

# Module Interface Specification for Slope Stability Analysis Program (SSP)

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# 1 Revision History

Date	Version	Notes
11/12/18	1.0	Initial updates based on template
11/21/18	1.1	Finished updating all of the modules

## 2 Symbols, Abbreviations and Acronyms

See Section [2](#) of the Software Requirements Specification (SRS) document, available in [the GitHub repository for the project](#).

# Contents

<b>1</b>	<b>Revision History</b>	<b>i</b>
<b>2</b>	<b>Symbols, Abbreviations and Acronyms</b>	<b>ii</b>
<b>3</b>	<b>Introduction</b>	<b>1</b>
<b>4</b>	<b>Notation</b>	<b>1</b>
<b>5</b>	<b>Numerical Algorithms</b>	<b>2</b>
<b>6</b>	<b>Module Decomposition</b>	<b>2</b>
<b>7</b>	<b>MIS of the Control Module</b>	<b>3</b>
7.1	Module . . . . .	3
7.2	Uses . . . . .	3
7.3	Syntax . . . . .	3
7.3.1	Exported Constants . . . . .	3
7.3.2	Exported Data Types . . . . .	4
7.3.3	Exported Access Programs . . . . .	4
7.4	Semantics . . . . .	4
7.4.1	State Variables . . . . .	4
7.4.2	Environment Variables . . . . .	4
7.4.3	Assumptions . . . . .	4
7.4.4	Access Routine Semantics . . . . .	4
7.4.5	Local Functions . . . . .	4
<b>8</b>	<b>MIS of the Input Module</b>	<b>4</b>
8.1	Module . . . . .	4
8.2	Uses . . . . .	5
8.3	Syntax . . . . .	5
8.3.1	Exported Constants . . . . .	5
8.3.2	Exported Data Types . . . . .	5
8.3.3	Exported Access Programs . . . . .	6
8.4	Semantics . . . . .	7
8.4.1	State Variables . . . . .	7
8.4.2	Environment Variables . . . . .	7
8.4.3	Assumptions . . . . .	7
8.4.4	Access Routine Semantics . . . . .	7
8.4.5	Local Functions . . . . .	11

<b>9</b>	<b>MIS of the Output Module</b>	<b>11</b>
9.1	Module . . . . .	11
9.2	Uses . . . . .	11
9.3	Syntax . . . . .	11
9.3.1	Exported Constants . . . . .	11
9.3.2	Exported Data Types . . . . .	11
9.3.3	Exported Access Programs . . . . .	11
9.4	Semantics . . . . .	11
9.4.1	State Variables . . . . .	11
9.4.2	Environment Variables . . . . .	11
9.4.3	Assumptions . . . . .	12
9.4.4	Access Routine Semantics . . . . .	12
9.4.5	Local Functions . . . . .	12
<b>10</b>	<b>MIS of the Genetic Algorithm Module</b>	<b>12</b>
10.1	Module . . . . .	12
10.2	Uses . . . . .	12
10.2.1	Imported Access Programs . . . . .	12
10.3	Syntax . . . . .	13
10.3.1	Exported Constants . . . . .	13
10.3.2	Exported Data Types . . . . .	13
10.3.3	Exported Access Programs . . . . .	13
10.4	Semantics . . . . .	13
10.4.1	State Variables . . . . .	13
10.4.2	Environment Variables . . . . .	13
10.4.3	Assumptions . . . . .	13
10.4.4	Access Routine Semantics . . . . .	13
10.4.5	Local Functions . . . . .	14
<b>11</b>	<b>MIS of the Kinematic Admissibility Module</b>	<b>14</b>
11.1	Module . . . . .	14
11.2	Uses . . . . .	14
11.3	Syntax . . . . .	14
11.3.1	Exported Constants . . . . .	14
11.3.2	Exported Data Types . . . . .	15
11.3.3	Exported Access Programs . . . . .	15
11.4	Semantics . . . . .	15
11.4.1	State Variables . . . . .	15
11.4.2	Environment Variables . . . . .	15
11.4.3	Assumptions . . . . .	15
11.4.4	Access Routine Semantics . . . . .	15
11.4.5	Local Functions . . . . .	16

<b>12 MIS of the Slip Weighting Module</b>	<b>16</b>
12.1 Module . . . . .	16
12.2 Uses . . . . .	16
12.3 Syntax . . . . .	16
12.3.1 Exported Constants . . . . .	16
12.3.2 Exported Data Types . . . . .	17
12.3.3 Exported Access Programs . . . . .	17
12.4 Semantics . . . . .	17
12.4.1 State Variables . . . . .	17
12.4.2 Environment Variables . . . . .	17
12.4.3 Assumptions . . . . .	17
12.4.4 Access Routine Semantics . . . . .	17
12.4.5 Local Functions . . . . .	17
<b>13 MIS of the Slip Slicing Module</b>	<b>18</b>
13.1 Module . . . . .	18
13.2 Uses . . . . .	18
13.3 Syntax . . . . .	18
13.3.1 Exported Constants . . . . .	18
13.3.2 Exported Data Types . . . . .	18
13.3.3 Exported Access Programs . . . . .	18
13.4 Semantics . . . . .	18
13.4.1 State Variables . . . . .	18
13.4.2 Environment Variables . . . . .	18
13.4.3 Assumption . . . . .	18
13.4.4 Access Routine Semantics . . . . .	19
13.4.5 Local Functions . . . . .	19
<b>14 MIS of the Morgenstern-Price Calculation Module</b>	<b>19</b>
14.1 Module . . . . .	19
14.2 Uses . . . . .	19
14.3 Syntax . . . . .	19
14.3.1 Exported Constants . . . . .	19
14.3.2 Exported Data Types . . . . .	20
14.3.3 Exported Access Programs . . . . .	20
14.4 Semantics . . . . .	20
14.4.1 State Variables . . . . .	20
14.4.2 Environment Variables . . . . .	20
14.4.3 Assumptions . . . . .	20
14.4.4 Access Routine Semantics . . . . .	20
14.4.5 Local Functions . . . . .	21

