > y_{wt,i} > y_{slip,i} \\ 0, & y_{wt,i} \leq y_{slip,i} \end{cases}$
Description	i is the index representing a single slice. H is the interslice water force ($\mathrm{N\,m^{-1}}$). y_{us} is the y-ordinate of a point on the slope (m). y_{slip} is the y-ordinate of a point on a slip surface (m). γ_{Sat} is the soil saturated unit weight. ($\mathrm{N\,m^{-3}}$). y_{wt} is the y-ordinate of a point on the water table (m).
Sources	Fredlund and J.Krahn (4 April 1977)
Ref. By	GD1, GD2, GD9, GD6, GD7, IM1, IM2, IM3

Number	DD5
Label	**Base Angle**
Symbol	α
SI Units	$^\circ$
Equation	$\alpha_{\mathrm{i}} = \arctan\left(\frac{y_{\mathrm{slip,i}}-y_{\mathrm{slip,i\text{-}1}}}{x_{\mathrm{slip,i}}-x_{\mathrm{slip,i\text{-}1}}}\right)$
Description	i is the index representing a single slice. α is the angle between the base of a slice and the horizontal ($^\circ$). y_{slip} is the y-ordinate of a point on a slip surface (m). x_{slip} is the x-ordinate of a point on a slip surface (m).
Sources	Fredlund and J.Krahn (4 April 1977)
Ref. By	GD1, GD2, GD9, DD8, GD6, GD7, IM1, IM2, IM3

Number	DD6
Label	**Surface Angle**
Symbol	β
SI Units	$^{\circ}$
Equation	$\beta_{\text{i}} = \arctan\left(\frac{y_{\text{us,i}} - y_{\text{us,i-1}}}{x_{\text{us,i}} - x_{\text{us,i-1}}}\right)$
Description	i is the index representing a single slice. β is the angle between the surface of a slice and the horizontal ($^{\circ}$). y_{us} is the y-ordinate of a point on the slope (m). x_{us} is the x-ordinate of a point on the slope (m).
Sources	Fredlund and J.Krahn (4 April 1977)
Ref. By	GD1, GD2, GD9, DD9, GD6, GD7, IM1, IM2, IM3

Number	DD7
Label	**Base x-Direction Width of a Slice**
Symbol	b
SI Units	m
Equation	$b_i = x_{slip,i} - x_{slip,i-1}$
Description	i is the index representing a single slice. b is the width of the base of a slice in the x direction (m). x_{slip} is the x-ordinate of a point on a slip surface (m).
Sources	Fredlund and J.Krahn (4 April 1977)
Ref. By	GD9, DD1, DD8, DD9, GD6, GD7, IM1, IM2, IM3

Number	DD8
Label	**Total Base Length of a Slice**
Symbol	ℓ_b
SI Units	m
Equation	$\ell_{b,i} = b_i \sec(\alpha_i)$
Description	i is the index representing a single slice. ℓ_b is the base length of a slice in the direction parallel to the slope of the base (m). b is the width of the base of a slice in the x direction (m). α is the angle between the base of a slice and the horizontal (°).
Sources	Fredlund and J.Krahn (4 April 1977)
Ref. By	GD3, GD4, DD2

Number	DD9
Label	**Total Surface Length of a Slice**
Symbol	ℓ_s
SI Units	m
Equation	$\ell_{s,i} = b_i \sec(\beta_i)$
Description	i is the index representing a single slice. ℓ_s is the surface length of a slice in the direction parallel to the slope of the surface (m). b is the width of the base of a slice in the x direction (m). β is the angle between the surface of a slice and the horizontal (°).
Sources	Fredlund and J.Krahn (4 April 1977)
Ref. By	DD3

Number	DD10
Label	**Interslice Normal to Shear Force Ratio Variation Function**
Symbol	f
SI Units	unitless
Equation	$f_i = \begin{cases} 1 & const\_f \\ \sin\left(\pi \frac{x_{\text{slip},i} - x_{\text{slip},0}}{x_{\text{slip},n} - x_{\text{slip},0}}\right) & \neg const\_f \end{cases}$
Description	i is the index representing a single slice. f is a function describing variation of the interslice normal to shear force ratio; can be constant or half-sine. $const\_f$ is a boolean decision on which form of f the user desires: constant if true, or half-sine if false. x_{slip} is the x-ordinate of a point on a slip surface (m).
Sources	Fredlund and J.Krahn (4 April 1977)
Ref. By	GD8, DD11, DD12, IM1, IM2, IM3

Number	DD11
Label	**First Function for Incorporating Interslice Forces into Shear Force**
Symbol	Φ
SI Units	unitless
Equation	$\Phi_{\text{i}} = \left[\lambda \cdot f_{\text{i}} \cos\left(\alpha_{\text{i}}\right) - \sin\left(\alpha_{\text{i}}\right)\right]\left[\tan\left(\varphi'_{\text{i}}\right)\right] - \left[\lambda \cdot f_{\text{i}} \sin\left(\alpha_{\text{i}}\right) + \cos\left(\alpha_{\text{i}}\right)\right]\left(F_{\text{S}}\right)$
Description	i is the index representing a single slice. Φ is the first function used to convert shear without the influence of interslice forces to shear with the influence of interslice forces. λ is the proportionality constant for the interslice normal to shear force ratio. f is a function describing variation of the interslice normal to shear force ratio; can be constant or half-sine. α is the angle between the base of a slice and the horizontal (°). φ' is the effective angle of friction (°). F_{S} is the factor of safety. The equation for Φ arises as part of the derivation for IM1, so that derivation should be consulted for information relating to the derivation of Φ.
Sources	Zhu et al. (19 February 2005), Karchewski et al. (2012)
Ref. By	IM1, IM3

Number	DD12
Label	**Second Function for Incorporating Interslice Forces into Shear Force**
Symbol	Ψ
SI Units	unitless
Equation	$\Psi_{\text{i-1}} = \frac{[\lambda \cdot f_{\text{i-1}} \cos(\alpha_{\text{i}}) - \sin(\alpha_{\text{i}})][\tan(\varphi')] - [\lambda \cdot f_{\text{i-1}} \sin(\alpha_{\text{i}}) + \cos(\alpha_{\text{i}})](F_{\text{S}})}{\Phi_{i-1}}$
Description	i is the index representing a single slice. Ψ is the second function used to convert shear without the influence of interslice forces to shear with the influence of interslice forces. λ is the proportionality constant for the interslice normal to shear force ratio. f is a function describing variation of the interslice normal to shear force ratio; can be constant or half-sine. α is the angle between the base of a slice and the horizontal (°). φ' is the effective angle of friction (°). F_{S} is the factor of safety. Φ is the first function used to convert shear without the influence of interslice forces to shear with the influence of interslice forces. The equation for Ψ arises as part of the derivation for IM1, so that derivation should be consulted for information relating to the derivation of Ψ.
Sources	Zhu et al. (19 February 2005), Karchewski et al. (2012)
Ref. By	IM1, IM3

5.2.5 Instance Models

This section transforms the problem defined in the Section 5.1 into one which is expressed in mathematical terms. It uses concrete symbols defined in Section 5.2.4 to replace the abstract symbols in the models identified in the Sections 5.2.2 and 5.2.3.

