Software Requirements Specification for Slope Stability Analysis

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Contents

1 Reference Material

This section records information for easy reference.

1.1 Table of Units

The unit system used throughout is SI (Système International d'Unités). In addition to the basic units, several derived units are also used. For each unit, the table lists the symbol, a description and the SI name.

Physical Property	Name	Symbol
force	Newton	N
length	meter	m
pressure	Pascal	$\mathrm{Pa} = \mathrm{N\,m^{-2}}$
angle	degree	$^\circ$

1.2 Table of Symbols

The table that follows summarizes the symbols used in this document along with their units. Throughout the document, values with a subscript i implies that the value will be taken at and analyzed at a slice or slice interface composing the total slip mass.

Symbol	Unit	Description
$\{x_{cs}, y_{cs}\}$	m	The Set of X and Y Coordinates: describe the vertices of the critical slip surface
(x, y)	m	Cartesian Position Coordinates: y is considered parallel to the direction of the force of gravity and x is considered perpendicular to y
a	m	Constant: FIXME: missing description
A	m	Constant: FIXME: missing description
b	m	Base Width of a Slice: in the x-ordinate direction only for slice index i
c'	Pa	Effective Cohesion: internal pressure that sticks particles of soil together
$C1$	Nm	Interslice Normal Force Function: FIXME: missing description
$C2$	Nm	Interslice Shear Force Function: FIXME: missing description

E	Pa	Elastic Modulus: The ratio of the stress exerted on a body to the resulting strain.
E	N	Interslice Normal Force: exerted between adjacent slices for interslice index i
F	N	Force: An interaction that tends to produce change in the motion of an object
F_x	N	X-Component of the Net Force: FIXME: missing description
F_y	N	Y-Component of the Net Force: FIXME: missing description
f		Scaling Function: magnitude of interslice forces as a function of the x coordinate for interslice index i; can be constant or a half-sine
FS		Factor of Safety: The global stability of a surface in a slope
$FS_{Loc,i}$		Local Factor of Safety: for slice index i
H	N	Interslice Water Force: exerted in the x-ordinate direction between adjacent slices for interslice index i
ΔH	N	Difference Between Interslice Forces: exerted in the x-ordinate direction between adjacent slices for interslice index i
h	m	Midpoint Height: distance from the slip base to the slope surface in a vertical line from the midpoint of the slice for slice index i
i		Index: used to show a quantity applies to only one slice
K_{bA}	$\mathrm{Pa\,m^{-1}}$	Effective Base Stiffness a: for rotated coordinates of a slice base surface, for slice index i
K_{bB}	$\mathrm{Pa\,m^{-1}}$	Effective Base Stiffness a: for rotated coordinates of a slice base surface, for slice index i
K	$\mathrm{Pa\,m^{-1}}$	Stiffness: The extent a body resists strain.
K_{bn}	$\mathrm{Pa\,m^{-1}}$	Normal Stiffness: for a slice base surface, without length adjustment for slice index i
K_{bt}	$\mathrm{Pa\,m^{-1}}$	Shear Stiffness: for a slice base surface, without length adjustment for slice index i

K_c		Earthquake Load Factor: proportionality factor of force that weight pushes outwards; caused by seismic earth movements
K_{no}	$\mathrm{Pa\,m^{-1}}$	Normal Stiffness: residual strength
K_{sn}	$\mathrm{Pa\,m^{-1}}$	Normal Stiffness: for an interslice surface, without length adjustment for interslice index i
K_{st}	$\mathrm{Pa\,m^{-1}}$	Shear Stiffness: for interslice surface, without length adjustment for interslice index i
K_{tr}	$\mathrm{Pa\,m^{-1}}$	Shear Stiffness: residual strength
M	N m	Moment of a Body: assumed 2D allowing a scalar
N	N	Normal Force: total reactive force for a soil surface subject to a body resting on it
N'	N	Effective Normal Force: for a soil surface, subtracting pore water reactive force from total reactive force
n		Number of Slices: the slip mass has been divided into
$N*$	N	Effective Normal Force: for a soil surface, without the influence of interslice forces
p	Pa	Pressure: A force exerted over an area
P	N	Resistive Shear Force: Mohr Coulomb frictional force that describes the limit of mobilized shear force the slice i can withstand before failure
Q	N	Imposed Surface Load: a downward force acting into the surface from midpoint of slice i
R	N	Resistive Shear Force: without the influence of interslice forces for slice index i
S	N	Mobilized Shear Force: for slice index i
s	Pa	Mobilized Shear Stress: acting on the base of a slice
T	N	Mobilized Shear Force: without the influence of interslice forces for slice index i
U_b	N	Base Hydrostatic Force: from water pressure within the slice for slice index i

U_t	N	Surface Hydrostatic Force: from water pressure acting into the slice from standing water on the slope surface for slice index i
u		Local Index: used as a bound variable index in calculations
v		Local Index: used as a bound variable index in calculations
W	N	Weight: downward force caused by gravity on slice i
x	m	X Ordinate: refers to either slice i midpoint, or slice interface i
x_{slip}	m	X Ordinate: distance of the slip surface at i, refers to either slice i midpoint, or slice interface i
x_{us}	m	X Ordinate: distance of the edge of the slope at i, refers to either slice i midpoint, or slice interface i
X	N	Interslice Shear Force: exerted between adjacent slices for interslice index i
y_{slip}	m	Y Ordinate: height of the slip surface at i, refers to either slice i midpoint, or slice interface i
y_{us}	m	Y Ordinate: height of the top of the slope at i, refers to either slice i midpoint, or slice interface i
y_{wt}	m	Y Ordinate: height of the water table at i, refers to either slice i midpoint, or slice interface i
y	m	Y Ordinate: refers to either slice i midpoint, or slice interface i
z	m	Center of Slice Height: the distance from the lowest part of the slice to the height of the centers of slice
α	$^\circ$	Angle: base of the mass relative to the horizontal for slice index i
β	$^\circ$	Angle: surface of the mass relative to the horizontal for slice index i

γ	$\mathrm{Pa\,N^{-3}}$	Dry Unit Weight: The weight of a dry soil/ground layer divided by the volume of the layer.
γ_{Sat}	$\mathrm{Pa\,N^{-3}}$	Saturated Unit Weight: The weight of saturated soil/ground layer divided by the volume of the layer.
γ_w	$\mathrm{Pa\,N^{-3}}$	Unit Weight of Water: The weight of one cubic meter of water.
δ	m	Displacement: generic displacement of a body
δn	m	Displacement: for the element parallel to the surface for slice index i
δt	m	Displacement: for the element normal to the surface for slice index i
δu	m	Displacement: shear displacement for slice index i
δv	m	Displacement: normal displacement for slice index i
δx	m	Displacement: in the x-ordinate direction for slice index i
δy	m	Displacement: in the y-ordinate direction for slice index i
ε	m	Displacement: in rotated coordinate system
κ	Pa	Constant: FIXME: missing description
λ		Interslice Normal/shear Force Ratio: applied to all interslices
μ	Pa	Pore Pressure: from water within the soil
ν		Poisson's Ratio: The ratio of perpendicular strain to parallel strain.
σ	Pa	Normal Stress: The stress exerted perpendicular to the plain of the object
τ	Pa	Resistive Shear Stress: acting on the base of a slice
Υ		Function: generic minimization function or algorithm

φ'	$^\circ$	Effective Angle of Friction: The angle of inclination with respect to the horizontal axis of the Mohr-Coulomb shear resistance line
Φ	N	Constant: converts resistive shear without the influence of interslice forces, to a calculation considering the interslice forces
Ψ	N	Constant: converts mobile shear without the influence of interslice forces, to a calculation considering the interslice forces
ω	$^\circ$	Angle: of imposed surface load acting into the surface relative to the vertical for slice index i
ℓ_b	m	Total Base Length of a Slice: for slice index i
ℓ_s	m	Length of an Interslice Surface: from slip base to slope surface in a vertical line from an interslice vertex for interslice index i

1.3 Abbreviations and Acronyms

symbol	**description**
A	Assumption
DD	Data Definition
GD	General Definition
GS	Goal Statement
IM	Instance Model
LC	Likely Change
PS	Physical System Description
R	Requirement
SRS	Software Requirements Specification
SSP	Slope Stability Analysis Program
T	Theoretical Model
TU	Typical Uncertainty

2 Introduction

A slope of geological mass, composed of soil and rock, is subject to the influence of gravity on the mass. For an unstable slope this can cause instability in the form of soil/rock movement. The effects of soil/rock movement can range from inconvenient to seriously hazardous, resulting in significant life and economic losses. Slope stability is of interest both when analyzing natural slopes, and when designing an excavated slope. Slope stability analysis is the assessment of the safety of a slope, identifying the surface most likely to experience slip and an index of its relative stability known as the factor of safety.

The following section provides an overview of the Software Requirements Specification (SRS) for a slope stability analysis problem. The developed program will be referred to as the Slope Stability Analysis Program (SSP) program. This section explains the purpose of this document, the scope of the system, the organization of the document, and the characteristics of the intended readers.

2.1 Purpose of Document

The SSP determines the critical slip surface, and its respective factor of safety as a method of assessing the stability of a slope design. The program is intended to be used as an educational tool for introducing slope stability issues, and will facilitate the analysis and design of a safe slope.

This document will be used as a starting point for subsequent development phases, including writing the design specification and the software verification and validation plan. The design document will show how the requirements are to be realized, including decisions on the numerical algorithms and programming environment. The verification and validation plan will show the steps that will be used to increase confidence in the software documentation and the implementation. Although the SRS fits in a series of documents that follow the so-called waterfall model, the actual development process is not constrained in any way. Even when the waterfall model is not followed, as Parnas and Clements point out, the most logical way to present the documentation is still to "fake" a rational design process.

2.2 Scope of Requirements

The scope of the requirements includes stability analysis of a 2 dimensional slope, composed of homogeneous soil layers. Given the appropriate inputs, the code for SSP is intended to identify the most likely failure surface within the possible input range, and find the factor of safety for the slope as well as displacement of soil that will occur on the slope.

2.3 Characteristics of Intended Reader

Reviewers of this documentation should have a strong knowledge in solid mechanics. The reviewers should also have an understanding of undergraduate level 4 physics. The users of SSP can have a lower level of expertise, as explained in Section **??**.

2.4 Organization of Document

The organization of this document follows the template for an SRS for scientific computing software proposed by [2] and [4]. The presentation follows the standard pattern of presenting goals, theories, definitions, and assumptions. For readers that would like a more bottom up approach, they can start reading the instance models in Section 4.2.5 and trace back to find any additional information they require. The goal statements are refined to the theoretical models, and theoretical models (Section 4.2.2) to the instance models (Section 4.2.5). The instance models provide the set of algebraic equations that must be solved iteratively to perform a Morgenstern Price Analysis, and the system of equations that must be solved for Rigid Finite Element Analysis.

