Software Requirements Specification for Slope Stability Analysis

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1 Revision History

Date	Version	Notes
09/24/18	1.0	Removed RFEM
09/25/18	1.1	Notes

2 Reference Material

This section records information for easy reference.

2.1 Table of Units

The unit system used throughout is SI (Système International d'Unités). In addition to the basic units, several derived units are also used. For each unit, the table lists the symbol, a description and the SI name.

Symbol	Unit	SI
N	force	Newton
m	length	meter
$\mathrm{Pa} = \mathrm{N\,m^{-2}}$	pressure	Pascal
$^\circ$	angle	degree

[How is it decided which units show derivation information? —BM]

2.2 Table of Symbols

The table that follows summarizes the symbols used in this document along with their units. Throughout the document, the subscript i implies that the value will be taken and analyzed at a slice or slice interface composing the total slip mass.

Symbol	Unit	Description
$\{x_{cs}, y_{cs}\}$	m	The Set of X and Y Coordinates: describe the vertices of the critical slip surface
(x, y)	m	Cartesian Position Coordinates: y is considered parallel to the direction of the force of gravity and x is considered perpendicular to y
a	m	Constant: FIXME: missing description
A	m	Constant: FIXME: missing description
b	m	Base Width of a Slice: in the x-ordinate direction only for slice index i
c'	Pa	Effective Cohesion: internal pressure that sticks particles of soil together
$C1_i$	Nm	Interslice Normal Force Function: the normal force at the interslice interface for slice i
$C2_i$	Nm	Interslice Shear Force Function: the shear force at the interslice interface for slice i
E	Pa	Elastic Modulus: The ratio of the stress exerted on a body to the resulting strain.

F	N	Force: An interaction that tends to produce change in the motion of an object
F_x	N	x-component of the net force
F_y	N	y-component of the net force
f		Scaling Function: magnitude of interslice forces as a function of the x coordinate for interslice index i; can be constant or a half-sine
FS		Factor of Safety: The global stability of a surface in a slope
$FS_{Loc,i}$		Local Factor of Safety: for slice index i
G	N	Interslice Normal Force: exerted between adjacent slices for interslice index i
H	N	Interslice Water Force: exerted in the x-ordinate direction between adjacent slices for interslice index i
ΔH	N	Difference Between Interslice Forces: exerted in the x-ordinate direction between adjacent slices for interslice index i
h	m	Midpoint Height: distance from the slip base to the slope surface in a vertical line from the midpoint of the slice for slice index i
i		Index: used to show a quantity applies to only one slice
K_{bA}	$\mathrm{Pa\,m^{-1}}$	Effective Base Stiffness a: for rotated coordinates of a slice base surface, for slice index i
K_{bB}	$\mathrm{Pa\,m^{-1}}$	Effective Base Stiffness a: for rotated coordinates of a slice base surface, for slice index i
K	$\mathrm{Pa\,m^{-1}}$	Stiffness: The extent a body resists strain.
K_{bn}	$\mathrm{Pa\,m^{-1}}$	Normal Stiffness: for a slice base surface, without length adjustment for slice index i
K_{bt}	$\mathrm{Pa\,m^{-1}}$	Shear Stiffness: for a slice base surface, without length adjustment for slice index i
K_c		Earthquake Load Factor: proportionality factor of force that weight pushes outwards; caused by seismic earth movements
K_{no}	$\mathrm{Pa\,m^{-1}}$	Normal Stiffness: residual strength

K_{sn}	$\mathrm{Pa\,m^{-1}}$	Normal Stiffness: for an interslice surface, without length adjustment for interslice index i
K_{st}	$\mathrm{Pa\,m^{-1}}$	Shear Stiffness: for interslice surface, without length adjustment for interslice index i
K_{tr}	$\mathrm{Pa\,m^{-1}}$	Shear Stiffness: residual strength
M	Nm	Moment: a measure of the tendency of a body to rotate about a specific point or axis
N	N	Normal Force: total reactive force for a soil surface subject to a body resting on it
N'	N	Effective Normal Force: for a soil surface, subtracting pore water reactive force from total reactive force
n		Number of Slices: the slip mass has been divided into
$N*$	N	Effective Normal Force: for a soil surface, without the influence of interslice forces
p	Pa	Pressure: A force exerted over an area
P	N	Resistive Shear Force: Mohr Coulomb frictional force that describes the limit of mobilized shear force the slice i can withstand before failure
Q	N	Imposed Surface Load: a downward force acting into the surface from midpoint of slice i
R	N	Resistive Shear Force: without the influence of interslice forces for slice index i
S	N	Mobilized Shear Force: for slice index i
s	Pa	Mobilized Shear Stress: acting on the base of a slice
T	N	Mobilized Shear Force: without the influence of interslice forces for slice index i
U_b	N	Base Hydrostatic Force: from water pressure within the slice for slice index i
U_t	N	Surface Hydrostatic Force: from water pressure acting into the slice from standing water on the slope surface for slice index i
u		Local Index: used as a bound variable index in calculations

v		Local Index: used as a bound variable index in calculations
W	N	Weight: downward force caused by gravity on slice i
x	m	X Ordinate: refers to either slice i midpoint, or slice interface i
x_{slip}	m	X Ordinate: distance of the slip surface at i, refers to either slice i midpoint, or slice interface i
x_{us}	m	X Ordinate: distance of the edge of the slope at i, refers to either slice i midpoint, or slice interface i
X	N	Interslice Shear Force: exerted between adjacent slices for interslice index i
y	m	Y Ordinate: refers to either slice i midpoint, or slice interface i
y_{slip}	m	Y Ordinate: height of the slip surface at i, refers to either slice i midpoint, or slice interface i
y_{us}	m	Y Ordinate: height of the top of the slope at i, refers to either slice i midpoint, or slice interface i
y_{wt}	m	Y Ordinate: height of the water table at i, refers to either slice i midpoint, or slice interface i
z	m	Center of Slice Height: the distance from the lowest part of the slice to the height of the centers of slice
α	$^\circ$	Angle: base of the mass relative to the horizontal for slice index i
β	$^\circ$	Angle: surface of the mass relative to the horizontal for slice index i
γ	$\mathrm{Pa\,N^{-3}}$	Dry Unit Weight: The weight of a dry soil/ground layer divided by the volume of the layer.
γ_{Sat}	$\mathrm{Pa\,N^{-3}}$	Saturated Unit Weight: The weight of saturated soil/ground layer divided by the volume of the layer.