The goals GS1, GS3, and GS4 are met by the simultaneous solution of IM1, IM2, and IM3. The goal GS2 is met by IM4.

The Morgenstern-Price Method is a vertical slice, limit equilibrium slope stability analysis method. Analysis is performed by breaking the assumed slip surface into a series of vertical slices of mass. Static equilibrium analysis is performed, using two force equations and one moment equation as in T2. The problem is statically indeterminate with only these 3 equations and one constitutive equation (the Mohr-Coulomb shear strength of T3), so the assumption A6 and corresponding equation GD8 are used. The force equilibrium equations can be modified to be expressed only in terms of known physical values, as done in GD6 and GD7.

Number	IM1
Label	**Factor of Safety**
Input	$\{(x_{\text{us}}, y_{\text{us}})\}$, $\{(x_{\text{wt}}, y_{\text{wt}})\}$, c', φ', γ, γ_{Sat}, γ_{w}, $\{(x_{\text{slip}}, y_{\text{slip}})\}$
Output	$F_{\text{S}} = \dfrac{\sum_{i=1}^{n-1}\left[R_i \prod_{c=i}^{n-1} \Psi_c\right] + R_n}{\sum_{i=1}^{n-1}\left[T_i \prod_{c=i}^{n-1} \Psi_c\right] + T_n}$
Description	i is the index representing a single slice. n is the total number of slices. F_{S} is the factor of safety. R is the resistive shear force without the influence of interslice forces ($\text{N}\,\text{m}^{-1}$), defined in GD6 Ψ is the second function used to convert shear without the influence of interslice forces to shear with the influence of interslice forces, defined in GD12 T is the mobile shear force without the influence of interslice forces ($\text{N}\,\text{m}^{-1}$), defined in GD7
Sources	Zhu et al. (19 February 2005), Karchewski et al. (2012)
Ref. By	IM2, IM3

Factor of Safety Derivation

The mobile shear force defined in GD2 can be substituted into the definition of mobile shear force based on the factor of safety, from GD4, yielding Equation 1 below.

$$\left(\begin{array}{l}[W_{\text{i}} - X_{\text{i-1}} + X_{\text{i}} + U_{\text{t,i}} \cos(\beta_{\text{i}}) + Q_{\text{i}} \cos(\omega_{\text{i}})] \sin(\alpha_{\text{i}}) \\ -[-K_{\text{c}}\, W_{\text{i}} - G_{\text{i}} + G_{\text{i-1}} - H_{\text{i}} + H_{\text{i-1}} + U_{\text{t,i}} \sin(\beta_{\text{i}}) + Q_{\text{i}} \cos(\omega_{\text{i}})] \cos(\alpha_{\text{i}})\end{array}\right) = \frac{N_{\text{i}}' \cdot \tan(\varphi_{\text{i}}') + c' \cdot \ell_{b,i}}{F_{\text{S}}} \tag{1}$$

An expression for the effective normal force, N_i', can be derived by substituting the normal force equilibrium from GD1 into the definition for effective normal force from GD5. This results in Equation 2.

$$\begin{aligned} N_{\text{i}}' = \; & [W_{\text{i}} - X_{\text{i-1}} + X_{\text{i}} + U_{\text{t,i}} \cos(\beta_{\text{i}}) + Q_{\text{i}} \cos(\omega_{\text{i}})] \cos(\alpha_{\text{i}}) \\ & + [-K_{\text{c}}\, W_{\text{i}} - G_{\text{i}} + G_{\text{i-1}} - H_{\text{i}} + H_{\text{i-1}} + U_{\text{t,i}} \sin(\beta_{\text{i}}) + Q_{\text{i}} \sin(\omega_{\text{i}})] \sin(\alpha_{\text{i}}) \\ & - U_{\text{b,i}} \end{aligned} \tag{2}$$

Substituting Equation 2 into Equation 1 gives

$$\frac{\begin{pmatrix} [W_{\text{i}} - X_{\text{i-1}} + X_{\text{i}} + U_{\text{t,i}}\cos(\beta_{\text{i}}) + Q_{\text{i}}\cos(\omega_{\text{i}})]\sin(\alpha_{\text{i}}) \\ -[-K_{\text{c}}\,W_{\text{i}} - G_{\text{i}} + G_{\text{i-1}} - H_{\text{i}} + H_{\text{i-1}} + U_{\text{t,i}}\sin(\beta_{\text{i}}) + Q_{\text{i}}\cos(\omega_{\text{i}})]\cos(\alpha_{\text{i}}) \end{pmatrix} = \begin{pmatrix} [W_{\text{i}} - X_{\text{i-1}} + X_{\text{i}} + U_{\text{t,i}}\cos(\beta_{\text{i}}) + Q_{\text{i}}\cos(\omega_{\text{i}})]\cos(\alpha_{\text{i}}) \\ +[-K_{\text{c}}\,W_{\text{i}} - G_{\text{i}} + G_{\text{i-1}} - H_{\text{i}} + H_{\text{i-1}} + U_{\text{t,i}}\sin(\beta_{\text{i}}) + Q_{\text{i}}\sin(\omega_{\text{i}})]\sin(\alpha_{\text{i}}) - U_{\text{b,i}} \end{pmatrix} \cdot \tan(\varphi'_{\text{i}}) + c' \cdot \ell_{b,i}}{F_{\text{S}}}$$

Since the interslice shear force X and interslice normal force G are unknown, they are separated from the other terms as follows:

$$\frac{\begin{pmatrix} [W_{\text{i}} + U_{\text{t,i}}\cos(\beta_{\text{i}}) + Q_{\text{i}}\cos(\omega_{\text{i}})]\sin(\alpha_{\text{i}}) + (-X_{\text{i-1}} + X_{\text{i}})\sin(\alpha_{\text{i}}) \\ -[-K_{\text{c}}\,W_{\text{i}} - H_{\text{i}} + H_{\text{i-1}} + U_{\text{t,i}}\sin(\beta_{\text{i}}) + Q_{\text{i}}\cos(\omega_{\text{i}})]\cos(\alpha_{\text{i}}) - (-G_{\text{i}} + G_{\text{i-1}})\cos(\alpha_{\text{i}}) \end{pmatrix} = \begin{pmatrix} [W_{\text{i}} + U_{\text{t,i}}\cos(\beta_{\text{i}}) + Q_{\text{i}}\cos(\omega_{\text{i}})]\cos(\alpha_{\text{i}}) + (-X_{\text{i-1}} + X_{\text{i}})\cos(\alpha_{\text{i}}) \\ +[-K_{\text{c}}\,W_{\text{i}} - H_{\text{i}} + H_{\text{i-1}} + U_{\text{t,i}}\sin(\beta_{\text{i}}) + Q_{\text{i}}\sin(\omega_{\text{i}})]\sin(\alpha_{\text{i}}) + (-G_{\text{i}} + G_{\text{i-1}})\sin(\alpha_{\text{i}}) \\ -U_{\text{b,i}} \end{pmatrix} \cdot \tan(\varphi'_{\text{i}}) + c' \cdot \ell_{b,i}}{F_{\text{S}}}$$