3 General System Description

This section provides general information about the system including identifying the interfaces between the system and its environment (system context), describing the user characteristics, and listing the system constraints.

3.1 System Context

Figure 1 shows the system context. A circle represents an external entity outside the software, the user in this case. A rectangle represents the software system itself (SSP). Arrows are used to show the data flow between the system and its environment.

Figure 1: System Context

The interaction between the product and the user is through a user interface. The responsibilities of the user and the system are as follows:

- User Responsibilities:
  - Provide the input data related to the soil layer(s) and water table (if applicable), ensuring no errors in the data entry
  - Ensure that consistent units are used for input variables
  - Ensure required software assumptions (Section 4.2.1) are appropriate for any particular problem input to the software
- SSP Responsibilities:
  - Detect data type mismatch, such as a string of characters input instead of a floating point number
  - Determine if the inputs satisfy the required physical constraints
  - Identify the most likely failure surface within the possible input range
  - Find the factor of safety for the slope
  - Find the displacement of soil that will occur on the slope

3.2 User Characteristics

The end user of SSP should have an understanding of undergraduate Level 1 Calculus and Physics, and be familiar with soil and material properties.

3.3 System Constraints

There are no system constraints.

4 Specific System Description

This section first presents the problem description, which gives a high-level view of the problem to be solved. This is followed by the solution characteristics specification, which presents the assumptions, theories, definitions and finally the instance models that model the slope.

4.1 Problem Description

SSP is a computer program developed to evaluate the factor of safety of a slope's slip surface and, calculate the displacement the slope will experience.

4.1.1 Terminology

- *Factor of safety:* The global stability of a surface in a slope.
- *Critical slip surface:* Slip surface of the slope that has the lowest global factor of safety, and therefore most likely to experience failure.
- *Stress:* Forces that are exerted between planes internal to a larger body subject to external loading.
- *Strain:* Stress forces that result in deformation of the body/plane.
- *Normal Force:* A force applied perpendicular to the plane of the material.
- *Shear Force:* A force applied parallel to the plane of the material.
- *Tension:* A stress the causes displacement of the body away from its center.
- *Compression:* A stress the causes displacement of the body towards its center.
- *Plane Strain:* The resultant stresses in one of the directions of a 3 dimensional material can be approximated as 0. Results when the length of one dimension of the body dominates the others. Stresses in the dominant dimensions direction are the ones that can be approximated as 0.

4.1.2 Physical System Description

Analysis of the slope is performed by looking at properties of the slope as a series of slice elements. Some properties are interslice properties, and some are slice or slice base properties. The index convention for referencing which interslice or slice is being used is shown in Fig 1.

- Interslice properties convention is noted by j. The end Interslice properties are usually not of interest, therefore use the interslice properties from $1 \leq \mathrm{i} \leq n-1$.
- Slice properties convention is noted by i.

A free body diagram of the forces acting on the slice is displayed in Fig 2.

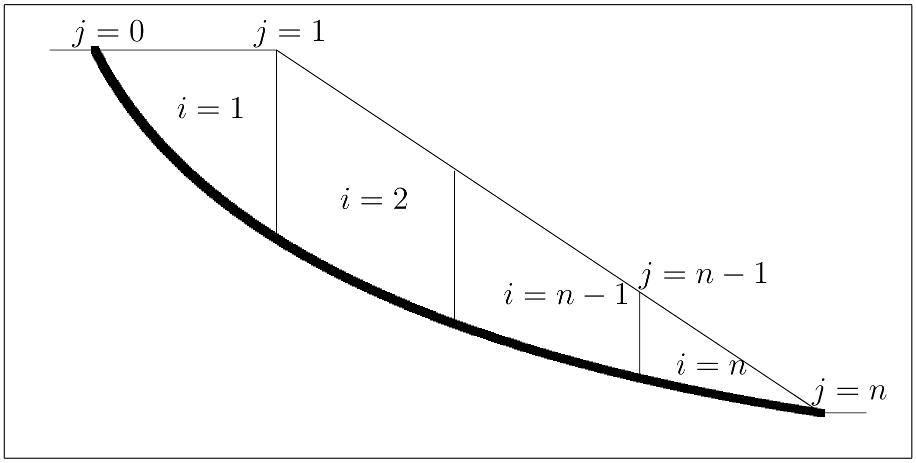

Figure 2: Index convention for numbering slice and interslice force variables

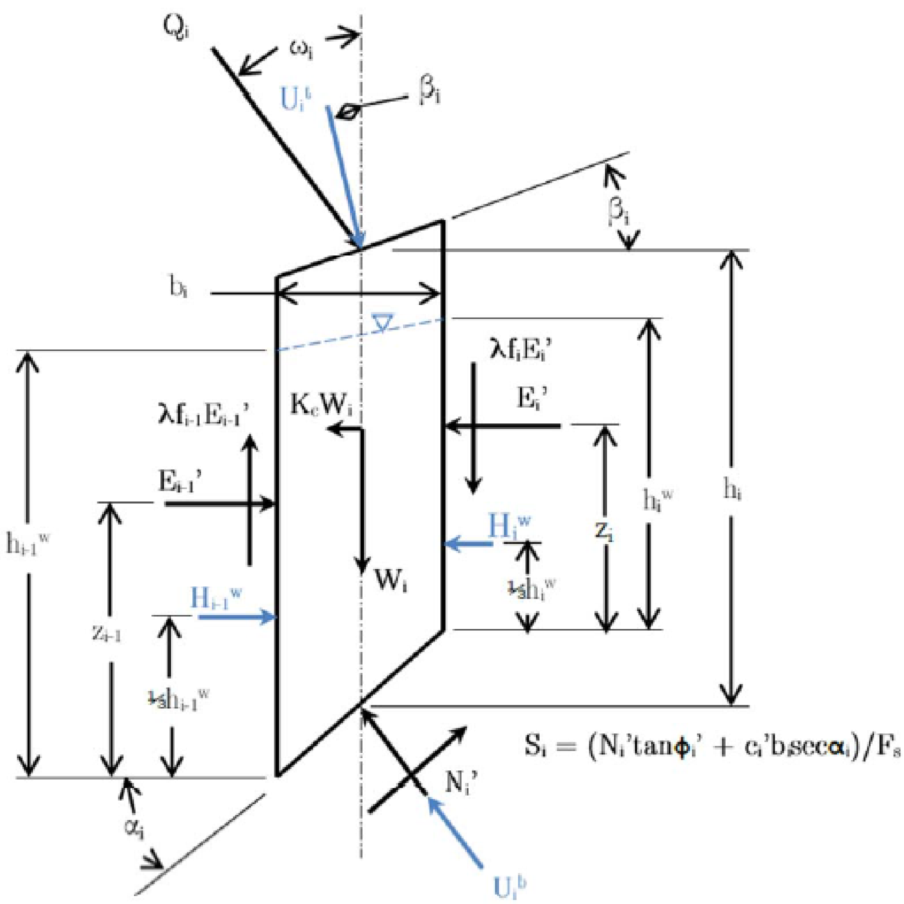

Figure 3: Forces acting on a slice

4.1.3 Goal statements

Given the geometry of the water table, the geometry of the layers composing the plane of a slope, and the material properties of the layers, the goal statements are:

G1: Evaluate local and global factors of safety along a given slip surface.

G2: Identify the critical slip surface for the slope, with the lowest factor of safety.

G3: Determine the displacement of the slope.

4.2 Solution Characteristics Specification

The instance models that govern SSP are presented in Section 4.2.5. The information to understand the meaning of the instance models and their derivation is also presented, so that the instance models can be verified.

4.2.1 Assumptions

This section simplifies the original problem and helps in developing the theoretical model by filling in the missing information for the physical system. The numbers given in the square brackets refer to the data definition, or the instance model, in which the respective assumption is used.

A1: The slip surface is concave with respect to the slope surface. The (x,y) coordinates of the failure surface follow a monotonic function.

A2: The geometry of the slope, and the material properties of the soil layers are given as inputs.

A3: The different layers of the soil are homogeneous, with consistent soil properties throughout, and independent of dry or saturated conditions, with the exception of unit weight.

A4: Soil layers are treated as if they have isotropic properties.

A5: Interslice normal and shear forces have a linear relationship, proportional to a constant (λ) and an interslice force function (f) depending on x position.

A6: Slice to base normal and shear forces have a linear relationship, dependent on the factor of safety (FS), and the Coulomb sliding law.

A7: The stress-strain curve for interslice relationships is linear with a constant slope.

A8: The slope and slip surface extends far into and out of the geometry (z coordinate). This implies plane strain conditions, making 2D analysis appropriate.

A9: The effective normal stress is large enough that the resistive shear to effective normal stress relationship can be approximated as a linear relationship.

A10: The surface and base of a slice between interslice nodes are approximated as straight lines.

4.2.2 Theoretical Models

This section focuses on the general equations and laws that SSP is based on.

Number	T1
Label	**Factor of Safety**
Equation	$\mathrm{FS} = \frac{P}{S}$
Description	The stability metric of the slope, known as the factor of safety FS, is determined by the ratio of the shear force at the base of the slope S, and the resistive shear P.
Source	[1]
Ref. By	IM1, GD4

Number	T2
Label	**Equilibrium**
Equation	$\sum F_{\mathrm{x}} = \sum F_{\mathrm{y}} = \sum M = 0$
Description	For a body in static equilibrium the net forces and net moments acting on the body will cancel out. Assuming a 2D problem (A8) the x-component of the net force F_x and y-component of the net force F_y will be equal to 0. All forces and their distance from the chosen point of rotation will create a net moment equal to 0.
Source	[1]
Ref. By	GD1, GD2, GD6, IM2

Number	T3
Label	**Mohr-Coulomb Shear Strength**
Equation	$P = \sigma \cdot \tan(\varphi') + c'$
Description	For a soil under stress it will exert a shear resistive strength based on the Coulomb sliding law. The resistive shear is the maximum amount of shear a surface can experience while remaining rigid, analogous to a maximum normal force. In this model the shear force P is proportional to the product of the normal stress on the plane σ with it's static friction, in the angular form $\tan(\varphi') = U_{\mathrm{s}}$. The P versus σ relationship is not truly linear, but assuming the effective normal force is strong enough it can be approximated with a linear fit (A9), where the cohesion c represents the P intercept of the fitted line.
Source	[1]
Ref. By	GD3, GD4, DD13, DD14, IM5

Number	T4
Label	**Effective Stress**
Equation	$\sigma' = \sigma - \mu$
Description	σ is the total stress a soil mass needs to maintain itself as a rigid collection of particles. The source of the stress can be provided by the soil skeleton σ', or by the pore pressure from water within the soil μ. The stress from the soil skeleton is known as the effective stress σ' and is the difference between the total stress σ and the pore stress μ.
Source	[1]
Ref. By	GD3, GD4, DD13, DD14, IM3

Number	T5
Label	**Hooke's Law**
Equation	$F = K \cdot \delta$
Description	Description Stiffness K is the resistance a body others to deformation by displacement δ when subject to a force F, along the same direction. A body with high stiffness will experience little deformation when subject to a force.
Source	[5]
Ref. By	GD8, GD9, DD19, IM4, IM5

4.2.3 General Definitions

This section collects the laws and equations that will be used in deriving the data definitions, which in turn are used to build the instance models.

