Software Requirements Specification for Slope Stability Analysis

Henry Frankis
McMaster University

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Contents

1 Reference Material

1.1 Table of Units

Units of the Physical properties of the soil that are of interest when examining slope stability problems.

Physical Property	Name	Symbol
force	Newton	N
length	meter	m
pressure	Pascal	$\mathrm{Pa} = \mathrm{N\,m^{-2}}$
angle	degree	$^\circ$

1.2 Table of Symbols

A collection of the symbols, that will be used in the models and equations of the program are summarized in the table below. Values with a subscript i implies that the value will be taken at and analyzed at a slice or slice interface composing the total slip mass.

Symbol	Unit	Description
φ'	$^\circ$	Effective angle of friction.
c'	Pa	Effective cohesion.
γ	$\mathrm{kN\,m^{-3}}$	Unit weight of dry soil / ground layer.
γ_{Sat}	$\mathrm{kN\,m^{-3}}$	Unit weight of saturated soil / ground layer.
γ_{w}	$\mathrm{kN\,m^{-3}}$	Unit weight of water.
E	Pa	Elastic modulus.
ν	/	Poisson's ratio.
(x, y)	m	Cartesian position coordinates. y is considered parallel to the direction of the force of gravity and x is considered perpendicular to y.
$y_{\mathrm{wt,i}}$	m	The y-ordinate, or height of the water table at x_{i}. Can refer to either slice i midpoint, or slice interface i.
$y_{\mathrm{us,i}}$	m	The y-ordinate, or height of the top of the slope at x_{i}. Can refer to either slice i midpoint, or slice interface i.
$y_{\mathrm{slip,i}}$	m	The y-ordinate, or height of the slip surface at x_{i}. Can refer to either slice i midpoint, or slice interface i.

x_{i}	m	The x-ordinate. Can refer to either slice i midpoint, or slice interface i.
$(\{x_{\mathrm{cs}}\}, \{y_{\mathrm{cs}}\})$	m	The set of x and y coordinates that describe the vertexes of the critical slip surface.
FS	/	Global Factor of Safety. Metric describing the stability of a surface in a slope.
$\mathrm{FS}_{\mathrm{Loc,i}}$	/	Local Factor of Safety. Factor of Safety specific to a slice. For slice index i.
S_{i}	N	Mobilized shear force. Shear forces that cause instability in a slice. For slice index i.
P_{i}	N	Shear resistance. Mohr Coulomb frictional force that describes the limit of mobilized shear force the slice can withstand before failure. For slice index i.
T_{i}	N	Mobilized shear force, without the influence of interslice forces. For slice index i.
R_{i}	N	Shear resistance, without the influence of interslice forces. For slice index i.
W_{i}	N	Weight. Downward force caused by gravity the mass of slice i exerts. For slice index i.
K_{c}	/	Earthquake load factor. Proportionality factor of force that weight pushes outwards. Caused by seismic earth movements.
H_{i}	N	Interslice water force exerted in the x-ordinate direction between adjacent slices. For interslice index i
ΔH_{i}	N	Difference between interslice forces on acting in the x-ordinate direction of the slice on each side. For slice index i. Refers to net force $H_{\mathrm{i}} - H_{\mathrm{i\text{-}1}}$
E_{i}	N	Interslice normal force being exerted between adjacent slices. For interslice index i.
X_{i}	N	Interslice shear force being exerted between adjacent slices. For interslice index i.
$U_{\mathrm{b,i}}$	N	Base hydrostatic force. Force from water pressure within the slice. For slice index i.

$U_{\mathrm{t,i}}$	N	Surface hydrostatic force. Force from water pressure acting into the slice from standing water on the slope surface. For slice index i.
N_{i}	N	Total reactive force for a soil surface subject to a body resting on it.
N'_{i}	N	Effective normal force of a soil surface, subtracting pore water reactive force from total reactive force.
$N{*}_{\mathrm{i}}$	N	Effective normal force of a soil surface, neglecting the influence of interslice forces.
Q_{i}	N	An imposed surface load. A downward force acting into the surface from midpoint of slice i.
α_{i}	$^\circ$	Angle of the base of the mass relative to the horizontal. For slice index i.
β_{i}	$^\circ$	Angle of the surface of the mass relative to the horizontal. For slice index i.
ω_{i}	$^\circ$	Angle of imposed surface load acting into the surface relative to the vertical. For slice index i.
λ	/	Ratio between Interslice normal and shear forces. Applied to all interslices.
f_{i}	/	function for inclination of interslice forces. A scaling function for magnitude of interslice forces as a function of the x coordinate. Can be constant or a half-sine. Value of function at interslice index i.
b_{i}	m	Base width of the slice in the x-ordinate direction only. For slice i.
$\ell_{\mathrm{b,i}}$	m	Total length of the base of a slice. For slice index i.
$\ell_{\mathrm{s,i}}$	m	Length of an interslice surface, from slip base to slope surface in a vertical line from an interslice vertex. For interslice index i.
h_{i}	m	Midpoint height. Distance from the slip base to the slope surface in a vertical line from the midpoint of the slice. For slice i.
n	/	Number of slices the slip mass has been divided into.

F	N	A generic force. Assumed 1D allowing a scalar.
M	N m	Moment of a body. Assumed 2D allowing a scalar.
Υ	/	A generic minimization function or algorithm.
δ	m	Generic displacement of a body.
K	newton/m	Stiffness. How much a body resists displacement when subject to a force.
$K_{\text{st,i}}$	Pa	Shear stiffness of an interslice surface, without length adjustment. For interslice index i.
$K_{\text{bt,i}}$	Pa	Shear stiffness of a slice base surface, without length adjustment. For slice index i.
$K_{\text{sn,i}}$	Pa	Normal stiffness of an interslice surface, without length adjustment. For interslice index i.
$K_{\text{bn,i}}$	Pa	Normal stiffness of a slice base surface, without length adjustment .For slice index i.
K_{tr}	Pa	Residual shear stiffness.
K_{no}	Pa	Residual normal stiffness.
δu_{i}	m	Shear displacement of a slice. For slice index i.
δv_{i}	m	Normal displacement of a slice. For slice index i.
δx_{i}	m	Displacement of a slice in the x-ordinate direction. For slice index i.
δy_{i}	m	Displacement of a slice in the y-ordinate direction. For slice index i.

1.3 Abbreviations and Acronyms

symbol	description
A	Assumption
DD	Data Definition
GD	General Definition
GS	Goal Statement
IM	Instance Model
LC	Likely Change
PS	Physical System Description
R	Requirement
SRS	Software Requirements Specification
SSA	Slope Stability Analysis
T	Theoretical Model

2 Introduction

A slope of Geological mass, composed of soil and rock, is subject to the influence of gravity on the mass. For an instable slope this can cause instability in the form of soil/rock movement. The effects of soil/rock movement can range from inconvenient to seriously hazardous, resulting in significant life and economic loses. Slope stability is of interest both when analyzing natural slopes, and when designing an excavated slope. Slope stability analysis is the assessment of the safety of a slope, identifying the surface most likely to experience slip and an index of it's relative stability known as the factor of safety.

The following section provides an overview of the Software Requirements Specification (SRS) for a slope stability analysis problem. The developed program will be referred to as Slope Stability Analysis program (SSA). This section explains the purpose of this document, the scope of the system, the organization of the document and the characteristics of the intended readers.

2.1 Purpose

The SSA program is slope stability analysis program. The program determines the critical slip surface, and it's respective factor of safety as a method of assessing the stability of a slope design. The program is intended to be used as an educational tool, introducing slope stability issues, analysis software, and the design of a safe slope.

This document will be used as a starting point for subsequent development phases, including writing the design specification and the software verification and validation plan. The design document will show how the requirements are to be realized, including decisions on the numerical algorithms and programming environment. The verification and validation plan will show the steps that will be used to increase confidence in the software documentation and the implementation. Although the SRS fits in a series of documents that follow the so-called waterfall model, the actual development process is not constrained in any way. Even when the process is not waterfall, as Parnas and Clements [4] point out, the most logical way to present the documentation is still to "fake" a rational design process.

2.2 Scope of Requirements

The scope of the requirements is limited to stability analysis of a 2 dimensional slope, composed of homogeneous soil layers. Given appropriate inputs the code for SSA will identify the most likely failure surface within the possible input range, and find the factor of safety for the slope and displacement of soil that will occur on the slope.

2.3 Organization of Document

The organization of this document follows the template for an SRS for scientific computing software proposed by [2] and [5]. The presentation follows the standard pattern of presenting goals, theories, definitions, and assumptions. For readers that would like a more bottom up approach, they can start reading the instance models in Section 4.2.5 and trace back to find any additional information they require. The instance models provide the set of algebraic equations that must be solved iteratively to perform a Morgenstern Price Analysis, and the system of equations that must be solved for Rigid Finite Element Analysis.

The goal statements are refined to the theoretical models, and theoretical models (Section 4.2.2) to the instance models (Section 4.2.5).

3 General System Description

This section provides general information about the system, identifies the interfaces between the system and its environment, and describes the user characteristics and the system constraints.

3.1 User Characteristics

The end user of SSA should have an understanding of undergraduate Level 1 Calculus and Physics, and be familiar with soil and material properties.

3.2 System Constraints

There are no system constraints.

4 Specific System Description

This section first presents the problem description, which gives a high-level view of the problem to be solved. This is followed by the solution characteristics specification, which presents the assumptions, theories, definitions and finally the instance models that model the slope. The program implements two solution methods to analyze the slope with. Models with **purple** table headings refer to a Morgenstern Price solution. Models with **blue** table headings refer to a RFEM solution. Models with **brass** table headings are models that refer to both solutions.

4.1 Problem Description

SSA is a computer program developed to evaluate the factor of safety of a slopes slip surface and, calculate the displacement the slope will experience.

4.1.1 Terminology

- *Factor of safety:* Stability metric. How likely a slip surface is to experience failure through slipping.
- *Critical slip surface:* Slip surface of the slope that has the lowest global factor of safety, and therefore most likely to experience failure.
- *Stress:* Forces that are exerted between planes internal to a larger body subject to external loading.
- *Strain:* Stress forces that result in deformation of the body/plane.
- *Normal Force:* A force applied perpendicular to the plane of the material.
- *Shear Force:* A force applied parallel to the plane of the material.
- *Tension:* A stress the causes displacement of the body away from it's center.
- *Compression:* A stress the causes displacement of the body towards it's center.
- *Plane Strain:* The resultant stresses in one of the directions of a 3 dimensional material can be approximated as 0. Results when the length of one dimension of the body dominates the others. Stresses in the dominate dimensions direction are the ones that can be approximated as 0.

4.1.2 Physical System Description

Analysis of the slope is performed by looking at properties of the slope as a series of slice elements. Some properties are interslice properties, and some are slice or slice base properties. The index convention for referencing which interslice or slice is being used is shown in Fig 1.

- Interslice properties convention is noted by j. The end Interslice properties are usually not of interest, therefore use the interslice properties from $1 \leq \mathrm{i} \leq n-1$.
- Slice properties convention is noted by i.

A free body diagram of the forces acting on the slice is displayed in Fig 2.

4.1.3 Goal statements

Given the geometry of the water table, the geometry of the layers composing the plane of a slope, and the material properties of the layers.

G1: Evaluate local and global factors of safety along a given slip surface.

G2: Identify the critical slip surface for the slope, with the lowest Factor of Safety.

G3: Determine the displacement of the slope.