Applying assumptions A11 and A12, which state that the seismic coefficient and the external force, respectively, are zero, allows for further simplification as shown below.

$$\frac{\begin{pmatrix} [W_{\text{i}} + U_{\text{t,i}}\cos(\beta_{\text{i}})]\sin(\alpha_{\text{i}}) + (-X_{\text{i-1}} + X_{\text{i}})\sin(\alpha_{\text{i}}) \\ -[-H_{\text{i}} + H_{\text{i-1}} + U_{\text{t,i}}\sin(\beta_{\text{i}})]\cos(\alpha_{\text{i}}) - (-G_{\text{i}} + G_{\text{i-1}})\cos(\alpha_{\text{i}}) \end{pmatrix} = \begin{pmatrix} [W_{\text{i}} + U_{\text{t,i}}\cos(\beta_{\text{i}})]\cos(\alpha_{\text{i}}) + (-X_{\text{i-1}} + X_{\text{i}})\cos(\alpha_{\text{i}}) \\ +[-H_{\text{i}} + H_{\text{i-1}} + U_{\text{t,i}}\sin(\beta_{\text{i}})]\sin(\alpha_{\text{i}}) + (-G_{\text{i}} + G_{\text{i-1}})\sin(\alpha_{\text{i}}) - U_{\text{b,i}} \end{pmatrix} \cdot \tan(\varphi'_{\text{i}}) + c' \cdot \ell_{b,i}}{F_{\text{S}}}$$

The definitions of GD6 and GD7 are present in this equation, and can thus be replaced by R_i and T_i, respectively.

[DDs referenced here will be changed to GDs as discussed —BM]

$$\frac{(T_{\text{i}} + (-X_{\text{i-1}} + X_{\text{i}})\sin(\alpha_{\text{i}}) - (-G_{\text{i}} + G_{\text{i-1}})\cos(\alpha_{\text{i}})) = R_{\text{i}} + ((-X_{\text{i-1}} + X_{\text{i}})\cos(\alpha_{\text{i}}) + (-G_{\text{i}} + G_{\text{i-1}})\sin(\alpha_{\text{i}})) \cdot \tan(\varphi'_{\text{i}}) + c' \cdot \ell_{b,i}}{F_{\text{S}}}$$

The interslice shear force X can be expressed in terms of the interslice normal force G using GD8, resulting in

$$\frac{(T_{\text{i}} + (-\lambda f_{\text{i-1}} G_{\text{i-1}} + \lambda f_{\text{i}} G_{\text{i}})\sin(\alpha_{\text{i}}) - (-G_{\text{i}} + G_{\text{i-1}})\cos(\alpha_{\text{i}})) = R_{\text{i}} + ((-\lambda f_{\text{i-1}} G_{\text{i-1}} + \lambda f_{\text{i}} G_{\text{i}})\cos(\alpha_{\text{i}}) + (-G_{\text{i}} + G_{\text{i-1}})\sin(\alpha_{\text{i}})) \cdot \tan(\varphi'_{\text{i}})}{F_{\text{S}}}$$

Rearranging yields the following:

$$G_{\text{i}} \begin{bmatrix} [\lambda \cdot f_{\text{i}}\cos(\alpha_{\text{i}}) - \sin(\alpha_{\text{i}})]\tan({\varphi'}_{i}) \\ -[\lambda \cdot f_{\text{i}}\sin(\alpha_{\text{i}}) + \cos(\alpha_{\text{i}})](F_{\text{S}}) \end{bmatrix} = G_{\text{i-1}} \begin{bmatrix} [\lambda \cdot f_{\text{i-1}}\cos(\alpha_{\text{i}}) - \sin(\alpha_{\text{i}})]\tan({\varphi'}_{i}) \\ -[\lambda \cdot f_{\text{i-1}}\sin(\alpha_{\text{i}}) + \cos(\alpha_{\text{i}})](F_{\text{S}}) \end{bmatrix} + (F_{\text{S}}) \cdot T_{\text{i}} - R_{\text{i}}$$

The definitions for Φ and Ψ from GD11 and GD12 simplify the above to Equation 3.

$$G_{\text{i}}\Phi_{\text{i}} = \Psi_{\text{i-1}} G_{\text{i-1}} \Phi_{\text{i-1}} + F_{\text{S}} T_{\text{i}} - R_{\text{i}} \tag{3}$$

Versions of Equation 3 instantiated for slices 1 to n are shown below.

$$G_1\Phi_1 = \Psi_0 G_0 \Phi_0 + F_S T_1 - R_1$$

$$G_2\Phi_2 = \Psi_1 G_1 \Phi_1 + F_S T_2 - R_2 \tag{4}$$

$$G_3\Phi_3 = \Psi_2 G_2 \Phi_2 + F_S T_3 - R_3 \tag{5}$$

$$...$$

$$G_{\text{n-2}}\Phi_{\text{n-2}} = \Psi_{\text{n-3}} G_{\text{n-3}} \Phi_{\text{n-3}} + F_S T_{\text{n-2}} - R_{\text{n-2}} \tag{6}$$

$$G_{\text{n-1}}\Phi_{\text{n-1}} = \Psi_{\text{n-2}} G_{\text{n-2}} \Phi_{\text{n-2}} + F_S T_{\text{n-1}} - R_{\text{n-1}} \tag{7}$$

$$G_{\text{n}}\Phi_{\text{n}} = \Psi_{\text{n-1}} G_{\text{n-1}} \Phi_{\text{n-1}} + F_S T_{\text{n}} - R_{\text{n}}$$

Applying A10, which says that G_0 and G_n are zero, results in the following special cases: Equation 8 for the first slice and Equation 9 for the nth slice.

$$G_1\Phi_1 = F_S T_1 - R_1 \tag{8}$$

$$-\frac{F_S T_{\text{n}} - R_{\text{n}}}{\Psi_{\text{n-1}}} = G_{\text{n-1}}\Phi_{\text{n-1}} \tag{9}$$

Substituting Equation 8 into Equation 4 yields Equation 10, which can be substituted into Equation 5 to get Equation 11, and so on until Equation 12 is obtained from Equation 7.

