# Software Requirements Specification for Tamias2D: A 2D Rigid Body Physics Library

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# 1 Revision History

# [added new page for revision history —OO] $\,$

Date	Version	Notes
Oct. 2, 2018	1.0	Initial updated version
Oct. 4, 2018	1.1	Formated the entire document to match blank template sample
Oct. 8, 2018	1.2	Updates throughout the document
Nov. 2, 2018	1.3	Updated System Context Diagram; Off the Shelf solution
Nov. 17, 2018	1.4	Updated NonFunctional Requirements
Nov. 20, 2018	1.5	Moved Chasles Theorem from Theoretical model section to Data definition section; updated from T4 to DD9
Nov. 20, 2018	1.6	Moved from General Definition to Data definition section; updated from GD1 to DD10
Dec 22, 2018	1.7	Updated TM5 and added IM4

# 2 Reference Material

[reformatted entire page to match blank template—OO]

This section records information for easy reference.

#### 2.1 Table of Units

Throughout this document, SI (Système International d'Unités) is employed as the unit system. For each unit, the symbol is given followed by a description of the unit with the SI name.

Symbol	Description
m	length (metre)
kg	mass (kilogram)
S	time (second)
N	force (newton)
rad	angle (radian)
J	energy (joules)

# 2.2 Table of Symbols

The table that follows summarizes the symbols used in this document along with their units. More specific instances of these symbols will be described in their respective sections. Throughout the document, symbols in **bold** will represent vectors, and scalars otherwise. The symbols are listed in alphabetical order.

symbol	unit	description
a	${ m ms^{-2}}$	Acceleration
$\alpha$	$\rm rads^{-2}$	Angular acceleration
$C_{\mathrm{R}}$	unitless	Coefficient of restitution
${f F}$	N	Force
g	${\rm ms^{-2}}$	Gravitational acceleration (9.81 $\rm ms^{-2})$
G	${ m m^3kg^{-1}s^{-2}}$	Gravitational constant $(6.673 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2})$
h	m	Height
I	${ m kg}{ m m}^2$	Moment of inertia
î	m	Horizontal unit vector
$\hat{\mathbf{j}}$	m	Vertical unit vector
j	Ns	Impulse (scalar)
J	Ns	Impulse (vector)

KE	J	Kinetic Energy
L	m	Length
m	kg	Mass
n	unitless	Number of particles in a rigid body
$\mathbf{n}$	m	Collision normal vector
$\omega$	$\rm rads^{-1}$	Angular velocity
PE	J	Potential Energy
p	m	Position
$oldsymbol{\phi}$	rad	Orientation
r	m	Distance
$\mathbf{r}$	m	Displacement
t	$\mathbf{s}$	Time
au	N m	Torque
$oldsymbol{ heta}$	rad	Angular displacement
$\mathbf{v}$	$\rm ms^{-1}$	Velocity
w	m	Width

# 2.3 Abbreviations and Acronyms

symbol	description
A	Assumption
CM	Center of Mass
DD	Data Definition
GD	General Definition
GS	Goal Statement
IM	Instance Model
LC	Likely Change
ODE	Ordinary Differential Equation
R	Requirement
SRS	Software Requirements Specification
Τ	Theoretical Model
Tamias2D	Software name
2D	Two-dimensional

[added program name —OO]

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# 10 Off the Shelf Solutions

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[moved TOC to a new page to match blank template —OO]  $\,$ 

## 3 Introduction

Due to the rising cost of developing video games, developers are looking for ways to save time and money on their projects. Using an open source physics library that is reliable and free will cut down development costs and lead to better quality products.

The following section provides an overview of the Software Requirements Specification (SRS) for Tamias2D, an open source 2D rigid body physics library. It explains the purpose of this document, the scope of the system, and the organization of the document.

## 3.1 Purpose of Document

This document describes the modeling of an open source 2D rigid body physics library used for games. The goals and theoretical models used in Tamias2D are provided. This document is intended to be used as a reference to provide all necessary information to understand and verify the model.

This document will be used as a starting point for subsequent development phases, including the writing of the design specification and the software verification and validation plan. The design document will show how the requirements are to be realized. The verification and validation plan will show the steps that will be taken to increase confidence in the software documentation and implementation.