γ_w	$\mathrm{Pa\,N^{-3}}$	Unit Weight of Water: The weight of one cubic meter of water.
δ	m	Displacement: generic displacement of a body
δn	m	Displacement: for the element parallel to the surface for slice index i
δt	m	Displacement: for the element normal to the surface for slice index i
δu	m	Displacement: shear displacement for slice index i
δv	m	Displacement: normal displacement for slice index i
δx	m	Displacement: in the x-ordinate direction for slice index i
δy	m	Displacement: in the y-ordinate direction for slice index i
ε	m	Displacement: in rotated coordinate system
κ	Pa	Constant: FIXME: missing description
λ		Interslice Normal/shear Force Ratio: applied to all interslices
μ	Pa	Pore Pressure: from water within the soil
ν		Poisson's Ratio: The ratio of perpendicular strain to parallel strain.
σ	Pa	Normal Stress: The stress exerted perpendicular to the plane of the object
τ	Pa	Resistive Shear Stress: acting on the base of a slice
Υ		Function: generic minimization function or algorithm
φ'	$^\circ$	Effective Angle of Friction: The angle of inclination with respect to the horizontal axis of the Mohr-Coulomb shear resistance line
Φ	N	Constant: converts resistive shear without the influence of interslice forces, to a calculation considering the interslice forces

Ψ	N	Constant: converts mobile shear without the influence of interslice forces, to a calculation considering the interslice forces
ω	$^\circ$	Angle: of imposed surface load acting into the surface relative to the vertical for slice index i
ℓ_b	m	Total Base Length of a Slice: for slice index i
ℓ_s	m	Length of an Interslice Surface: from slip base to slope surface in a vertical line from an interslice vertex for interslice index i

2.3 Abbreviations and Acronyms

Symbol	Description
A	Assumption
DD	Data Definition
GD	General Definition
GS	Goal Statement
IM	Instance Model
LC	Likely Change
PS	Physical System Description
R	Requirement
SRS	Software Requirements Specification
SSP	Slope Stability Analysis Program
T	Theoretical Model
TU	Typical Uncertainty
UC	Unlikely Change

Contents

3 Introduction

A slope of geological mass, composed of soil and rock, is subject to the influence of gravity on the mass. For an unstable slope this can cause instability in the form of soil/rock movement. The effects of soil/rock movement can range from inconvenient to seriously hazardous, resulting in significant life and economic losses. Slope stability is of interest both when analyzing natural slopes, and when designing an excavated slope. Slope stability analysis is the assessment of the safety of a slope, identifying the surface most likely to experience slip and an index of its relative stability known as the factor of safety.

The following section provides an overview of the Software Requirements Specification (SRS) for a slope stability analysis problem. The developed program will be referred to as the Slope Stability Analysis Program (SSP) program. This section explains the purpose of this document, the scope of the system, the organization of the document, and the characteristics of the intended readers.

3.1 Purpose of Document

The SSP determines the critical slip surface, and its respective factor of safety as a method of assessing the stability of a slope design. The program is intended to be used as an educational tool for introducing slope stability issues, and will facilitate the analysis and design of a safe slope.

This document will be used as a starting point for subsequent development phases, including writing the design specification and the software verification and validation plan. The design document will show how the requirements are to be realized, including decisions on the numerical algorithms and programming environment. The verification and validation plan will show the steps that will be used to increase confidence in the software documentation and the implementation. Although the SRS fits in a series of documents that follow the so-called waterfall model, the actual development process is not constrained in any way. Even when the waterfall model is not followed, as Parnas and Clements point out [4], the most logical way to present the documentation is still to "fake" a rational design process.

3.2 Scope of Requirements

The scope of the requirements includes stability analysis of a 2 dimensional slope, composed of homogeneous soil layers. Given the appropriate inputs, SSP identifies the most likely failure surface within the possible input range, and finds the factor of safety for the slope as well as displacement of soil that will occur on the slope.

3.3 Characteristics of Intended Reader

Reviewers of this documentation should have a strong knowledge in solid mechanics. The reviewers should also have an understanding of undergraduate level 4 physics. The users of SSP can have a lower level of expertise, as explained in Section 4.2.

3.4 Organization of Document

The organization of this document follows the template for an SRS for scientific computing software proposed by [2] and [5]. The presentation follows the standard pattern of presenting goals, theories, definitions, and assumptions. For readers that would like a more bottom up approach, they can start reading the instance models in Section 5.2.5 and trace back to find any additional information they require.

The goal statements are refined to the theoretical models, and the theoretical models (Section 5.2.2) to the instance models (Section 5.2.5). The instance models provide the set of algebraic equations that must be solved iteratively to perform a Morgenstern Price Analysis, and the system of equations that must be solved for Rigid Finite Element Analysis.

4 General System Description

This section provides general information about the system including identifying the interfaces between the system and its environment (system context), describing the user characteristics, and listing the system constraints.

4.1 System Context

Figure 1 shows the system context. A circle represents an external entity outside the software, the user in this case. A rectangle represents the software system itself (SSP). Arrows are used to show the data flow between the system and its environment.

Figure 1: System Context

The interaction between the product and the user is through a user interface. The responsibilities of the user and the system are as follows:

- User Responsibilities:
  - Provide the input data related to the soil layer(s) and water table (if applicable), ensuring no errors in the data entry
  - Ensure that consistent units are used for input variables
  - Ensure required software assumptions (Section 5.2.1) are appropriate for any particular problem input to the software
- SSP Responsibilities:
  - Detect data type mismatch, such as a string of characters input instead of a floating point number
  - Determine if the inputs satisfy the required physical constraints

- Identify the most likely failure surface within the possible input range
- Find the factor of safety for the slope

4.2 User Characteristics

The end user of SSP should have an understanding of undergraduate Level 1 Calculus and Physics, and be familiar with soil and material properties.

4.3 System Constraints

There are no system constraints.

5 Specific System Description

This section first presents the problem description, which gives a high-level view of the problem to be solved. This is followed by the solution characteristics specification, which presents the assumptions, theories, definitions and finally the instance models that model the slope.

5.1 Problem Description

SSP is a computer program developed to evaluate the factor of safety of a slope's slip surface and identify the critical slip surface of the slope.

5.1.1 Terminology

This subsection provides a list of terms that are used in the subsequent sections and their meaning, with the purpose of reducing ambiguity and making it easier to correctly understand the requirements.

- *Factor of safety:* The global stability of a surface in a slope.
- *Critical slip surface:* Slip surface of the slope that has the lowest global factor of safety, and therefore most likely to experience failure.
- *Stress:* Forces that are exerted between planes internal to a larger body subject to external loading.
- *Strain:* Stress forces that result in deformation of the body/plane.
- *Normal Force:* A force applied perpendicular to the plane of the material.
- *Shear Force:* A force applied parallel to the plane of the material.
- *Tension:* A stress that causes displacement of the body away from its center.
- *Compression:* A stress that causes displacement of the body towards its center.

- *Plane Strain:* The resultant stresses in one of the directions of a 3 dimensional material can be approximated as 0. Results when the length of one dimension of the body dominates the others. Stresses in the dominant dimensions direction are the ones that can be approximated as 0.

5.1.2 Physical System Description

Analysis of the slope is performed by looking at properties of the slope as a series of slice elements. Some properties are interslice properties, and some are slice or slice base properties. The index convention for referencing which interslice or slice is being used is shown in Fig 1.

- Interslice properties convention is noted by j. The end interslice properties are usually not of interest, therefore use the interslice properties from $1 \leq \text{i} \leq n-1$.
- Slice properties convention is noted by i.