$$G_2\Phi_2 = \Psi_1 \left(F_S T_1 - R_1\right) + F_S T_2 - R_2 \tag{10}$$

$$G_3\Phi_3 = \Psi_2 \left(\Psi_1 \left(F_S T_1 - R_1\right) + F_S T_2 - R_2\right) + F_S T_3 - R_3 \tag{11}$$

$$\ldots$$

$$\begin{aligned} G_{\text{n-1}}\Phi_{\text{n-1}} =& \Psi_{\text{n-2}} \left(\Psi_{\text{n-3}} \left(\ldots \left(\Psi_1 \left(F_S T_1 - R_1\right) + F_S T_2 - R_2\right) \ldots\right) + F_S T_{\text{n-2}} - R_{\text{n-2}}\right) \\ &+ F_S T_{\text{n-1}} - R_{\text{n-1}} \end{aligned} \tag{12}$$

Equation 9 can then be substituted into the left-hand side of Equation 12, resulting in:

$$\begin{aligned} -\frac{F_S T_{\text{n}} - R_{\text{n}}}{\Psi_{\text{n-1}}} =& \Psi_{\text{n-2}} \left(\Psi_{\text{n-3}} \left(\ldots \left(\Psi_1 \left(F_S T_1 - R_1\right) + F_S T_2 - R_2\right) \ldots\right) + F_S T_{\text{n-2}} - R_{\text{n-2}}\right) \\ &+ F_S T_{\text{n-1}} - R_{\text{n-1}} \end{aligned}$$

This can be rearranged by multiplying both sides by $\Psi_{\text{n-1}}$ and then distributing the multiplication of each Ψ over addition to obtain:

$$-\left(F_S T_{\text{n}} - R_{\text{n}}\right) = \Psi_{\text{n-1}}\Psi_{\text{n-2}} \ldots \Psi_1 \left(F_S T_1 - R_1\right) + \Psi_{\text{n-1}}\Psi_{\text{n-2}} \ldots \Psi_2 \left(F_S T_2 - R_2\right) + \cdots + \Psi_{\text{n-1}} \left(F_S T_{\text{n-1}} - R_{\text{n-1}}\right)$$

The multiplication of the Ψ terms can be further distributed over the subtractions, resulting in the equation having terms that each either contain an R or a T. The equation can then be rearranged so terms containing an R are on one side of the equality, and terms containing a T are on the other. The multiplication by the factor of safety is common to all of the T terms, and thus can be factored out, resulting in:

$$F_{\mathrm{S}}\left(\Psi_{\text{n-1}}\Psi_{\text{n-2}}\ldots\Psi_1 T_1+\Psi_{\text{n-1}}\Psi_{\text{n-2}}\ldots\Psi_2 T_2+\ldots\Psi_{\text{n-1}}T_{\text{n-1}}+T_{\text{n}}\right)=$$
$$\Psi_{\text{n-1}}\Psi_{\text{n-2}}\ldots\Psi_1 R_1+\Psi_{\text{n-1}}\Psi_{\text{n-2}}\ldots\Psi_2 R_2+\cdots+\Psi_{\text{n-1}}R_{\text{n-1}}+R_{\text{n}}$$

Isolating the factor of safety on the left-hand side and using compact notation for the products and sums yields Equation 13, which can also be seen in IM1. F_{S} depends on the unknowns λ (IM2) and G (IM3).

$$F_{\mathrm{S}}=\frac{\displaystyle\sum_{i=1}^{n-1}\left[R_i\prod_{c=i}^{n-1}\Psi_c\right]+R_n}{\displaystyle\sum_{i=1}^{n-1}\left[T_i\prod_{c=i}^{n-1}\Psi_c\right]+T_n} \tag{13}$$

Number	IM2
Label	**Normal and Shear Force Proportionality Constant**
Input	$\{(x_{\text{us}}, y_{\text{us}})\}$, $\{(x_{\text{wt}}, y_{\text{wt}})\}$, γ_{w}, $\{(x_{\text{slip}}, y_{\text{slip}})\}$
Output	$C_{\text{num},i} = \begin{cases} b_1 \left[G_1 + H_1\right] \tan\left(\alpha_1\right) & \text{i} = 1 \\ b_{\text{i}} \left[\left(G_{\text{i}} + G_{\text{i-1}}\right) + \left(H_{\text{i}} + H_{\text{i-1}}\right)\right] \tan\left(\alpha_{\text{i}}\right) + h_{\text{i}} \left(-2\, U_{\text{t,i}} \sin\left(\beta_{\text{i}}\right)\right) & 2 \leq \text{i} \leq \text{n-1} \\ b_{\text{n}} \left[G_{\text{n-1}} + H_{\text{n-1}}\right] \tan\left(\alpha_{\text{n-1}}\right) & \text{i} = \text{n} \end{cases}$ $C_{\text{den},i} = \begin{cases} b_1 G_1 f_1 & \text{i} = 1 \\ b_{\text{i}} \left(f_{\text{i}} G_{\text{i}} + f_{\text{i-1}} G_{\text{i-1}}\right) & 2 \leq \text{i} \leq \text{n-1} \\ b_{\text{n}} G_{\text{n-1}} f_{\text{n-1}} & \text{i} = \text{n} \end{cases}$ $\lambda = \frac{\sum_{i=1}^{n} C_{\text{num},i}}{\sum_{i=1}^{n} C_{\text{den},i}}$
Description	i is the index representing a single slice. n is the total number of slices. b is the width of the base of a slice in the x direction (m). G is the interslice normal force ($\text{N}\,\text{m}^{-1}$). H is the interslice water force ($\text{N}\,\text{m}^{-1}$). α is the angle between the base of a slice and the horizontal (°). h is the height in the y-direction from the base of a slice to the slope surface, at the x-direction midpoint on the slice (m). U_t is the surface hydrostatic force ($\text{N}\,\text{m}^{-1}$). β is the angle between the surface of a slice and the horizontal (°). f is a function describing variation of the interslice normal to shear force ratio; can be constant or a half-sine. λ is the proportionality constant.
Sources	Zhu et al. (19 February 2005)
Ref. By	IM1, IM3

Normal/Shear Force Ratio Derivation

From the moment equilibrium of GD9, with the primary assumption for the Morgenstern-Price method of A6 and associated definition GD8, Equation (14) can be derived.