Number	GD1
Label	**Normal Force Equilibrium**
Equation	$N_{\text{i}} = [W_{\text{i}} - X_{\text{i-1}} + X_{\text{i}} + U_{\text{t,i}} \cos(\beta_{\text{i}}) + Q_{\text{i}} \cos(\omega_{\text{i}})] \cos(\alpha_{\text{i}}) + [-K_{\text{c}} W_{\text{i}} - E_{\text{i}} + E_{\text{i-1}} - H_{\text{i}} + H_{\text{i-1}} + U_{\text{t,i}} \sin(\beta_{\text{i}}) + Q_{\text{i}} \sin(\omega_{\text{i}})] \sin(\alpha_{\text{i}})$
Description	For a slice of mass in the slope the force equilibrium to satisfy T2 in the direction perpendicular to the base surface of the slice. Rearranged to solve for the normal force of the surface N. Force equilibrium is derived from the free body diagram of Fig 2 in section 4.1.2. Index i refers to the values of the properties for slice/interslices following convention in Fig 1 in section 4.1.2. Force variable definitions can be found in DD1 to DD12.
Source	[6]
Ref. By	DD13, DD14, IM3

Number	GD2
Label	**Base Shear Force Equilibrium**
Equation	$S_{\text{i}} = [W_{\text{i}} - X_{\text{i-1}} + X_{\text{i}} + U_{\text{t,i}} \cos(\beta_{\text{i}}) + Q_{\text{i}} \cos(\omega_{\text{i}})] \sin(\alpha_{\text{i}}) - [-K_{\text{c}} W_{\text{i}} - E_{\text{i}} + E_{\text{i-1}} - H_{\text{i}} + H_{\text{i-1}} + U_{\text{t,i}} \sin(\beta_{\text{i}}) + Q_{\text{i}} \cos(\omega_{\text{i}})] \cos(\alpha_{\text{i}})$
Description	For a slice of mass in the slope the force equilibrium to satisfy T2 in the direction parallel to the base surface of the slice. Rearranged to solve for the shear force acting on the base S_{i}. Force equilibrium is derived from the free body diagram of Fig 2 in section 4.1.2. Index i refers to the values of the properties for slice/interslices following convention in Fig 1 in section 4.1.2. Force variable definitions can be found in DD1 to DD12.
Source	[6]
Ref. By	DD13, DD14,IM3

Number	GD3
Label	**Resistive Shear Force**
Equation	$P_{\mathrm{i}}=N'_{\mathrm{i}}\cdot\tan\left(\varphi'_{\mathrm{i}}\right)+c'\cdot b_{\mathrm{i}}\cdot\sec\left(\alpha_{\mathrm{i}}\right)$
Description	The Mohr-Coulomb resistive shear strength of a slice τ from T3 is multiplied by the area $b\sec\left(\alpha\right)\cdot 1$ to obtain the resistive shear force P. Note the extra 1 is to represent a unit of width which is multiplied by the total base length of a slice ℓ_b of the plane where the normal occurs, where $\ell_b=b\sec\left(\alpha\right)$ and b is the x width of the base. This accounts for the effective normal force $N'=N-U_b$ of a soil from T4 where the normal stress is multiplied by the same area to obtain the effective normal force $\sigma b\sec\left(\alpha\right)\cdot 1=N'$.
Source	[6]
Ref. By	GD4, DD13, DD14

Number	GD4
Label	**Mobile Shear**
Equation	$S_{\mathrm{i}}=\frac{P_{\mathrm{i}}}{\mathrm{FS}}=\frac{N'_{\mathrm{i}}\cdot\tan\left(\varphi'_{\mathrm{i}}\right)+c'\cdot b_{\mathrm{i}}\cdot\sec\left(\alpha_{\mathrm{i}}\right)}{\mathrm{FS}}$
Description	From the definition of the Factor of Safety in T1, and the new definition of P, a new relation for the net mobile shear force of the slice T is found as the resistive shear P (GD3) divided by the factor of safety FS.
Source	[6]
Ref. By	DD13, DD14

Number	GD5
Label	**Interslice Normal/Shear Relationship**
Equation	$X=\lambda\cdot f\cdot E$
Description	The assumption for the Morgenstern Price method (A5) that the interslice shear force X is proportional to the interslice normal force E by a proportionality constant λ, and a predetermined scaling function f, that changes the proportionality as a function of the x-ordinate position of the interslice. f is typically either a half-sine along the slip surface, or a constant.
Source	[6]
Ref. By	DD13, DD14, IM1, IM2, IM3

Number	GD6
Label	**Moment Equilibrium**
Equation	$0 = \begin{array}{l} -E_{\text{i}}\left[z_{\text{i}} + \frac{b_{\text{i}}}{2}\tan\left(\alpha_{\text{i}}\right)\right] + E_{\text{i-1}}\left[z_{\text{i-1}} - \frac{b_{\text{i}}}{2}\tan\left(\alpha_{\text{i}}\right)\right] - H_{\text{i}}\left[z_{\text{w,i}} + \frac{b_{\text{i}}}{2}\tan\left(\alpha_{\text{i}}\right)\right] \\ +H_{\text{i-1}}\left[z_{\text{w,i-1}} - \frac{b_{\text{i}}}{2}\tan\left(\alpha_{\text{i}}\right)\right] + \frac{b_{\text{i}}}{2}\left(X_{\text{i}} + X_{\text{i-1}}\right) - K_{\text{c}}W_{\text{i}}\frac{h_{\text{i}}}{2} + U_{\text{t,i}}\sin\left(\beta_{\text{i}}\right)h_{\text{i}} \\ +Q_{\text{i}}\sin\left(\omega_{\text{i}}\right)h_{\text{i}} \end{array}$
Description	For a slice of mass in the slope the moment equilibrium to satisfy T2 in the direction perpendicular to the base surface of the slice. Moment equilibrium is derived from the free body diagram of Fig 2 in section 4.1.2. Index i refers to the values of the properties for slice/interslices following convention in Fig 1 in section 4.1.2. Variable definitions can be found in DD1 to DD12.
Source	[6]
Ref. By	IM2

Number	GD7
Label	**Net X-Component Force**
Equation	$F_{\text{x,i}} = -\Delta H_{\text{i}} - K_{\text{c}} \cdot W_{\text{i}} - U_{\text{b,i}}\sin\left(\alpha_{\text{i}}\right) + U_{\text{t,i}}\sin\left(\beta\right) + Q_{\text{i}}\sin\left(\omega_{\text{i}}\right)$
Description	
Source	
Ref. By	

Number	GD8
Label	**Net Y-Component Force**
Equation	$F_{\text{y,i}} = -W_{\text{i}} + U_{\text{b,i}}\cos\left(\alpha_{\text{i}}\right) - U_{\text{t,i}}\cos\left(\beta_{\text{i}}\right) - Q_{\text{i}}\cos\left(\omega_{\text{i}}\right)$
Description	These equations show the net sum of the forces acting on a slice for the RFEM model and the forces that create an applied load on the slice. F_x refers to the load in the direction perpendicular to the direction of the force of gravity for slice i, while F_y refers to the load in the direction parallel to the force of gravity for slice i. Forces are found in the free body diagram of Fig 2 in section 4.1.2. In this model the elements are not exerting forces on each other, so the interslice forces E and X are not a part of the model. Index i refers to the values of the properties for slice/interslices following convention in Fig 1 in section 4.1.2. Force variable definitions can be found in DD1 to DD11.
Source	[6]
Ref. By	DD17, IM4

Number	GD9
Label	**Hooke's Law 2D**
Equation	$\begin{bmatrix} p_{\text{t,i}} \\ p_{\text{n,i}} \end{bmatrix} = \begin{bmatrix} K_{\text{t,i}} & 0 \\ 0 & K_{\text{n,i}} \end{bmatrix} \begin{bmatrix} \delta t_{\text{i}} \\ \delta n_{\text{i}} \end{bmatrix}$
Description	A 2D component implementation of Hooke's law as seen in T5. δn_{i} is the displacement of the element normal to the surface and δt_{i} is the displacement of the element parallel to the surface. $p_{\text{n,i}}$, is the net pressure acting normal to the surface, and $p_{\text{t,i}}$ is the net pressure acting parallel to the surface. Pressure is used in place of force as the surface has not been normalized for it's length. The stiffness values $K_{\text{n,i}}$ and $K_{\text{t,i}}$ are then the resistance to displacement in the respective directions defined as in DD19. The pressure forces would be the result of applied loads on the mass, the product of the stiffness elements with the displacement would be the mass's reactive force that creates equilibrium with the applied forces after reaching the equilibrium displacement.
Source	[5]
Ref. By	DD16, IM4

Number	GD10
Label	**Displacement Vectors**
Equation	$\varepsilon_i = \begin{bmatrix} \delta u_i \\ \delta v_i \end{bmatrix} = \begin{bmatrix} \cos(\alpha_i) & \sin(\alpha_i) \\ (-\sin(\alpha_i)) & \cos(\alpha_i) \end{bmatrix} \delta_i = \begin{bmatrix} \cos(\alpha_i) & \sin(\alpha_i) \\ (-\sin(\alpha_i)) & \cos(\alpha_i) \end{bmatrix} \begin{bmatrix} \delta x_i \\ \delta y_i \end{bmatrix}$
Description	Vectors describing the displacement of slice i. δ is the displacement in the unrotated coordinate system, where δx is the displacement of the slice perpendicular to the direction of gravity, and δy is the displacement of the slice parallel to the force of gravity. ε is the displacement in the rotated coordinate system, where δu is the displacement of the slice parallel to the slice base, and δy is the displacement of the slice perpendicular to the slice base. ε can also be found by rotating δ clockwise by the base angle, α through a rotation matrix as shown.
Source	[5]
Ref. By	DD16, IM4, IM5

4.2.4 Data Definition

This section collects and defines all the data needed to build the instance models. Definitions DD1 to DD11 are the force variables that can be solved by direct analysis of given inputs. The interslice forces DD12 are force variables that must be written in terms of DD1 to DD11 to solve.