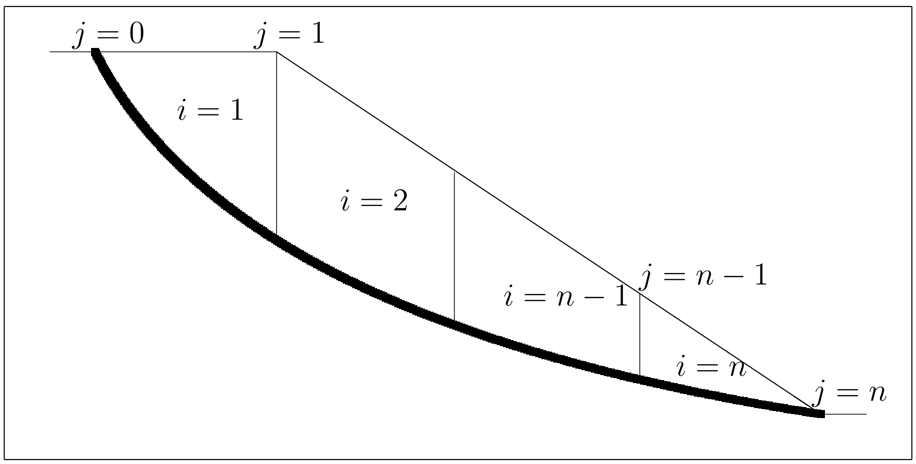

Figure 2: Index convention for numbering slice and interslice force variables

A free body diagram of the forces acting on the slice is displayed in Fig 2.

5.1.3 Goal statements

Given the geometry of the layers composing the plane of a slope, the material properties of the layers, and optionally the geometry of a water table, the goal statements are:

GS1: Evaluate the factor of safety along a given slip surface.

GS2: Identify the critical slip surface for the slope, with the lowest factor of safety.

5.2 Solution Characteristics Specification

The instance models that govern SSP are presented in Section 5.2.5. The information to understand the meaning of the instance models and their derivation is also presented, so that the instance models can be verified.

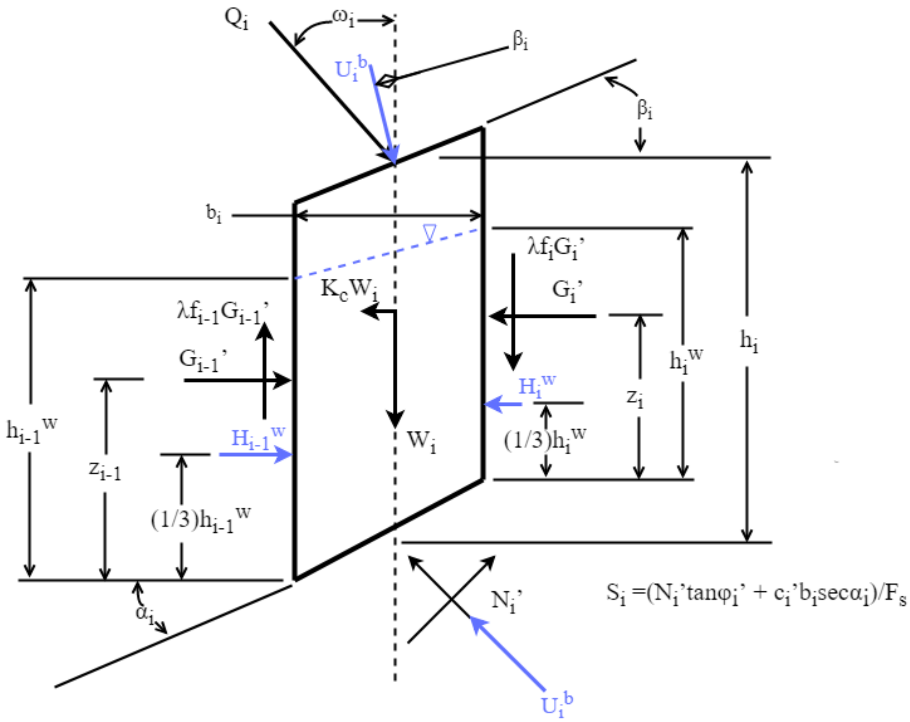

Figure 3: Forces acting on a slice

5.2.1 Assumptions

This section simplifies the original problem and helps in developing the theoretical model by filling in the missing information for the physical system. The numbers given in the square brackets refer to the data definition, or the instance model, in which the respective assumption is used.

A1: The slip surface is concave with respect to the slope surface. The (x, y) coordinates of the failure surface follow a monotonic function.

A2: The geometry of the slope, and the material properties of the soil layers are given as inputs. [IM**??**]

A3: The different layers of the soil are homogeneous, with consistent soil properties throughout, and independent of dry or saturated conditions, with the exception of unit weight.

A4: Soil layers are treated as if they have isotropic properties.

A5: Interslice normal and shear forces have a linear relationship, proportional to a constant (λ) and an interslice force function (f) depending on x position.

A6: Slice to base normal and shear forces have a linear relationship, dependent on the factor of safety (FS), and the Coulomb sliding law.

A7: The stress-strain curve for interslice relationships is linear with a constant slope.

A8: The slope and slip surface extends far into and out of the geometry (z coordinate). This implies plane strain conditions, making 2D analysis appropriate.

A9: The effective normal stress is large enough that the resistive shear to effective normal stress relationship can be approximated as a linear relationship.

A10: The surface and base of a slice between interslice nodes are approximated as straight lines.

5.2.2 Theoretical Models

This section focuses on the general equations and laws that SSP is based on.

Number	T1
Label	**Factor of Safety**
Equation	$\text{FS} = \frac{P}{S}$
Description	The stability metric of the slope, known as the factor of safety FS, is determined by the ratio of the shear force at the base of the slope S, and the resistive shear P.
Source	[1]
Ref. By	IM1, GD4

Number	T2
Label	**Equilibrium**
Equation	$\sum F_{\text{x}} = \sum F_{\text{y}} = \sum M = 0$
Description	For a body in static equilibrium the net forces and net moments acting on the body will cancel out. Assuming a 2D problem (A8) the x-component of the net force F_x and y-component of the net force F_y will be equal to 0. All forces and their distance from the chosen point of rotation will create a net moment equal to 0.
Source	[1]
Ref. By	GD1, GD2, GD6, IM2

Number	T3
Label	**Mohr-Coulomb Shear Strength**
Equation	$P = \sigma \cdot \tan(\varphi') + c'$
Description	For a soil under stress it will exert a shear resistive strength based on the Coulomb sliding law. The resistive shear is the maximum amount of shear a surface can experience while remaining rigid, analogous to a maximum normal force. In this model the shear force P is proportional to the product of the normal stress on the plane σ with it's static friction, in the angular form $\tan(\varphi') = U_{\mathrm{s}}$. The P versus σ relationship is not truly linear, but assuming the effective normal force is strong enough, it can be approximated with a linear fit (A9), where the cohesion c' represents the P intercept of the fitted line.
Source	[1]
Ref. By	GD3, GD4, DD13, DD14

Number	T4
Label	**Effective Stress**
Equation	$\sigma' = \sigma - \mu$
Description	σ is the total stress a soil mass needs to maintain itself as a rigid collection of particles. The source of the stress can be provided by the soil skeleton σ', or by the pore pressure from water within the soil μ. The stress from the soil skeleton is known as the effective stress σ' and is the difference between the total stress σ and the pore stress μ.
Source	[1]
Ref. By	GD3, GD4, DD13, DD14, IM3

5.2.3 General Definitions

This section collects the laws and equations that will be used in deriving the data definitions, which in turn are used to build the instance models.