<b>15 MIS of the Slice Property Calculation Module</b>	<b>22</b>
15.1 Module . . . . .	22
15.2 Uses . . . . .	22
15.3 Syntax . . . . .	22
15.3.1 Exported Constants . . . . .	22
15.3.2 Exported Data Types . . . . .	22
15.3.3 Exported Access Programs . . . . .	22
15.4 Semantics . . . . .	23
15.4.1 State Variables . . . . .	23
15.4.2 Environment Variables . . . . .	23
15.4.3 Assumptions . . . . .	23
15.4.4 Access Routine Semantics . . . . .	23
15.4.5 Local Functions . . . . .	25
<b>16 MIS of the Sequence Data Structure Module</b>	<b>25</b>
16.1 Module . . . . .	25
16.2 Uses . . . . .	25
16.3 Syntax . . . . .	25
16.3.1 Exported Constants . . . . .	25
16.3.2 Exported Data Types . . . . .	25
16.3.3 Exported Access Programs . . . . .	25
16.4 Semantics . . . . .	25
16.4.1 State Variables . . . . .	25
16.4.2 Environment Variables . . . . .	25
16.4.3 Assumptions . . . . .	26
16.4.4 Access Routine Semantics . . . . .	26
16.4.5 Local Functions . . . . .	26
16.4.6 Considerations . . . . .	26
<b>17 MIS of the Random Number Generation Module</b>	<b>26</b>
17.1 Module . . . . .	26
17.2 Uses . . . . .	26
17.3 Syntax . . . . .	26
17.3.1 Exported Constants . . . . .	26
17.3.2 Exported Data Types . . . . .	27
17.3.3 Exported Access Programs . . . . .	27
17.4 Semantics . . . . .	27
17.4.1 State Variables . . . . .	27
17.4.2 Environment Variables . . . . .	27
17.4.3 Assumptions . . . . .	27
17.4.4 Access Routine Semantics . . . . .	27
17.4.5 Local Functions . . . . .	27
17.4.6 Considerations . . . . .	27

<b>18 MIS of the Plotting Module</b>	<b>27</b>
18.1 Module . . . . .	27
18.2 Uses . . . . .	28
18.3 Syntax . . . . .	28
18.3.1 Exported Constants . . . . .	28
18.3.2 Exported Data Types . . . . .	28
18.3.3 Exported Access Programs . . . . .	28
18.4 Semantics . . . . .	28
18.4.1 State Variables . . . . .	28
18.4.2 Environment Variables . . . . .	28
18.4.3 Assumptions . . . . .	28
18.4.4 Access Routine Semantics . . . . .	28
18.4.5 Local Functions . . . . .	29
18.4.6 Considerations . . . . .	29
<b>19 Appendix</b>	<b>31</b>
19.1 Parameter Tables . . . . .	31
19.1.1 Layer Parameters . . . . .	31
19.1.2 Piezometric Parameter . . . . .	31
19.1.3 Search Range Parameters . . . . .	31
19.1.4 Solution Parameters . . . . .	32
19.1.5 Internal Force Parameters . . . . .	33
19.1.6 Angle Parameters . . . . .	33



### 3 Introduction

The following document details the Module Interface Specifications for SSP, a program for determining the critical slip surface and corresponding factor of safety for a given sloped mass of soil. The document is intended to ease understanding of the design of SSP and should be used as a resource for any maintenance of SSP.

Complementary documents include the System Requirement Specifications and Module Guide. The full documentation and implementation can be found at [the GitHub repository for the project](#).

### 4 Notation

The structure of the MIS for modules comes from [Hoffman and Strooper \(1995\)](#), with the addition that template modules have been adapted from [Ghezzi et al. \(2003\)](#). The mathematical notation comes from Chapter 3 of [Hoffman and Strooper \(1995\)](#). For instance, the symbol  $:=$  is used for a multiple assignment statement and conditional rules follow the form  $(c_1 \Rightarrow r_1 | c_2 \Rightarrow r_2 | \dots | c_n \Rightarrow r_n)$ . The notation for quantifiers is from [Gries and Schneider \(1993\)](#).

The following table summarizes the primitive data types used by SSP.

Data Type	Notation	Description
character	char	a single symbol or digit
boolean	$\mathbb{B}$	a value from the set {true, false}
real	$\mathbb{R}$	any number in $(-\infty, \infty)$
integer	$\mathbb{Z}$	a number without a fractional component in $(-\infty, \infty)$

The specification of SSP uses some derived data types: sequences, strings, and tuples. Sequences are ordered lists of elements of the same data type, denoted by brackets enclosing the type of the data elements. If a sequence has fixed dimensions, the notation of the type will include the dimensions in superscript. Strings are sequences of characters. Tuples contain a list of values, potentially of different types, each associated with a field identifier. When a tuple is referenced in this document, a link to an appendix section that specifies the fields of the tuple will be provided. In addition, SSP uses functions, which are defined by the data types of their inputs and outputs. Local functions are described by giving their type signature followed by their specification.

## 5 Numerical Algorithms

### Morgenstern-Price (Section 14)

The non-linear nature of the systems of equations in the Morgenstern-Price solver algorithm requires that the equations for the factor of safety (IM1), the interslice normal-to-shear force ratio (IM2), and the interslice normal forces (IM3) are solved iteratively, with an initial guess for two of the values, typically the factor of safety and interslice normal-to-shear force ratio.

### Genetic Algorithm (Section 10)

SSP uses a genetic algorithm to find the coordinates of the critical slip surface vertices that minimize the factor of safety, as described in IM4. The genetic algorithm generates a set of initial potential slip surfaces, and subsequent generations are created by merging and mutating slip surfaces with low factors of safety from the previous generation. The minimum factor of safety after several generations is assumed to correspond to the critical slip surface.

[This section is not on the template. I've left it in for now because the information does seem useful, but maybe this is not the right place for it? Maybe this should go to an appendix? —BM]

## 6 Module Decomposition

The following table is taken directly from the Module Guide document for this project.

Level 1	Level 2
Hardware-Hiding	
	Control
	Input
	Output
Behaviour-Hiding	Genetic Algorithm
	Kinematic Admissibility
	Slip Weighting
	Slip Slicing
	Morgenstern-Price Calculation
	Slice Property Calculation
Software Decision	Sequence Data Structure
	Random Number Generation
	Plotting

Table 1: Module Hierarchy

## 7 MIS of the Control Module

### 7.1 Module

Control

### 7.2 Uses

Input (Section 8), Output (Section 9), GenAlg (Section 10), Sequence (Section 16)

### 7.3 Syntax

#### 7.3.1 Exported Constants

N/A

### 7.3.2 Exported Data Types

N/A

### 7.3.3 Exported Access Programs

Name	In	Out	Exceptions
Control	string	-	-

## 7.4 Semantics

### 7.4.1 State Variables

N/A

### 7.4.2 Environment Variables

N/A

### 7.4.3 Assumptions

The access program is called with a string parameter.