Number	DD1
Label	**Weight**
Equation	$W = b_i \begin{cases} (y_{us,i} - y_{slip,i})\,\gamma_{Sat}, & y_{wt,i} \geq y_{us,i} \\ (y_{us,i} - y_{wt,i})\,\gamma + (y_{wt,i} - y_{slip,i})\,\gamma_{Sat}, & y_{us,i} > y_{wt,i} > y_{slip,i} \\ (y_{us,i} - y_{slip,i})\,\gamma, & y_{wt,i} \leq y_{slip,i} \end{cases}$
Description	W is the weight (N) b is the base width of a slice (m) i is the index y_{us} is the y ordinate (m) y_{slip} is the y ordinate (m) γ_{Sat} is the saturated unit weight ($\frac{\text{N}}{\text{m}^3}$) y_{wt} is the y ordinate (m) γ is the dry unit weight ($\frac{\text{N}}{\text{m}^3}$)
Sources	[1]
Ref. By	DD13, DD14, IM1, IM2, IM3

Number	DD2
Label	**Base Water Force**
Equation	$U_b = \ell_{b,i} \begin{cases} (y_{wt,i} - y_{slip,i})\,\gamma_w, & y_{wt,i} > y_{slip,i} \\ 0, & y_{wt,i} \leq y_{slip,i} \end{cases}$
Description	U_b is the base hydrostatic force (N) ℓ_b is the total base length of a slice (m) i is the index y_{wt} is the y ordinate (m) y_{slip} is the y ordinate (m) γ_w is the unit weight of water ($\frac{\mathrm{N}}{\mathrm{m}^3}$)
Sources	[1]
Ref. By	DD13, DD14, IM1, IM2, IM3

Number	DD3
Label	**Surface Hydrostatic Force**
Equation	$U_t = \ell_{s,i} \begin{cases} (y_{wt,i} - y_{us,i})\,\gamma_w, & y_{wt,i} > y_{us,i} \\ 0, & y_{wt,i} \leq y_{us,i} \end{cases}$
Description	U_t is the surface hydrostatic force (N) ℓ_s is the length of an interslice surface (m) i is the index y_{wt} is the y ordinate (m) y_{us} is the y ordinate (m) γ_w is the unit weight of water ($\frac{\mathrm{N}}{\mathrm{m}^3}$)
Sources	[1]
Ref. By	DD13, DD14, IM1, IM2, IM3

Number	DD4
Label	**Interslice Water Force**
Equation	$H = \begin{cases} \frac{\left[y_{us,i} - y_{slip,i}\right]^2}{2} \gamma_{Sat} + \left[y_{wt,i} - y_{us,i}\right]^2 \gamma_{Sat}, & y_{wt,i} \geq y_{us,i} \\ \frac{\left[y_{wt,i} - y_{slip,i}\right]^2}{2} \gamma_{Sat}, & y_{us,i} > y_{wt,i} > y_{slip,i} \\ 0, & y_{wt,i} \leq y_{slip,i} \end{cases}$
Description	H is the interslice water force (N) y_{us} is the y ordinate (m) i is the index y_{slip} is the y ordinate (m) γ_{Sat} is the saturated unit weight $\left(\frac{\mathrm{N}}{\mathrm{m}^3}\right)$ y_{wt} is the y ordinate (m)
Sources	[1]
Ref. By	DD13, DD14, IM1, IM2, IM3

Number	DD5
Label	**Angle**
Equation	$\alpha_{\mathrm{i}} = \frac{y_{\mathrm{slip,i}} - y_{\mathrm{slip,i\text{-}1}}}{x_{\mathrm{slip,i}} - x_{\mathrm{slip,i\text{-}1}}}$
Description	α is the angle (°) y_{slip} is the y ordinate (m) i is the index x_{slip} is the x ordinate (m)
Sources	[1]
Ref. By	DD13, DD14, IM1, IM2, IM3

Number	DD6
Label	**Angle**
Equation	$\beta_{\mathrm{i}} = \frac{y_{\mathrm{us,i}} - y_{\mathrm{us,i\text{-}1}}}{x_{\mathrm{us,i}} - x_{\mathrm{us,i\text{-}1}}}$
Description	β is the angle (°) y_{us} is the y ordinate (m) i is the index x_{us} is the x ordinate (m)
Sources	[1]
Ref. By	DD13, DD14, IM1, IM2, IM3

Number	DD7
Label	**Base Width of a Slice**
Equation	$b = x_{slip,i} - x_{slip,i-1}$
Description	b is the base width of a slice (m) x_{slip} is the x ordinate (m) i is the index
Sources	[1]
Ref. By	DD13, DD14, IM1, IM2, IM3

Number	DD8
Label	**Total Base Length of a Slice**
Equation	$\ell_b = b_i \sec(\alpha_i)$
Description	ℓ_b is the total base length of a slice (m) b is the base width of a slice (m) i is the index α is the angle (°)
Sources	[1]
Ref. By	DD13, DD14, IM1, IM2, IM3

Number	DD9
Label	**Length of an Interslice Surface**
Equation	$\ell_s = b_i \sec(\beta_i)$
Description	ℓ_s is the length of an interslice surface (m) b is the base width of a slice (m) i is the index β is the angle (°)
Sources	[1]
Ref. By	DD13, DD14, IM1, IM2, IM3

Number	DD10
Label	**Earthquake Load Factor**
Equation	$K_c = K_c W_i$
Description	K_c is the earthquake load factor W is the weight (N) i is the index
Sources	[1]
Ref. By	DD13, DD14, IM1, IM2, IM3

Number	DD11
Label	**Imposed Surface Loads**
Equation	$Q = Q_i \omega_i$
Description	Q is the imposed surface load (N) i is the index ω is the angle (°)
Sources	[6]
Ref. By	DD13, DD14, IM1, IM2, IM3

Number	DD12
Label	**Interslice ShearForces**
Equation	$X = \lambda f_i E_i$
Description	X is the interslice shear force (N) λ is the interslice normal/shear force ratio f is the scaling function i is the index E is the interslice normal force (N)
Sources	[6]
Ref. By	DD13, DD14, IM1, IM2, IM3

Number	DD13
Label	**Resistive Shear, Without Interslice Forces**
Equation	$R = \left(\begin{array}{l}[W_{\text{i}} + U_{\text{t,i}} \cos(\beta_{\text{i}}) + Q_{\text{i}} \cos(\omega_{\text{i}})] \cos(\alpha_{\text{i}}) \\ + [-K_{\text{c}} W_{\text{i}} - \Delta H_{\text{i}} + U_{\text{t,i}} \sin(\beta_{\text{i}}) + Q_{\text{i}} \sin(\omega_{\text{i}})] \sin(\alpha_{\text{i}}) - U_{\text{b,i}}\end{array}\right) \cdot \tan(\varphi') + c'_{\text{i}} \cdot b_{\text{i}} \cdot \sec(\alpha_{\text{i}})$
Description	R is the resistive shear force (N) W is the weight (N) i is the index U_t is the surface hydrostatic force (N) β is the angle (°) Q is the imposed surface load (N) ω is the angle (°) α is the angle (°) K_c is the earthquake load factor ΔH is the difference between interslice forces (N) U_b is the base hydrostatic force (N) φ' is the effective angle of friction (°) c' is the effective cohesion (Pa) b is the base width of a slice (m)
Sources	[6]
Ref. By	IM1

Resistive Shear Force, Without the Influence of Interslice Forces Derivation

The resistive shear force of a slice is defined as P_{i} in GD3. The effective normal in the equation for P_{i} of the soil is defined in the perpendicular force equilibrium of a slice from GD2, Using the effective normal N'_{i} of T4 shown in equation (1).

$$
\begin{aligned}
N'_{\text{i}} \;=\; & [W_{\text{i}} - X_{\text{i-1}} + X_{\text{i}} + U_{\text{t,i}} \cos(\beta_{\text{i}}) + Q_{\text{i}} \cos(\omega_{\text{i}})] \cos(\alpha_{\text{i}}) \\
& + [-K_{\text{c}}\, W_{\text{i}} - E_{\text{i}} + E_{\text{i-1}} - H_{\text{i}} + H_{\text{i-1}} + U_{\text{t,i}} \sin(\beta_{\text{i}}) + Q_{\text{i}} \sin(\omega_{\text{i}})] \sin(\alpha_{\text{i}}) \\
& - U_{\text{b,i}}
\end{aligned} \tag{1}
$$

The values of the interslice forces E and X in the equation are unknown, while the other values are found from the physical force definitions of DD1 to DD12. Consider a force equilibrium without the affect of interslice forces, to obtain a solvable value as done for N^*_{i} in equation (2).

$$
\begin{aligned}
N^*_{\text{i}} \;=\; & [W_{\text{i}} + U_{\text{t,i}} \cos(\beta_{\text{i}}) + Q_{\text{i}} \cos(\omega_{\text{i}})] \cos(\alpha_{\text{i}}) \\
& + [-K_{\text{c}}\, W_{\text{i}} - H_{\text{i}} + H_{\text{i-1}} + U_{\text{t,i}} \sin(\beta_{\text{i}}) + Q_{\text{i}} \sin(\omega_{\text{i}})] \sin(\alpha_{\text{i}}) - U_{\text{b,i}}
\end{aligned} \tag{2}
$$

Using N^*_{i}, a resistive shear force neglecting the influence of interslice forces can be solved for in terms of all known values as done in equation (3).

$$R_{\text{i}} = N^*_{\text{i}} \tan(\varphi') + c'_{\text{i}} \cdot b'_{\text{i}} \sec(\alpha_t exti')$$

$$R_{\mathrm{i}} = \left(\begin{array}{l} [W_{\mathrm{i}} + U_{\mathrm{t,i}} \cos(\beta_{\mathrm{i}}) + Q_{\mathrm{i}} \cos(\omega_{\mathrm{i}})] \cos(\alpha_{\mathrm{i}}) \\ + [-K_{\mathrm{c}} W_{\mathrm{i}} - \Delta H_{\mathrm{i}} + U_{\mathrm{t,i}} \sin(\beta_{\mathrm{i}}) + Q_{\mathrm{i}} \sin(\omega_{\mathrm{i}})] \sin(\alpha_{\mathrm{i}}) - U_{\mathrm{b,i}} \end{array} \right) \cdot \tan(\varphi') + c'_{\mathrm{i}} \cdot b_{\mathrm{i}} \cdot \sec(\alpha_{\mathrm{i}}) \quad (3)$$

Number	DD14
Label	**Mobile Shear, Without Interslice Forces**
Equation	$T = (W_i + U_{t,i} \cos(\beta_i) + Q_i \cos(\omega_i)) \sin(\alpha_i) - (-K_c W_i - \Delta H_i + U_{t,i} \sin(\beta_i) + Q_i \sin(\omega_i)) \cos(\alpha_i)$
Description	T is the mobilized shear force (N) W is the weight (N) i is the index U_t is the surface hydrostatic force (N) β is the angle (°) Q is the imposed surface load (N) ω is the angle (°) α is the angle (°) K_c is the earthquake load factor ΔH is the difference between interslice forces (N)
Sources	[6]
Ref. By	IM1

Mobile Shear Force, Without the Influence of Interslice Forces Derivation

The mobile shear force acting on a slice is defined as S_{i} from the force equilibrium in GD2,also shown in equation (4).