Number	GD1
Label	**Normal Force Equilibrium**
Equation	$N_{\text{i}} = \begin{aligned}&[W_{\text{i}} - X_{\text{i-1}} + X_{\text{i}} + U_{\text{t,i}} \cos(\beta_{\text{i}}) + Q_{\text{i}} \cos(\omega_{\text{i}})] \cos(\alpha_{\text{i}}) \\ &+ [-K_{\text{c}}\, W_{\text{i}} - G_{\text{i}} + G_{\text{i-1}} - H_{\text{i}} + H_{\text{i-1}} + U_{\text{t,i}} \sin(\beta_{\text{i}}) + Q_{\text{i}} \sin(\omega_{\text{i}})] \sin(\alpha_{\text{i}})\end{aligned}$
Description	For a slice of mass in the slope the force equilibrium to satisfy T2 in the direction perpendicular to the base surface of the slice. Rearranged to solve for the normal force of the surface N. Force equilibrium is derived from the free body diagram of Fig 2 in section 5.1.2. Index i refers to the values of the properties for slice/interslices following convention in Fig 1 in section 5.1.2. Force variable definitions can be found in DD1 to DD12.
Source	[7]
Ref. By	DD13, DD14, IM3

Number	GD2
Label	**Base Shear Force Equilibrium**
Equation	$S_{\text{i}} = \begin{aligned}&[W_{\text{i}} - X_{\text{i-1}} + X_{\text{i}} + U_{\text{t,i}} \cos(\beta_{\text{i}}) + Q_{\text{i}} \cos(\omega_{\text{i}})] \sin(\alpha_{\text{i}}) \\ &- [-K_{\text{c}}\, W_{\text{i}} - G_{\text{i}} + G_{\text{i-1}} - H_{\text{i}} + H_{\text{i-1}} + U_{\text{t,i}} \sin(\beta_{\text{i}}) + Q_{\text{i}} \cos(\omega_{\text{i}})] \cos(\alpha_{\text{i}})\end{aligned}$
Description	For a slice of mass in the slope the force equilibrium to satisfy T2 in the direction parallel to the base surface of the slice. Rearranged to solve for the shear force acting on the base S_{i}. Force equilibrium is derived from the free body diagram of Fig 2 in section 5.1.2. Index i refers to the values of the properties for slice/interslices following convention in Fig 1 in section 5.1.2. Force variable definitions can be found in DD1 to DD12.
Source	[7]
Ref. By	DD13, DD14,IM3

Number	GD3
Label	**Resistive Shear Force**
Equation	$P_\text{i} = N'_\text{i} \cdot \tan\left(\varphi'_\text{i}\right) + c' \cdot b_\text{i} \cdot \sec\left(\alpha_\text{i}\right)$
Description	The Mohr-Coulomb resistive shear strength of a slice τ from T3 is multiplied by the area $b\sec\left(\alpha\right) \cdot 1$ to obtain the resistive shear force P. Note the extra 1 is to represent a unit of width which is multiplied by the total base length of a slice ℓ_b of the plane where the normal occurs, where $\ell_b = b\sec\left(\alpha\right)$ and b is the x width of the base. This accounts for the effective normal force $N' = N - U_b$ of a soil from T4 where the normal stress is multiplied by the same area to obtain the effective normal force $\sigma b\sec\left(\alpha\right) \cdot 1 = N'$.
Source	[7]
Ref. By	GD4, DD13, DD14

Number	GD4
Label	**Mobile Shear Force**
Equation	$S_\text{i} = \frac{P_\text{i}}{\text{FS}} = \frac{N'_\text{i} \cdot \tan\left(\varphi'_\text{i}\right) + c' \cdot b_\text{i} \cdot \sec\left(\alpha_\text{i}\right)}{\text{FS}}$
Description	From the definition of the factor of safety in T1, and the new definition of P, a new relation for the net mobile shear force of the slice T is found as the resistive shear P (GD3) divided by the factor of safety FS.
Source	[7]
Ref. By	DD13, DD14

Number	GD5
Label	**Interslice Normal/Shear Relationship**
Equation	$X = \lambda \cdot f \cdot G$
Description	The assumption for the Morgenstern Price method (A5) that the interslice shear force X is proportional to the interslice normal force G by a proportionality constant λ, and a predetermined scaling function f, that changes the proportionality as a function of the x-ordinate position of the interslice. f is typically either a half-sine along the slip surface, or a constant.
Source	[7]
Ref. By	DD13, DD14, IM1, IM2, IM3

Number	GD6
Label	**Moment Equilibrium**
Equation	$0 = \begin{array}{l} -G_{\mathrm{i}}\left[z_{\mathrm{i}}+\frac{b_{\mathrm{i}}}{2}\tan\left(\alpha_{\mathrm{i}}\right)\right]+G_{\mathrm{i\text{-}1}}\left[z_{\mathrm{i\text{-}1}}-\frac{b_{\mathrm{i}}}{2}\tan\left(\alpha_{\mathrm{i}}\right)\right]-H_{\mathrm{i}}\left[z_{\mathrm{w,i}}+\frac{b_{\mathrm{i}}}{2}\tan\left(\alpha_{\mathrm{i}}\right)\right] \\ +H_{\mathrm{i\text{-}1}}\left[z_{\mathrm{w,i\text{-}1}}-\frac{b_{\mathrm{i}}}{2}\tan\left(\alpha_{\mathrm{i}}\right)\right]+\frac{b_{\mathrm{i}}}{2}\left(X_{\mathrm{i}}+X_{\mathrm{i\text{-}1}}\right)-K_{\mathrm{c}}W_{\mathrm{i}}\frac{h_{\mathrm{i}}}{2}+U_{\mathrm{t,i}}\sin\left(\beta_{\mathrm{i}}\right)h_{\mathrm{i}} \\ +Q_{\mathrm{i}}\sin\left(\omega_{\mathrm{i}}\right)h_{\mathrm{i}} \end{array}$
Description	For a slice of mass in the slope the moment equilibrium to satisfy T2 in the direction perpendicular to the base surface of the slice. Moment equilibrium is derived from the free body diagram of Fig 2 in section 5.1.2. Index i refers to the values of the properties for slice/interslices following convention in Fig 1 in section 5.1.2. Variable definitions can be found in DD1 to DD12.
Source	[7]
Ref. By	IM2

5.2.4 Data Definition

This section collects and defines all the data needed to build the instance models. Definitions DD1 to DD11 are the force variables that can be solved by direct analysis of given inputs. The interslice forces DD12 are force variables that must be written in terms of DD1 to DD11 to solve.