### 7.4.4 Access Routine Semantics

`control(fname):`

- transition:

Modifies the state of the Input Module, Genetic Algorithm Module, and Output Module.

### 7.4.5 Local Functions

N/A

## 8 MIS of the Input Module

### 8.1 Module

Input

## 8.2 Uses

Sequence (Section 16)

## 8.3 Syntax

### 8.3.1 Exported Constants

N/A

### 8.3.2 Exported Data Types

`coord` = tuple of ( $x : \mathbb{R}$ ,  $y : \mathbb{R}$ )

`coords` = [`coord`]

`paramsLayers` = tuple of (`strat` : `coords`,  $\phi : \mathbb{R}$ ,  $\text{coh} : \mathbb{R}$ ,  $\text{gam} : \mathbb{R}$ ,  $\text{gams} : \mathbb{R}$ ) (Appendix 19.1.1)

`paramsPiez` = tuple of (`piez` : `coords`,  $\text{gamw} : \mathbb{R}$ ) (Appendix 19.1.2)

`paramsSearch` = tuple of (`Xext`, `Xetr`,  $\text{Ylim} : [\mathbb{R}]^{1,2}$ ) (Appendix 19.1.3)

`paramsSoln` = tuple of (`ltor`, `ftype`, `evnslc`, `cncvu`, `obtu` :  $\mathbb{B}$ ) (Appendix 19.1.4)

### 8.3.3 Exported Access Programs

Name	In	Out	Exceptions
load_params	string	-	fileNotExist, badFileExtension, unexpectedInput
verify_params	-	-	badSlopeGeometry, badEffAngleFriction, badCohesion, badDryUnitWeight, badSatUnitWeight, badPiezGeometry, badWatUnitWeight
strat	-	coords	-
slopeX	-	$[\mathbb{R}]$	-
slopeY	-	$[\mathbb{R}]$	-
phi	-	$\mathbb{R}$	-
coh	-	$\mathbb{R}$	-
gam	-	$\mathbb{R}$	-
gams	-	$\mathbb{R}$	-
piez	-	coords	-
piezX	-	$[\mathbb{R}]$	-
piezY	-	$[\mathbb{R}]$	-
gamw	-	$\mathbb{R}$	-
xExt	-	$[\mathbb{R}]^{1 \times 2}$	-
xEtr	-	$[\mathbb{R}]^{1 \times 2}$	-
yLim	-	$[\mathbb{R}]^{1 \times 2}$	-
ltor	-	$\mathbb{B}$	-
ftype	-	$\mathbb{B}$	-
evnslc	-	$\mathbb{B}$	-
cncvu	-	$\mathbb{B}$	-
obtu	-	$\mathbb{B}$	-

## 8.4 Semantics

### 8.4.1 State Variables

*slope* : paramsLayers  
*piez* : paramsPiez  
*search* : paramsSearch  
*soln* : paramsSoln

### 8.4.2 Environment Variables

*in\_file* : String

- *in\_file* represents a file stored in the file system of the hardware running SSP.

### 8.4.3 Assumptions

- load\_params is called before any of the other access programs.
- The guesses for potential minimum and maximum  $x$  and  $y$  values of the critical slip surface, as described in *in\_file*, lie within the boundaries of the given slope geometry.

### 8.4.4 Access Routine Semantics

load\_params(*fname*):

- transition:

$slope, piez, search, soln := slope', piez', search', soln'$

where  $slope'$ ,  $piez'$ ,  $search'$ , and  $soln'$  are populated based on the contents of *in\_file*.

- exceptions:

$exc := (fname \text{ does not exist in file system} \Rightarrow \text{fileNotExist}$   
 $| fname[(|fname| - 5)..(|fname| - 1)] = \text{"out"} \Rightarrow \text{badFileExtension}$   
 $| in\_file \text{ is not formatted correctly} \Rightarrow \text{unexpectedInput})$

verify\_params():

- exceptions:

$exc := (\neg(\forall(i : \mathbb{Z} | i \in [0..|slope.strat| - 2] : slope.strat[i].x - slope.strat[i + 1].x \leq 0)) \Rightarrow \text{badSlopeGeometry}$   
 $| \neg(0 < slope.phi < 90) \Rightarrow \text{badEffAngleFriction})$

$\neg(0 < slope.coh) \Rightarrow badCohesion$   
 $\neg(0 < slope.gam) \Rightarrow badDryUnitWeight$   
 $\neg(0 < slope.gams) \Rightarrow badSatUnitWeight$   
 $\neg(\forall(i : \mathbb{Z} | i \in [0..|piez.piez| - 2] : piez.piez[i].x - piez.piez[i + 1].x \leq 0))$   
 $\vee piez.piez[0].x \neq slope.strat[0].x$   
 $\vee piez.piez[|piez.piez| - 1].x \neq slope.strat[|slope.strat| - 1].x$   
 $\Rightarrow badPiezGeometry$

strat():

- output:

$out := slope.strat$

slopeX():

- output:

$out := slope.strat[0].x || slope.strat[1].x ||$   
 $\dots || slope.strat[|slope.strat| - 1].x$

slopeY():

- output:

$out := slope.strat[0].y || slope.strat[1].y ||$   
 $\dots || slope.strat[|slope.strat| - 1].y$

phi():

- output:

$out := slope.phi$

coh():

- output:

$out := slope.coh$

gam():



- output:

$out := slope.gam$

gams():

- output:

$out := slope.gams$

piez():

- output:

$out := piez.piez$

piezX():

- output:

$out := piez.piez[0].x || piez.piez[1].x ||$   
 $\dots || piez.piez[|piez.piez| - 1].x$

piezY():

- output:

$out := piez.piez[0].y || piez.piez[1].y ||$   
 $\dots || piez.piez[|piez.piez| - 1].y$

gamw():

- output:

$out := piez.gamw$

xExt():

- output:

$out := search.Xext$

xEtr():

- output:

*out := search.Xetr*

yLim():

- output:

*out := search.Ylim*

ltor():

- output:

*out := soln.ltor*

ftype():

- output:

*out := soln.ftype*

evnslc():

- output:

*out := soln.evnslc*

cncvu():

- output:

*out := soln.cncvu*

obtu():

- output:

*out := soln.obtu*

#### 8.4.5 Local Functions

N/A

## 9 MIS of the Output Module

### 9.1 Module

Output

### 9.2 Uses

Sequence (Section 16), Plot (Section 18)

### 9.3 Syntax

#### 9.3.1 Exported Constants

N/A

#### 9.3.2 Exported Data Types

N/A

#### 9.3.3 Exported Access Programs

Name	In	Out	Exceptions
verify_output	$\mathbb{R}$	-	negativeFS
output	$\mathbb{R}$ , coords, coords, coords, string	-	-

### 9.4 Semantics

#### 9.4.1 State Variables

N/A

#### 9.4.2 Environment Variables

*out\_file* : String

- *out\_file* represents a file stored in the file system of the hardware running SSP.

*screen* :  $[\mathbb{Z}]$

- *screen* represents the colour values for each pixel on the screen of the hardware running SSP.

### 9.4.3 Assumptions

N/A

### 9.4.4 Access Routine Semantics

verify\_output(*Fs*):

- exceptions:

$exc := Fs < 0 \Rightarrow \text{negativeFS}$

output(*Fs*, *crit\_slip*, *G*, *X*, *fname*):

- transition:

*out\_file* is created at path *fname* || “.out”. The outputs of xEtr(), xExt(), yLim(), ftype(), *Fs*, *crit\_slip*, *G*, and *X* are written to *out\_file*. *screen* is modified to display the outputs of plot(*crit\_slip.x*, *crit\_slip.y*), plot(*G.x*, *G.y*), and plot(*X.x*, *X.y*).

### 9.4.5 Local Functions

N/A

## 10 MIS of the Genetic Algorithm Module

### 10.1 Module

GenAlg

### 10.2 Uses

#### 10.2.1 Imported Access Programs

Input (Section 8), MorgPriceSolver (Section 14), Slicer (Section 13), KinAdm (Section 11), SlipWeighter (Section 12), Sequence (Section 16), Rand (Section 17)

## 10.3 Syntax

### 10.3.1 Exported Constants

MIN\_GENS = 100

NUM\_SLIPS = 20

REL\_DIFF = 0.00005

### 10.3.2 Exported Data Types

slip = tuple of (surf : coords, Fs :  $\mathbb{R}$ , G : coords, X : coords, wt :  $\mathbb{R}$ )

slips = [slip]

### 10.3.3 Exported Access Programs

Name	In	Out	Exceptions
genetic_alg	-	$\mathbb{R}$ , coords, coords, coords	-

## 10.4 Semantics

### 10.4.1 State Variables

N/A

### 10.4.2 Environment Variables

N/A

### 10.4.3 Assumptions

N/A

### 10.4.4 Access Routine Semantics

genetic\_alg():

- output:

$out := \text{weighter}(slip\_surfs)[0].surf, \text{weighter}(slip\_surfs)[0].Fs, \text{weighter}(slip\_surfs)[0].G,$   
and  $\text{weighter}(slip\_surfs)[0].X$ , where  $slip\_surfs$ , of type slips, is developed by:

- \* using `rand` to randomly generate coordinates for `NUM_SLIPS` potential slip surfaces, where the entry and exit x-coordinate for each slip surface are computed according to `generate_slips(xEtr)` and `generate_slips(xExt)`. Corresponding y-coordinates are determined by interpolating on the slope geometry.
- \* using `kinAdm` to verify that the geometry of each potential slip surface is physically realizable. If any are not, new slip surfaces are randomly generated until `NUM_SLIPS` valid slip surfaces have been generated,
- \* using `slicer` to redefine each slip surface's coordinates based on the desired number of slices
- \* using `morg_price` to determine the  $Fs$ ,  $G$ , and  $X$  fields of each slip surface
- \* using `weighter` to determine the  $wt$  field of each slip surface
- \* using `rand` to generate a new pool of `NUM_SLIPS` slip surfaces by applying crossovers and mutations to the previous generation, with the more highly-weighted members having a greater likelihood of contributing to the subsequent generations
- \* applying `kinAdm`, `slicer`, `morg_price`, and `weighter` to the new generation
- \* repeating until at least `MIN_GENS` have occurred and the relative difference between subsequent generations is less than `REL_DIFF`.

#### 10.4.5 Local Functions

`generate_slips(Xrange) : [ℝ] → ℝ`

`generate_slips(Xrange) = (Xrange[0] + rand() * (Xrange[1] - Xrange[0]))`

## 11 MIS of the Kinematic Admissibility Module

### 11.1 Module

`KinAdm`

### 11.2 Uses

Input (Section 8), Sequence (Section 16)

### 11.3 Syntax

#### 11.3.1 Exported Constants

N/A

### 11.3.2 Exported Data Types

N/A

### 11.3.3 Exported Access Programs

Name	In	Out	Exceptions
kinAdm	slip	$\mathbb{B}$	-

## 11.4 Semantics

### 11.4.1 State Variables

N/A

### 11.4.2 Environment Variables

N/A

### 11.4.3 Assumptions

- The *surf* field is populated for every member of the input sequence of slip data.

### 11.4.4 Access Routine Semantics

kinAdm(*slip\_surf*):

- output:

$$\begin{aligned} out := & ( \neg(\forall(i : \mathbb{Z} | i \in [0..|slip\_surf.surf| - 2] : slip\_surf.surf[i].x - slip\_surf.surf[i + \\ & 1].x \leq 0)) \\ & \vee \neg is\_on\_slope(slip\_surf.surf[0]) \\ & \vee \neg is\_on\_slope(slip\_surf.surf[|slip\_surf.surf| - 1]) \\ & \vee \neg is\_in\_slope(slip\_surf.surf) \\ & \vee (cncvu() \wedge \neg(is\_concave\_up(slip\_surf.surf))) \\ & \vee (obtu() \wedge \neg(has\_no\_sharp\_angles(slip\_surf.surf))) \\ & \Rightarrow false \\ & | else \Rightarrow true \end{aligned}$$