$$0 = \begin{aligned} &-G_{\mathrm{i}}\left[z_{\mathrm{i}} - \tfrac{b_{\mathrm{i}}}{2}\tan\left(\alpha_{\mathrm{i}}\right)\right] + G_{\text{i-1}}\left[z_{\text{i-1}} + \tfrac{b_{\mathrm{i}}}{2}\tan\left(\alpha_{\mathrm{i}}\right)\right] - H_{\mathrm{i}}\left[z_{\mathrm{w,i}} - \tfrac{b_{\mathrm{i}}}{2}\tan\left(\alpha_{\mathrm{i}}\right)\right] \\ &+H_{\text{i-1}}\left[z_{\text{w,i-1}} + \tfrac{b_{\mathrm{i}}}{2}\tan\left(\alpha_{\mathrm{i}}\right)\right] - \lambda\tfrac{b_{\mathrm{i}}}{2}\left(G_{\mathrm{i}}f_{\mathrm{i}} + G_{\text{i-1}}f_{\text{i-1}}\right) + K_{\mathrm{c}}W_{\mathrm{i}}\tfrac{h_{\mathrm{i}}}{2} - U_{\mathrm{t,i}}\sin\left(\beta_{\mathrm{i}}\right)h_{\mathrm{i}} - Q_{\mathrm{i}}\sin\left(\omega_{\mathrm{i}}\right)h_{\mathrm{i}} \end{aligned} \tag{14}$$

Rearranging the equation in terms of λ leads to Equation (15).

$$\lambda = \frac{\begin{aligned} &-G_{\mathrm{i}}\left[z_{\mathrm{i}} - \tfrac{b_{\mathrm{i}}}{2}\tan\left(\alpha_{\mathrm{i}}\right)\right] + G_{\text{i-1}}\left[z_{\text{i-1}} + \tfrac{b_{\mathrm{i}}}{2}\tan\left(\alpha_{\mathrm{i}}\right)\right] - H_{\mathrm{i}}\left[z_{\mathrm{i}} - \tfrac{b_{\mathrm{i}}}{2}\tan\left(\alpha_{\mathrm{i}}\right)\right] \\ &+H_{\text{i-1}}\left[z_{\text{i-1}} + \tfrac{b_{\mathrm{i}}}{2}\tan\left(\alpha_{\mathrm{i}}\right)\right] + K_{\mathrm{c}}W_{\mathrm{i}}\tfrac{h_{\mathrm{i}}}{2} - U_{\mathrm{t,i}}\sin\left(\beta_{\mathrm{i}}\right)h_{\mathrm{i}} - Q_{\mathrm{i}}\sin\left(\omega_{\mathrm{i}}\right)h_{\mathrm{i}} \end{aligned}}{\tfrac{b_{\mathrm{i}}}{2}\left[G_{\mathrm{i}}f_{\mathrm{i}} + G_{\text{i-1}}f_{\text{i-1}}\right]} \tag{15}$$

This equation can be simplified by applying assumptions A11 and A12, which state that the seismic and external forces, respectively are zero.

$$\lambda = \frac{\begin{aligned} &-G_{\mathrm{i}}\left[z_{\mathrm{i}} - \tfrac{b_{\mathrm{i}}}{2}\tan\left(\alpha_{\mathrm{i}}\right)\right] + G_{\text{i-1}}\left[z_{\text{i-1}} + \tfrac{b_{\mathrm{i}}}{2}\tan\left(\alpha_{\mathrm{i}}\right)\right] - H_{\mathrm{i}}\left[z_{\mathrm{i}} - \tfrac{b_{\mathrm{i}}}{2}\tan\left(\alpha_{\mathrm{i}}\right)\right] \\ &+H_{\text{i-1}}\left[z_{\text{i-1}} + \tfrac{b_{\mathrm{i}}}{2}\tan\left(\alpha_{\mathrm{i}}\right)\right] - U_{\mathrm{t,i}}\sin\left(\beta_{\mathrm{i}}\right)h_{\mathrm{i}} \end{aligned}}{\tfrac{b_{\mathrm{i}}}{2}\left[G_{\mathrm{i}}f_{\mathrm{i}} + G_{\text{i-1}}f_{\text{i-1}}\right]}$$

Taking the summation of all slices, and applying A10 to set G_0, G_n, H_0, and H_n equal to zero, a general equation for the constant λ is developed in Equation (16), also found in IM2.

$$\lambda = \frac{\displaystyle\sum_{i=1}^{n} b_{\mathrm{i}}\left[\left(G_{\mathrm{i}} + G_{\text{i-1}}\right) + \left(H_{\mathrm{i}} + H_{\text{i-1}}\right)\right]\tan\left(\alpha_{\mathrm{i}}\right) + h_{\mathrm{i}}\ \left[-2\ U_{\mathrm{t,i}}\sin\left(\beta_{\mathrm{i}}\right)\right]}{\displaystyle\sum_{i=1}^{n} b_{\mathrm{i}}\left[f_{i}G_{\mathrm{i}} + f_{\text{i-1}}G_{\text{i-1}}\right]} \tag{16}$$

Equation (16) for λ, is a function of the unknown interslice normal force, G (IM3), which itself depends on the unknown factor of safety, F_{S} (IM1).

Number	IM3
Label	**Interslice Normal Forces**
Input	FS, T_{i}, R_{i}, Ψ_i, Φ_i
Output	$G_{\text{i}} = \begin{cases} \frac{(\text{FS})T_1 - R_1}{\Phi_{\text{i}}} & \text{i=1} \\ \frac{\Psi_{\text{i-1}} \cdot G_{\text{i-1}} + (\text{FS}) \cdot T_{\text{i}} - R_{\text{i}}}{\Phi_{\text{i}}} & 2 \leq \text{i} \leq \text{n-1} \\ 0 & \text{i=0} \vee \text{i=n} \end{cases}$
Description	i is the index representing a single slice. n is the total number of slices. G is the interslice normal force ($\mathrm{N\,m^{-1}}$). FS is the factor of safety. T is the mobile shear force without the influence of interslice forces ($\mathrm{N\,m^{-1}}$). R is the resistive shear force without the influence of interslice forces ($\mathrm{N\,m^{-1}}$). Φ is the first constant used to convert shear without the influence of interslice forces to shear with the influence of interslice forces. Ψ is the second constant used to convert shear without the influence of interslice forces to shear with the influence of interslice forces.
Sources	Zhu et al. (19 February 2005)
Ref. By	IM1, IM2

Interslice Force Derivation

Substituting the normal force equilibrium of GD1 and the assumption A6 represented by GD8 into the effective normal force definition from GD5 yields Equation (17).

$$N'_{\text{i}} = \begin{aligned} &[W_{\text{i}} - \lambda \cdot f_{\text{i-1}} \cdot G_{\text{i-1}} + \lambda \cdot f_{\text{i}} \cdot G_{\text{i}} + U_{\text{t,i}} \cos(\beta_{\text{i}}) + Q_{\text{i}} \cos(\omega_{\text{i}})] \cos(\alpha_{\text{i}}) \\ &+ [-K_{\text{c}} W_{\text{i}} - G_{\text{i}} + G_{\text{i-1}} - H_{\text{i}} + H_{\text{i-1}} + U_{\text{t,i}} \sin(\beta_{\text{i}}) + Q_{\text{i}} \sin(\omega_{\text{i}})] \sin(\alpha_{\text{i}}) - U_{\text{b,i}} \end{aligned} \tag{17}$$