Number	DD1
Label	**Weight**
Equation	$W = b_i \begin{cases} \left(y_{us,i}-y_{slip,i}\right)\gamma_{Sat}, & y_{wt,i} \geq y_{us,i} \\ \left(y_{us,i}-y_{wt,i}\right)\gamma+\left(y_{wt,i}-y_{slip,i}\right)\gamma_{Sat}, & y_{us,i} > y_{wt,i} > y_{slip,i} \\ \left(y_{us,i}-y_{slip,i}\right)\gamma, & y_{wt,i} \leq y_{slip,i} \end{cases}$
Description	W is the weight (N) b is the base width of a slice (m) i is the index y_{us} is the y ordinate (m) y_{slip} is the y ordinate (m) γ_{Sat} is the saturated unit weight $\left(\frac{\mathrm{N}}{\mathrm{m}^3}\right)$ y_{wt} is the y ordinate (m) γ is the dry unit weight $\left(\frac{\mathrm{N}}{\mathrm{m}^3}\right)$
Sources	[1]
Ref. By	DD13, DD14, IM1, IM2, IM3

Number	DD2
Label	**Base Water Force**
Equation	$U_b = \ell_{b,i} \begin{cases} (y_{wt,i} - y_{slip,i})\,\gamma_w, & y_{wt,i} > y_{slip,i} \\ 0, & y_{wt,i} \leq y_{slip,i} \end{cases}$
Description	U_b is the base hydrostatic force (N) ℓ_b is the total base length of a slice (m) i is the index y_{wt} is the y ordinate (m) y_{slip} is the y ordinate (m) γ_w is the unit weight of water ($\frac{\text{N}}{\text{m}^3}$)
Sources	[1]
Ref. By	DD13, DD14, IM1, IM2, IM3

Number	DD3
Label	**Surface Hydrostatic Force**
Equation	$U_t = \ell_{s,i} \begin{cases} (y_{wt,i} - y_{us,i})\,\gamma_w, & y_{wt,i} > y_{us,i} \\ 0, & y_{wt,i} \leq y_{us,i} \end{cases}$
Description	U_t is the surface hydrostatic force (N) ℓ_s is the length of an interslice surface (m) i is the index y_{wt} is the y ordinate (m) y_{us} is the y ordinate (m) γ_w is the unit weight of water ($\frac{\text{N}}{\text{m}^3}$)
Sources	[1]
Ref. By	DD13, DD14, IM1, IM2, IM3

Number	DD4
Label	**Interslice Water Force**
Equation	$H = \begin{cases} \frac{\left[y_{us,i} - y_{slip,i}\right]^2}{2} \gamma_{Sat} + \left[y_{wt,i} - y_{us,i}\right]^2 \gamma_{Sat}, & y_{wt,i} \geq y_{us,i} \\ \frac{\left[y_{wt,i} - y_{slip,i}\right]^2}{2} \gamma_{Sat}, & y_{us,i} > y_{wt,i} > y_{slip,i} \\ 0, & y_{wt,i} \leq y_{slip,i} \end{cases}$
Description	H is the interslice water force (N) y_{us} is the y ordinate (m) i is the index y_{slip} is the y ordinate (m) γ_{Sat} is the saturated unit weight ($\frac{\mathrm{N}}{\mathrm{m}^3}$) y_{wt} is the y ordinate (m)
Sources	[1]
Ref. By	DD13, DD14, IM1, IM2, IM3

Number	DD5
Label	**Angle**
Equation	$\alpha_{\mathrm{i}} = \frac{y_{\mathrm{slip,i}} - y_{\mathrm{slip,i-1}}}{x_{\mathrm{slip,i}} - x_{\mathrm{slip,i-1}}}$
Description	α is the angle (°) y_{slip} is the y ordinate (m) i is the index x_{slip} is the x ordinate (m)
Sources	[1]
Ref. By	DD13, DD14, IM1, IM2, IM3

Number	DD6
Label	**Angle**
Equation	$\beta_{\mathrm{i}} = \frac{y_{\mathrm{us,i}} - y_{\mathrm{us,i-1}}}{x_{\mathrm{us,i}} - x_{\mathrm{us,i-1}}}$
Description	β is the angle (°) y_{us} is the y ordinate (m) i is the index x_{us} is the x ordinate (m)
Sources	[1]
Ref. By	DD13, DD14, IM1, IM2, IM3

Number	DD7
Label	**Base Width of a Slice**
Equation	$b = x_{slip,i} - x_{slip,i-1}$
Description	b is the base width of a slice (m) x_{slip} is the x ordinate (m) i is the index
Sources	[1]
Ref. By	DD13, DD14, IM1, IM2, IM3

Number	DD8
Label	**Total Base Length of a Slice**
Equation	$\ell_b = b_i \sec(\alpha_i)$
Description	ℓ_b is the total base length of a slice (m) b is the base width of a slice (m) i is the index α is the angle (°)
Sources	[1]
Ref. By	DD13, DD14, IM1, IM2, IM3

Number	DD9
Label	**Length of an Interslice Surface**
Equation	$\ell_s = b_i \sec(\beta_i)$
Description	ℓ_s is the length of an interslice surface (m) b is the base width of a slice (m) i is the index β is the angle (°)
Sources	[1]
Ref. By	DD13, DD14, IM1, IM2, IM3

Number	DD10
Label	**Earthquake Load Factor**
Equation	$K_c = K_c W_i$
Description	K_c is the earthquake load factor W is the weight (N) i is the index
Sources	[1]
Ref. By	DD13, DD14, IM1, IM2, IM3

Number	DD11
Label	**Imposed Surface Loads**
Equation	$Q = Q_i \omega_i$
Description	Q is the imposed surface load (N) i is the index ω is the angle (°)
Sources	[7]
Ref. By	DD13, DD14, IM1, IM2, IM3

Number	DD12
Label	**Interslice ShearForces**
Equation	$X = \lambda f_i G_i$
Description	X is the interslice shear force (N) λ is the interslice normal/shear force ratio f is the scaling function i is the index G is the interslice normal force (N)
Sources	[7]
Ref. By	DD13, DD14, IM1, IM2, IM3

Number	DD13
Label	**Resistive Shear, Without Interslice Forces**
Equation	$R = \begin{pmatrix} [W_{\text{i}} + U_{\text{t,i}} \cos(\beta_{\text{i}}) + Q_{\text{i}} \cos(\omega_{\text{i}})] \cos(\alpha_{\text{i}}) \\ + [-K_{\text{c}} W_{\text{i}} - \Delta H_{\text{i}} + U_{\text{t,i}} \sin(\beta_{\text{i}}) + Q_{\text{i}} \sin(\omega_{\text{i}})] \sin(\alpha_{\text{i}}) - U_{\text{b,i}} \end{pmatrix} \cdot \tan(\varphi')$ $+ c'_{\text{i}} \cdot b_{\text{i}} \cdot \sec(\alpha_{\text{i}})$
Description	R is the resistive shear force (N) W is the weight (N) i is the index U_t is the surface hydrostatic force (N) β is the angle (°) Q is the imposed surface load (N) ω is the angle (°) α is the angle (°) K_c is the earthquake load factor ΔH is the difference between interslice forces (N) U_b is the base hydrostatic force (N) φ' is the effective angle of friction (°) c' is the effective cohesion (Pa) b is the base width of a slice (m)
Sources	[7]
Ref. By	IM1

Resistive Shear Force, Without the Influence of Interslice Forces Derivation

The resistive shear force of a slice is defined as P_{i} in GD3. The effective normal in the equation for P_{i} of the soil is defined in the perpendicular force equilibrium of a slice from GD2, Using the effective normal N'_{i} of T4 shown in equation (1).