# 3.2 Scope of Requirements

The scope of the requirements includes the physical simulation of 2D rigid bodies acted on by forces. [removed the above sentence due to repetition —OO]

#### 3.3 Characteristics of Intended Reader

Reviewers of this documentation should have a strong knowledge of Physics, which is at the level covered in a second year engineering mechanics course. The reviewers should also have an understanding of Calculus 1 which is at the level covered in first year engineering or science calculus. The users of Tamias2D can have a lower level of expertise, as explained in Section 4.2. [reformatted paragraph —OO]

# 3.4 Organization of Document

The organization of this document follows the template for an SRS for scientific computing software proposed by [1] and [2]. The presentation follows the standard pattern of presenting goals, theories, definitions, and assumptions. For readers that would like a more bottom-up approach, they can start reading the instance models in Section 5.2.5 and trace back to find any additional information they require.

The goal statements are refined to the theoretical models, and theoretical models to the instance models.

# 4 General System Description

This section provides general information about the system including identifying the interfaces between the system and its environment (system context), describing the user characteristics and listing the system constraints.

## 4.1 System Context

Figure 1 shows the system context. A circle represents an external entity outside the software, the user in this case. A rectangle represents the software system itself (Tamias2D). Arrows are used to show the data flow between the system and its environment.



Figure 1: System Context

The interaction between the product and the user is through an application programming interface. The responsibilities of the user and the system are as follows:

#### • User Responsibilities:

- Provide initial conditions of the physical state of the simulation, rigid bodies present, and forces applied to them.
- Ensure application programming interface use complies with the user guide.
- Ensure required software assumptions (Section 5.2.1) are appropriate for any particular problem the software addresses.

#### • Tamias2D Responsibilities:

- Determine if the inputs and simulation state satisfy the required physical and software constraints. (Section 5.2.6)
- Calculate the new state of all rigid bodies within the simulation at each simulation step.
- Provide updated physical state of all rigid bodies at the end of simulation step.

#### 4.2 User Characteristics

The end user of Tamias2D should have an understanding of first year programming concepts and high school Physics. [updated with program name, fixed sentence —OO]

#### 4.3 System Constraints

There are no system constraints.

# 5 Specific System Description

This section first presents the problem description, which provides a high-level view of the problem to be solved. This is followed by the solution characteristics specification, which presents the assumptions, theories, and definitions that are used for the physics library.

## 5.1 Problem Description

Creating a gaming physics library is a difficult task. Games need physics libraries that can simulate objects acting under various physical conditions, while simultaneously being fast and efficient enough to work in soft real-time during the game. Developing a physics library from scratch takes a long period of time and is very costly, presenting barriers of entry which make it difficult for game developers to include physics in their products. There are a few free, open-source and high quality physics libraries available to be used for consumer products (Section 10). By creating a simple, lightweight, fast, and portable 2D rigid body physics library, game development will be more accessible to the masses and higher quality products will be produced.

#### 5.1.1 Terminology and Definitions

This subsection provides a list of terms that are used in the subsequent sections and their meanings, with the purpose of reducing ambiguity and making it easier to correctly understand the requirements:

- Rigid Body: a solid body in which deformation is neglected.
- Elasticity: ratio of the velocities of two colliding objects after and before the collision.
- Center of Mass: the mean location of the distribution of mass of the object.
- Cartesian coordinates: a coordinate system that specifies each point uniquely in a plane by a pair of numerical coordinates.
- Right-handed coordinate system: a coordinate system where the positive z-axis comes out of the screen.

#### 5.1.2 Physical System Description

This project is to implement a set of physics libraries. Hence a physical system description might not be applicable??? [Not sure if I need to have any information in this section, since it is about implementing a set of libraries —OO]

#### 5.1.3 Goal Statements

Given the kinematic properties, and forces (including any collision forces) applied on a set of rigid bodies:

- GS1: Determine their new position and velocities over a period of time (IM1)(IM4).
- GS2: Determine their new orientations and angular velocities over a period of time (IM2) (IM4).