[Not sure if I'm allowed to use "else" here but don't know how else to express the "else" case succinctly —BM]

### 11.4.5 Local Functions

$\text{linSlope}(\text{point1}, \text{point2}) : \text{coord} \times \text{coord} \rightarrow \mathbb{R}$

$$\text{linSlope}(\text{point1}, \text{point2}) = \frac{\text{point2.y} - \text{point1.y}}{\text{point2.x} - \text{point1.x}}$$

$\text{is\_on\_slope}(\text{point}) : \text{coord} \rightarrow \mathbb{B}$

$$\begin{aligned} \text{is\_on\_slope}(\text{point}) = & (\exists(i : \mathbb{Z} | i \in [0..|\text{slope.strat}| - 1] : \text{point} = \text{slope.strat}[i])) \\ \vee & (\exists(i : \mathbb{Z} | i \in [0..|\text{slope.strat}| - 2] : \text{point.y} = \text{linSlope}(\text{slope.strat}[i], \text{slope.strat}[i + 1]) * \\ & \text{point.x} + \frac{\text{slope.strat}[i].y}{\text{linSlope}(\text{slope.strat}[i], \text{slope.strat}[i+1]) * \text{slope.strat}[i].x})) \end{aligned}$$

$\text{is\_in\_slope}(\text{surf}) : \text{coords} \rightarrow \mathbb{B}$

$$\begin{aligned} \text{is\_in\_slope}(\text{surf}) = & (\forall(i : \mathbb{Z} | i \in [1..|\text{surf}| - 2] : (\forall(j : \mathbb{Z} | j \in [0..|\text{slope.strat}| - 2] \wedge \text{slope.strat}[j].x \leq \\ & \text{surf}[i].x < \text{slope.strat}[j + 1].x : \text{surf}[i].y < (\text{slope.strat}[j].y + (\text{surf}[i].x - \text{slope.strat}[j].x) * \\ & \text{linSlope}(\text{slope.strat}[j], \text{slope.strat}[j + 1]))))) \end{aligned}$$

$\text{is\_concave\_up}(\text{surf}) : \text{coords} \rightarrow \mathbb{B}$

$$\text{is\_concave\_up}(\text{surf}) = (\forall(i : \mathbb{Z} | i \in [0..|\text{surf}| - 3] : \text{linSlope}(\text{surf}[i+1], \text{surf}[i+2]) \geq \text{linSlope}(\text{surf}[i], \text{surf}[i+1])))$$

$\text{distance}(\text{point1}, \text{point2}) : \text{coord} \times \text{coord} \rightarrow \mathbb{R}$

$$\text{distance}(\text{point1}, \text{point2}) = \sqrt{(\text{point1.x} - \text{point2.x})^2 + (\text{point1.y} - \text{point2.y})^2}$$

$\text{has\_no\_sharp\_angles}(\text{surf}) : \text{coords} \rightarrow \mathbb{B}$

$$\begin{aligned} \text{has\_no\_sharp\_angles}(\text{surf}) = & (\forall(i : \mathbb{Z} | i \in [0..|\text{surf}| - 3] : \\ & \arccos \frac{(\text{distance}(\text{surf}[i], \text{surf}[i+1]))^2 + (\text{distance}(\text{surf}[i+1], \text{surf}[i+2]))^2 - (\text{distance}(\text{surf}[i], \text{surf}[i+2]))^2}{2 * \text{distance}(\text{surf}[i], \text{surf}[i+1]) * \text{distance}(\text{surf}[i+1], \text{surf}[i+2])} \geq 1.9199)) \end{aligned}$$

## 12 MIS of the Slip Weighting Module

### 12.1 Module

SlipWeighter

### 12.2 Uses

Sequence (Section 16)

### 12.3 Syntax

#### 12.3.1 Exported Constants

N/A



### 12.3.2 Exported Data Types

N/A

### 12.3.3 Exported Access Programs

Name	In	Out	Exceptions
weighter	slips	slips	-

## 12.4 Semantics

### 12.4.1 State Variables

N/A

### 12.4.2 Environment Variables

N/A

### 12.4.3 Assumptions

- The  $Fs$  field is populated for every member of the input sequence of slip data.

### 12.4.4 Access Routine Semantics

weighter( $slip\_surfs$ ):

- output:

$out := slip\_surfs'$  such that  $slip\_surfs' = assign\_weights(sort\_Fs(slip\_surfs))$

### 12.4.5 Local Functions

$sort\_Fs(unsorted) : slips \rightarrow slips$   $sort\_Fs(unsorted) = sorted$  such that

$\forall(a : slip | a \in unsorted : \exists(b : slip | b \in sorted : b = a \wedge count(a, A) = count(b, B))) \wedge \forall(i : \mathbb{Z} | i \in [0..|unsorted| - 1] : sorted[i].Fs \leq sorted[i + 1].Fs)$

$count(a, A) : slip \times slips \rightarrow \mathbb{Z}$

$count(a, A) = +(x : slip | x \in A \wedge x = a : 1)$

$assign\_weights(s) : slips \rightarrow slips$

$assign\_weights(s) = s'$  such that

$s'[0].wt = s[0].Fs - s[|s| - 1].Fs$  and

$\forall(i : \mathbb{Z} | i \in [1..|s| - 1] : s'[i].wt = s'[i - 1].wt + \frac{s[i].Fs - s[|s| - 1].Fs}{+(j : \mathbb{Z} | j \in [0..|s| - 1] : s[j].Fs - s[|s| - 1].Fs)})$

## 13 MIS of the Slip Slicing Module

### 13.1 Module

Slicer

### 13.2 Uses

Sequence (Section 16)

### 13.3 Syntax

#### 13.3.1 Exported Constants

N/A

#### 13.3.2 Exported Data Types

N/A

#### 13.3.3 Exported Access Programs

Name	In	Out	Exceptions
slicer	coords, $\mathbb{Z}$	slip	-

### 13.4 Semantics

#### 13.4.1 State Variables

N/A

#### 13.4.2 Environment Variables

N/A

#### 13.4.3 Assumption

- The integer input to *slicer* is greater than the size of the slip input to *slicer*.