$$S_{\mathrm{i}} = \begin{array}{l} [W_{\mathrm{i}} - X_{\text{i-1}} + X_{\mathrm{i}} + U_{\mathrm{t,i}} \cos(\beta_{\mathrm{i}}) + Q_{\mathrm{i}} \cos(\omega_{\mathrm{i}})] \sin(\alpha_{\mathrm{i}}) \\ - [-K_{\mathrm{c}} W_{\mathrm{i}} - E_{\mathrm{i}} + E_{\text{i-1}} - H_{\mathrm{i}} + H_{\text{i-1}} + U_{\mathrm{t,i}} \sin(\beta_{\mathrm{i}}) + Q_{\mathrm{i}} \cos(\omega_{\mathrm{i}})] \cos(\alpha_{\mathrm{i}}) \end{array} \quad (4)$$

The equation is unsolvable, containing the unknown interslice normal force E and shear force X. Consider a force equilibrium without the affect of interslice forces, to obtain the mobile shear force without the influence of interslice forces T, as done in equation (5).n

$$T_{\mathrm{i}} = \begin{array}{l} [W_{\mathrm{i}} + U_{\mathrm{t,i}} \cos(\beta_{\mathrm{i}}) + Q_{\mathrm{i}} \cos(\omega_{\mathrm{i}})] \sin(\alpha_{\mathrm{i}}) \\ - [-K_{\mathrm{c}} W_{\mathrm{i}} - \Delta H_{\mathrm{i}} + U_{\mathrm{t,i}} \sin(\beta_{\mathrm{i}}) + Q_{\mathrm{i}} \sin(\omega_{\mathrm{i}})] \cos(\alpha_{\mathrm{i}}) \end{array} \quad (5)$$

The values of R_{i} and T_{i} are now defined completely in terms of the known force property values of DD1 to DD12.

Number	DD15
Label	**Force**
Equation	$p = \begin{bmatrix} K_{st,i} & 0 \\ 0 & K_{bn,i} \end{bmatrix} \begin{bmatrix} \delta x_i \\ \delta y_i \end{bmatrix}$
Description	p is the pressure (Pa) K_{st} is the shear stiffness ($\frac{\text{Pa}}{\text{m}}$) i is the index K_{bn} is the normal stiffness ($\frac{\text{Pa}}{\text{m}}$) δx is the displacement (m) δy is the displacement (m)
Sources	[5]
Ref. By	IM4

Number	DD16
Label	**Force**
Equation	$p = \begin{bmatrix} K_{bA,i} & K_{bB,i} \\ K_{bB,i} & K_{bA,i} \end{bmatrix} \begin{bmatrix} \delta x_i \\ \delta y_i \end{bmatrix}$
Description	p is the pressure (Pa) K_{bA} is the effective base stiffness A ($\frac{\text{Pa}}{\text{m}}$) i is the index K_{bB} is the effective base stiffness A ($\frac{\text{Pa}}{\text{m}}$) δx is the displacement (m) δy is the displacement (m)
Sources	[5]
Ref. By	IM4

Derivation of Stifness Matrixes

Using the force-displacement relationship of GD9 to define stiffness matrix K_{st}, as seen in equation (6).

$$K_{\text{st,i}} = \begin{bmatrix} K_{\text{st,i}} & 0 \\ 0 & K_{\text{bn,i}} \end{bmatrix} \tag{6}$$

For interslice surfaces the stiffness constants and displacements refer to an unrotated coordinate system, δ of GD10. The interslice elements are left in their standard coordinate system, and therefore are described by the same equation from GD9. Seen as K_{st} in DD16. K_{st} is the shear element in the matrix, and K_{sn} is the normal element in the matrix, calculated as in DD19.

For basal surfaces the stiffness constants and displacements refer to a system rotated for the base angle alpha (DD5). To analyze the effect of force-displacement relationships occurring on both basal and interslice surfaces of an element i they must reference the same coordinate system. The basal stiffness matrix must be rotated counter clockwise to align with the angle of the basal surface. The base stiffness counter clockwise rotation is applied in equation (7) to the new matrix $\bar{K_{\mathrm{i}}^{*}}$.

$$\begin{aligned} \bar{K_{\mathrm{i}}^{*}} &= \begin{bmatrix} \cos(\alpha_{\mathrm{i}}) & -\sin(\alpha_{\mathrm{i}}) \\ \sin(\alpha_{\mathrm{i}}) & \cos(\alpha_{\mathrm{i}}) \end{bmatrix} \bar{K_{\mathrm{i}}} \\ &= \begin{bmatrix} K_{\mathrm{bt,i}}\cos(\alpha_{\mathrm{i}}) & -K_{\mathrm{bn,i}}\sin(\alpha_{\mathrm{i}}) \\ K_{\mathrm{bt,i}}\sin(\alpha_{\mathrm{i}}) & K_{\mathrm{bn,i}}\cos(\alpha_{\mathrm{i}}) \end{bmatrix} \end{aligned} \tag{7}$$

The Hooke's law force displacement relationship of GD9 applied to the base also references a displacement vector $\bar{\epsilon}_{\mathrm{i}}$ of GD10 rotated for the base angle angle of the slice α_{i}. The basal displacement vector $\bar{\epsilon}_{\mathrm{i}}$ is rotated clockwise to align with the interslice displacement vector $\bar{\delta}_{\mathrm{i}}$, applying the definition of $\bar{\epsilon}_{\mathrm{i}}$ in terms of $\bar{\delta}_{\mathrm{i}}$ as seen in GD10. Using this with base stiffness matrix $\bar{K^{*}}_{\mathrm{i}}$, a basal force displacement relationship in the same coordinate system as the interslice relationship can be derived as done in equation (8).

$$\begin{aligned} \begin{bmatrix} p_{\mathrm{bx,i}} \\ p_{\mathrm{by,i}} \end{bmatrix} &= \bar{K_{\mathrm{i}}^{*}}\ \bar{\epsilon} \\ &= \begin{bmatrix} K_{\mathrm{bt,i}}\cos(\alpha_{\mathrm{i}}) & -K_{\mathrm{bn,i}}\sin(\alpha_{\mathrm{i}}) \\ K_{\mathrm{bt,i}}\sin(\alpha_{\mathrm{i}}) & K_{\mathrm{bn,i}}\cos(\alpha_{\mathrm{i}}) \end{bmatrix} \begin{bmatrix} \cos(\alpha_{\mathrm{i}}) & \sin(\alpha_{\mathrm{i}}) \\ -\sin(\alpha_{\mathrm{i}}) & \cos(\alpha_{\mathrm{i}}) \end{bmatrix} \begin{bmatrix} \delta x_{\mathrm{i}} \\ \delta y_{\mathrm{i}} \end{bmatrix} \\ &= \begin{bmatrix} K_{\mathrm{bt,i}}\cos^2(\alpha_{\mathrm{i}}) + K_{\mathrm{bn,i}}\sin^2(\alpha_{\mathrm{i}}) & (K_{\mathrm{bt,i}} - K_{\mathrm{bn,i}})\sin(\alpha_{\mathrm{i}})\cos(\alpha_{\mathrm{i}}) \\ (K_{\mathrm{bt,i}} - K_{\mathrm{bn,i}})\sin(\alpha_{\mathrm{i}})\cos(\alpha_{\mathrm{i}}) & K_{\mathrm{bt,i}}\cos^2(\alpha_{\mathrm{i}}) + K_{\mathrm{bn,i}}\sin^2(\alpha_{\mathrm{i}}) \end{bmatrix} \begin{bmatrix} \delta x_{\mathrm{i}} \\ \delta y_{\mathrm{i}} \end{bmatrix} \end{aligned} \tag{8}$$

The new effective base stiffness matrix $\bar{K_{\mathrm{i}}'}$,as derived in equation (7) is defined in equation (9). This is seen as matrix $\bar{K}_{\mathrm{b,i}}$ in GD16. $K_{\mathrm{bt,i}}$ is the shear element in the matrix, and $K_{\mathrm{bn,i}}$ is the normal element in the matrix, calculated as in DD19. The notation is simplified by the introduction of the constants $K_{\mathrm{bA,i}}$ and $K_{\mathrm{bB,i}}$, defined in equations (10) and (11) respectively.

$$\begin{aligned} \bar{K_{\mathrm{i}}'} &= \begin{bmatrix} K_{\mathrm{bt,i}}\cos^2(\alpha_{\mathrm{i}}) + K_{\mathrm{bn,i}}\sin^2(\alpha_{\mathrm{i}}) & (K_{\mathrm{bt,i}} - K_{\mathrm{bn,i}})\sin(\alpha_{\mathrm{i}})\cos(\alpha_{\mathrm{i}}) \\ (K_{\mathrm{bt,i}} - K_{\mathrm{bn,i}})\sin(\alpha_{\mathrm{i}})\cos(\alpha_{\mathrm{i}}) & K_{\mathrm{bt,i}}\cos^2(\alpha_{\mathrm{i}}) + K_{\mathrm{bn,i}}\sin^2(\alpha_{\mathrm{i}}) \end{bmatrix} \\ \bar{K_{\mathrm{i}}'} &= \begin{bmatrix} K_{\mathrm{bA,i}} & K_{\mathrm{bB,i}} \\ K_{\mathrm{bB,i}} & K_{\mathrm{bA,i}} \end{bmatrix} \end{aligned} \tag{9}$$

$$K_{\mathrm{bA,i}} = K_{\mathrm{bt,i}}\cos^2(\alpha_{\mathrm{i}}) + K_{\mathrm{bn,i}}\sin^2(\alpha_{\mathrm{i}}) \tag{10}$$

$$K_{\mathrm{bB,i}} = (K_{\mathrm{bt,i}} - K_{\mathrm{bn,i}})\sin(\alpha_{\mathrm{i}})\cos(\alpha_{\mathrm{i}}) \tag{11}$$

A force-displacement relationship for an element i can be written in terms of displacements occurring in the unrotated coordinate system $\bar{\delta}_{\mathrm{i}}$ of GD10 using the matrix $K_{\mathrm{s,i}}$, and $K_{\mathrm{b,i}}$ as seen in DD16.