[in Goal statement section, GS3 and GS4 were merged into GS1 and GS2—OO]

## 5.2 Solution Characteristics Specification

#### 5.2.1 Assumptions

This section simplifies the original problem and helps in developing the theoretical model by filling in the missing information for the physical system. The numbers given in the square brackets refer to the Theoretical Models [Section 5.2.2], General Definitions [Section 5.2.3], Data Definitions [Section 5.2.4], Instance Models [Section 5.2.5], or Likely Changes [Section 7], in which the respective assumption is used.

- A1: All objects are rigid bodies.
- A2: All objects are 2D (two-dimensional).
- A3: The library uses a Cartesian coordinate system.
- A4: The axes are defined using a right-handed coordinate system.
- A5: All rigid body collisions are vertex-to-edge collisions.
- A6: There is no damping involved throughout the simulation. This implies that there are no friction forces. [Add another sentence: "This implies that there are no friction forces." —SS] [added sentence —OO]
- A7: There are no constraints and joints involved throughout the simulation.

# 5.2.2 Theoretical Models

This section focuses on the general equations and laws that the physics library is based on.

Number	T1
Label	Newton's second law of motion
Equation	$\mathbf{F} = m\mathbf{a}$
Description	The net force $\mathbf{F}$ (N) on a body is proportional to the acceleration $\mathbf{a}$ (m s <sup>-2</sup> ) of the body, where $m$ (kg) denotes the mass of the body as the constant of proportionality.
Source	
Ref. By	DD10, GD2 IM1

Number	T2
Label	Newton's third law of motion
Equation	$\mathbf{F}_1 = -\mathbf{F}_2$
Description	Every action has an equal and opposite reaction. In other words, the force $\mathbf{F}_1$ (N) exerted on the second body by the first is equal in magnitude and in the opposite direction to the force $\mathbf{F}_2$ (N) exerted on the first body by the second.
Source	
Ref. By	GD1

Number	T3
Label	Newton's law of universal gravitation
Equation	$\mathbf{F} = G \frac{m_1 m_2}{  \mathbf{r}  ^2} \hat{\mathbf{r}} = G \frac{m_1 m_2}{  \mathbf{r}  ^2} \frac{\mathbf{r}}{  \mathbf{r}  }$
Description	Two bodies in the universe attract each other with a force $\mathbf{F}$ (N) that is directly proportional to the product of their masses, $m_1$ and $m_2$ (kg), and inversely proportional to the square of the distance $  \mathbf{r}  ^2$ (m <sup>2</sup> ) between them.
	The vector $\mathbf{r}$ (m) is the displacement between the centers of the bodies and $  \mathbf{r}  $ (m) represents the norm, or absolute distance between the two. $\hat{\mathbf{r}}$ denotes the unit displacement vector, equivalent to $\frac{\mathbf{r}}{  \mathbf{r}  }$ . Finally, $G$ is the gravitational constant $6.673 \times 10^{-11}$ m <sup>3</sup> kg <sup>-1</sup> s <sup>-2</sup> .
Source	
Ref. By	GD2

Number	T4
Label	Newton's second law for rotational motion
Equation	$\tau = \mathbf{I}\alpha$
Description	The net torque $\tau$ (N m) on a body (DD13) is proportional to its angular acceleration $\alpha$ (rad s <sup>-2</sup> ). Here, <b>I</b> (kg m <sup>2</sup> ) denotes the moment of inertia of the body (DD14). We also assume that all rigid bodies involved are two-dimensional (A2).
Source	
Ref. By	IM2

Number	T5
Label	Collision Detection
Equation	If p is a point in space and A and B are 2 bodies moving in space; if: $p \in A$ and $p \in B$ then collision has occurred between A and B.
Description	The problem of collision detection is detecting the intersection of two or more colliding objects. The computation of collision detection is used to determine whether two objects have collided. We also assume that all rigid bodies involved are two-dimensional (A2). When 2 objects $A_{x,y}$ and $B_{x,y}$ occupy the same position $P_{x,y}$ in space then they have collided. If an object A intersects with an object
Source	https://en.wikipedia.org/wiki/Collision_detection
Ref. By	IM4

#### 5.2.3 General Definitions

This section collects the laws and equations that will be used in deriving the data definitions, which in turn will be used to build the instance models.