### 13.4.4 Access Routine Semantics

$\text{slicer}(\text{slip\_surf}, \text{num\_slices})$ :

- output:

$\text{out} := (\text{evnslc} \Rightarrow \text{slip\_surf}'$  obtained by repeatedly applying  $\text{slip\_surf}[\text{large\_segment}(\text{slip\_surf})]$   
 $\parallel \text{midpoint}(\text{slip\_surf}[\text{large\_segment}(\text{slip\_surf})], \text{slip\_surf}[\text{large\_segment}(\text{slip\_surf})+1])$   
 $\parallel \text{slip\_surf}[\text{large\_segment}(\text{slip\_surf})+1]$  until  $|\text{slip\_surf}'| = \text{num\_slices}$   
 $\parallel \neg \text{evnslc} \Rightarrow \text{slip\_surf}'$  such that  $\forall(i : \mathbb{Z} \mid i \in [0..|\text{slip\_surf}'| - 2] : \text{slip\_surf}'[i] * \frac{\text{num\_slices}}{|\text{slip\_surf}'|-1} \cdot (i + 1) * \frac{\text{num\_slices}}{|\text{slip\_surf}'|-1} = \text{subslice}(\frac{\text{num\_slices}}{|\text{slip\_surf}'|-1}, \text{slip\_surf}[i], \text{slip\_surf}[i + 1]))$

### 13.4.5 Local Functions

$\text{large\_segment}(\text{surf}) : \text{coords} \rightarrow \mathbb{Z}$

$\text{large\_segment}(\text{surf}) = \text{index}$  such that

$\forall(i : \mathbb{Z} \mid i \in [0..|\text{surf}| - 2] : \text{surf}[\text{index} + 1] - \text{surf}[\text{index}] \geq \text{surf}[i + 1] - \text{surf}[i])$

$\text{midpoint}(\text{point1}, \text{point2}) : \text{coord} \times \text{coord} \rightarrow \text{coord}$

$\text{midpoint}(\text{point1}, \text{point2}) = \langle \frac{\text{point1}.x + \text{point2}.x}{2}, \frac{\text{point1}.y + \text{point2}.y}{2} \rangle$

$\text{subslice}(n, \text{point1}, \text{point2}) : \text{int} \times \text{coord} \times \text{coord} \rightarrow \text{coords}$

$\text{subslice}(n, \text{point1}, \text{point2}) = \text{subslices}$  such that

$\forall(i : \mathbb{Z} \mid i \in [0..n] : \text{subslices}[i].x = \text{point1}.x + \frac{i}{n} * (\text{point2}.x - \text{point1}.x) \wedge \text{subslices}[i].y = \text{point1}.y + \frac{i}{n} * (\text{point2}.y - \text{point1}.y))$

## 14 MIS of the Morgenstern-Price Calculation Module

### 14.1 Module

MorgPriceSolver

### 14.2 Uses

Input (Section 8), PropertyCalc (Section 15), Sequence (Section 16)

### 14.3 Syntax

#### 14.3.1 Exported Constants

$\text{MAX\_DIFF} = 0.000001$

$\text{MAX\_ITER} = 20$

$\text{MIN\_FS} = 0.5$

### 14.3.2 Exported Data Types

N/A

### 14.3.3 Exported Access Programs

Name	In	Out	Exceptions
morg_price	slip	slip	-

## 14.4 Semantics

### 14.4.1 State Variables

N/A

### 14.4.2 Environment Variables

N/A

### 14.4.3 Assumptions

N/A

### 14.4.4 Access Routine Semantics

morg\_price(*slip\_surf*):

- output:

*out* := *slip\_surf*' where *slip\_surf*'.*Fs*, *slip\_surf*'.*G*, and *slip\_surf*'.*X* satisfy the following system of equations, taken from the SRS document. The equations are presented with the symbols from the SRS document for readability, though the symbols *F<sub>S</sub>*, *G*, and *X* correspond to *slip\_surf*'.*Fs*, *slip\_surf*'.*G*, and *slip\_surf*'.*X*, respectively.

$$(IM1): F_S = \frac{\sum_{i=1}^{n-1} \left[ R_i \prod_{c=i}^{n-1} \Psi_c \right] + R_n}{\sum_{i=1}^{n-1} \left[ T_i \prod_{c=i}^{n-1} \Psi_c \right] + T_n}$$

$$\begin{aligned}
(\text{IM2}): C1_i &= \begin{cases} b_1 [G_1 + H_1] \tan(\alpha_1) & i = 1 \\ b_i [(G_i + G_{i-1}) + (H_i + H_{i-1})] \tan(\alpha_i) & 2 \leq i \leq n-1 \\ + h_i (-2 U_{t,i} \sin(\beta_i)) \\ b_n [G_{n-1} + H_{n-1}] \tan(\alpha_{n-1}) & i = n \end{cases} \\
C2_i &= \begin{cases} b_1 G_1 f_1 & i = 1 \\ b_i (f_i G_i + f_{i-1} G_{i-1}) & 2 \leq i \leq n-1 \\ b_n G_{n-1} f_{n-1} & i = n \end{cases} \\
\lambda &= \frac{\sum_{i=1}^n C1_i}{\sum_{i=1}^n C2_i} \\
(\text{IM3}): G_i &= \begin{cases} \frac{(FS)T_1 - R_1}{\Phi_i} & i=1 \\ \frac{\Psi_{i-1} \cdot G_{i-1} + (FS) \cdot T_i - R_i}{\Phi_i} & 2 \leq i \leq n-1 \\ 0 & i=0 \vee i=n \end{cases}
\end{aligned}$$

$$(\text{GD6}): X = \lambda \cdot f \cdot G$$

The solution method is to start with initial guesses  $F_S = 1$  and  $\lambda = 0$  and use them to compute  $F_S$  using IM1 and  $G$  using IM3, then use these values to compute a new guess for  $\lambda$  using IM2. This iteration continues until the absolute difference between  $F_S$  in the current iteration and in the previous iteration is less than MAX\_DIFF, or until the absolute difference between  $\lambda$  in the current iteration and in the previous iteration is less than MAX\_DIFF. When this occurs,  $X$  is computed using GD6, and  $\text{slip\_surf}'.Fs, \text{slip\_surf}'.G, \text{slip\_surf}'.X := F_S, G, X$ . If MAX\_ITER iterations occur, the solution is considered to be non-converging. If the solution converges but  $F_S < \text{MIN\_FS}$ , the solution is considered to be spurious. In either case,  $\text{slip\_surf}'.Fs, \text{slip\_surf}'.G, \text{slip\_surf}'.X := 1000, [], []$ .