$$N'_{\text{i}} = \begin{aligned} &[W_{\text{i}} - X_{\text{i-1}} + X_{\text{i}} + U_{\text{t,i}} \cos(\beta_{\text{i}}) + Q_{\text{i}} \cos(\omega_{\text{i}})] \cos(\alpha_{\text{i}}) \\ &+ [-K_{\text{c}} W_{\text{i}} - G_{\text{i}} + G_{\text{i-1}} - H_{\text{i}} + H_{\text{i-1}} + U_{\text{t,i}} \sin(\beta_{\text{i}}) + Q_{\text{i}} \sin(\omega_{\text{i}})] \sin(\alpha_{\text{i}}) \\ &- U_{\text{b,i}} \end{aligned} \tag{1}$$

The values of the interslice forces G and X in the equation are unknown, while the other values are found from the physical force definitions of DD1 to DD12. Consider a force equilibrium without the affect of interslice forces, to obtain a solvable value as done for N^*_{i} in equation (2).

$$N^*_{\text{i}} = \begin{aligned} &[W_{\text{i}} + U_{\text{t,i}} \cos(\beta_{\text{i}}) + Q_{\text{i}} \cos(\omega_{\text{i}})] \cos(\alpha_{\text{i}}) \\ &+ [-K_{\text{c}} W_{\text{i}} - H_{\text{i}} + H_{\text{i-1}} + U_{\text{t,i}} \sin(\beta_{\text{i}}) + Q_{\text{i}} \sin(\omega_{\text{i}})] \sin(\alpha_{\text{i}}) - U_{\text{b,i}} \end{aligned} \tag{2}$$

Using N^*_{i}, a resistive shear force neglecting the influence of interslice forces can be solved for in terms of all known values as done in equation (3).

$$R_{\text{i}} = N^*_{\text{i}} \tan(\varphi') + c'_{\text{i}} \cdot b'_{\text{i}} \sec(\alpha_t exti')$$

$$R_i = \begin{pmatrix} [W_i + U_{t,i} \cos(\beta_i) + Q_i \cos(\omega_i)] \cos(\alpha_i) \\ + [-K_c W_i - \Delta H_i + U_{t,i} \sin(\beta_i) + Q_i \sin(\omega_i)] \sin(\alpha_i) - U_{b,i} \end{pmatrix} \cdot \tan(\varphi') + c_i' \cdot b_i \cdot \sec(\alpha_i) \quad (3)$$

Number	DD14
Label	**Mobile Shear, Without Interslice Forces**
Equation	$T = (W_i + U_{t,i} \cos(\beta_i) + Q_i \cos(\omega_i)) \sin(\alpha_i) - (-K_c W_i - \Delta H_i + U_{t,i} \sin(\beta_i) + Q_i \sin(\omega_i)) \cos(\alpha_i)$
Description	T is the mobilized shear force (N) W is the weight (N) i is the index U_t is the surface hydrostatic force (N) β is the angle (°) Q is the imposed surface load (N) ω is the angle (°) α is the angle (°) K_c is the earthquake load factor ΔH is the difference between interslice forces (N)
Sources	[7]
Ref. By	IM1

Mobile Shear Force, Without the Influence of Interslice Forces Derivation

The mobile shear force acting on a slice is defined as S_i from the force equilibrium in GD2,also shown in equation (4).

$$S_i = \begin{array}{l} [W_i - X_{i-1} + X_i + U_{t,i} \cos(\beta_i) + Q_i \cos(\omega_i)] \sin(\alpha_i) \\ - [-K_c W_i - G_i + G_{i-1} - H_i + H_{i-1} + U_{t,i} \sin(\beta_i) + Q_i \cos(\omega_i)] \cos(\alpha_i) \end{array} \quad (4)$$

The equation is unsolvable, containing the unknown interslice normal force G and shear force X. Consider a force equilibrium without the affect of interslice forces, to obtain the mobile shear force without the influence of interslice forces T, as done in equation (5).n

$$T_i = \begin{array}{l} [W_i + U_{t,i} \cos(\beta_i) + Q_i \cos(\omega_i)] \sin(\alpha_i) \\ - [-K_c W_i - \Delta H_i + U_{t,i} \sin(\beta_i) + Q_i \sin(\omega_i)] \cos(\alpha_i) \end{array} \quad (5)$$

The values of R_i and T_i are now defined completely in terms of the known force property values of DD1 to DD12.

5.2.5 Instance Models

This section transforms the problem defined in the Section 5.1 into one which is expressed in mathematical terms. It uses concrete symbols defined in Section 5.2.4 to replace the abstract symbols in the models identified in the Sections 5.2.2 and 5.2.3.

The Morgenstern Price Method is a vertical slice, limit equilibrium slope stability analysis method. Analysis is performed by breaking the assumed failure surface into a series of vertical slices of mass. Static equilibrium analysis using two force equilibrium, and one moment equation as in T2. The problem is statically indeterminate with only these 3 equations and one constitutive equation (the Mohr Coulomb shear strength of T3) so the assumption of GD5 is used. Solving for force equilibrium allows definitions of all forces in terms of the physical properties of DD1 to DD12, as done in DD13, DD14.

The values of the interslice normal force G the interslice normal/shear force magnitude ratio λ, and the Factor of Safety FS, are unknown. Equations for the unknowns are written in terms of only the values in DD1 to DD12, the values of R, and T in DD13 and DD14, and each other. The relationships between the unknowns are non linear, and therefore explicit equations cannot be derived and an iterative solution method is required.

Number	IM1
Label	**Factor of Safety**
Input	Ψ_{v} , Φ_{v} , T_{v} , R_{v}
Output	$FS = \dfrac{\sum\limits_{v=1}^{n-1}\left[R_v \prod\limits_{c=i}^{n-1} \dfrac{\Psi_u}{\Phi_u}\right] + R_n}{\sum\limits_{v=1}^{n-1}\left[T_v \prod\limits_{c=i}^{n-1} \dfrac{\Psi_u}{\Phi_u}\right] + T_n}$
Description	Equation for the Factor of Safety, the ratio between resistive and mobile shear the slip surface. The sum of values from each slice is taken to find the total resistive and mobile shear for the slip surface. The constants Φ and Ψ convert the resistive and mobile shear without the influence of interslice forces, to a calculation considering the interslice forces.
Sources	[7]
Ref. By	IM2, IM3

Factor of Safety Derivation

Using equation (15) from section 5.2.5, rearranging, and applying the boundary condition that E_0 and E_{n} are equal to 0 an equation for the factor of safety is found as equation (6), also seen in IM1.

$$\mathrm{FS} = \frac{\sum_{v=1}^{n-1} \left[R_v \prod_{c=v}^{n-1} \frac{\Psi_u}{\Phi_u} \right] + R_n}{\sum_{v=1}^{n-1} \left[T_v \prod_{c=v}^{n-1} \frac{\Psi_u}{\Phi_u} \right] + T_n} \tag{6}$$

The constants Ψ and Φ described in equations 14 and 13 are functions of the unknowns: the interslice normal/shear force ratio λ (IM2) and the Factor of Safety FS (IM1).