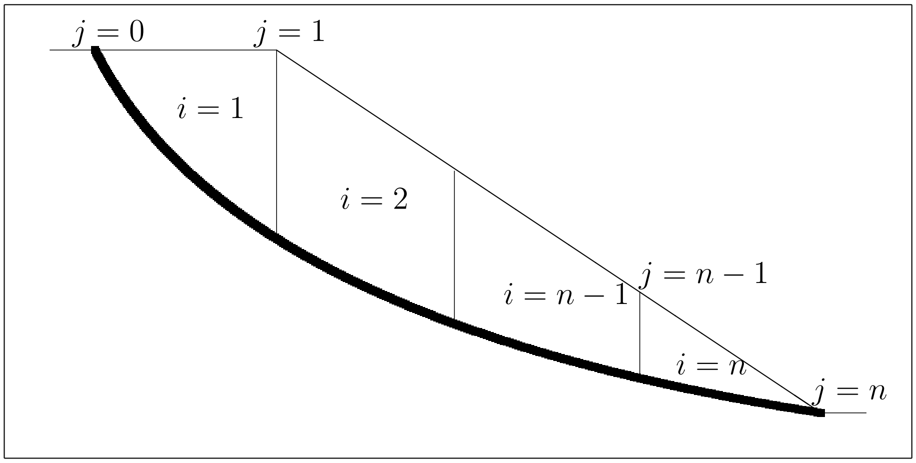

Figure 1: Index convention for numbering slice and interslice force variables

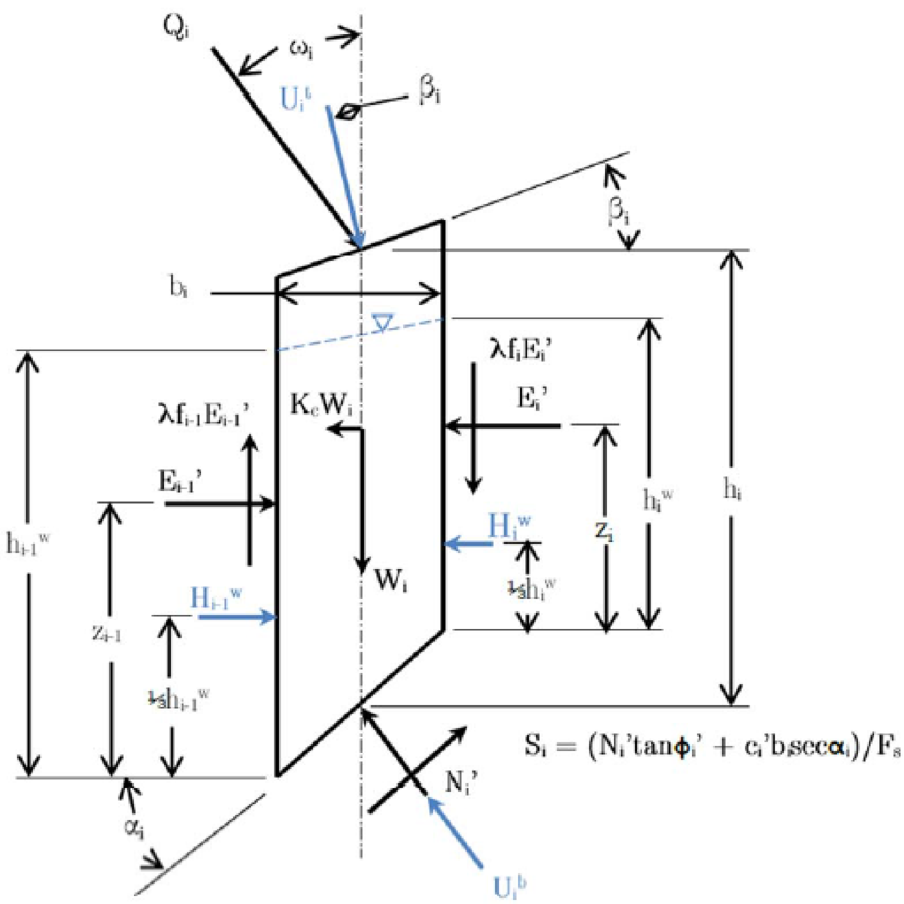

Figure 2: Forces acting on a slice

4.2 Solution Characteristics Specification

The instance models that govern SSA are presented in Subsection 4.2.5. The information to understand the meaning of the instance models and their derivation is also presented, so that the instance models can be verified.

4.2.1 Assumptions

This section simplifies the original problem and helps in developing the theoretical model by filling in the missing information for the physical system. The numbers given in the square brackets refer to the data definition, or the instance model, in which the respective assumption is used.

A1: The slip surface is concave with respect to the slope surface. The (x,y) coordinates of the failure surface follow a monotonic function.

A2: The geometry of the slope, and the material properties of the soil layers are given as inputs.

A3: The different layers of the soil are homogeneous, with consistent soil properties throughout, and independent of dry or saturated conditions, with the exception of unit weight.

A4: Soil layers are treated as if they have isotropic properties.

A5: Interslice normal and shear forces have a linear relationship, proportional to a constant (λ) and an interslice force function (f) depending on x position.

A6: Slice to base normal and shear forces have a linear relationship, dependent on the factor of safety (FS), and the Coulomb sliding law.

A7: The stress-strain curve for interslice relationships is linear with a constant slope.

A8: The slope and slip surface extends far into and out of the geometry (z coordinate). This implies plane strain conditions, making 2D analysis appropriate.

A9: The effective normal stress is large enough that the resistive shear to effective normal stress relationship can be approximated as a linear relationship.

A10: The surface and base of a slice between interslice nodes are approximated as straight lines.

4.2.2 Theoretical Models

This section focuses on the general equations and laws that SSA is based on.

Number	T1
Label	**Factor of Safety**
Equation	$\mathrm{FS} = \frac{P}{S}$
Description	The stability metric of the slope, known as the factor of safety FS, is determined by the ratio of the shear force at the base of the slope S (GD4), and the resistive shear P (T3).
Source	[1]
Ref. By	IM1, GD4

Number	T2
Label	**Equilibrium**
Equation	$\sum F_{\text{x}} = 0$ $\sum F_{\text{y}} = 0$ $\sum M = 0$
Description	For a body in static equilibrium the net forces, and net moments acting on the body will cancel out. Assuming a 2D problem (A8) the net x-ordinate (F_{x}) and y-ordinate (F_{y}) scalar components will be equal to 0. All forces and their distance from the chosen point of rotation will create a net moment equal to 0, also able to be analyzed as a scalar in a 2D problem.
Source	[1]
Ref. By	GD1, GD2, GD6, IM2

Number	T3
Label	**Mohr-Coulomb Shear Strength**
Equation	$P = \sigma \cdot \tan(\varphi') + c$
Description	For a soil under stress it will exert a shear resistive strength based on the Coulomb sliding law. The resistive shear is the maximum amount of shear a surface can experience while remaining rigid, analogous to a maximum normal force. In this model the shear force P is proportional to the product of the normal stress on the plane σ with it's static friction, in the angular form $\tan(\varphi') = U_{\text{s}}$. The P versus σ relationship is not truly linear, but assuming the effective normal force is strong enough it can be approximated with a linear fit (A9), where the cohesion c represents the P intercept of the fitted line.
Source	[1]
Ref. By	GD3, GD4, DD10, DD11, IM5

Number	T4
Label	**Effective Stress**
Equation	$\sigma' = \sigma - \mu$
Description	σ is the total stress a soil mass needs to maintain itself as a rigid collection of particles. The source of the stress can be provided by the soil skeleton σ', or by the pore pressure from water within the soil μ. The stress from the soil skeleton is known as the effective stress σ' and is the difference between the total stress σ and the pore stress μ.
Source	[1]
Ref. By	GD3, GD4, DD10, DD11, IM3

Number	T5
Label	**Hooke's Law**
Equation	$F = K \cdot \delta$
Description	Stiffness K is the resistance a body offers to deformation by displacement δ when subject to a force F, along the same direction. A body with high stiffness will experience little deformation when subject to a force.
Source	[6]
Ref. By	GD7, GD8, DD14, IM4, IM5

4.2.3 General Definitions

This section collects the laws and equations that will be used in deriving the data definitions, which in turn are used to build the instance models.

Number	GD1
Label	**Normal Force Equilibrium**
Equation	$N_{\text{i}} = [W_{\text{i}} - X_{\text{i-1}} + X_{\text{i}} + U_{\text{t,i}} \cos(\beta_{\text{i}}) + Q_{\text{i}} \cos(\omega_{\text{i}})] \cos(\alpha_{\text{i}}) + [-K_{\text{c}}\, W_{\text{i}} - E_{\text{i}} + E_{\text{i-1}} - H_{\text{i}} + H_{\text{i-1}} + U_{\text{t,i}} \sin(\beta_{\text{i}}) + Q_{\text{i}} \sin(\omega_{\text{i}})] \sin(\alpha_{\text{i}})$
Description	For a slice of mass in the slope the force equilibrium to satisfy T2 in the direction perpendicular to the base surface of the slice. Rearranged to solve for the normal force of the surface N_{i}. Force equilibrium is derived from the free body diagram of Fig 2 in section 4.1.2. Index i refers to the values of the properties for slice/interslices following convention in Fig 1 in section 4.1.2. Force variable definitions can be found in DD1 to DD9.
Source	[7]
Ref. By	DD10, DD11, IM3

Number	GD2
Label	**Base Shear Force Equilibrium**
Equation	$S_{\text{i}} = [W_{\text{i}} - X_{\text{i-1}} + X_{\text{i}} + U_{\text{t,i}} \cos(\beta_{\text{i}}) + Q_{\text{i}} \cos(\omega_{\text{i}})] \sin(\alpha_{\text{i}}) - [-K_{\text{c}}\, W_{\text{i}} - E_{\text{i}} + E_{\text{i-1}} - H_{\text{i}} + H_{\text{i-1}} + U_{\text{t,i}} \sin(\beta_{\text{i}}) + Q_{\text{i}} \cos(\omega_{\text{i}})] \cos(\alpha_{\text{i}})$
Description	For a slice of mass in the slope the force equilibrium to satisfy T2 in the direction parallel to the base surface of the slice. Rearranged to solve for the shear force acting on the base S_{i}. Force equilibrium is derived from the free body diagram of Fig 2 in section 4.1.2. Index i refers to the values of the properties for slice/interslices following convention in Fig 1 in section 4.1.2. Force variable definitions can be found in DD1 to DD9.
Source	[7]
Ref. By	DD10, DD11,IM3

Number	GD3
Label	**Resistive Shear**
Equation	$P_\mathrm{i} = N'_\mathrm{i} \cdot \tan\left(\varphi'_\mathrm{i}\right) + c' \cdot b_\mathrm{i} \cdot \sec\left(\alpha_\mathrm{i}\right)$
Description	The Mohr-Coulomb resistive shear strength of a slice P_i is adjusted to account for the effective normal $\sigma' = N' = N - U_\mathrm{b}$ of a soil from T4. Also and the cohesion is adjusted to account for the length ℓ of the plane where the normal occurs, where $\ell_{\mathrm{b,i}} = b_\mathrm{i} \cdot \sec\left(\alpha\right)$, and b_i is the x width of the base. therefore $c = c' \cdot b_\mathrm{i} \cdot \sec\left(\alpha_\mathrm{i}\right)$.
Source	[7]
Ref. By	GD4, DD10, DD11