Number	DD17
Label	**Force**
Equation	$F = (-\ell_{s,i-1}) K_{sn,i-1} \delta_{i-1} + (\ell_{s,i-1} K_{sn,i-1} + \ell_{b,i} K_{bn,i} + \ell_{s,i} K_{sn,i}) \delta_i - \ell_{s,i} K_{sn,i} \delta_{i+1}$
Description	F is the force (N) ℓ_s is the length of an interslice surface (m) i is the index K_{sn} is the normal stiffness ($\frac{\text{Pa}}{\text{m}}$) δ is the displacement (m) ℓ_b is the total base length of a slice (m) K_{bn} is the normal stiffness ($\frac{\text{Pa}}{\text{m}}$)
Sources	[5]
Ref. By	IM4

Number	DD18
Label	**Shear Stiffness**
Input	E , ν , b , c , σ , ϕ , κ a , A , u , v
Output	$K_{bt} = \frac{E}{2(1+\nu)} \frac{0.1}{b} + \frac{{c'}_i - \sigma \tan({\varphi'}_i)}{\lvert \delta u \rvert + a}$
Description	K_{bt} is the shear stiffness ($\frac{\text{Pa}}{\text{m}}$) E is the interslice normal force (N) ν is the Poisson's ratio b is the base width of a slice (m) c' is the effective cohesion (Pa) i is the index σ is the normal stress (Pa) φ' is the effective angle of friction (°) δu is the displacement (m) a is the constant (m)
Sources	[5]
Ref. By	IM4, IM5

Number	DD19
Label	**Normal Stiffness**
Input	E , ν , b , c , σ , ϕ , κ a , A , u , v
Output	$K_{bn} = \begin{cases} \frac{E(1-\nu)}{(1+\nu)(1-2\nu+b)}, & \nu < 0 \\ 0.01\frac{E(1-\nu)}{(1+\nu)(1-2\nu+b)} + \frac{\kappa}{\delta v+A}, & \nu \geq 0 \end{cases}$
Description	K_{bn} is the normal stiffness ($\frac{\text{Pa}}{\text{m}}$) E is the interslice normal force (N) ν is the Poisson's ratio b is the base width of a slice (m) κ is the constant (Pa) δv is the displacement (m) A is the constant (m)
Sources	[5]
Ref. By	IM4, IM5

4.2.5 Instance Models

This section transforms the problem defined in the Section 4.1 into one which is expressed in mathematical terms. It uses concrete symbols defined in Section 4.2.4 to replace the abstract symbols in the models identified in the Sections 4.2.2 and 4.2.3.

The Morgenstern Price Method is a vertical slice, limit equilibrium slope stability analysis method. Analysis is performed by breaking the assumed failure surface into a series of vertical slices of mass. Static equilibrium analysis using two force equilibrium, and one moment equation as in T2. The problem is statically indeterminate with only these 3 equations and one constitutive equation (the Mohr Coulomb shear strength of T3) so the assumption of GD5 is used. Solving for force equilibrium allows definitions of all forces in terms of the physical properties of DD1 to DD12, as done in DD13, DD14.

The values of the interslice normal force E the interslice normal/shear force magnitude ratio λ, and the Factor of Safety FS, are unknown. Equations for the unknowns are written in terms of only the values in DD1 to DD12, the values of R, and T in DD13 and DD14, and each other. The relationships between the unknowns are non linear, and therefore explicit equations cannot be derived and an iterative solution method is required.

Number	IM1
Label	**Factor of Safety**
Input	Ψ_{v} , Φ_{v} , T_{v} , R_{v}
Output	$FS = \frac{\sum_{v=1}^{n-1}\left[R_v \prod_{c=i}^{n-1} \frac{\Psi_u}{\Phi_u}\right] + R_n}{\sum_{v=1}^{n-1}\left[T_v \prod_{c=i}^{n-1} \frac{\Psi_u}{\Phi_u}\right] + T_n}$
Description	Equation for the Factor of Safety, the ratio between resistive and mobile shear the slip surface. The sum of values from each slice is taken to find the total resistive and mobile shear for the slip surface. The constants Φ and Ψ convert the resistive and mobile shear without the influence of interslice forces, to a calculation considering the interslice forces.
Sources	[6]
Ref. By	IM2, IM3

Factor of Safety Derivation

Using equation (21) from section 4.2.5, rearranging, and applying the boundary condition that E_0 and E_{n} are equal to 0 an equation for the factor of safety is found as equation (12), also seen in IM1.

$$\text{FS} = \frac{\sum_{v=1}^{n-1}\left[R_v \prod_{c=v}^{n-1} \frac{\Psi_u}{\Phi_u}\right] + R_n}{\sum_{v=1}^{n-1}\left[T_v \prod_{c=v}^{n-1} \frac{\Psi_u}{\Phi_u}\right] + T_n} \tag{12}$$

The constants Ψ and Φ described in equations 20 and 19 are functions of the unknowns: the interslice normal/shear force ratio λ (IM2) and the Factor of Safety FS (IM1).

Number	IM2
Label	**Normal/Shear Force Ratio**
Input	b_{v} , E_{v} , H_{v} , α_{v} , h_{v} , W_{v} , $U_{\text{t,v}}$, β_{v} , f_{v} , K_{c}
Output	$C1_{\text{i}} = \begin{cases} b_1 \left[E_1 + H_1\right] \tan\left(\alpha_1\right) & \text{i} = 1 \\ \begin{array}{l} b_{\text{i}} \left[\left(E_{\text{i}} + E_{\text{i-1}}\right) + \left(H_{\text{i}} + H_{\text{i-1}}\right)\right] \tan\left(\alpha_{\text{i}}\right) \\ + h_{\text{i}} \left(K_{\text{c}}\, W_{\text{i}} - 2\, U_{\text{t,i}} \sin\left(\beta_{\text{i}}\right) - 2\, Q_{\text{i}} \cos\left(\omega_{\text{i}}\right)\right) \end{array} & 2 \leq \text{i} \leq \text{n-1} \\ b_{\text{n}} \left[E_{\text{n-1}} + H_{\text{n-1}}\right] \tan\left(\alpha_{\text{n-1}}\right) & \text{i} = \text{n} \end{cases}$ $C2_{\text{i}} = \begin{cases} b_1 E_1 f_1 & \text{i} = 1 \\ b_{\text{i}} \left(f_{\text{i}} E_{\text{i}} + f_{\text{i-1}} E_{\text{i-1}}\right) & 2 \leq \text{i} \leq \text{n-1} \\ b_{\text{n}} E_{\text{n-1}} f_{\text{n-1}} & \text{v} = \text{n} \end{cases}$ $\lambda = \dfrac{\displaystyle\sum_{i=1}^{n} C1_{\text{i}}}{\displaystyle\sum_{i=1}^{n} C2_{\text{i}}}$
Description	λ is the magnitude ratio between shear and normal forces at the interslice interfaces as the assumption of the Morgenstern Price method in GD5. The inclination function f determines the relative magnitude ratio between the different interslices, while λ determines the magnitude. λ uses the sum of interslice normal and shear forces taken from each interslice.
Sources	[6]
Ref. By	IM1, IM3

Normal/Shear Force Ratio Derivation

The last static equation of T2 the moment equilibrium of GD6 about the midpoint of the base is taken, with the assumption of GD5. Results in equation (13).

$$0 = \begin{array}{l} -E_{\text{i}} \left[z_{\text{i}} - \frac{b_{\text{i}}}{2} \tan\left(\alpha_{\text{i}}\right)\right] + E_{\text{i-1}} \left[z_{\text{i-1}} + \frac{b_{\text{i}}}{2} \tan\left(\alpha_{\text{i}}\right)\right] - H_{\text{i}} \left[z_{\text{w,i}} - \frac{b_{\text{i}}}{2} \tan\left(\alpha_{\text{i}}\right)\right] \\ + H_{\text{i-1}} \left[z_{\text{w,i-1}} + \frac{b_{\text{i}}}{2} \tan\left(\alpha_{\text{i}}\right)\right] - \lambda \frac{b_{\text{i}}}{2} \left(E_{\text{i}} f_{\text{i}} + E_{\text{i-1}} f_{\text{i-1}}\right) + K_{\text{c}} W_{\text{i}} \frac{h_{\text{i}}}{2} - U_{\text{t,i}} \sin\left(\beta_{\text{i}}\right) h_{\text{i}} - Q_{\text{i}} \sin\left(\omega_{\text{i}}\right) h_{\text{i}} \end{array} \tag{13}$$

Rearranging the equation in terms of λ leads to equation (14).

$$\lambda = \frac{\begin{array}{c} -E_{\text{i}} \left[z_{\text{i}} - \frac{b_{\text{i}}}{2} \tan\left(\alpha_{\text{i}}\right)\right] + E_{\text{i-1}} \left[z_{\text{i-1}} + \frac{b_{\text{i}}}{2} \tan\left(\alpha_{\text{i}}\right)\right] - H_{\text{i}} \left[z_{\text{i}} - \frac{b_{\text{i}}}{2} \tan\left(\alpha_{\text{i}}\right)\right] \\ + H_{\text{i-1}} \left[z_{\text{i-1}} + \frac{b_{\text{i}}}{2} \tan\left(\alpha_{\text{i}}\right)\right] + K_{\text{c}} W_{\text{i}} \frac{h_{\text{i}}}{2} - U_{\text{t,i}} \sin\left(\beta_{\text{i}}\right) h_{\text{i}} - Q_{\text{i}} \sin\left(\omega_{\text{i}}\right) h_{\text{i}} \end{array}}{\frac{b_{\text{i}}}{2} \left[E_{\text{i}} f_{\text{i}} + E_{\text{i-1}} f_{\text{i-1}}\right]} \tag{14}$$

Taking a summation of each slice, and considering the boundary conditions that E_0 and E_n are equal to zero, a general equation for the constant λ is developed in equation (15), also found in IM2.

$$\lambda = \frac{\sum_{i=1}^{n} b_{\mathrm{i}} \left[(E_{\mathrm{i}} + E_{\mathrm{i\text{-}1}}) + (H_{\mathrm{i}} + H_{\mathrm{i\text{-}1}}) \right] \tan(\alpha_{\mathrm{i}}) + h_{\mathrm{i}} \left[K_{\mathrm{c}}\, W_{\mathrm{i}} - 2\, U_{\mathrm{t,i}} \sin(\beta_{\mathrm{i}}) - 2\, Q_{\mathrm{i}} \sin(\omega_{\mathrm{i}}) \right]}{\sum_{i=1}^{n} b_{\mathrm{i}} \left[f_i E_{\mathrm{i}} + f_{\mathrm{i\text{-}1}} E_{\mathrm{i\text{-}1}} \right]} \tag{15}$$

Equation (15) for λ, is a function of the unknown interslice normal force E (IM3).