Number	IM2
Label	**Normal/Shear Force Ratio**
Input	b_{v} , E_{v} , H_{v} , α_{v} , h_{v} , W_{v} , $U_{\text{t,v}}$, β_{v} , f_{v} , K_{c}
Output	$C1_{\text{i}} = \begin{cases} b_1 \left[E_1 + H_1\right] \tan\left(\alpha_1\right) & \text{i} = 1 \\ b_{\text{i}} \left[\left(E_{\text{i}} + E_{\text{i-1}}\right) + \left(H_{\text{i}} + H_{\text{i-1}}\right)\right] \tan\left(\alpha_{\text{i}}\right) \\ +h_{\text{i}} \left(K_{\text{c}}\, W_{\text{i}} - 2\, U_{\text{t,i}} \sin\left(\beta_{\text{i}}\right) - 2\, Q_{\text{i}} \cos\left(\omega_{\text{i}}\right)\right) & 2 \leq \text{i} \leq \text{n-1} \\ b_{\text{n}} \left[E_{\text{n-1}} + H_{\text{n-1}}\right] \tan\left(\alpha_{\text{n-1}}\right) & \text{i} = \text{n} \end{cases}$ $C2_{\text{i}} = \begin{cases} b_1 E_1 f_1 & \text{i} = 1 \\ b_{\text{i}} \left(f_{\text{i}} E_{\text{i}} + f_{\text{i-1}} E_{\text{i-1}}\right) & 2 \leq \text{i} \leq \text{n-1} \\ b_{\text{n}} E_{\text{n-1}} f_{\text{n-1}} & \text{v} = \text{n} \end{cases}$ $\lambda = \frac{\sum_{i=1}^{n} C1_{\text{i}}}{\sum_{i=1}^{n} C2_{\text{i}}}$
Description	λ is the magnitude ratio between shear and normal forces at the interslice interfaces as the assumption of the Morgenstern Price method in GD5. The inclination function f determines the relative magnitude ratio between the different interslices, while λ determines the magnitude. λ uses the sum of interslice normal and shear forces taken from each interslice.
Sources	[7]
Ref. By	IM1, IM3

Normal/Shear Force Ratio Derivation

The last static equation of T2 the moment equilibrium of GD6 about the midpoint of the base is taken, with the assumption of GD5. Results in equation (7).

$$0 = \begin{array}{l} -G_i\left[z_i - \frac{b_i}{2}\tan(\alpha_i)\right] + G_{i\text{-}1}\left[z_{i\text{-}1} + \frac{b_i}{2}\tan(\alpha_i)\right] - H_i\left[z_{w,i} - \frac{b_i}{2}\tan(\alpha_i)\right] \\ +H_{i\text{-}1}\left[z_{w,i\text{-}1} + \frac{b_i}{2}\tan(\alpha_i)\right] - \lambda\frac{b_i}{2}\left(G_i f_i + G_{i\text{-}1} f_{i\text{-}1}\right) + K_c W_i \frac{h_i}{2} - U_{t,i}\sin(\beta_i)\, h_i - Q_i\sin(\omega_i) h_i \end{array} \tag{7}$$

Rearranging the equation in terms of λ leads to equation (8).

$$\lambda = \frac{\begin{array}{l} -G_i\left[z_i - \frac{b_i}{2}\tan(\alpha_i)\right] + G_{i\text{-}1}\left[z_{i\text{-}1} + \frac{b_i}{2}\tan(\alpha_i)\right] - H_i\left[z_i - \frac{b_i}{2}\tan(\alpha_i)\right] \\ +H_{i\text{-}1}\left[z_{i\text{-}1} + \frac{b_i}{2}\tan(\alpha_i)\right] + K_c W_i \frac{h_i}{2} - U_{t,i}\sin(\beta_i)\, h_i - Q_i\sin(\omega_i) h_i \end{array}}{\frac{b_i}{2}\left[G_i f_i + G_{i\text{-}1} f_{i\text{-}1}\right]} \tag{8}$$

Taking a summation of each slice, and considering the boundary conditions that G_0 and G_n are equal to zero, a general equation for the constant λ is developed in equation (9), also found in IM2.

$$\lambda = \frac{\sum_{i=1}^{n} b_i\left[(G_i + G_{i\text{-}1}) + (H_i + H_{i\text{-}1})\right]\tan(\alpha_i) + h_i\ \left[K_c\, W_i - 2\, U_{t,i}\sin(\beta_i) - 2\, Q_i\sin(\omega_i)\right]}{\sum_{i=1}^{n} b_i\ \left[f_i G_i + f_{i\text{-}1} G_{i\text{-}1}\right]} \tag{9}$$

Equation (9) for λ, is a function of the unknown interslice normal force G (IM3).

Number	IM3
Label	**Interslice Forces**
Input	FS, T_i, R_i, Ψ, Φ
Output	$G_i = \begin{cases} \frac{(\text{FS})T_1 - R_1}{\Phi_i} & \text{i=1} \\ \frac{\Psi_{i\text{-}1}\cdot G_{i\text{-}1} + (\text{FS})\cdot T_i - R_i}{\Phi_i} & 2 \leq \text{i} \leq \text{n-1} \\ 0 & \text{i=0} \vee \text{i=n} \end{cases}$
Description	The value of the interslice normal force G_i at interface i. The net force the weight of the slices adjacent to interface i exert horizontally on each other.
Sources	[7]
Ref. By	IM1, IM2

Interslice Force Derivation

Taking the perpendicular force equilibrium of GD1 with the effective stress definition from T4 that $N_i = N'_i - U_{b,i}$, and the assumption of GD5 the equilibrium equation can be rewritten as equation (10).

$$N'_i = \begin{array}{l} \left[W_i - \lambda \cdot f_{i\text{-}1} \cdot G_{i\text{-}1} + \lambda \cdot f_i \cdot G_i + U_{t,i}\cos(\beta_i) + Q_i\cos(\omega_i)\right]\cos(\alpha_i) \\ +\left[-K_c W_i - G_i + G_{i\text{-}1} - H_i + H_{i\text{-}1} + U_{t,i}\sin(\beta_i) + Q_i\sin(\omega_i)\right]\sin(\alpha_i) - U_{b,i} \end{array} \tag{10}$$

Taking the base shear force equilibrium of GD2 with the definition of mobilized shear from GD4 and the assumption of GD5, the equilibrium equation can be rewritten as equation (11).

$$\frac{N_{\text{i}} \tan(\varphi'_i) + c'_{\text{i}} \cdot b'_{\text{i}} \cdot \sec(\alpha_{\text{i}})}{\text{FS}} = \begin{array}{l} [W_{\text{i}} - \lambda \cdot f_{\text{i-1}} \cdot G_{\text{i-1}} + \lambda \cdot f_{\text{i}} \cdot G_{\text{i}} + U_{\text{t,i}} \cos(\beta_{\text{i}}) + Q_{\text{i}} \cos(\omega_{\text{i}})] \sin(\alpha_{\text{i}}) \\ - [-K_{\text{c}} W_{\text{i}} - G_{\text{i}} + G_{\text{i-1}} - H_{\text{i}} + H_{\text{i-1}} + U_{\text{t,i}} \cdot \sin(\beta_{\text{i}}) + Q_{\text{i}} \sin(\omega_{\text{i}})] \cos(\alpha_{\text{i}}) \end{array} \tag{11}$$