Next, substituting the base shear force equilibrium of GD2 and the assumption of GD8 into the definition of mobilized shear from GD4 results in Equation (18).

$$\frac{N'_{\text{i}} \tan(\varphi'_i) + c'_{\text{i}} \cdot b'_{\text{i}} \cdot \sec(\alpha_{\text{i}})}{\text{FS}} = \begin{aligned} &[W_{\text{i}} - \lambda \cdot f_{\text{i-1}} \cdot G_{\text{i-1}} + \lambda \cdot f_{\text{i}} \cdot G_{\text{i}} + U_{\text{t,i}} \cos(\beta_{\text{i}}) + Q_{\text{i}} \cos(\omega_{\text{i}})] \sin(\alpha_{\text{i}}) \\ &- [-K_{\text{c}} W_{\text{i}} - G_{\text{i}} + G_{\text{i-1}} - H_{\text{i}} + H_{\text{i-1}} + U_{\text{t,i}} \cdot \sin(\beta_{\text{i}}) + Q_{\text{i}} \sin(\omega_{\text{i}})] \cos(\alpha_{\text{i}}) \end{aligned} \tag{18}$$

Substituting Equation (17), GD6, and GD7 into Equation (18) and rearranging results in Equation (19)

$$G_{\mathrm{i}}\begin{bmatrix}[\lambda\cdot f_{\mathrm{i}}\cos(\alpha_{\mathrm{i}})-\sin(\alpha_{\mathrm{i}})]\tan(\varphi'_{i})\\-[\lambda\cdot f_{\mathrm{i}}\sin(\alpha_{\mathrm{i}})+\cos(\alpha_{\mathrm{i}})]\,(\mathrm{FS})\end{bmatrix}=G_{\mathrm{i\text{-}1}}\begin{bmatrix}[\lambda\cdot f_{\mathrm{i\text{-}1}}\cos(\alpha_{\mathrm{i}})-\sin(\alpha_{\mathrm{i}})]\tan(\varphi'_{i})\\-[\lambda\cdot f_{\mathrm{i\text{-}1}}\sin(\alpha_{\mathrm{i}})+\cos(\alpha_{\mathrm{i}})]\,(\mathrm{FS})\end{bmatrix}+(\mathrm{FS})\cdot T_{\mathrm{i}}-R_{\mathrm{i}} \tag{19}$$

Where R and T are the resistive and mobile shear of the slice, without the influence of interslice forces G and X, as defined in GD6 and GD7. The constants Φ and Ψ are defined below in Equations (20) and (21) respectively.

$$\Phi_{\mathrm{i}}=[\lambda\cdot f_{\mathrm{i}}\cos(\alpha_{\mathrm{i}})-\sin(\alpha_{\mathrm{i}})]\,[\tan(\varphi'_{\mathrm{i}})]-[\lambda\cdot f_{\mathrm{i}}\sin(\alpha_{\mathrm{i}})+\cos(\alpha_{\mathrm{i}})]\,(\mathrm{FS}) \tag{20}$$

$$\Psi_{\mathrm{i}}=[\lambda\cdot f_{\mathrm{i}}\cos(\alpha_{\mathrm{i+1}})-\sin(\alpha_{\mathrm{i+1}})]\,[\tan(\varphi')]-[\lambda\cdot f_{\mathrm{i}}\sin(\alpha_{\mathrm{i+1}})+\cos(\alpha_{\mathrm{i+1}})]\,(\mathrm{FS}) \tag{21}$$

These constants can be used to simplify Equation (19) to obtain Equation (22), which can be found in IM3.

$$G_{\mathrm{i}}=\frac{\Psi_{\mathrm{i\text{-}1}}\,G_{\mathrm{i\text{-}1}}+(FS)\;T_{\mathrm{i}}-R_{\mathrm{i}}}{\Phi_{\mathrm{i}}} \tag{22}$$

The constants Ψ and Φ in Equation (22) for are functions of the unknowns: the proportionality constant, λ (IM2), and the factor of safety, FS (IM1).

Number	IM4
Label	**Critical Slip Surface Identification**
Input	$\{(x_{\mathrm{us}}, y_{\mathrm{us}})\}$
Output	$(\mathrm{FS}_{\mathrm{Min}}, \{(x_{\mathrm{cs}}, y_{\mathrm{cs}})\}) = \Upsilon(\{(x_{\mathrm{us}}, y_{\mathrm{us}})\})$
Description	FS_{Min} is the minimum factor of safety associated with the critical slip surface. x_{cs} is the x-ordinate of a point on the critical slip surface (m). x_{cs} is the y-ordinate of a point on the critical slip surface (m). Υ is a minimization algorithm or function. x_{us} is the x-ordinate of a point on the slope (m). y_{us} is the y-ordinate of a point on the slope (m).
Sources	Li et al. (25 June 2010)

[Should this IM exist? It doesn't arise from any T —BM]

[We need something to explain that we pick the slip surface with the minimum factor of safety. I'll give this some further thought on whether this is the best way to say it. —SS]

5.2.6 Data Constraints

Tables 1 and 2 show the data constraints on the input and output variables, respectively. The column for physical constraints gives the physical limitations on the range of values that can be taken by the variable. The uncertainty column provides an estimate of the confidence with which the physical quantities can be measured. This information would be part of the input if one were performing an uncertainty quantification exercise. The constraints are conservative, to give the user of the model the flexibility to experiment with unusual situations. The column of typical values is intended to provide a feel for a common scenario.

Table 1: Input Variables

Var	Physical Constraints	Typical Value	Uncertainty
$\{(x_{wt}, y_{wt})\}$ (*)	At least two ordered pairs must be specified. Starts and ends on the same x values as soil layers.	N/A	10%
$\{(x_{us}, y_{us})\}$ (**)	At least two ordered pairs must be specified. y values of consecutive vertices are always increasing or always decreasing. All layers start and end on the same x values.	N/A	10%
x_{slip}^{maxEnd}	$x_{slip}^{maxEnd} > x_{slip}^{minEnd}$ and must be less than or equal to the maximum x value of the slope.	N/A	10%
x_{slip}^{minEnd}	$x_{slip}^{minEnd} \geq x_{slip}^{maxStart}$	N/A	10%
$x_{slip}^{maxStart}$	$x_{slip}^{maxStart} > x_{slip}^{minStart}$	N/A	10%
$x_{slip}^{minStart}$	must be greater than or equal to the minimum x value of the slope.	N/A	10%
y_{slip}^{max}	$y_{slip}^{max} > y_{slip}^{min}$	N/A	10%
y_{slip}^{min}	$y_{slip}^{min} < y_{slip}^{max}$	N/A	10%
c' (**)	$c > 0$	10	10%
φ' (**)	$0 < \varphi < 90$	25	10%
γ (**)	$\gamma > 0$	20	10%
γ_{Sat} (**)	$\gamma_{\text{Sat}} > 0$	20	10%
γ_{w}	$\gamma_{\text{w}} > 0$	9.8	10%

(*) Optional input.