Number	GD1
Label	Conservation of momentum
Equation	$\sum_{k=0}^{n} m_k \mathbf{v}_{i_k} = \sum_{k=0}^{n} m_k \mathbf{v}_{f_k}$
Description	In an isolated system, where the sum of external impulses acting on the system is zero, the total momentum of the bodies is constant (conserved).
	$m_k$ is the mass of the $k$ -th body (kg).
	$\mathbf{v}_{i_k}$ is the initial velocity of the $k$ -th body (m s <sup>-1</sup> ).
	$\mathbf{v}_{\mathrm{f}_k}$ is the final velocity of the k-th body (m s <sup>-1</sup> ).
Source	
Ref. By	IM4

#### Derivation of the Conservation of Momentum

When bodies collide, they exert an equal force on each other in opposite directions. This is Newton's third law (T2):

$$\mathbf{F}_1 = -\mathbf{F}_2$$

The objects collide with each other for the exact same amount of time t:

$$\mathbf{F}_1 t = -\mathbf{F}_2 t \tag{1}$$

The above equation is equal to the impulse (DD10):

$$\mathbf{F}_1 t = \int \mathbf{F}_1 \, \mathrm{d}t = \mathbf{J}$$

The impulse is equal to the change in momentum:

$$\mathbf{J} = \Delta \mathbf{P} = m\Delta \mathbf{v} \tag{2}$$

Substituting 2 into 1 yields:

$$m_1 \Delta \mathbf{v}_1 = -m_2 \Delta \mathbf{v}_2$$

Expanding and rearranging the above formula gives:

$$m_1 \mathbf{v}_{i_1} + m_2 \mathbf{v}_{i_2} = m_1 \mathbf{v}_{f_1} + m_2 \mathbf{v}_{f_2}$$

Generalizing for multiple (k) colliding objects:

$$\sum_{k=0}^{n} m_k \mathbf{v}_{\mathbf{i}_k} = \sum_{k=0}^{n} m_k \mathbf{v}_{\mathbf{f}_k}$$

Number	GD2
Label	Acceleration due to gravity
Units	$\mathrm{ms^{-2}}$
Equation	$\mathbf{F}_{\mathrm{g}} = m\mathbf{g}$ , where $\mathbf{g} = [-g_{\mathrm{x}}, -g_{\mathrm{y}}]$
Description	$\mathbf{F}_{\mathrm{g}}$ is the force due to gravity (N).
	m is the mass of a rigid body (kg).
	${f g}$ is the acceleration due to gravity (m s <sup>-2</sup> ).
Source	
Ref. By	IM <mark>1</mark>

#### **Derivation of Gravitational Acceleration**

From Newton's law of universal gravitation (T3), we have:

$$\mathbf{F} = G \frac{m_1 m_2}{||\mathbf{r}||^2} \hat{\mathbf{r}} \tag{3}$$

Equation 3 governs the gravitational attraction between two bodies. Suppose that one of the bodies is significantly more massive than the other, so that we concern ourselves with the force the massive body exerts on the lighter body. Further suppose that the coordinate system is chosen such that this force acts on a line which lies along one of the principal axes (A2). Then our unit vector  $\hat{\mathbf{r}} = \frac{\mathbf{r}}{||\mathbf{r}||} = \hat{\mathbf{i}}$  or  $\hat{\mathbf{j}}$  for the x or y axes (A3), respectively.

Given the above assumptions, let M and m be the mass of the massive and light body, respectively. Using 3 and equating this with Newton's second law (T1) for the force experienced by the light body, we get:

$$\mathbf{F}_{g} = G \frac{Mm}{||\mathbf{r}||^{2}} \hat{\mathbf{r}} = m\mathbf{g} \tag{4}$$

where **g** is gravitational acceleration. Dividing 4 by m, and resolving this into separate x and y components:

$$G \frac{M}{||r_{\mathbf{x}}||^2} \hat{\mathbf{i}} = -g_{\mathbf{x}} \hat{\mathbf{i}}$$
$$G \frac{M}{||r_{\mathbf{y}}||^2} \hat{\mathbf{j}} = -g_{\mathbf{y}} \hat{\mathbf{j}}$$

Thus:

$$\mathbf{g} = [-g_{\mathbf{x}}, -g_{\mathbf{y}}]$$

#### 5.2.4 Data Definitions

This section collects and defines all the data needed to build the instance models. The dimension of each quantity is also given.