Substituting the equation for N'_{i} from equation (10) into equation (11) and rearranging results in equation (12)

$$G_{\text{i}} \begin{bmatrix} [\lambda \cdot f_{\text{i}} \cos(\alpha_{\text{i}}) - \sin(\alpha_{\text{i}})] \tan(\varphi'_i) \\ - [\lambda \cdot f_{\text{i}} \sin(\alpha_{\text{i}}) + \cos(\alpha_{\text{i}})] \, (\text{FS}) \end{bmatrix} = G_{\text{i-1}} \begin{bmatrix} [\lambda \cdot f_{\text{i-1}} \cos(\alpha_{\text{i}}) - \sin(\alpha_{\text{i}})] \tan(\varphi'_i) \\ - [\lambda \cdot f_{\text{i-1}} \sin(\alpha_{\text{i}}) + \cos(\alpha_{\text{i}})] \, (\text{FS}) \end{bmatrix} + (\text{FS}) \cdot T_{\text{i}} - R_{\text{i}} \tag{12}$$

Where R and T are the resistive and mobile shear of the slice, without the influence of interslice forces G and X, as defined in DD13 and DD14. Making use of the constants ϕ and Ψ with full equations found below in equations (13) and (14) respectively, then equation (12) can be simplified to equation (15), also seen in IM3.

$$\Phi_{\text{i}} = [\lambda \cdot f_{\text{i}} \cos(\alpha_{\text{i}}) - \sin(\alpha_{\text{i}})] \, [\tan(\varphi'_{\text{i}})] - [\lambda \cdot f_{\text{i}} \sin(\alpha_{\text{i}}) + \cos(\alpha_{\text{i}})] \, (\text{FS}) \tag{13}$$

$$\Psi_{\text{i}} = [\lambda \cdot f_{\text{i}} \cos(\alpha_{\text{i+1}}) - \sin(\alpha_{\text{i+1}})] \, [\tan(\varphi')] - [\lambda \cdot f_{\text{i}} \sin(\alpha_{\text{i+1}}) + \cos(\alpha_{\text{i+1}})] \, (\text{FS}) \tag{14}$$

$$G_{\text{i}} = \frac{\Psi_{\text{i-1}} \, G_{\text{i-1}} + (FS) \; T_{\text{i}} - R_{\text{i}}}{\Phi_{\text{i}}} \tag{15}$$

The constants Ψ and Φ in equation (15) for are functions of the unknowns: the interslice normal/shear force ratio λ (IM2), and the Factor of Safety FS (IM1).

Number	IM4
Label	**Critical Slip Identification**
Input	The geometry of the water table, the geometry of the layers composing the plane of a slope, and the material properties of the layers.
Output	$\text{FS}_{\text{Min}} = \Upsilon(\{x_{\text{cs}}, y_{\text{cs}}\}, \text{Input})$
Description	Given the necessary slope inputs, a minimization algorithm or function Υ, will identify the critical slip surface of the slope, with the critical slip coordinates $\{x_{\text{cs}}, y_{\text{cs}}\}$ and the minimum factor of safety FS_{Min} that results.
Sources	[3]

[Should this IM exist? It doesn't arise from any T or A —BM]

5.2.6 Data Constraints

Table 2 and 3 shows the data constraints on the input and output variables, respectively. The column for physical constraints gives the physical limitations on the range of values that can be taken by the variable. The uncertainty column provides an estimate of the confidence with which the physical quantities can be measured. This information would be part of the input if one were performing an uncertainty quantification exercise. The constraints are conservative, to give the user of the model the flexibility to experiment with unusual situations. The column of typical values is intended to provide a feel for a common scenario.

Var	Physical Constraints	Typical Value	Uncertainty
(x, y) of water table vertices'	Consecutive vertices have increasing x values. All layers start and end vertices' go to the same x values.	N/A	10%
(x, y) of slip vertices'	Consecutive vertices have increasing x values. All layers start and end vertices' go to the same x values.	N/A	10%
(x, y) of slope vertices' (*)	Consecutive vertices have increasing x values. All layers start and end vertices' go to the same x values.	N/A	10%
E (*)	$E > 0$	15000	10%
c (*)	$c > 0$	10	10%
v (*)	$0 < v < 1$	0.4	10%
φ' (*)	$0 < \varphi < 90$	25	10%
γ (*)	$\gamma > 0$	20	10%
γ_{Sat} (*)	$\gamma_{\text{Sat}} > 0$	20	10%
γ_{w}	$\gamma_{\text{w}} > 0$	9.8	10%

(*) Input coordinates needed for each layer.

Var	Physical Constraints
FS	$FS > 0$
(x, y) Slip vertices'	Vertices are monotonic

6 Requirements

This section provides the functional requirements, the business tasks that the software is expected to complete, and the nonfunctional requirements, the qualities that the software is expected to exhibit.

6.1 Functional Requirements

R1: Read the input file, and store the data. Necessary input data summarized in Table1. [A2, A3]

symbol	unit	description
(x, y)	m	x and y coordinates for vertices of the slope layers, and for the water table if one exists. Assumed straight line fits between vertices.
E	kPa	Young's modulus for each layer of the slope.
c'	kPa	Cohesion for each slope layer.
v	/	Poisson's ratio for each soil layer.
φ'	$\circ$	Effective angle of friction for each slope layer.
γ	$\frac{\text{kN}}{\text{m}^3}$	Unit weight of dry soil / ground layer for each slope layer.
γ_{Sat}	$\frac{\text{kN}}{\text{m}^3}$	Unit weight of saturated soil / ground layer for each slope layer.
γ_{w}	$\frac{\text{kN}}{\text{m}^3}$	Unit weight of water.

R2: Generate potential critical slip surface's for the input slope.

R3: Test the slip surfaces to determine if they are physically realizable based on a set of pass or fail criteria. [A1]

R4: Prepare the slip surfaces for a method of slices or limit equilibrium analysis.

R5: Calculate the factors of safety of the slip surfaces.

R6: Rank and weight the slopes based on their factor of safety, such that a slip surface with a smaller factor of safety has a larger weighting.

R7: Generate new potential critical slip surfaces based on previously analysed slip surfaces with low factors of safety.

R8: Repeat requirements R3 to R7 until the minimum factor of safety remains approximately the same over a predetermined number of repetitions. Identify the slip surface that generates the minimum factor of safety as the critical slip surface.

[Can we reference requirements in other requirements like this? This reads like a procedure, not a list of requirements. —BM]

R9: Prepare the critical slip surface for method of slices or limit equilibrium analysis.

R10: Calculate the factor of safety of the critical slip surface using the Morgenstern price method.

R11: Display the critical slip surface graphically. Display the values of the factors of safety.

6.2 Nonfunctional Requirements

SSP is intended to be an educational tool, therefore accuracy and performance speed are secondary program priorities to correctness, understandability, reusability, and maintainability.

7 Likely Changes

LC1: The system currently assumes the different layers of the soil are homogeneous. In the future, implementation can be added for inconsistent soil properties throughout.

8 Unlikely Changes

If changes were to be made with regard to the following, a different algorithm would be needed.

UC1: Changes related to A5 and A6 are not possible due to the dependency of the calculations on the linear relationship between interslice normal and shear forces.

UC2: A8 allows for 2D analysis with these models only because stress along z-direction is zero. These models do not take into account stress in the z-direction, and therefore cannot be used without manipulation to attempt 3D analysis.

9 Values of Auxiliary Constants

There are no auxiliary constants.

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