Number	GD4
Label	**Mobile Shear**
Equation	$S_\mathrm{i} = \frac{P_\mathrm{i}}{\mathrm{FS}} = \frac{N'_\mathrm{i} \cdot \tan\left(\varphi'_\mathrm{i}\right) + c' \cdot b_\mathrm{i} \cdot \sec\left(\alpha_\mathrm{i}\right)}{\mathrm{FS}}$
Description	From the definition of the Factor of Safety in T1, and the new definition of P_i, a new relation for the net mobile shear force of the slice T_i is found as the resistive shear P_i (GD3) divided by the factor of safety FS.
Source	[7]
Ref. By	DD10, DD11

Number	GD5
Label	**Interslice Normal/Shear Relationship**
Equation	$X_\mathrm{i} = \lambda \cdot f_\mathrm{i} \cdot E_\mathrm{i}$
Description	The assumption for the Morgenstern Price method (A5) that the interslice shear force X_i is proportional to the interslice normal force E_i by a proportionality constant λ, and a predetermined scaling function f, that changes the proportionality as a function of the x-ordinate position of the interslice. f is typically either a half-sine along the slip surface, or a constant.
Source	[7]
Ref. By	DD10, DD11, IM1, IM2, IM3

Number	GD6
Label	**Moment Equilibrium**
Equation	$0 = \begin{array}{l} -E_{\text{i}}\left[z_{\text{i}}+\frac{b_{\text{i}}}{2}\tan\left(\alpha_{\text{i}}\right)\right]+E_{\text{i-1}}\left[z_{\text{i-1}}-\frac{b_{\text{i}}}{2}\tan\left(\alpha_{\text{i}}\right)\right]-H_{\text{i}}\left[z_{\text{w,i}}+\frac{b_{\text{i}}}{2}\tan\left(\alpha_{\text{i}}\right)\right] \\ +H_{\text{i-1}}\left[z_{\text{w,i-1}}-\frac{b_{\text{i}}}{2}\tan\left(\alpha_{\text{i}}\right)\right]+\frac{b_{\text{i}}}{2}\left(X_{\text{i}}+X_{\text{i-1}}\right)-K_{\text{c}}W_{\text{i}}\frac{h_{\text{i}}}{2}+U_{\text{t,i}}\sin\left(\beta_{\text{i}}\right)h_{\text{i}} \\ +Q_{\text{i}}\sin\left(\omega_{\text{i}}\right)h_{\text{i}} \end{array}$
Description	The equation shows the moment equilibrium (T2) for a slice of mass in the slope. Moment equilibrium is derived from the free body diagram of Fig 2 in section 4.1.2. Index i refers to the values of the properties for slice/interslices following convention in Fig 1 in section 4.1.2. Variable definitions can be found in DD1 to DD9. The signs shown in the equation consider a clockwise moment as positive and a counter clockwise moment as negative. (The opposite can be achieved simply by multiplying the equation by -1.) The moment arm for the force $K_{\text{c}}W_{\text{i}}$ is the equivalent moment arm that results from considering a uniformly distributed load over the length h_{i}. The equation shown matches with [7, Eq. 7], except that H and U values are not included there and the subscripts for i and i-1 have been swapped.
Source	[7]
Ref. By	IM2

Number	GD7
Label	**Net Force**
Equation	$F_{\text{x,i}} = -\Delta H_{\text{i}} - K_{\text{c}} \cdot W_{\text{i}} - U_{\text{b,i}}\sin\left(\alpha_{\text{i}}\right) + U_{\text{t,i}}\sin\left(\beta\right) + Q_{\text{i}}\sin\left(\omega_{\text{i}}\right)$ $F_{\text{y,i}} = -W_{\text{i}} + U_{\text{b,i}}\cos\left(\alpha_{\text{i}}\right) - U_{\text{t,i}}\cos\left(\beta_{\text{i}}\right) - Q_{\text{i}}\cos\left(\omega_{\text{i}}\right)$
Description	The net sum of forces acting on a slice for the RFEM model. The forces that create an applied load on the slice. $F_{\text{x,i}}$ refers to the load in the direction perpendicular to the direction of the force of gravity for slice i, while $F_{\text{y,i}}$ refers to the load in the direction parallel to the force of gravity for slice i. Forces are found in the free body diagram of Fig 2 in section 4.1.2. In this model the elements are not exerting forces on each other, so the interslice forces E and X are not a part of the model. Index i refers to the values of the properties for slice/interslices following convention in Fig 1 in section 4.1.2. Force variable definitions can be found in DD1 to DD8.
Source	[7]
Ref. By	DD13, IM4

Number	GD8
Label	**Hooke's Law 2D**
Equation	$\begin{bmatrix} p_{\text{t,i}} \\ p_{\text{n,i}} \end{bmatrix} = \begin{bmatrix} K_{\text{t,i}} & 0 \\ 0 & K_{\text{n,i}} \end{bmatrix} \begin{bmatrix} \delta t_{\text{i}} \\ \delta n_{\text{i}} \end{bmatrix}$
Description	A 2D component implementation of Hooke's law as seen in T5. δn_{i} is the displacement of the element normal to the surface and δt_{i} is the displacement of the element parallel to the surface. $p_{\text{n,i}}$, is the net pressure acting normal to the surface, and $p_{\text{t,i}}$ is the net pressure acting parallel to the surface. Pressure is used in place of force as the surface has not been normalized for it's length. The stiffness values $K_{\text{n,i}}$ and $K_{\text{t,i}}$ are then the resistance to displacement in the respective directions defined as in DD14. The pressure forces would be the result of applied loads on the mass, the product of the stiffness elements with the displacement would be the mass's reactive force that creates equilibrium with the applied forces after reaching the equilibrium displacement.
Source	[6]
Ref. By	DD12, IM4

Number	GD9
Label	**Displacement Vectors**
Equation	$\bar{\delta}_{\text{i}} = \begin{bmatrix} \delta x_{\text{i}} \\ \delta y_{\text{i}} \end{bmatrix}$ $\bar{\epsilon}_{\text{i}} = \begin{bmatrix} \delta u_{\text{i}} \\ \delta v_{\text{i}} \end{bmatrix} = \begin{bmatrix} \cos(\alpha_{\text{i}}) & \sin(\alpha_{\text{i}}) \\ -\sin(\alpha_{\text{i}}) & \cos(\alpha_{\text{i}}) \end{bmatrix} \bar{\delta}_{\text{i}}$
Description	Vectors describing the displacement of slice i. $\bar{\delta}_{\text{i}}$ is the displacement in the unrotated coordinate system, where δx_{i} is the displacement of the slice perpendicular to the direction of gravity, and δy_{i} is the displacement of the slice parallel to the force of gravity. $\bar{\epsilon}_{\text{i}}$ is the displacement in the rotated coordinate system, where δu_{i} is the displacement of the slice parallel to the slice base, and δy_{i} is the displacement of the slice perpendicular to the slice base. $\bar{\epsilon}_{\text{i}}$ can also be found by rotating $\bar{\delta}_{\text{i}}$ clockwise by the base angle α through a rotation matrix as shown.
Source	[6]
Ref. By	DD12, IM4, IM5

4.2.4 Data Definition

This section collects and defines all the data needed to build the instance models. Definitions DD1 to DD8 are the force variables that can be solved by direct analysis of given inputs. The interslice forces DD9 are force variables that must be written in terms of DD1 to DD8 to solve.

Number	DD1
Label	**Slice Weight**
Equation	$W_{\text{i}} = b_{\text{i}} \cdot \begin{cases} [y_{\text{us,i}} - y_{\text{slip,i}}] \cdot \gamma_{\text{Sat}} & \text{If } y_{\text{wt,i}} \geq y_{\text{us,i}} \\ [y_{\text{us,i}} - y_{\text{wt,i}}] \cdot \gamma + [y_{\text{wt,i}} - y_{\text{slip,i}}] \cdot \gamma_{\text{Sat}} & \text{If } y_{\text{us,i}} > y_{\text{wt,i}} > y_{\text{slip,i}} \\ [y_{\text{us,i}} - y_{\text{slip,i}}] \cdot \gamma & \text{If } y_{\text{wt,i}} \leq y_{\text{slip,i}} \end{cases}$
Description	The weight of the slice W is how much force the soil mass exerts into the slip surface. The piecewise function is the unit weight of the slice. Wet regions are determined by $(y_{\text{wt,i}} > y_{\text{slip,i}})$, where $y_{\text{wt,i}}$ is the height of the water table and $y_{\text{slip,i}}$ is the height of the slip surface, at the midpoint of slice i. Dry regions are determined by $(y_{\text{us,i}} > y_{\text{slip,i}})$, where $y_{\text{us,i}}$ is the height of the slice surface at the midpoint of slice i. The height of weight regions is multiplied by weight of wet soil γ_{Sat}, and dry regions are multiplied by the weight of dry soil γ. the sum is the total unit weight of the slice. The unit weight is multiplied with the width of the slice b_{i}(DD6) to get the total weight of the slice.
Sources	[1]
Ref. By	DD10, DD11, IM1, IM2, IM3

Number	DD2
Label	**Base Water Force**
Equation	$U_{\text{b,i}} = \ell_{\text{b,i}} \cdot \begin{cases} [y_{\text{wt,i}} - y_{\text{slip,i}}] \cdot \gamma_{\text{w}} & \text{If } y_{\text{wt,i}} > y_{\text{slip,i}} \\ 0 & \text{If } y_{\text{wt,i}} \leq y_{\text{slip,i}} \end{cases}$
Description	The base water force $U_{\text{b,i}}$ is how much force the water contributes to the total normal force, the variable μ from T4. If the slice contains water $y_{\text{wt,i}} > y_{\text{slip,i}}$, where $y_{\text{wt,i}}$ is the height of the water table and $y_{\text{slip,i}}$ is the height of the slip surface at the midpoint of slice i,then the water will contribute to the normal force of the slice. The unit base water force is the product of the height of the water above the slip surface with the unit weight of water γ_{w}. The unit base water force is normalized by multiplying by the length of the base of the slice $\ell_{\text{b,i}}$, to get the total base water force.
Sources	[1]
Ref. By	DD10, DD11, IM1, IM2, IM3

Number	DD3
Label	**Surface Water Force**
Equation	$U_{\mathrm{t,i}} = \ell_{\mathrm{s,i}} \cdot \begin{cases} [y_{\mathrm{wt,i}} - y_{\mathrm{us,i}}] \cdot \gamma_{\mathrm{w}} & \text{If } y_{\mathrm{wt,i}} > y_{\mathrm{us,i}} \\ 0 & \text{If } y_{\mathrm{wt,i}} \leq y_{\mathrm{us,i}} \end{cases}$
Description	The surface water force $U_{\mathrm{t,i}}$, exerting a force downwards into the slice. Occurs when standing water is resting on the surface of the slope, such that. $y_{\mathrm{wt,i}} > y_{\mathrm{us,i}}$, where $y_{\mathrm{wt,i}}$ is the height of the water table and $y_{\mathrm{us,i}}$ is the height of the slope surface at the midpoint of slice i. The unit surface water force is the product of the height the water is above the surface with the unit weight of water γ_{w}. The unit surface water force is normalized by multiplying by the length of the surface of the slice $\ell_{\mathrm{s,i}}$, to get the total base water force.
Sources	[1]
Ref. By	DD10, DD11, IM1, IM2, IM3