Number	IM3
Label	**Interslice Forces**
Input	FS, T_{i}, R_{i}, Ψ, Φ
Output	$E_{\mathrm{i}} = \begin{cases} \frac{(\mathrm{FS})T_1 - R_1}{\Phi_{\mathrm{i}}} & \mathrm{i{=}1} \\ \frac{\Psi_{\mathrm{i\text{-}1}} \cdot E_{\mathrm{i\text{-}1}} + (\mathrm{FS}) \cdot T_{\mathrm{i}} - R_{\mathrm{i}}}{\Phi_{\mathrm{i}}} & 2 \leq \mathrm{i} \leq \mathrm{n\text{-}1} \\ 0 & \mathrm{i{=}0} \vee \mathrm{i{=}n} \end{cases}$
Description	The value of the interslice normal force E_{i} at interface i. The net force the weight of the slices adjacent to interface i exert horizontally on each other.
Sources	[6]
Ref. By	IM1, IM2

Interslice Force Derivation

Taking the perpendicular force equilibrium of GD1 with the effective stress definition from T4 that $N_{\mathrm{i}} = N'_{\mathrm{i}} - U_{\mathrm{b,i}}$, and the assumption of GD5 the equilibrium equation can be rewritten as equation (16).

$$N'_{\mathrm{i}} = \begin{array}{l} \left[W_{\mathrm{i}} - \lambda \cdot f_{\mathrm{i\text{-}1}} \cdot E_{\mathrm{i\text{-}1}} + \lambda \cdot f_{\mathrm{i}} \cdot E_{\mathrm{i}} + U_{\mathrm{t,i}} \cos(\beta_{\mathrm{i}}) + Q_{\mathrm{i}} \cos(\omega_{\mathrm{i}}) \right] \cos(\alpha_{\mathrm{i}}) \\ + \left[-K_{\mathrm{c}} W_{\mathrm{i}} - E_{\mathrm{i}} + E_{\mathrm{i\text{-}1}} - H_{\mathrm{i}} + H_{\mathrm{i\text{-}1}} + U_{\mathrm{t,i}} \sin(\beta_{\mathrm{i}}) + Q_{\mathrm{i}} \sin(\omega_{\mathrm{i}}) \right] \sin(\alpha_{\mathrm{i}}) - U_{\mathrm{b,i}} \end{array} \tag{16}$$

Taking the base shear force equilibrium of GD2 with the definition of mobilized shear from GD4 and the assumption of GD5, the equilibrium equation can be rewritten as equation (17).

$$\frac{N_{\mathrm{i}} \tan(\varphi'_{i}) + c'_{\mathrm{i}} \cdot b'_{\mathrm{i}} \cdot \sec(\alpha_{\mathrm{i}})}{\mathrm{FS}} = \begin{array}{l} \left[W_{\mathrm{i}} - \lambda \cdot f_{\mathrm{i\text{-}1}} \cdot E_{\mathrm{i\text{-}1}} + \lambda \cdot f_{\mathrm{i}} \cdot E_{\mathrm{i}} + U_{\mathrm{t,i}} \cos(\beta_{\mathrm{i}}) + Q_{\mathrm{i}} \cos(\omega_{\mathrm{i}}) \right] \sin(\alpha_{\mathrm{i}}) \\ - \left[-K_{\mathrm{c}} W_{\mathrm{i}} - E_{\mathrm{i}} + E_{\mathrm{i\text{-}1}} - H_{\mathrm{i}} + H_{\mathrm{i\text{-}1}} + U_{\mathrm{t,i}} \cdot \sin(\beta_{\mathrm{i}}) + Q_{\mathrm{i}} \sin(\omega_{\mathrm{i}}) \right] \cos(\alpha_{\mathrm{i}}) \end{array} \tag{17}$$

Substituting the equation for N'_{i} from equation (16) into equation (17) and rearranging results in equation (18)

$$E_{\text{i}}\begin{bmatrix} [\lambda \cdot f_{\text{i}} \cos(\alpha_{\text{i}}) - \sin(\alpha_{\text{i}})] \tan({\varphi'}_i) \\ -[\lambda \cdot f_{\text{i}} \sin(\alpha_{\text{i}}) + \cos(\alpha_{\text{i}})]\,(\text{FS}) \end{bmatrix} = E_{\text{i-1}}\begin{bmatrix} [\lambda \cdot f_{\text{i-1}} \cos(\alpha_{\text{i}}) - \sin(\alpha_{\text{i}})] \tan({\varphi'}_i) \\ -[\lambda \cdot f_{\text{i-1}} \sin(\alpha_{\text{i}}) + \cos(\alpha_{\text{i}})]\,(\text{FS}) \end{bmatrix} + (\text{FS}) \cdot T_{\text{i}} - R_{\text{i}} \tag{18}$$

Where R and T are the resistive and mobile shear of the slice, without the influence of interslice forces E and X, as defined in DD13 and DD14. Making use of the constants ϕ and Ψ with full equations found below in equations (19) and (20) respectively, then equation (18) can be simplified to equation (21), also seen in IM3.

$$\Phi_{\text{i}} = [\lambda \cdot f_{\text{i}} \cos(\alpha_{\text{i}}) - \sin(\alpha_{\text{i}})]\,[\tan(\varphi'_{\text{i}})] - [\lambda \cdot f_{\text{i}} \sin(\alpha_{\text{i}}) + \cos(\alpha_{\text{i}})]\,(\text{FS}) \tag{19}$$

$$\Psi_{\text{i}} = [\lambda \cdot f_{\text{i}} \cos(\alpha_{\text{i+1}}) - \sin(\alpha_{\text{i+1}})]\,[\tan(\varphi')] - [\lambda \cdot f_{\text{i}} \sin(\alpha_{\text{i+1}}) + \cos(\alpha_{\text{i+1}})]\,(\text{FS}) \tag{20}$$

$$E_{\text{i}} = \frac{\Psi_{\text{i-1}}\ E_{\text{i-1}} + (FS)\ T_{\text{i}} - R_{\text{i}}}{\Phi_{\text{i}}} \tag{21}$$

The constants Ψ and Φ in equation (21) for are functions of the unknowns: the interslice normal/shear force ratio λ (IM2), and the Factor of Safety FS (IM1).

Number	IM4
Label	**Force Displacement Equilibrium**
Input	E , ν , b , c , σ , ϕ , κ a , A , u , v
Output	**X Equilibrium** $$\begin{aligned} &-\Delta H_{\text{i}} - K_{\text{c}} \cdot W_{\text{i}} - U_{\text{b,i}} \sin(\alpha_{\text{i}}) + U_{\text{t,i}} \sin(\beta_{\text{i}}) + Q_{\text{i}} \sin(\omega_{\text{i}}) = [\delta x_{\text{i-1}}](-\ell_{\text{s,i-1}} K_{\text{sn,i-1}}) \\ &+ [\delta x_{\text{i}}](-\ell_{\text{s,i-1}} K_{\text{sn,i-1}} + \ell_{\text{s,i}} K_{\text{sn,i}} + \ell_{\text{b,i}} K_{\text{bA,i}}) + [\delta x_{\text{i+1}}](-\ell_{\text{s,i}} K_{\text{sn,i}}) + [\delta y_{\text{i}}](-\ell_{\text{b,i}} K_{\text{bB,i}}) \end{aligned}$$ **Y Equilibrium** $$\begin{aligned} &-W_{\text{i}} + U_{\text{b,i}} \cos(\alpha_{\text{i}}) - U_{\text{t,i}} \cos(\beta_{\text{i}}) - Q_{\text{i}} \cos(\omega_{\text{i}}) = [\delta y_{\text{i-1}}](-\ell_{\text{s,i-1}} K_{\text{st,i-1}}) \\ &+ [\delta y_{\text{i}}](-\ell_{\text{s,i-1}} K_{\text{st,i-1}} + \ell_{\text{s,i}} K_{\text{st,i}} + \ell_{\text{b,i}} K_{\text{bA,i}}) + [\delta y_{\text{i+1}}](-\ell_{\text{s,i}} K_{\text{st,i}}) + [\delta x_{\text{i}}](-\ell_{\text{b,i}} K_{\text{bB,i}}) \end{aligned}$$
Description	One set of force displacement equilibrium equations in the x and y directions. There is of equations for each element. System of equations solved for displacements (δx, and δy) $\Delta H_{\text{i}} = H_{\text{i}} - H_{\text{i-1}}$ is the net hydrostatic force across a slice. K_{c} is the earthquake load factor. W_{i} is the weight of the slice. $U_{\text{b,i}}$ is the pore water pressure acting on the slice base. $U_{\text{t,i}}$ is the pore water pressure acting on the slice surface. α_{i} is the angle of the base with the horizontal. β_{i} is the angle of the surface with the horizontal δx_{i} is the x displacement of slice i δy_{i} is the y displacement of slice i $\ell_{\text{s,i}}$ is the length of the interslice surface i $\ell_{\text{s,i}}$ is the length of the base surface i $K_{\text{st,i}}$ is the interslice shear stiffness at surface i. $K_{\text{st,i-1}}$ is the interslice normal stiffness at surface i. $K_{\text{bA,i}}$, and $K_{\text{bB,i}}$ are the base stiffness values for slice i.
Sources	[5]
Ref. By	IM5

Rigid Finite Element Displacement Derivation

Using the net force-displacement equilibrium equation of a slice from DD17, with the definitions of the stiffness matrices from DD16, and the force definitions from GD8, a broken down force-displacement equilibrium equation can be derived. Equation (22) gives the broken down equation in the x direction, and equation (23) gives the broken down equation in the y direction.

$$-\Delta H_{\text{i}} - K_{\text{c}} \cdot W_{\text{i}} - U_{\text{b,i}} \sin(\alpha_{\text{i}}) + U_{\text{t,i}} \sin(\beta_{\text{i}}) + Q_{\text{i}} \sin(\omega_{\text{i}}) = \begin{array}{l} [\delta x_{\text{i-1}}](-\ell_{\text{s,i-1}} K_{\text{sn,i-1}}) \\ + [\delta x_{\text{i}}](-\ell_{\text{s,i-1}} K_{\text{sn,i-1}} + \ell_{\text{s,i}} K_{\text{sn,i}} + \ell_{\text{b,i}} K_{\text{bA,i}}) \\ + [\delta x_{\text{i+1}}](-\ell_{\text{s,i}} K_{\text{sn,i}}) + [\delta y_{\text{i}}](-\ell_{\text{b,i}} K_{\text{bB,i}}) \end{array} \tag{22}$$

$$-W_{\text{i}} + U_{\text{b,i}} \cos(\alpha_{\text{i}}) - U_{\text{t,i}} \cos(\beta_{\text{i}}) - Q_{\text{i}} \cos(\omega_{\text{i}}) = \begin{array}{l} [\delta y_{\text{i-1}}](-\ell_{\text{s,i-1}} K_{\text{st,i-1}}) \\ + [\delta y_{\text{i}}](-\ell_{\text{s,i-1}} K_{\text{st,i-1}} + \ell_{\text{s,i}} K_{\text{st,i}} + \ell_{\text{b,i}} K_{\text{bA,i}}) \\ + [\delta y_{\text{i+1}}](-\ell_{\text{s,i}} K_{\text{st,i}}) + [\delta x_{\text{i}}](-\ell_{\text{b,i}} K_{\text{bB,i}}) \end{array} \tag{23}$$

Using the known input assumption of A2, the force variable definitions of DD1 to DD11 on the left side of the equations can be solved for. The only unknown in the variables to solve for the stiffness values from DD19 is the displacements. Therefore taking the equation from each slice a set of $2 \cdot n$ equations, with $2 \cdot n$ unknown displacements in the x and y directions of each slice can be derived. Solutions for the displacements of each slice can then be found. The use of displacement in the definition of the stiffness values makes the equation implicit, which means an iterative solution method, with an initial guess for the displacements in the stiffness values is required.