#### 14.4.5 Local Functions

N/A

## 15 MIS of the Slice Property Calculation Module

### 15.1 Module

PropertyCalc

### 15.2 Uses

Input (Section 8), Sequence (Section 16)

### 15.3 Syntax

#### 15.3.1 Exported Constants

N/A

#### 15.3.2 Exported Data Types

paramsInternalForce = tuple of (Ub, Ut, W, H :  $[\mathbb{R}]$ ) (Appendix 19.1.5)

paramsAngles = tuple of (alpha, beta :  $[\mathbb{R}]$ ) (Appendix 19.1.6)

#### 15.3.3 Exported Access Programs

Name	In	Out	Exceptions
prop_calc	slip	-	-
ub	-	$[\mathbb{R}]$	-
ut	-	$[\mathbb{R}]$	-
w	-	$[\mathbb{R}]$	-
h	-	$[\mathbb{R}]$	-
alpha	-	$[\mathbb{R}]$	-
beta	-	$[\mathbb{R}]$	-
hts	-	$[\mathbb{R}]$	-

## 15.4 Semantics

### 15.4.1 State Variables

*force* : paramsInternalForce

*angles* : paramsAngles

*heights* :  $[\mathbb{R}]$

### 15.4.2 Environment Variables

N/A

### 15.4.3 Assumptions

- prop\_calc is called before any of the other access programs.

### 15.4.4 Access Routine Semantics

prop\_calc(*slip\_surf*):

- transition:

The equations used below contain symbols from the SRS document for this project for the sake of brevity. The SRS should be consulted for the definitions of these symbols.

*force*, *angles*, *heights* := *force*', *angles*', *soil*'*heights*' where

$\forall(i : \mathbb{Z} | i \in [1..|slip\_surf| - 1]) :$

$$\begin{aligned}
 force.Ub[i] &= \ell_{b,i} \begin{cases} (y_{wt,i} - y_{slip,i}) \gamma_w, & y_{wt,i} > y_{slip,i} \\ 0, & y_{wt,i} \leq y_{slip,i} \end{cases} \\
 \wedge force.Ut[i] &= \ell_{s,i} \begin{cases} (y_{wt,i} - y_{us,i}) \gamma_w, & y_{wt,i} > y_{us,i} \\ 0, & y_{wt,i} \leq y_{us,i} \end{cases} \\
 \wedge force.W[i] &= b_i \begin{cases} (y_{us,i} - y_{slip,i}) \gamma_{Sat}, & y_{wt,i} \geq y_{us,i} \\ (y_{us,i} - y_{wt,i}) \gamma + (y_{wt,i} - y_{slip,i}) \gamma_{Sat}, & y_{us,i} > y_{wt,i} > y_{slip,i} \\ (y_{us,i} - y_{slip,i}) \gamma, & y_{wt,i} \leq y_{slip,i} \end{cases} \\
 \wedge force.H[i] &= \begin{cases} \frac{[y_{us,i} - y_{slip,i}]^2}{2} \gamma_{Sat} + [y_{wt,i} - y_{us,i}]^2 \gamma_{Sat}, & y_{wt,i} \geq y_{us,i} \\ \frac{[y_{wt,i} - y_{slip,i}]^2}{2} \gamma_{Sat}, & y_{us,i} > y_{wt,i} > y_{slip,i} \\ 0, & y_{wt,i} \leq y_{slip,i} \end{cases} \\
 \wedge angles.alpha[i] &= \arctan \left( \frac{y_{slip,i} - y_{slip,i-1}}{x_{slip,i} - x_{slip,i-1}} \right)
 \end{aligned}$$

$$\wedge \text{angles.beta}[i] = \arctan \left( \frac{y_{us,i} - y_{us,i-1}}{x_{us,i} - x_{us,i-1}} \right)$$

$$\wedge \text{heights}[i] = 0.5 * ((y_{us,i} - y_{slip,i}) + (y_{us,i+1} - y_{slip,i+1}))$$

ub():

- output:

$$out := force.Ub$$

ut():

- output:

$$out := force.Ut$$

w():

- output:

$$out := force.W$$

h():

- output:

$$out := force.H$$

alpha():

- output:

$$out := angles.alpha$$

beta():

- output:

$$out := angles.beta$$

hts():

- output:

$$out := heights$$



#### 15.4.5 Local Functions

N/A

## 16 MIS of the Sequence Data Structure Module

### 16.1 Module

Sequence

### 16.2 Uses

N/A

### 16.3 Syntax

#### 16.3.1 Exported Constants

N/A

#### 16.3.2 Exported Data Types

[T] = sequence of T, where T is any type

#### 16.3.3 Exported Access Programs

Name	In	Out	Exceptions
[ $\_$ ]	Any number of values of type T	[T]	-
-( $\_$ )	[T], $\mathbb{Z}$	T	
-( $\_$ :- $\_$ )	[T], $\mathbb{Z}$ , $\mathbb{Z}$	[T]	-

### 16.4 Semantics

#### 16.4.1 State Variables

N/A

#### 16.4.2 Environment Variables

N/A

### 16.4.3 Assumptions

N/A

### 16.4.4 Access Routine Semantics

$[-]$ (Any number of values):

- output:

$out :=$  A sequence containing the arguments passed to the function.

$[-](list, int)$ :

- output:

$out := list[int]$

$[-:](list, int1, int2)$ :

- output:

$out := list[int1..int2]$

### 16.4.5 Local Functions

N/A

### 16.4.6 Considerations

This module is the sequence data type and operations on sequences implemented by Matlab.

## 17 MIS of the Random Number Generation Module

### 17.1 Module

Rand

### 17.2 Uses

N/A

### 17.3 Syntax

#### 17.3.1 Exported Constants

N/A

### 17.3.2 Exported Data Types

N/A

### 17.3.3 Exported Access Programs

Name	In	Out	Exceptions
rand	-	$\mathbb{R}$	-

## 17.4 Semantics

### 17.4.1 State Variables

N/A

### 17.4.2 Environment Variables

N/A

### 17.4.3 Assumptions

N/A

### 17.4.4 Access Routine Semantics

rand():

- output:

*out coloneqq* A random number in the interval (0,1).