Number	DD4
Label	**Interslice Water Forces**
Equation	$H_{\mathrm{i}} = \begin{cases} \frac{[y_{\mathrm{us,i}} - y_{\mathrm{slip,i}}]^2}{2} \cdot \gamma_{\mathrm{Sat}} + [y_{\mathrm{wt,i}} - y_{\mathrm{us,i}}]^2 \cdot \gamma_{\mathrm{Sat}} & \text{If } y_{\mathrm{wt,i}} \geq y_{\mathrm{us,i}} \\ \frac{[y_{\mathrm{wt,i}} - y_{\mathrm{slip,i}}]^2}{2} \cdot \gamma_{\mathrm{Sat}} & \text{If } y_{\mathrm{us,i}} > y_{\mathrm{wt,i}} > y_{\mathrm{slip,i}} \\ 0 & \text{If } y_{\mathrm{wt,i}} \leq y_{\mathrm{slip,i}} \end{cases}$
Description	The interslice water force H_{i} is a force created by water acting horizontally from an interslice i onto adjacent slice.
Sources	[1]
Ref. By	DD10, DD11, IM1, IM2, IM3

Number	DD5
Label	**Angles**
Equation	$\alpha_{\text{i}} = \frac{y_{\text{slip,i}} - y_{\text{slip,i-1}}}{x_{\text{slip,i}} - x_{\text{slip,i-1}}}$, $\beta_{\text{i}} = \frac{y_{\text{us,i}} - y_{\text{us,i-1}}}{x_{\text{us,i}} - x_{\text{us,i-1}}}$
Description	The angle the slip surface (slice base) and slope surface (slice surface) make with the horizontal. Uses approximation A10 that the base and surface of slices between nodes are straight lines.
Sources	[1]
Ref. By	DD10, DD11, IM1, IM2, IM3

Number	DD6
Label	**Lengths**
Equation	$b_{\text{i}} = x_{\text{slip,i}} - x_{\text{slip,i-1}}$, $\ell_{\text{b,i}} = b_{\text{i}} \cdot \sec(\alpha_{\text{i}})$ $\ell_{\text{s,i}} = b_{\text{i}} \cdot \sec(\beta_{\text{i}})$
Description	The width of the slice b_{i} is the difference in x-ordinates between x_{slip} nodes surrounding the slice. Using the slice base angle α_{i} and surface angle β_{i} from DD5, and Pythagoreans theorem the total length of the slice base $\ell_{\text{b,i}}$ or slice surface $\ell_{\text{s,i}}$ can be calculated.
Sources	[1]
Ref. By	DD10, DD11, IM1, IM2, IM3

Number	DD7
Label	**Seismic Load Factor**
Equation	$K_{\text{E,i}} = K_{\text{c}} W_{\text{i}}$
Description	Input K_{c} is the seismic load factor. It represents the proportion of weight that slice i will exert outward as a result of horizontal motion of the earth dues to earthquakes. The force $K_{\text{E,i}}$ will be the product of the load factor K_{c} and the weight of the slice W_{i} from DD1.
Sources	[1]
Ref. By	DD10, DD11, IM1, IM2, IM3

Number	DD8
Label	**Surface Loads**
Equation	$Q_{\mathrm{i}}, \omega_{\mathrm{i}}$
Description	Inputs Q_{i} and ω_{i} are the results of a slope surface load from an external weight such as a building on the slope. Q_{i} is the magnitude of the surface load being exerted on slice i and ω is the angle the force is being exerted at relative to the vertical (line parallel to the direction of the force of gravity).
Sources	[7]
Ref. By	DD10, DD11, IM1, IM2, IM3

Number	DD9
Label	**Interslice Forces**
Equation	$E_{\mathrm{i}}, X_{\mathrm{i}} = \lambda \cdot f_{\mathrm{i}} \cdot E_{\mathrm{i}}$
Description	The interslice forces are the normal and shear forces occurring on a slice at it's interfaces as a result of the internal stress on the slices of mass from the weight of the adjacent slice. E_{i} is the normal force exerted by interslice i, and X_{i} is the shear force exerted by interslice i. The value of X_{i} is determined by it's relationship to E_{i} in GD5. E_{i} is one of the three solution variables, determined by IM3.
Sources	[7]
Ref. By	DD10, DD11, IM1, IM2, IM3

Number	DD10
Label	**Resistive Shear, Without Interslice Forces**
Equation	$R_{\mathrm{i}} = \begin{pmatrix} [W_{\mathrm{i}} + U_{\mathrm{t,i}} \cos(\beta_{\mathrm{i}}) + Q_{\mathrm{i}} \cos(\omega_{\mathrm{i}})] \cos(\alpha_{\mathrm{i}}) \\ + [-K_{\mathrm{c}} W_{\mathrm{i}} - \Delta H_{\mathrm{i}} + U_{\mathrm{t,i}} \sin(\beta_{\mathrm{i}}) + Q_{\mathrm{i}} \sin(\omega_{\mathrm{i}})] \sin(\alpha_{\mathrm{i}}) - U_{\mathrm{b,i}} \end{pmatrix} \cdot \tan(\varphi') + c'_{\mathrm{i}} \cdot b_{\mathrm{i}} \cdot \sec(\alpha_{\mathrm{i}})$
Description	The resistive shear of GD3, with the normal force N_{i} defined in terms of the physical properties of DD1 to DD9, without considering the effects of the interslice forces E and X.
Sources	[7]
Ref. By	IM1

Resistive Shear Force, Without the Influence of Interslice Forces Derivation

The resistive shear force of a slice is defined as P_{i} in GD3. The effective normal in the equation for P_{i} of the soil is defined in the perpendicular force equilibrium of a slice from GD2, Using the effective normal N'_{i} of T4 shown in equation (1).

$$N'_{\mathrm{i}} = \begin{array}{l} [W_{\mathrm{i}} - X_{\mathrm{i\text{-}1}} + X_{\mathrm{i}} + U_{\mathrm{t,i}} \cos(\beta_{\mathrm{i}}) + Q_{\mathrm{i}} \cos(\omega_{\mathrm{i}})] \cos(\alpha_{\mathrm{i}}) \\ + [-K_{\mathrm{c}}\, W_{\mathrm{i}} - E_{\mathrm{i}} + E_{\mathrm{i\text{-}1}} - H_{\mathrm{i}} + H_{\mathrm{i\text{-}1}} + U_{\mathrm{t,i}} \sin(\beta_{\mathrm{i}}) + Q_{\mathrm{i}} \sin(\omega_{\mathrm{i}})] \sin(\alpha_{\mathrm{i}}) \\ -U_{\mathrm{b,i}} \end{array} \quad (1)$$

The values of the interslice forces E and X in the equation are unknown, while the other values are found from the physical force definitions of DD1 to DD9. Consider a force equilibrium without the affect of interslice forces, to obtain a solvable value as done for N^*_{i} in equation (2).

$$N^*_{\mathrm{i}} = \begin{array}{l} [W_{\mathrm{i}} + U_{\mathrm{t,i}} \cos(\beta_{\mathrm{i}}) + Q_{\mathrm{i}} \cos(\omega_{\mathrm{i}})] \cos(\alpha_{\mathrm{i}}) \\ + [-K_{\mathrm{c}}\, W_{\mathrm{i}} - H_{\mathrm{i}} + H_{\mathrm{i\text{-}1}} + U_{\mathrm{t,i}} \sin(\beta_{\mathrm{i}}) + Q_{\mathrm{i}} \sin(\omega_{\mathrm{i}})] \sin(\alpha_{\mathrm{i}}) - U_{\mathrm{b,i}} \end{array} \quad (2)$$

Using N^*_{i}, a resistive shear force neglecting the influence of interslice forces can be solved for in terms of all known values as done in equation (3).

$$R_{\mathrm{i}} = N^*_{\mathrm{i}} \tan(\varphi') + c'_{\mathrm{i}} \cdot b'_{\mathrm{i}} \sec(\alpha_t exti')$$

$$R_{\mathrm{i}} = \left(\begin{array}{l} [W_{\mathrm{i}} + U_{\mathrm{t,i}} \cos(\beta_{\mathrm{i}}) + Q_{\mathrm{i}} \cos(\omega_{\mathrm{i}})] \cos(\alpha_{\mathrm{i}}) \\ + [-K_{\mathrm{c}} W_{\mathrm{i}} - \Delta H_{\mathrm{i}} + U_{\mathrm{t,i}} \sin(\beta_{\mathrm{i}}) + Q_{\mathrm{i}} \sin(\omega_{\mathrm{i}})] \sin(\alpha_{\mathrm{i}}) - U_{\mathrm{b,i}} \end{array} \right) \cdot \tan(\varphi') + c'_{\mathrm{i}} \cdot b_{\mathrm{i}} \cdot \sec(\alpha_{\mathrm{i}}) \quad (3)$$

Number	DD11
Label	**Mobile Shear, Without Interslice Forces**
Equation	$T_{\mathrm{i}} = \begin{array}{l} [W_{\mathrm{i}} + U_{\mathrm{t,i}} \cos(\beta_{\mathrm{i}}) + Q_{\mathrm{i}} \cos(\omega_{\mathrm{i}})] \sin(\alpha_{\mathrm{i}}) \\ - [-K_{\mathrm{c}} W_{\mathrm{i}} - \Delta H_{\mathrm{i}} + U_{\mathrm{t,i}} \sin(\beta_{\mathrm{i}}) + Q_{\mathrm{i}} \sin(\omega_{\mathrm{i}})] \cos(\alpha_{\mathrm{i}}) \end{array}$
Description	The mobile shear of GD2, defined in terms of the physical properties of DD1, to DD9 without considering the effects of the interslice forces E and X.
Sources	[7]
Ref. By	IM1

Mobile Shear Force, Without the Influence of Interslice Forces Derivation

The mobile shear force acting on a slice is defined as S_{i} from the force equilibrium in GD2,also shown in equation (4).

$$S_{\mathrm{i}} = \begin{array}{l} [W_{\mathrm{i}} - X_{\mathrm{i\text{-}1}} + X_{\mathrm{i}} + U_{\mathrm{t,i}} \cos(\beta_{\mathrm{i}}) + Q_{\mathrm{i}} \cos(\omega_{\mathrm{i}})] \sin(\alpha_{\mathrm{i}}) \\ - [-K_{\mathrm{c}}\, W_{\mathrm{i}} - E_{\mathrm{i}} + E_{\mathrm{i\text{-}1}} - H_{\mathrm{i}} + H_{\mathrm{i\text{-}1}} + U_{\mathrm{t,i}} \sin(\beta_{\mathrm{i}}) + Q_{\mathrm{i}} \cos(\omega_{\mathrm{i}})] \cos(\alpha_{\mathrm{i}}) \end{array} \quad (4)$$

The equation is unsolvable, containing the unknown interslice normal force E and shear force X. Consider a force equilibrium without the affect of interslice forces, to obtain the mobile shear force without the influence of interslice forces T, as done in equation (5).n

$$T_{\mathrm{i}} = \begin{array}{l} [W_{\mathrm{i}} + U_{\mathrm{t,i}} \cos(\beta_{\mathrm{i}}) + Q_{\mathrm{i}} \cos(\omega_{\mathrm{i}})] \sin(\alpha_{\mathrm{i}}) \\ - [-K_{\mathrm{c}} W_{\mathrm{i}} - \Delta H_{\mathrm{i}} + U_{\mathrm{t,i}} \sin(\beta_{\mathrm{i}}) + Q_{\mathrm{i}} \sin(\omega_{\mathrm{i}})] \cos(\alpha_{\mathrm{i}}) \end{array} \tag{5}$$

The values of R_{i} and T_{i} are now defined completely in terms of the known force property values of DD1 to DD9.