Number	IM5
Label	**RFEM Factor of Safety**
Input	c, ℓ_{b}, δu , δv, φ', $K_{\text{bt,i}}$, $K_{\text{bn,i}}$
Output	$FS_{\text{Loc,i}} = \frac{c - K_{\text{bn,i}} \cdot \delta v_{\text{i}} \cdot \tan(\varphi'_{\text{i}})}{K_{\text{bt,i}} \cdot \delta u_{\text{i}}}$ $FS = \frac{\sum_{i=1}^{n} \ell_{\text{b,i}} \left[c - K_{\text{bn,i}} \cdot \delta v_{\text{i}} \cdot \tan(\varphi'_{\text{i}})\right]}{\sum_{i=1}^{n} \ell_{\text{b,i}} \left[K_{\text{bt,i}} \cdot \delta u_{\text{i}}\right]}$
Description	$FS_{\text{Loc,i}}$ Factor of Safety for slice i. FS Factor of Safety for the entire slip surface. c is the cohesion of slice i's base. φ'_{i} is the effective angle of friction of slice i's base. δv_{i} is the normal displacement of slice i δu_{i} is the shear displacement of slice i $\ell_{\text{b,i}}$ is the length of the base of slice i $K_{\text{bt,i}}$ is the base shear stiffness at surface i. $K_{\text{bn,i}}$ is the base normal stiffness at surface i. n is the number of slices in the slip surface.
Sources	[5]

Rigid Finite Element Factor of Safety Derivation

RFEM analysis can also be used to calculate the Factor of safety for the slope. For a slice element i the displacements δx_{i} and δy_{i}, are solved from the system of equations in IM4. The definition of $\bar{\epsilon}_{\text{i}}$ as the rotation of the displacement vector $\bar{\delta}_{\text{i}}$ is seen inGD10 .

This is used to find the displacements of the slice parallel to the base of the slice δu in equation (24) and normal to the base of the slice δv in equation (25).

$$\delta u_{\text{i}} = \cos(\alpha_{\text{i}})\,\delta x_{\text{i}} + \sin(\alpha_{\text{i}})\,\delta y_{\text{i}} \tag{24}$$

$$\delta v_{\text{i}} = -\sin(\alpha_{\text{i}})\,\delta x_{\text{i}} + \cos(\alpha_{\text{i}})\,\delta y_{\text{i}} \tag{25}$$

With the definition of normal stiffness from DD19 to find the normal stiffness of the base $K_{\text{bn,i}}$, and the now known base displacement perpendicular to the surface δv_{i} from equation (25), the normal base stress can be calculated from the force-displacement relationship of T5. Stress σ is used in place of force F as the stiffness hasn't been normalized for the length of the base. Results in equation (26).

$$\sigma_{\mathrm{i}} = K_{\mathrm{bn,i}} \cdot \delta v_{\mathrm{i}} \tag{26}$$

The resistive shear to calculate the factor of safety FS in is found from the Mohr Coulomb resistive strength of soil in T3. Using the normal stress σ from equation (26) as the stress the resistive shear of the slice can be calculated from calculated in equation (27).

$$S_{\mathrm{i}} = c - \sigma_{\mathrm{i}} \cdot \tan\left(\varphi'\right) \tag{27}$$

previously the value of the base shear stiffness $K_{\mathrm{bt,i}}$ as seen in equation (28) was unsolvable because the normal stress σ_{i} was unknown. With the definition of σ_{i} from equation (26) and the definition of displacement shear to the base δu_{i} from equation (25), the value of $K_{\mathrm{bt,i}}$ becomes solvable.

$$K_{\mathrm{bt,i}} = \frac{E_{\mathrm{i}}}{2\left[1+\nu_{\mathrm{i}}\right]} \frac{0.1}{b_{\mathrm{i}}} + \frac{c_{\mathrm{i}} - \sigma_{\mathrm{i}} \cdot \tan\left(\phi_{\mathrm{i}}\right)}{\left|\delta u_{\mathrm{i}}\right| + a} \tag{28}$$

With shear stiffness $K_{\mathrm{bt,i}}$ calculated in equation (28) and shear displacement δu_{i} calculated in equation (24) values now known the shear stress acting on the base of a slice τ can be calculated using T5, as done in equation (29). Again stress τ is used in place of force F as the stiffness hasn't been normalized for the length of the base.

$$\tau_{\mathrm{i}} = K_{\mathrm{bt,i}} \cdot \delta u_{\mathrm{i}} \tag{29}$$

The shear stress on the base τ acts as the mobile shear acting on the base. Using the definition Factor of Safety equation from T1, with the definitions of resistive shear strength of a slice S_{i} from equation (27) and mobile shear on a slice τ from equation (29) the factor of safety for a slice $FS_{\mathrm{Loc,i}}$ can be found from as seen in equation (30), and IM5.

$$FS_{\mathrm{Loc,i}} = \frac{S_{\mathrm{i}}}{\tau_{\mathrm{i}}} = \frac{c - K_{\mathrm{bn,i}} \cdot \delta v_{\mathrm{i}} \cdot \tan\left(\varphi'_{\mathrm{i}}\right)}{K_{\mathrm{bt,i}} \cdot \delta u_{\mathrm{i}}} \tag{30}$$

The global Factor of Safety is then the ratio of the summation of the resistive and mobile shears for each slice, with a weighting for the length of the slices base. Shown in equation (31), and IM5.

$$FS = \frac{\sum_{i=1}^{n} \ell_{\mathrm{i}} \cdot S_{\mathrm{i}}}{\sum_{i=1}^{n} \ell_{\mathrm{i}} \cdot \tau_{\mathrm{i}}} = \frac{\sum_{i=1}^{n} \ell_{\mathrm{b,i}} \left[c - K_{\mathrm{bn,i}} \cdot \delta v_{\mathrm{i}} \cdot \tan\left(\varphi'_{\mathrm{i}}\right)\right]}{\sum_{i=1}^{n} \ell_{\mathrm{b,i}} \left[K_{\mathrm{bt,i}} \cdot \delta u_{\mathrm{i}}\right]} \tag{31}$$

Number	IM6
Label	**Critical Slip Identification**
Input	The geometry of the water table, the geometry of the layers composing the plane of a slope, and the material properties of the layers.
Output	$\mathrm{FS}_{\mathrm{Min}} = \Upsilon\left(\left\{x_{\mathrm{cs}}, y_{\mathrm{cs}}\right\}, \mathrm{Input}\right)$
Description	Given the necessary slope inputs, a minimization algorithm or function Υ, will identify the critical slip surface of the slope, with the critical slip coordinates $\left\{x_{\mathrm{cs}}, y_{\mathrm{cs}}\right\}$ and the minimum factor of safety $\mathrm{FS}_{\mathrm{Min}}$ that results.
Sources	[3]

4.2.6 Data Constraints

Table 2 and 3 shows the data constraints on the input and output variables, respectively. The column for physical constraints gives the physical limitations on the range of values that can be taken by the variable. The uncertainty column provides an estimate of the confidence with which the physical quantities can be measured. This information would be part of the input if one were performing an uncertainty quantification exercise. The constraints are conservative, to give the user of the model the flexibility to experiment with unusual situations. The column of typical values is intended to provide a feel for a common scenario.

Var	Physical Constraints	Typical Value	Uncertainty
(x, y) of water table vertices's	Consecutive vertexes have increasing x values. All layers start and end vertices's go to the same x values.	N/A	10%
(x, y) of slip vertices's	Consecutive vertexes have increasing x values. All layers start and end vertices's go to the same x values.	N/A	10%
(x, y) of slope vertices's (*)	Consecutive vertexes have increasing x values. All layers start and end vertices's go to the same x values.	N/A	10%
E (*)	$E > 0$	15000	10%
c (*)	$c > 0$	10	10%
v (*)	$0 < v < 1$	0.4	10%
φ' (*)	$0 < \varphi < 90$	25	10%
γ (*)	$\gamma > 0$	20	10%
γ_{Sat} (*)	$\gamma_{\text{Sat}} > 0$	20	10%
γ_{Wat}	$\gamma_{\text{Wat}} > 0$	9.8	10%

(*) Input coordinates needed for each layer.

Var	Physical Constraints
FS	$FS > 0$
(x, y) Slip vertices's	Vertices's monotonic
δx	
δy	

5 Requirements

This section provides the functional requirements, the business tasks that the software is expected to complete, and the nonfunctional requirements, the qualities that the software is expected to exhibit.

5.1 Functional Requirements

R1: Read the input file, and store the data. Necessary input data summarized in Table1. [A2, A3]

symbol	unit	description
(x, y)	m	x and y coordinates for vertices of the slope layers, and for the water table if one exists. Assumed straight line fits between vertexes.
E	kPa	Young's modulus for each layer of the slope.
c	kPa	Cohesion for each slope layer.
v	/	Poisson's ratio for each soil layer.
φ	deg	Effective angle of friction for each slope layer.
γ	$\frac{\text{kN}}{\text{m}^3}$	Unit weight of dry soil / ground layer for each slope layer.
γ_{Sat}	$\frac{\text{kN}}{\text{m}^3}$	Unit weight of saturated soil / ground layer for each slope layer.
γ_{Wat}	$\frac{\text{kN}}{\text{m}^3}$	Unit weight of water.

R2: Generate potential critical slip surface's for the input slope.

R3: Test the slip surfaces to determine if they are physically realizable based on a set of pass or fail criteria. [A1]

R4: Prepare the slip surfaces for a method of slices or limit equilibrium analysis.

R5: Calculate the factors of safety of the slip surfaces.

R6: Rank and weight the slopes based on their factor of safety, such that a slip surface with a smaller factor of safety has a larger weighting.

R7: Generate new potential critical slip surfaces based on previously analysed slip surfaces with low factors of safety.

R8: Repeat requirements R3 to R7 until the minimum factor of safety remains approximately the same over a predetermined number of repetitions. Identify the slip surface that generates the minimum factor of safety as the critical slip surface.

R9: Prepare the critical slip surface for method of slices or limit equilibrium analysis.

R10: Calculate the factor of safety of the critical slip surface using the Morgenstern price method. Also calculate the local and global factors of safety for the critical slip using the RFEM method, and the displacement of the slice elements using the RFEM method.

R11: Display the critical slip surface and the slice element displacements graphically. Give the values of the factors of safety calculated by both methods, and the local factors of safety calculated by the RFEM method of analysis.

5.2 Nonfunctional Requirements

SSP is intended to be an educational tool, therefore accuracy and performance speed are secondary program priorities to correctness, understandability, reusability, and maintainability.

6 Likely Changes

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