(**) Input must be specified for each layer.

Table 2: Output Variables

Var	Physical Constraints
FS	$FS > 0$
$\{(x_{cs}, y_{cs})\}$	x values must be between the minimum and maximum x values of the slope. y values must not exceed the maximum y value of the slope. The slope between consecutive vertices must be always increasing as x increases.
G_i	$G_i > 0$
X_i	$X_i < 0$

5.2.7 Properties of a Correct Solution

A correct solution must exhibit conformation to the system of non-linear equations presented in IM1, IM2, and IM3. In addition, the factor of safety for the critical slip surface must be lower than the factors of safety calculated for other slip surfaces compared by IM4. The coordinates of the critical slip surface must form a concave up function.

[It seems to me that the content of this section could all be deleted, except for the last sentence. Saying that the solution agrees with the instance models is covered by the requirements. The shape of the critical slip surface seems to be the only new information here. What do you think BM? —SS]

[I agree that the information is covered elsewhere. Even the last sentence is mostly covered by A1. I struggled to come up with content for this section. —BM]

6 Requirements

This section provides the functional requirements, the business tasks that the software is expected to complete, and the nonfunctional requirements, the qualities that the software is expected to exhibit.

6.1 Functional Requirements

R1: Read the inputs, shown in the table below, and store the data.

R2: Verify that the input data lies within physical constraints shown in Table 1.

R3: Generate potential critical slip surfaces for the input slope (using IM4).

R4: Calculate the factors of safety for each of the potential critical slip surfaces (using IM1, IM2, and IM3).

R5: Compare the factor of safety for each potential critical slip surface to determine the minimum factor of safety, corresponding to the critical slip surface (using IM4).

R6: Verify that the factor of safety, critical slip surface, and interslice forces satisfy the physical constraints shown in Table 2 and Section 5.2.7.

symbol	unit	description
(x, y)	m	x and y-coordinates for vertices of the slope layers, for the water table if one exists, and for potential start and end points of a slip surface.
c'	Pa	Cohesion for each slope layer.
φ'	$^\circ$	Effective angle of friction for each slope layer.
γ	$\mathrm{N\,m^{-3}}$	Unit weight of dry soil for each slope layer.
γ_{Sat}	$\mathrm{N\,m^{-3}}$	Unit weight of saturated soil for each slope layer.
γ_{w}	$\mathrm{N\,m^{-3}}$	Unit weight of water.
const_f	N/A	Boolean decision on which form of f the user desires: constant if true, or half-sine if false.

R7: Display as output the user-supplied inputs listed in the table below:

symbol	description
x_{slip}^{maxEnd}	Maximum potential x-ordinate of the ending point of a slip surface
x_{slip}^{minEnd}	Minimum potential x-ordinate of the ending point of a slip surface
$x_{slip}^{maxStart}$	Maximum potential x-ordinate of the starting point of a slip surface
$x_{slip}^{minStart}$	Minimum potential x-ordinate of the starting point of a slip surface
y_{slip}^{max}	Maximum potential y-ordinate of the of a point on a slip surface
y_{slip}^{min}	Minimum potential y-ordinate of the of a point on a slip surface
const_f	Boolean decision on which form of f the user desires: constant if true, or half-sine if false.

R8: Display the critical slip surface of the 2D slope, as determined from IM4, graphically.

R9: Display the value of the factor of safety for the critical slip surface, as determined from IM1, IM2, and IM3.

R10: Using IM1, IM2, and IM3, calculate and graphically display the interslice normal forces.

R11: Using IM1, IM2, and IM3, calculate and graphically display the interslice shear forces.

6.2 Nonfunctional Requirements

SSP is intended to be an educational tool, therefore accuracy and performance speed are secondary program priorities. Instead, the following non-functional requirements are prioritized:

NFR1: Correctness, achieved if the outputs of the code have the properties described in 5.2.7.

NFR2: Understandability, achieved if the code is modularized with complete module guide and module interface specification.

NFR3: Reusability, achieved if the code is modularized.

NFR4: Maintainability, achieved if the traceability between requirements, assumptions, theoretical models, general definitions, data definitions, instance models, likely changes, and modules is completely recorded in traceability matrices in the SRS and module guide.

7 Likely Changes

LC1: The system currently assumes the different layers of the soil are homogeneous (A3). In the future, implementation can be added for inconsistent soil properties throughout.

LC2: The system currently assumes no seismic force (A11). In the future, implementation can be added for the presence of seismic force.

LC3: The system currently assumes no external force (A12). In the future, implementation can be added for an imposed surface load on the slope.

8 Unlikely Changes

If changes were to be made with regard to the following, a different algorithm would be needed.

UC1: Changes related to A6 are not possible due to the dependency of the calculations on the proportional relationship between interslice normal and shear forces.

UC2: A7 allows for 2D analysis with these models only because stress along z-direction is zero. These models do not take into account stress in the z-direction, and therefore cannot be used without manipulation to attempt 3-dimensional analysis.

[This section is not on the template, not sure if it should be kept —BM]
[I'm going to think about adding this section to the template. It is a way to show that some of the assumptions are critical to the identity of the problem. —SS]

9 Traceability Matrices and Graphs

The purpose of the traceability matrices is to provide easy references on what has to be additionally modified if a certain component is changed. Every time a component is changed, the items in the column of that component that are marked with an "X" may have to be modified as well. Tables 5 and 6 show the dependencies of theoretical models, general definitions, data definitions, and instance models with each other. Table 4 shows the dependencies of instance models, requirements, and data constraints on each other. Table 3 shows the dependencies of theoretical models, general definitions, data definitions, instance models, and likely changes on the assumptions.