Number	DD12
Label	**Displacement Reaction Force**
Equation	**Interslice** $\begin{bmatrix} p_{\mathrm{x,i}} \\ p_{\mathrm{y,i}} \end{bmatrix} = \bar{K}_{\mathrm{s,i}} \, \bar{\delta}_{\mathrm{i}} = \begin{bmatrix} K_{\mathrm{st,i}} & 0 \\ 0 & K_{\mathrm{sn,i}} \end{bmatrix} \begin{bmatrix} \delta x_{\mathrm{i}} \\ \delta y_{\mathrm{i}} \end{bmatrix}$ **Basal** $\begin{bmatrix} p_{\mathrm{x,i}} \\ p_{\mathrm{y,i}} \end{bmatrix} = \bar{K}_{\mathrm{b,i}} \, \bar{\delta}_{\mathrm{i}} = \begin{bmatrix} K_{\mathrm{bA,i}} & K_{\mathrm{bB,i}} \\ K_{\mathrm{bB,i}} & K_{\mathrm{bA,i}} \end{bmatrix} \begin{bmatrix} \delta x_{\mathrm{i}} \\ \delta y_{\mathrm{i}} \end{bmatrix}$
Description	The force displacement relationship of GD8 for a slice based on interslice or basal surface forces, generalized for displacement in the unrotated coordinate system $\bar{\delta}_{\mathrm{i}}$ from GD9. Uses the definitions of shear and normal stiffness from DD14. $K_{\mathrm{bA,i}}$, and $K_{\mathrm{bB,i}}$ are effective values for the rotated coordinate system found in equations (10) and (11) respectively.
Sources	[6]
Ref. By	IM4

Derivation of Stifness Matrixes

Using the force-displacement relationship of GD8 to define stiffness matrix $\bar{K}_{\mathrm{i}}$, as seen in equation (6).

$$\bar{K}_{\mathrm{i}} = \begin{bmatrix} K_{\mathrm{t,i}} & 0 \\ 0 & K_{\mathrm{n,i}} \end{bmatrix} \tag{6}$$

For interslice surfaces the stiffness constants and displacements refer to an unrotated coordinate system, $\bar{\delta}_{\mathrm{i}}$ of GD9. The interslice elements are left in their standard coordinate system, and therefore are described by the same equation from GD8. Seen as $\bar{K}_{\mathrm{s,i}}$ in DD12. $K_{\mathrm{st,i}}$ is the shear element in the matrix, and $K_{\mathrm{sn,i}}$ is the normal element in the matrix, calculated as in DD14.

For basal surfaces the stiffness constants and displacements refer to a system rotated for the base angle α (DD5). To analyze the effect of force-displacement relationships occurring on both basal and interslice surfaces of an element i they must reference the same coordinate system. The basal stiffness matrix must be rotated counter clockwise to align with the angle of the basal surface. The base stiffness counter clockwise rotation is applied in equation (7) to the new matrix $\bar{K_{\mathrm{i}}^*}$.

$$\begin{aligned}\bar{K_{\mathrm{i}}^*} &= \begin{bmatrix} \cos(\alpha_{\mathrm{i}}) & -\sin(\alpha_{\mathrm{i}}) \\ \sin(\alpha_{\mathrm{i}}) & \cos(\alpha_{\mathrm{i}}) \end{bmatrix} \bar{K_{\mathrm{i}}} \\ &= \begin{bmatrix} K_{\mathrm{bt,i}}\cos(\alpha_{\mathrm{i}}) & -K_{\mathrm{bn,i}}\sin(\alpha_{\mathrm{i}}) \\ K_{\mathrm{bt,i}}\sin(\alpha_{\mathrm{i}}) & K_{\mathrm{bn,i}}\cos(\alpha_{\mathrm{i}}) \end{bmatrix}\end{aligned} \tag{7}$$

The Hooke's law force displacement relationship of GD8 applied to the base also references a displacement vector $\bar{\epsilon}_{\mathrm{i}}$ of GD9 rotated for the base angle angle of the slice α_{i}. The basal displacement vector $\bar{\epsilon}_{\mathrm{i}}$ is rotated clockwise to align with the interslice displacement vector $\bar{\delta}_{\mathrm{i}}$, applying the definition of $\bar{\epsilon}_{\mathrm{i}}$ in terms of $\bar{\delta}_{\mathrm{i}}$ as seen in GD9. Using this with base stiffness matrix $\bar{K^*}_{\mathrm{i}}$, a basal force displacement relationship in the same coordinate system as the interslice relationship can be derived as done in equation (8).

$$\begin{aligned}\begin{bmatrix} p_{\mathrm{bx,i}} \\ p_{\mathrm{by,i}} \end{bmatrix} &= \bar{K_{\mathrm{i}}^*}\,\bar{\epsilon} \\ &= \begin{bmatrix} K_{\mathrm{bt,i}}\cos(\alpha_{\mathrm{i}}) & -K_{\mathrm{bn,i}}\sin(\alpha_{\mathrm{i}}) \\ K_{\mathrm{bt,i}}\sin(\alpha_{\mathrm{i}}) & K_{\mathrm{bn,i}}\cos(\alpha_{\mathrm{i}}) \end{bmatrix} \begin{bmatrix} \cos(\alpha_{\mathrm{i}}) & \sin(\alpha_{\mathrm{i}}) \\ -\sin(\alpha_{\mathrm{i}}) & \cos(\alpha_{\mathrm{i}}) \end{bmatrix} \begin{bmatrix} \delta x_{\mathrm{i}} \\ \delta y_{\mathrm{i}} \end{bmatrix} \\ &= \begin{bmatrix} K_{\mathrm{bt,i}}\cos^2(\alpha_{\mathrm{i}}) + K_{\mathrm{bn,i}}\sin^2(\alpha_{\mathrm{i}}) & (K_{\mathrm{bt,i}} - K_{\mathrm{bn,i}})\sin(\alpha_{\mathrm{i}})\cos(\alpha_{\mathrm{i}}) \\ (K_{\mathrm{bt,i}} - K_{\mathrm{bn,i}})\sin(\alpha_{\mathrm{i}})\cos(\alpha_{\mathrm{i}}) & K_{\mathrm{bt,i}}\cos^2(\alpha_{\mathrm{i}}) + K_{\mathrm{bn,i}}\sin^2(\alpha_{\mathrm{i}}) \end{bmatrix} \begin{bmatrix} \delta x_{\mathrm{i}} \\ \delta y_{\mathrm{i}} \end{bmatrix}\end{aligned} \tag{8}$$

The new effective base stiffness matrix K_{i}',as derived in equation (7) is defined in equation (9). This is seen as matrix $\bar{K}_{\mathrm{b,i}}$ in GD12. $K_{\mathrm{bt,i}}$ is the shear element in the matrix, and $K_{\mathrm{bn,i}}$ is the normal element in the matrix, calculated as in DD14. The notation is simplified by the introduction of the constants $K_{\mathrm{bA,i}}$ and $K_{\mathrm{bB,i}}$, defined in equations (10) and (11) respectively.

$$\begin{aligned}\bar{K_{\mathrm{i}}'} &= \begin{bmatrix} K_{\mathrm{bt,i}}\cos^2(\alpha_{\mathrm{i}}) + K_{\mathrm{bn,i}}\sin^2(\alpha_{\mathrm{i}}) & (K_{\mathrm{bt,i}} - K_{\mathrm{bn,i}})\sin(\alpha_{\mathrm{i}})\cos(\alpha_{\mathrm{i}}) \\ (K_{\mathrm{bt,i}} - K_{\mathrm{bn,i}})\sin(\alpha_{\mathrm{i}})\cos(\alpha_{\mathrm{i}}) & K_{\mathrm{bt,i}}\cos^2(\alpha_{\mathrm{i}}) + K_{\mathrm{bn,i}}\sin^2(\alpha_{\mathrm{i}}) \end{bmatrix} \\ \bar{K_{\mathrm{i}}'} &= \begin{bmatrix} K_{\mathrm{bA,i}} & K_{\mathrm{bB,i}} \\ K_{\mathrm{bB,i}} & K_{\mathrm{bA,i}} \end{bmatrix}\end{aligned} \tag{9}$$

$$K_{\mathrm{bA,i}} = K_{\mathrm{bt,i}}\cos^2(\alpha_{\mathrm{i}}) + K_{\mathrm{bn,i}}\sin^2(\alpha_{\mathrm{i}}) \tag{10}$$

$$K_{\mathrm{bB,i}} = (K_{\mathrm{bt,i}} - K_{\mathrm{bn,i}})\sin(\alpha_{\mathrm{i}})\cos(\alpha_{\mathrm{i}}) \tag{11}$$

A force-displacement relationship for an element i can be written in terms of displacements occurring in the unrotated coordinate system $\bar{\delta}_{\mathrm{i}}$ of GD9 using the matrix $K_{\mathrm{s,i}}$, and $K_{\mathrm{b,i}}$ as seen in DD12.

Number	DD13
Label	**Net Force-Displacement Equilibrium**
Equation	$-\ell_{\text{s,i-1}} \cdot \bar{K}_{\text{s,i-1}} \cdot \bar{\delta}_{\text{i-1}} + \left(\ell_{\text{s,i-1}} \cdot \bar{K}_{\text{s,i-1}} + \ell_{\text{b,i}} \cdot \bar{K}_{\text{b,i}} + \ell_{\text{s,i}} \cdot \bar{K}_{\text{s,i}}\right) \cdot \bar{\delta}_{\text{i}} - \ell_{\text{s,i}} \cdot \bar{K}_{\text{s,i}} \cdot \bar{\delta}_{\text{i+1}} = \begin{bmatrix} F_{\text{x,i}} \\ F_{\text{y,i}} \end{bmatrix}$
Description	The total force displacement equilibrium relationship on a slice, considering the effects of displacements by adjacent slices. The net loads acting on a slice ($F_{\text{x,i}}$ and $F_{\text{y,i}}$) are found from GD7. A net load on a slice will cause an opposing reaction force, therefore $\bar{\delta}_{\text{i}}$ is positively related to the net force. The reaction of the slice will be dampened by the movement of adjacent slices, therefore $\bar{\delta}_{\text{i-1}}$ and $\bar{\delta}_{\text{i+1}}$ are negatively related. $\bar{K}_{\text{s,i}}$ and $\bar{K}_{\text{b,i}}$ are defined as in DD12. The stiffnesses are normalized for the length of the surfaces ℓ.
Sources	[6]
Ref. By	IM4

Number	DD14
Label	**Soil Stiffness**
Input	E , ν , b , c , σ , ϕ , κ a , A , u , v
Output	$K_{\text{t}} = \frac{E}{2[1+\nu]}\frac{0.1}{b} + \frac{c-\sigma\cdot\tan(\phi)}{\lvert\delta u\rvert+a}$ $K_{\text{n}} = \begin{cases} \frac{E[1-\nu]}{[1+\nu][1-2\nu+b]} & \text{for v} \leq 0 \\ \frac{0.01\ E[1-\nu]}{[1+\nu][1-2\nu+b]} + \frac{\kappa}{\delta v+A} & \text{for v} \geq 0 \end{cases}$
Description	Calculations are applied to interslice and basal interfaces, which will have different material properties and displacements. Material constants are constant for a homogeneous layer, and a weighting scheme is used to interpolate a new material constant for surfaces that cross multiple layers with different material constants, noted as an effective material constant. K_{t} is the shear stiffness of a slice. K_{n} is the normal stiffness of a slice. E is the effective Young's modulus. c is the effective cohesion. v is the effective Poisson's ratio. $\tan(\phi)$ is the effective static friction. b is the length of the base for base stiffness, and the length between adjacent slice midpoints for interslice stiffness. σ is the normal stress on the surface. δu is the shear displacement of the surface. Relative for interslice surfaces, and absolute for base surfaces. δv is the normal displacement of the surface. Relative for interslice surfaces, and absolute for base surfaces κ, A, and a are constants.
Sources	[6]
Ref. By	IM4, IM5

4.2.5 Instance Models

This section transforms the problem defined in the Section 4.1 into one which is expressed in mathematical terms. It uses concrete symbols defined in Section 4.2.4 to replace the abstract symbols in the models identified in the Sections 4.2.2 and 4.2.3.