### 17.4.5 Local Functions

N/A

### 17.4.6 Considerations

This module is the rand function implemented by Matlab.

## 18 MIS of the Plotting Module

### 18.1 Module

Plot

## 18.2 Uses

N/A

## 18.3 Syntax

### 18.3.1 Exported Constants

N/A

### 18.3.2 Exported Data Types

N/A

### 18.3.3 Exported Access Programs

Name	In	Out	Exceptions
plot	$[\mathbb{R}], [\mathbb{R}]$	-	-

## 18.4 Semantics

### 18.4.1 State Variables

N/A

### 18.4.2 Environment Variables

*screen* :  $[\mathbb{Z}]$

- *screen* represents the colour values for each pixel on the screen of the hardware running SSP.

### 18.4.3 Assumptions

N/A

### 18.4.4 Access Routine Semantics

plot(*x*, *y*):

- transition:

Modifies *screen* to display a plot with *x* on the horizontal axis and *y* on the vertical axis.

### **18.4.5 Local Functions**

N/A

### **18.4.6 Considerations**

This module is the plot function implemented by Matlab.

## References

- Carlo Ghezzi, Mehdi Jazayeri, and Dino Mandrioli. *Fundamentals of Software Engineering*. Prentice Hall, Upper Saddle River, NJ, USA, 2nd edition, 2003.
- David Gries and Fred B. Schneider. *A logical approach to discrete math*. Springer-Verlag Inc., New York, 1993.
- Daniel M. Hoffman and Paul A. Strooper. *Software Design, Automated Testing, and Maintenance: A Practical Approach*. International Thomson Computer Press, New York, NY, USA, 1995. URL <http://citeseer.ist.psu.edu/428727.html>.

## 19 Appendix

### 19.1 Parameter Tables

#### 19.1.1 Layer Parameters

The elements in the `paramsLayers` structure, which describe the mass of soil on which slope stability analysis is to be performed, are explained in the table below.

Parameter	Description
<i>strat</i> : coords	Coordinates describing the vertices of the slope of soil.
<i>phi</i> : $\mathbb{R}$	The effective angle of friction of the soil.
<i>coh</i> : $\mathbb{R}$	The effective cohesion of the soil.
<i>gam</i> : $\mathbb{R}$	The dry unit weight of the soil.
<i>gams</i> : $\mathbb{R}$	The saturated unit weight of the soil.

#### 19.1.2 Piezometric Parameter

The elements in the `paramsPiez` structure, which describe the water table, are explained in the table below.

Parameter	Description
<i>piez</i> : <i>coords</i>	Coordinates describing the vertices of the water table. If there is no water table than <i>piez</i> is an empty array.
<i>gamw</i> : $\mathbb{R}$	The unit weight of water.

#### 19.1.3 Search Range Parameters

The elements in the `paramsSearch` structure, which are parameters relating to the range of coordinates between which the critical slip surface may exist, are described in the table below.

Parameter	Description
Xext : $[\mathbb{R}]^{1,2}$	The range of $x$ -ordinates between which the exit point of the critical slip surface may exist. Exit refers to the point of the slip at lower elevation toward which the mass of soil will move during failure.
Xetr : $[\mathbb{R}]^{1,2}$	The range of $x$ -ordinates between which the entry point of the critical slip surface may exist. Entry refers to the point of the slip at higher elevation away from which the mass of soil will move during failure.
Ylim : $[\mathbb{R}]^{1,2}$	The range of $y$ -ordinates between which the critical slip surface may exist. The larger value should be greater than the max $y$ -ordinate of the slope. The smaller value is the lowest elevation to which the critical slip surface may descend.

#### 19.1.4 Solution Parameters

The elements in the `paramsSoln` structure, which are parameters relating to the solution method, are described in the table below.

Parameter	Description
ltor : $\mathbb{B}$	Direction the slope is expected to experience failure in. If true then the side of the slope with a greater $x$ -ordinate value is at a lower elevation. If false then the side of the slope with a greater $x$ -ordinate is at a higher elevation.
ftype : $\mathbb{B}$	Function to use for interslice normal/shear force ratio variation function. If true then the function is a constant (Spencer's method). If false then the function is a half-sine (standard Morgenstern-Price method).
evnslc : $\mathbb{B}$	Method for slicing a slip surface prior to analysis. If true then slice slip surface into equal $x$ -ordinate widths. If false then slice distance between vertices into even number of slices.
cncvu : $\mathbb{B}$	Concave slip surface admissibility criterion. If true then an admissible slip surface must be concave upwards towards the surface. If false then an admissible slip surface does not need to pass this criterion.
obtu : $\mathbb{B}$	Angle slip surface admissibility criterion. If true then an admissible slip surface must have all interior angles greater than a set limit. If false then an admissible slip surface does not need to pass this criterion.



### 19.1.5 Internal Force Parameters

The elements in the `paramsInternalForce` structure, which are parameters relating to the forces acting on a slice, and water in the slope acting on itself, are described in the table below.  $n$  refers to the number of slices composing the slip surface under evaluation, and is defined by the Slicer module (section 13).

Parameter	Description
$U_b : [\mathbb{R}]^{1,n}$	Sequence of the force acting on the basal surface of a slice as a result of pore water pressure within the slice. From DD2 of the SRS.
$U_t : [\mathbb{R}]^{1,n}$	Sequence of the force acting on the upper surface of a slice as a result of pore water pressure from standing water on the surface. From DD3 of the SRS.
$W : [\mathbb{R}]^{1,n}$	Sequence of the downward force acting on the slice caused by the mass of the slice and the force of gravity. From DD1 of the SRS.
$H : [\mathbb{R}]^{1,n-1}$	Sequence of the force acting into the interslice surfaces as a result of pore water pressure within the adjacent slices. From DD4 of the SRS.

### 19.1.6 Angle Parameters

The elements in the `paramsAngles` structure, which are parameters relating to the angles of the slice surfaces.  $n$  refers to the number of slices composing the slip surface under evaluation, and is defined by the Slicer module (section 13).

Parameter	Description
$\alpha : [\mathbb{R}]^{1,n}$	Sequence of the angle that the basal surface of the slice makes with the horizontal. From DD5 of the SRS.
$\beta : [\mathbb{R}]^{1,n}$	Sequence of the angle that the upper surface of the slice makes with the horizontal. From DD6 of the SRS.