The purpose of the traceability graphs is also to provide easy references on what has to be additionally modified if a certain component is changed. The arrows in the graphs represent

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12
T1												
T2							X					
T3								X				
T4												
GD1												
GD2												
GD3			X	X	X							
GD4		X	X	X	X							
GD5								X				
GD8						X						
GD9												
DD1												
DD2									X			
DD3									X			
DD4												
DD5									X			
DD6									X			
DD7												
DD8									X			
DD9									X			
GD6			X	X	X						X	X
GD7			X	X	X						X	X
IM1		X				X				X	X	X
IM2						X				X	X	X
IM3		X				X				X	X	X
IM4	X											
LC1			X									
LC2											X	
LC3												X

Table 3: Traceability Matrix Showing the Connections Between Assumptions and Other Items

	IM1	IM2	IM3	IM4	5.2.6	R1
IM1						X
IM2						X
IM3						X
IM4						X
R1						
R2					X	
R3				X		
R4	X	X	X			
R5				X		
R6					X	
R7						X
R8					X	
R9	X	X	X			
R10	X	X	X			
R11	X	X	X			

Table 4: Traceability Matrix Showing the Connections Between Requirements and Instance Models

	T1	T2	T3	T4	GD1	GD2	GD3	GD4	GD5	GD8	GD9
T1											
T2											
T3											
T4											
GD1		X									
GD2		X									
GD3			X	X							
GD4	X		X	X			X				
GD5				X							
GD8											
GD9		X									
DD1											
DD2											
DD3											
DD4											
DD5											
DD6											
DD7											
DD8											
DD9											
GD6			X	X	X	X	X	X	X		
GD7			X	X	X	X	X				
IM1	X			X						X	
IM2		X		X						X	X
IM3				X	X	X		X		X	
IM4											

Table 5: Traceability Matrix Showing the Connections Between Items of Different Sections With Theory Models and General Definitions

	DD1	DD2	DD3	DD4	DD5	DD6	DD7	DD8	DD9	GD6	GD7	IM1	IM2	IM3	IM4
T1															
T2															
T3															
T4															
GD1	X		X	X	X	X									
GD2	X		X	X	X	X									
GD3								X							
GD4								X							
GD5															
GD8															
GD9	X		X	X	X	X	X								
DD1															
DD2								X							
DD3									X						
DD4															
DD5															
DD6															
DD7															
DD8					X		X								
DD9						X	X								
GD6	X	X	X	X	X	X	X								
GD7	X		X	X	X	X	X								
IM1	X	X	X	X	X	X	X			X	X		X	X	
IM2	X	X	X	X	X	X	X					X		X	
IM3	X	X	X	X	X	X	X			X		X	X		
IM4															

Table 6: Traceability Matrix Showing the Connections Between Items of Different Sections with Data Definitions and Instance Models

dependencies. The component at the tail of an arrow is depended on by the component at the head of that arrow. Therefore, if a component is changed, the components that it points to should also be changed. Figure 5 shows the dependencies of theoretical models, general definitions, data definitions, instance models, likely changes, and assumptions on each other. Figure 6 shows the dependencies of instance models, requirements, and data constraints on each other.

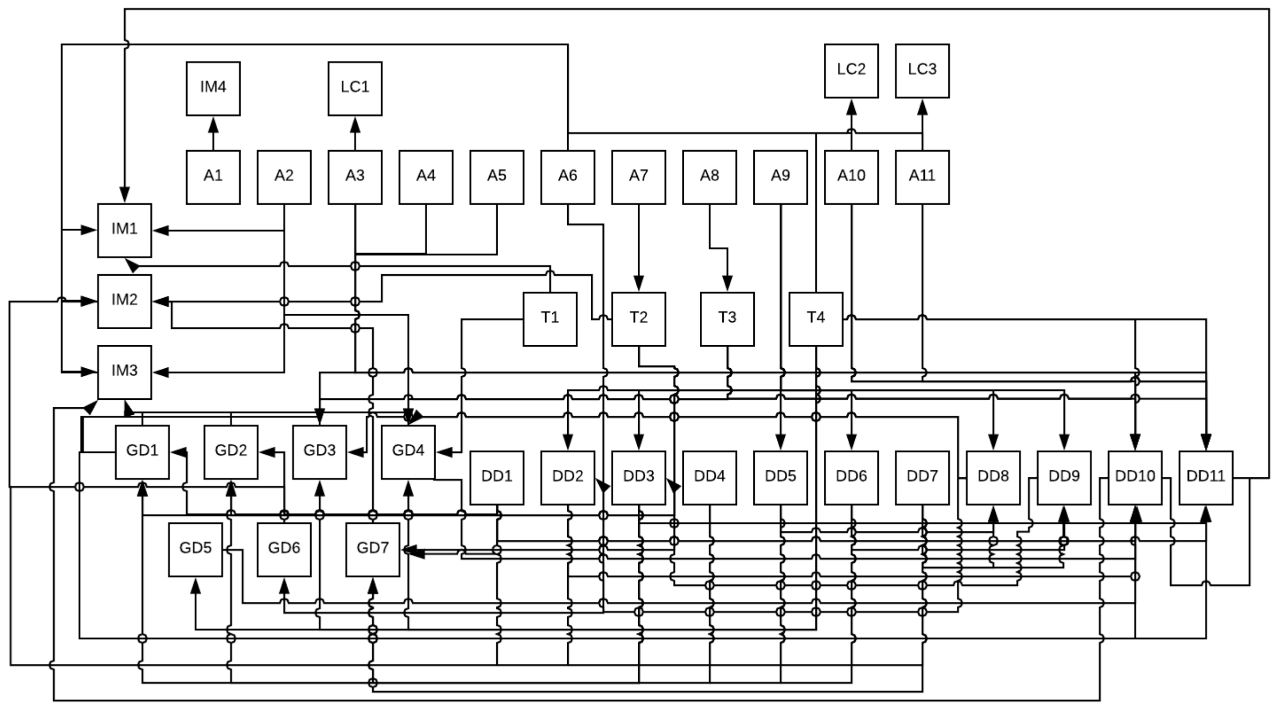

Figure 5: Traceability Matrix Showing the Connections Between Items of Different Sections

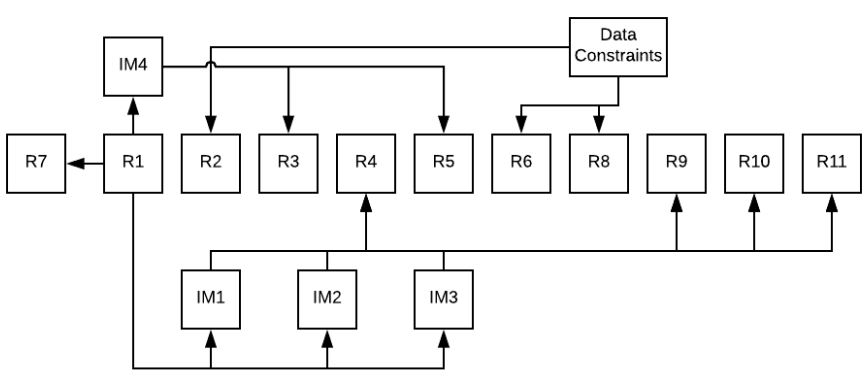

Figure 6: Traceability Matrix Showing the Connections Between Requirements, Instance Models, and Data Constraints

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11 Appendix

11.1 Symbolic Parameters

There are no symbolic parameters.