The Morgenstern Price Method is a vertical slice, limit equilibrium slope stability analysis method. Analysis is performed by breaking the assumed failure surface into a series of vertical slices of mass. Static equilibrium analysis using two force equilibrium, and one moment equation as in T2. The problem is statically indeterminate with only these 3 equations and one constitutive equation (the Mohr Coulomb shear strength of T3) so the assumption of GD5 is used. Solving for force equilibrium allows definitions of all forces in terms of the physical properties of DD1 to DD9, as done in DD10, DD11.

The values of the interslice normal force E the interslice normal/shear force magnitude ratio λ, and the Factor of Safety FS, are unknown. Equations for the unknowns are written in terms of only the values in DD1 to DD9, the values of R_{i}, and T_{i} in DD10 and DD11, and each other. The relationships between the unknowns are non linear, and therefore explicit equations cannot be derived and an iterative solution method is required.

Number	IM1
Label	**Factor of Safety**
Input	Ψ_{v} , Φ_{v} , T_{v} , R_{v}
Output	$FS = \dfrac{\sum_{v=1}^{n-1} \left[R_v \prod_{c=i}^{n-1} \frac{\Psi_c}{\Phi_c} \right] + R_n}{\sum_{v=1}^{n-1} \left[T_v \prod_{c=i}^{n-1} \frac{\Psi_c}{\Phi_c} \right] + T_n}$
Description	Equation for the Factor of Safety, the ratio between resistive and mobile shear the slip surface. The sum of values from each slice is taken to find the total resistive and mobile shear for the slip surface. The constants Φ and Ψ convert the resistive and mobile shear without the influence of interslice forces, to a calculation considering the interslice forces.
Sources	[7]
Ref. By	IM2, IM3

Factor of Safety Derivation

Using equation (21) from section 4.2.5, rearranging, and applying the boundary condition that E_0 and E_{n} are equal to 0 an equation for the factor of safety is found as equation (12), also seen in IM1.

$$\mathrm{FS} = \frac{\sum_{v=1}^{n-1} \left[R_v \prod_{c=v}^{n-1} \frac{\Psi_c}{\Phi_c} \right] + R_n}{\sum_{v=1}^{n-1} \left[T_v \prod_{c=v}^{n-1} \frac{\Psi_c}{\Phi_c} \right] + T_n} \tag{12}$$

The constants Ψ and Φ described in equations 20 and 19 are functions of the unknowns: the interslice normal/shear force ratio λ (IM2) and the Factor of Safety itself FS (IM1).

Number	IM2
Label	**Normal/Shear Force Ratio**
Input	b_{v} , E_{v} , H_{v} , α_{v} , h_{v} , W_{v} , $U_{\mathrm{t,v}}$, β_{v} , f_{v} , K_{c}
Output	$C1_{\mathrm{i}} = \begin{cases} b_1\left[E_1 + H_1\right]\tan\left(\alpha_1\right) & \text{for i} = 1 \\ b_{\mathrm{i}}\left[\left(E_{\mathrm{i}} + E_{\mathrm{i\text{-}1}}\right) + \left(H_{\mathrm{i}} + H_{\mathrm{i\text{-}1}}\right)\right]\tan\left(\alpha_{\mathrm{i}}\right) \\ +h_{\mathrm{i}}\left(K_{\mathrm{c}}\,W_{\mathrm{i}} - 2\,U_{\mathrm{t,i}}\,\sin\left(\beta_{\mathrm{i}}\right) - 2\,Q_{\mathrm{i}}\,\cos\left(\omega_{\mathrm{i}}\right)\right) & \text{for } 2 \leq \text{i} \leq \text{n-1} \\ b_{\mathrm{n}}\left[E_{\mathrm{n\text{-}1}} + H_{\mathrm{n\text{-}1}}\right]\tan\left(\alpha_{\mathrm{n\text{-}1}}\right) & \text{for i} = \text{n} \end{cases}$ $C2_{\mathrm{i}} = \begin{cases} b_1 E_1 f_1 & \text{for i} = 1 \\ b_{\mathrm{i}}\left(f_{\mathrm{i}} E_{\mathrm{i}} + f_{\mathrm{i\text{-}1}} E_{\mathrm{i\text{-}1}}\right) & \text{for } 2 \leq \text{i} \leq \text{n-1} \\ b_{\mathrm{n}} E_{\mathrm{n\text{-}1}} f_{\mathrm{n\text{-}1}} & \text{for v} = \text{n} \end{cases}$ $\lambda = \frac{\displaystyle\sum_{i=1}^{n} C1_{\mathrm{i}}}{\displaystyle\sum_{i=1}^{n} C2_{\mathrm{i}}}$
Description	λ is the magnitude ratio between shear and normal forces at the interslice interfaces as the assumption of the Morgenstern Price method in GD5. The inclination function f determines the relative magnitude ratio between the different interslices, while λ determines the magnitude. λ uses the sum of interslice normal and shear forces taken from each interslice.
Sources	[7]
Ref. By	IM1, IM3

Normal/Shear Force Ratio Derivation

The last static equation of T2 the moment equilibrium of GD6 about the midpoint of the base is taken, with the assumption of GD5. Results in equation (13).

$$0 = \begin{array}{l} -E_{\mathrm{i}}\left[z_{\mathrm{i}} - \frac{b_{\mathrm{i}}}{2}\tan\left(\alpha_{\mathrm{i}}\right)\right] + E_{\mathrm{i\text{-}1}}\left[z_{\mathrm{i\text{-}1}} + \frac{b_{\mathrm{i}}}{2}\tan\left(\alpha_{\mathrm{i}}\right)\right] - H_{\mathrm{i}}\left[z_{\mathrm{w,i}} - \frac{b_{\mathrm{i}}}{2}\tan\left(\alpha_{\mathrm{i}}\right)\right] \\ +H_{\mathrm{i\text{-}1}}\left[z_{\mathrm{w,i\text{-}1}} + \frac{b_{\mathrm{i}}}{2}\tan\left(\alpha_{\mathrm{i}}\right)\right] - \lambda\frac{b_{\mathrm{i}}}{2}\left(E_{\mathrm{i}} f_{\mathrm{i}} + E_{\mathrm{i\text{-}1}} f_{\mathrm{i\text{-}1}}\right) + K_{\mathrm{c}} W_{\mathrm{i}}\frac{h_{\mathrm{i}}}{2} - U_{\mathrm{t,i}}\sin\left(\beta_{\mathrm{i}}\right)h_{\mathrm{i}} - Q_{\mathrm{i}}\sin\left(\omega_{\mathrm{i}}\right)h_{\mathrm{i}} \end{array} \tag{13}$$

Rearranging the equation in terms of λ leads to equation (14).

$$\lambda = \frac{\begin{array}{c}-E_{\mathrm{i}}\left[z_{\mathrm{i}}-\frac{b_{\mathrm{i}}}{2}\tan\left(\alpha_{\mathrm{i}}\right)\right]+E_{\mathrm{i-1}}\left[z_{\mathrm{i-1}}+\frac{b_{\mathrm{i}}}{2}\tan\left(\alpha_{\mathrm{i}}\right)\right]-H_{\mathrm{i}}\left[z_{\mathrm{w,i}}-\frac{b_{\mathrm{i}}}{2}\tan\left(\alpha_{\mathrm{i}}\right)\right]\\+H_{\mathrm{i-1}}\left[z_{\mathrm{w,i-1}}+\frac{b_{\mathrm{i}}}{2}\tan\left(\alpha_{\mathrm{i}}\right)\right]+K_{\mathrm{c}}W_{\mathrm{i}}\frac{h_{\mathrm{i}}}{2}-U_{\mathrm{t,i}}\sin\left(\beta_{\mathrm{i}}\right)h_{\mathrm{i}}-Q_{\mathrm{i}}\sin\left(\omega_{\mathrm{i}}\right)h_{\mathrm{i}}\end{array}}{\frac{b_{\mathrm{i}}}{2}\left[E_{\mathrm{i}}f_{\mathrm{i}}+E_{\mathrm{i-1}}f_{\mathrm{i-1}}\right]} \tag{14}$$

Taking a summation of each slice, and considering the boundary conditions that E_0 and E_{n} are equal to zero, a general equation for the constant λ is developed in equation (15), also found in IM2.

$$\lambda = \frac{\sum_{i=1}^{n} b_{\mathrm{i}}\left[\left(E_{\mathrm{i}}+E_{\mathrm{i-1}}\right)+\left(H_{\mathrm{i}}+H_{\mathrm{i-1}}\right)\right]\tan\left(\alpha_{\mathrm{i}}\right)+h_{\mathrm{i}}\ \left[K_{\mathrm{c}}\ W_{\mathrm{i}}-2\ U_{\mathrm{t,i}}\sin\left(\beta_{\mathrm{i}}\right)-2\ Q_{\mathrm{i}}\sin\left(\omega_{\mathrm{i}}\right)\right]}{\sum_{i=1}^{n} b_{\mathrm{i}}\ \left[f_i E_{\mathrm{i}}+f_{\mathrm{i-1}}E_{\mathrm{i-1}}\right]} \tag{15}$$

Equation (15) for λ, is a function of the unknown interslice normal force E (IM3).

Number	IM3
Label	**Interslice Forces**
Input	FS, T_{i}, R_{i}, Ψ, Φ
Output	$E_{\mathrm{i}} = \begin{cases} \frac{(\mathrm{FS})T_{\mathrm{i}}-R_{\mathrm{i}}}{\Phi_{\mathrm{i}}} & \text{for i=1} \\ \frac{\Psi_{\mathrm{i-1}}\cdot E_{\mathrm{i-1}}+(\mathrm{FS})\cdot T_{\mathrm{i}}-R_{\mathrm{i}}}{\Phi_{\mathrm{i}}} & \text{for } 1\leq \text{i} \leq \text{n-1} \\ 0 & \text{for i=0 and i=n} \end{cases}$
Description	The value of the interslice normal force E_{i} at interface i. The net force the weight of the slices adjacent to interface i exert horizontally on each other.
Sources	[7]
Ref. By	IM1, IM2

Interslice Force Derivation

Taking the perpendicular force equilibrium of GD1 with the effective stress definition from T4 that $N_{\mathrm{i}} = N'_{\mathrm{i}} - U_{\mathrm{b,i}}$, and the assumption of GD5 the equilibrium equation can be rewritten as equation (16).

$$N'_{\mathrm{i}} = \begin{array}{l}\left[W_{\mathrm{i}}-\lambda\cdot f_{\mathrm{i-1}}\cdot E_{\mathrm{i-1}}+\lambda\cdot f_{\mathrm{i}}\cdot E_{\mathrm{i}}+U_{\mathrm{t,i}}\cos\left(\beta_{\mathrm{i}}\right)+Q_{\mathrm{i}}\cos\left(\omega_{\mathrm{i}}\right)\right]\cos\left(\alpha_{\mathrm{i}}\right)\\+\left[-K_{\mathrm{c}}W_{\mathrm{i}}-E_{\mathrm{i}}+E_{\mathrm{i-1}}-H_{\mathrm{i}}+H_{\mathrm{i-1}}+U_{\mathrm{t,i}}\sin\left(\beta_{\mathrm{i}}\right)+Q_{\mathrm{i}}\sin\left(\omega_{\mathrm{i}}\right)\right]\sin\left(\alpha_{\mathrm{i}}\right)-U_{\mathrm{b,i}}\end{array} \tag{16}$$

Taking the parallel force equilibrium of GD2 with the definition of mobile shear from GD4 and the assumption of GD5, the equilibrium equation can be rewritten as equation (17).

$$\frac{N_{\mathrm{i}}\tan\left(\varphi'\right)+c'_{\mathrm{i}}\cdot b'_{\mathrm{i}}\cdot\sec\left(\alpha_{\mathrm{i}}\right)}{\mathrm{FS}} = \begin{array}{l}\left[W_{\mathrm{i}}-\lambda\cdot f_{\mathrm{i-1}}\cdot E_{\mathrm{i-1}}+\lambda\cdot f_{\mathrm{i}}\cdot E_{\mathrm{i}}+U_{\mathrm{t,i}}\cos\left(\beta_{\mathrm{i}}\right)+Q_{\mathrm{i}}\cos\left(\omega_{\mathrm{i}}\right)\right]\sin\left(\alpha_{\mathrm{i}}\right)\\-\left[-K_{\mathrm{c}}W_{\mathrm{i}}-E_{\mathrm{i}}+E_{\mathrm{i-1}}-H_{\mathrm{i}}+H_{\mathrm{i-1}}+U_{\mathrm{t,i}}\cdot\sin\left(\beta_{\mathrm{i}}\right)+Q_{\mathrm{i}}\sin\left(\omega_{\mathrm{i}}\right)\right]\cos\left(\alpha_{\mathrm{i}}\right)\end{array} \tag{17}$$

Substituting the equation for N'_{i} from equation (16) into equation (17) and rearranging results in equation (18)

$$E_{\text{i}}\left[\begin{array}{c}[\lambda\cdot f_{\text{i}}\cos(\alpha_{\text{i}})-\sin(\alpha_{\text{i}})]\tan(\varphi')\\-[\lambda\cdot f_{\text{i}}\sin(\alpha_{\text{i}})+\cos(\alpha_{\text{i}})]\,(\text{FS})\end{array}\right]=E_{\text{i-1}}\left[\begin{array}{c}[\lambda\cdot f_{\text{i-1}}\cos(\alpha_{\text{i}})-\sin(\alpha_{\text{i}})]\tan(\varphi')\\-[\lambda\cdot f_{\text{i-1}}\sin(\alpha_{\text{i}})+\cos(\alpha_{\text{i}})]\,(\text{FS})\end{array}\right]+(\text{FS})\cdot T_{\text{i}}-R_{\text{i}} \tag{18}$$

Where R_{i} and T_{i} are the resistive and mobile shear of the slice, without the influence of interslice forces E and X, as defined in DD10 and DD11. Making use of the constants ϕ and Ψ with full equations found below in equations (19) and (20) respectively, then equation (18) can be simplified to equation (21), also seen in IM3.

$$\begin{gathered}\Phi_{\text{i}}=[\lambda\cdot f_{\text{i}}\cos(\alpha_{\text{i}})-\sin(\alpha_{\text{i}})]\,[\tan(\varphi'_{\text{i}})]-[\lambda\cdot f_{\text{i}}\sin(\alpha_{\text{i}})+\cos(\alpha_{\text{i}})]\,(\text{FS})\\\text{Where i is the local slice of mass for } 1\leq\text{i}\leq\text{n-1}\end{gathered} \tag{19}$$

$$\begin{gathered}\Psi_{\text{i}}=[\lambda\cdot f_{\text{i}}\cos(\alpha_{\text{i+1}})-\sin(\alpha_{\text{i+1}})]\,[\tan(\varphi')]-[\lambda\cdot f_{\text{i}}\sin(\alpha_{\text{i+1}})+\cos(\alpha_{\text{i+1}})]\,(\text{FS})\\\text{Where i is the local slice of mass for } 1\leq\text{i}\leq\text{n-1}\end{gathered} \tag{20}$$

$$E_{\text{i}}=\frac{\Psi_{\text{i-1}}\ E_{\text{i-1}}+(FS)\ T_{\text{i}}-R_{\text{i}}}{\Phi_{\text{i}}} \tag{21}$$

The constants Ψ and Φ in equation (21) for E_{i} is a function of the unknown values, the interslice normal/shear force ratio λ (IM2), and the Factor of Safety FS (IM1).

Number	IM4
Label	**Force Displacement Equilibrium**
Input	E , ν , b , c , σ , ϕ , κ a , A , u , v
Output	**X Equilibrium** $$\begin{array}{l} -\Delta H_{\mathrm{i}} - K_{\mathrm{c}} \cdot W_{\mathrm{i}} - U_{\mathrm{b,i}} \sin\left(\alpha_{\mathrm{i}}\right) \\ +U_{\mathrm{t,i}} \sin\left(\beta_{\mathrm{i}}\right) + Q_{\mathrm{i}} \sin\left(\omega_{\mathrm{i}}\right) \end{array} = \begin{array}{l} \left[\delta x_{\mathrm{i\text{-}1}}\right]\left(-\ell_{\mathrm{s,i\text{-}1}} K_{\mathrm{sn,i\text{-}1}}\right) \\ +\left[\delta x_{\mathrm{i}}\right]\left(-\ell_{\mathrm{s,i\text{-}1}} K_{\mathrm{sn,i\text{-}1}} + \ell_{\mathrm{s,i}} K_{\mathrm{sn,i}} + \ell_{\mathrm{b,i}} K_{\mathrm{bA,i}}\right) \\ +\left[\delta x_{\mathrm{i+1}}\right]\left(-\ell_{\mathrm{s,i}} K_{\mathrm{sn,i}}\right) + \left[\delta y_{\mathrm{i}}\right]\left(-\ell_{\mathrm{b,i}} K_{\mathrm{bB,i}}\right) \end{array}$$ **Y Equilibrium** $$\begin{array}{l} -W_{\mathrm{i}} + U_{\mathrm{b,i}} \cos\left(\alpha_{\mathrm{i}}\right) \\ -U_{\mathrm{t,i}} \cos\left(\beta_{\mathrm{i}}\right) - Q_{\mathrm{i}} \cos\left(\omega_{\mathrm{i}}\right) \end{array} = \begin{array}{l} \left[\delta y_{\mathrm{i\text{-}1}}\right]\left(-\ell_{\mathrm{s,i\text{-}1}} K_{\mathrm{st,i\text{-}1}}\right) \\ +\left[\delta y_{\mathrm{i}}\right]\left(-\ell_{\mathrm{s,i\text{-}1}} K_{\mathrm{st,i\text{-}1}} + \ell_{\mathrm{s,i}} K_{\mathrm{st,i}} + \ell_{\mathrm{b,i}} K_{\mathrm{bA,i}}\right) \\ +\left[\delta y_{\mathrm{i+1}}\right]\left(-\ell_{\mathrm{s,i}} K_{\mathrm{st,i}}\right) + \left[\delta x_{\mathrm{i}}\right]\left(-\ell_{\mathrm{b,i}} K_{\mathrm{bB,i}}\right) \end{array}$$
Description	One set of force displacement equilibrium equations in the x and y directions. There is of equations for each element. System of equations solved for displacements (δx, and δy) $\Delta H_{\mathrm{i}} = H_{\mathrm{i}} - H_{\mathrm{i\text{-}1}}$ is the net hydrostatic force across a slice. K_{c} is the earthquake load factor. W_{i} is the weight of the slice. $U_{\mathrm{b,i}}$ is the pore water pressure acting on the slice base. $U_{\mathrm{t,i}}$ is the pore water pressure acting on the slice surface. α_{i} is the angle of the base with the horizontal. β_{i} is the angle of the surface with the horizontal δx_{i} is the x displacement of slice i δy_{i} is the y displacement of slice i $\ell_{\mathrm{s,i}}$ is the length of the interslice surface i $\ell_{\mathrm{s,i}}$ is the length of the base surface i $K_{\mathrm{st,i}}$ is the interslice shear stiffness at surface i. $K_{\mathrm{st,i\text{-}1}}$ is the interslice normal stiffness at surface i. $K_{\mathrm{bA,i}}$, and $K_{\mathrm{bB,i}}$ are the base stiffness values for slice i.
Sources	[6]
Ref. By	IM5

Rigid Finite Element Displacement Derivation

Using the net force-displacement equilibrium equation of a slice from DD13, with the definitions of the stiffness matrices from DD12, and the force definitions from GD7, a broken down force-displacement equilibrium equation can be derived. Equation (22) gives the broken down equation in the x direction, and equation (23) gives the broken down equation in the y direction.

$$-\Delta H_{\text{i}} - K_{\text{c}} \cdot W_{\text{i}} - U_{\text{b,i}} \sin(\alpha_{\text{i}}) + U_{\text{t,i}} \sin(\beta_{\text{i}}) + Q_{\text{i}} \sin(\omega_{\text{i}}) = \begin{array}{l} [\delta x_{\text{i-1}}] (-\ell_{\text{s,i-1}} K_{\text{sn,i-1}}) \\ + [\delta x_{\text{i}}] (-\ell_{\text{s,i-1}} K_{\text{sn,i-1}} + \ell_{\text{s,i}} K_{\text{sn,i}} + \ell_{\text{b,i}} K_{\text{bA,i}}) \\ + [\delta x_{\text{i+1}}] (-\ell_{\text{s,i}} K_{\text{sn,i}}) + [\delta y_{\text{i}}] (-\ell_{\text{b,i}} K_{\text{bB,i}}) \end{array} \tag{22}$$

$$-W_{\text{i}} + U_{\text{b,i}} \cos(\alpha_{\text{i}}) - U_{\text{t,i}} \cos(\beta_{\text{i}}) - Q_{\text{i}} \cos(\omega_{\text{i}}) = \begin{array}{l} [\delta y_{\text{i-1}}] (-\ell_{\text{s,i-1}} K_{\text{st,i-1}}) \\ + [\delta y_{\text{i}}] (-\ell_{\text{s,i-1}} K_{\text{st,i-1}} + \ell_{\text{s,i}} K_{\text{st,i}} + \ell_{\text{b,i}} K_{\text{bA,i}}) \\ + [\delta y_{\text{i+1}}] (-\ell_{\text{s,i}} K_{\text{st,i}}) + [\delta x_{\text{i}}] (-\ell_{\text{b,i}} K_{\text{bB,i}}) \end{array} \tag{23}$$

Using the known input assumption of A2, the force variable definitions of DD1 to DD8 on the left side of the equations can be solved for. The only unknown in the variables to solve for the stiffness values from DD14 is the displacements. Therefore taking the equation from each slice a set of $2 \cdot n$ equations, with $2 \cdot n$ unknown displacements in the x and y directions of each slice can be derived. Solutions for the displacements of each slice can then be found. The use of displacement in the definition of the stiffness values makes the equation implicit, which means an iterative solution method, with an initial guess for the displacements in the stiffness values is required.

Number	IM5
Label	**RFEM Factor of Safety**
Input	c, ℓ_{b}, δu , δv, φ', $K_{\text{bt,i}}$, $K_{\text{bn,i}}$
Output	$FS_{\text{Loc,i}} = \dfrac{c - K_{\text{bn,i}} \cdot \delta v_{\text{i}} \cdot \tan\left(\varphi'_{\text{i}}\right)}{K_{\text{bt,i}} \cdot \delta u_{\text{i}}}$ $FS = \dfrac{\sum_{i=1}^{n} \ell_{\text{b,i}} \left[c - K_{\text{bn,i}} \cdot \delta v_{\text{i}} \cdot \tan\left(\varphi'_{\text{i}}\right)\right]}{\sum_{i=1}^{n} \ell_{\text{b,i}} \left[K_{\text{bt,i}} \cdot \delta u_{\text{i}}\right]}$
Description	$FS_{\text{Loc,i}}$ Factor of Safety for slice i. FS Factor of Safety for the entire slip surface. c is the cohesion of slice i's base. φ'_{i} is the effective angle of friction of slice i's base. δv_{i} is the normal displacement of slice i δu_{i} is the shear displacement of slice i $\ell_{\text{b,i}}$ is the length of the base of slice i $K_{\text{bt,i}}$ is the base shear stiffness at surface i. $K_{\text{bn,i}}$ is the base normal stiffness at surface i. n is the number of slices in the slip surface.
Sources	[6]

Rigid Finite Element Factor of Safety Derivation

RFEM analysis can also be used to calculate the Factor of safety for the slope. For a slice element i the displacements δx_{i} and δy_{i}, are solved from the system of equations in IM4. The definition of $\bar{\epsilon}_{\text{i}}$ as the rotation of the displacement vector $\bar{\delta}_{\text{i}}$ is seen inGD9 .

This is used to find the displacements of the slice parallel to the base of the slice δu in equation (24) and normal to the base of the slice δv in equation (25).

$$\delta u_{\text{i}} = \cos\left(\alpha_{\text{i}}\right) \delta x_{\text{i}} + \sin\left(\alpha_{\text{i}}\right) \delta y_{\text{i}} \tag{24}$$

$$\delta v_{\text{i}} = -\sin\left(\alpha_{\text{i}}\right) \delta x_{\text{i}} + \cos\left(\alpha_{\text{i}}\right) \delta y_{\text{i}} \tag{25}$$

With the definition of normal stiffness from DD14 to find the normal stiffness of the base $K_{\text{bn,i}}$, and the now known base displacement perpendicular to the surface δv_{i} from equation (25), the normal base stress can be calculated from the force-displacement relationship of T5. Stress σ is used in place of force F as the stiffness hasn't been normalized for the length of the base. Results in equation (26).

$$\sigma_{\mathrm{i}} = K_{\mathrm{bn,i}} \cdot \delta v_{\mathrm{i}} \tag{26}$$

The resistive shear to calculate the factor of safety FS in is found from the Mohr Coulomb resistive strength of soil in T3. Using the normal stress σ from equation (26) as the stress the resistive shear of the slice can be calculated from calculated in equation (27).

$$S_{\mathrm{i}} = c - \sigma_{\mathrm{i}} \cdot \tan(\varphi') \tag{27}$$

previously the value of the base shear stiffness $K_{\mathrm{bt,i}}$ as seen in equation (28) was unsolvable because the normal stress σ_{i} was unknown. With the definition of σ_{i} from equation (26) and the definition of displacement shear to the base δu_{i} from equation (25), the value of $K_{\mathrm{bt,i}}$ becomes solvable.

$$K_{\mathrm{bt,i}} = \frac{E_{\mathrm{i}}}{2\left[1 + \nu_{\mathrm{i}}\right]} \frac{0.1}{b_{\mathrm{i}}} + \frac{c_{\mathrm{i}} - \sigma_{\mathrm{i}} \cdot \tan(\phi_{\mathrm{i}})}{|\delta u_{\mathrm{i}}| + a} \tag{28}$$

With shear stiffness $K_{\mathrm{bt,i}}$ calculated in equation (28) and shear displacement δu_{i} calculated in equation (24) values now known the shear stress acting on the base of a slice τ can be calculated using T5, as done in equation (29). Again stress τ is used in place of force F as the stiffness hasn't been normalized for the length of the base.

$$\tau_{\mathrm{i}} = K_{\mathrm{bt,i}} \cdot \delta u_{\mathrm{i}} \tag{29}$$

The shear stress on the base τ acts as the mobile shear acting on the base. Using the definition Factor of Safety equation from T1, with the definitions of resistive shear strength of a slice S_{i} from equation (27) and mobile shear on a slice τ from equation (29) the factor of safety for a slice $FS_{\mathrm{Loc,i}}$ can be found from as seen in equation (30), and IM5.

$$FS_{\mathrm{Loc,i}} = \frac{S_{\mathrm{i}}}{\tau_{\mathrm{i}}} = \frac{c - K_{\mathrm{bn,i}} \cdot \delta v_{\mathrm{i}} \cdot \tan(\varphi'_{\mathrm{i}})}{K_{\mathrm{bt,i}} \cdot \delta u_{\mathrm{i}}} \tag{30}$$

The global Factor of Safety is then the ratio of the summation of the resistive and mobile shears for each slice, with a weighting for the length of the slices base. Shown in equation (31), and IM5.

$$FS = \frac{\sum_{i=1}^{n} \ell_{\mathrm{i}} \cdot S_{\mathrm{i}}}{\sum_{i=1}^{n} \ell_{\mathrm{i}} \cdot \tau_{\mathrm{i}}} = \frac{\sum_{i=1}^{n} \ell_{\mathrm{b,i}} \left[c - K_{\mathrm{bn,i}} \cdot \delta v_{\mathrm{i}} \cdot \tan(\varphi'_{\mathrm{i}})\right]}{\sum_{i=1}^{n} \ell_{\mathrm{b,i}} \left[K_{\mathrm{bt,i}} \cdot \delta u_{\mathrm{i}}\right]} \tag{31}$$

Number	IM6
Label	**Critical Slip Identification**
Input	The geometry of the water table, the geometry of the layers composing the plane of a slope, and the material properties of the layers.
Output	$\mathrm{FS_{Min}} = \Upsilon\left(\{x_{\mathrm{cs}}, y_{\mathrm{cs}}\}, \mathrm{Input}\right)$
Description	Given the necessary slope inputs, a minimization algorithm or function Υ, will identify the critical slip surface of the slope, with the critical slip coordinates $\{x_{\mathrm{cs}}, y_{\mathrm{cs}}\}$ and the minimum factor of safety $\mathrm{FS_{Min}}$ that results.
Sources	[3]

4.2.6 Data Constraints

Table 2 and 3 show the data constraints on the input and output variables, respectively. The column physical constraints gives the physical limitations on the range of values that can be taken by the variable. The constraints are conservative, to give the user of the model the flexibility to experiment with unusual situations. The column of typical values is intended to provide a feel for a common scenario. The uncertainty column provides an estimate of the confidence with which the physical quantities can be measured. This information would be part of the input if one were performing an uncertainty quantification exercise.

Var	Physical Constraints	Typical Value	Uncertainty
(x, y) of water table vertices's	Consecutive vertexes have increasing x values. All layers start and end vertices's go to the same x values.	N/A	/
(x, y) of slip vertices's	Consecutive vertexes have increasing x values. All layers start and end vertices's go to the same x values.	N/A	/
(x, y) of slope vertices's (*)	Consecutive vertexes have increasing x values. All layers start and end vertices's go to the same x values.	N/A	/
E (*)	$E > 0$	15000	/
c (*)	$c > 0$	10	/
v (*)	$0 < v < 1$	0.4	
φ' (*)	$0 < \varphi < 90$	25	/
γ (*)	$\gamma > 0$	20	/
γ_{Sat} (*)	$\gamma_{\text{Sat}} > 0$	20	/
γ_{Wat}	$\gamma_{\text{Wat}} > 0$	9.8	/

(*) Input coordinates needed for each layer.

Var	Physical Constraints
FS	$FS > 0$
(x, y) Slip vertices's	Vertices's monotonic
δx	
δy	

5 Requirements

This section provides the functional requirements, the business tasks that the software is expected to complete, and the nonfunctional requirements, the qualities that the software is expected to exhibit.

5.1 Functional Requirements

R1: Read the input file, and store the data. Necessary input data summarized in Table1. [A2, A3]

symbol	unit	description
(x, y)	m	x and y coordinates for vertices of the slope layers, and for the water table if one exists. Assumed straight line fits between vertexes.
E	kPa	Young's modulus for each layer of the slope.
c	kPa	Cohesion for each slope layer.
v	/	Poisson's ratio for each soil layer.
φ	deg	Effective angle of friction for each slope layer.
γ	$\frac{\text{kN}}{\text{m}^3}$	Unit weight of dry soil / ground layer for each slope layer.
γ_{Sat}	$\frac{\text{kN}}{\text{m}^3}$	Unit weight of saturated soil / ground layer for each slope layer.
γ_{Wat}	$\frac{\text{kN}}{\text{m}^3}$	Unit weight of water.

R2: Generate potential critical slip surface's for the input slope.

R3: Test the slip surfaces to determine if they are physically realizable based on a set of pass or fail criteria. [A1]

R4: Prepare the slip surfaces for a method of slices or limit equilibrium analysis.

R5: Calculate the factors of safety of the slip surfaces.

R6: Rank and weight the slopes based on their factor of safety, such that a slip surface with a smaller factor of safety has a larger weighting.

R7: Generate new potential critical slip surfaces based on previously analysed slip surfaces with low factors of safety.

R8: Repeat requirements R3 to R7 until the minimum factor of safety remains approximately the same over a predetermined number of repetitions. Identify the slip surface that generates the minimum factor of safety as the critical slip surface.

R9: Prepare the critical slip surface for method of slices or limit equilibrium analysis.

R10: Calculate the factor of safety of the critical slip surface using the Morgenstern price method. Also calculate the local and global factors of safety for the critical slip using the RFEM method, and the displacement of the slice elements using the RFEM method.

R11: Display the critical slip surface and the slice element displacements graphically. Give the values of the factors of safety calculated by both methods, and the local factors of safety calculated by the RFEM method of analysis.

5.2 Nonfunctional Requirements

SSA is intended to be an educational tool, therefore accuracy and performance speed are secondary program priorities to correctness, understandability, reusability, and maintainability.

6 Likely Changes

References

[1] D.G. Fredlund and J.Krahn. Comparison of slope stability methods of analysis. *Can. Geotech. J.*, (14):429–439, 4 April 1977.

[2] Nirmitha Koothoor. A document drive approach to certifying scientific computing software. Master's thesis, McMaster University, Hamilton, Ontario, Canada, 2013.

[3] Yu-Chao Li, Yun-Min Chen, Tony L.T Zhan, Dao-Sheng Ling, and Peter John Cleall. An efficient approach for locating the critical slip surface in slope stability analyses using a real-coded genetic algorithm. *Can. Geotech. J.*, (47):806–820, 25 June 2010.

[4] David L. Parnas and P.C. Clements. A rational design process: How and why to fake it. *IEEE Transactions on Software Engineering*, 12(2):251–257, February 1986.

[5] W. Spencer Smith and Lei Lai. A new requirements template for scientific computing. In J. Ralyté, P. Ȧgerfalk, and N. Kraiem, editors, *Proceedings of the First International Workshop on Situational Requirements Engineering Processes – Methods, Techniques and Tools to Support Situation-Specific Requirements Engineering Processes, SREP'05*, pages 107–121, Paris, France, 2005. In conjunction with 13th IEEE International Requirements Engineering Conference.

[6] Dieter Stolle and Peijun Guo. Limit equilibrum slope stability analysis using rigid finite elements. *Can. Geotech. J.*, (45):653–662, 20 May 2008.

[7] D.Y. Zhu, C.F. Lee, Q.H. Qian, and G.R. Chen. A concise algorithm for computing the factor of safety using the morgenstern–price method. *Can. Geotech. J.*, (42):272–278, 19 February 2005.