Number	DD1
Label	Center of mass
Symbol	$\mathbf{p}_{\mathrm{CM}}$
Units	m
Equation	$\mathbf{p}_{ ext{CM}} = rac{\sum_i m_i \mathbf{p}_i}{M}$
Description	The center of mass $\mathbf{p}_{\text{CM}}$ (m) of a rigid body (A1) is the mass-weighted average position of all its particles, or the unique point where all of its mass is concentrated.
	$m_i$ is the mass of the <i>i</i> -th particle (kg).
	$\mathbf{p}_i$ is the position vector (A2) of the <i>i</i> -th particle (m).
	M is the total mass of the body (kg).
Sources	
Ref. By	IM1, IM4

Number	DD2
Label	Linear displacement
Symbol	r
Units	m
Equation	$\mathbf{r}(t) = rac{\mathrm{d}\mathbf{p}(t)}{\mathrm{d}t}$
Description	$\mathbf{r}(t)$ is the linear displacement of a body (A1, A2), without damping (A6), as a function of time $t$ , also equal to the derivative of its linear position with respect to time $t$ (m).
Sources	
Ref. By	IM <mark>1</mark>

Number	DD3
Label	Linear velocity
Symbol	v
Units	$\mathrm{m}\mathrm{s}^{-1}$
Equation	$\mathbf{v}(t) = rac{\mathrm{d}\mathbf{r}(t)}{\mathrm{d}t}$
Description	$\mathbf{v}(t)$ is the linear velocity of a body (A1, A2), without damping (A6), as a function of time $t$ , also equal to the derivative of its linear displacement with respect to time $t$ (m s <sup>-1</sup> ).
Sources	
Ref. By	IM <mark>1</mark>

Number	DD4
Label	Linear acceleration
Symbol	a
Units	$\mathrm{ms^{-2}}$
Equation	$\mathbf{a}(t) = rac{\mathrm{d}\mathbf{v}(t)}{\mathrm{d}t}$
Description	$\mathbf{a}(t)$ is the linear acceleration of a body (A1, A2), without damping (A6), as a function of time $t$ , also equal to the derivative of its linear velocity with respect to time $t$ (m s <sup>-2</sup> ).
Sources	
Ref. By	IM <mark>1</mark>

Number	DD5
Label	Angular displacement
Symbol	$\theta$
Units	rad
Equation	$oldsymbol{ heta}(t) = rac{\mathrm{d}oldsymbol{\phi}(t)}{\mathrm{d}t}$
Description	$\theta(t)$ is the angular displacement of a body (A1, A2), without damping (A6), as a function of time $t$ , also equal to the derivative of its angular position with respect to time $t$ (rad).
Sources	
Ref. By	IM2

Number	DD6
Label	Angular velocity
Symbol	$\omega$
Units	$\mathrm{rad}\mathrm{s}^{-1}$
Equation	$oldsymbol{\omega}(t) = rac{\mathrm{d}oldsymbol{ heta}(t)}{\mathrm{d}t}$
Description	$\omega(t)$ is the angular velocity of a body (A1, A2), without damping (A6), as a function of time $t$ , also equal to the derivative of its angular displacement with respect to time $t$ (rad s <sup>-1</sup> ).
Sources	
Ref. By	IM2

Number	DD7
Label	Angular acceleration
Symbol	$\alpha$
Units	$ m rads^{-2}$
Equation	$oldsymbol{lpha}(t) = rac{\mathrm{d}oldsymbol{\omega}(t)}{\mathrm{d}t}$
Description	$\alpha(t)$ is the angular acceleration of a body (A1, A2), without damping (A6), as a function of time $t$ , also equal to the derivative of its angular velocity with respect to time $t$ (rad s <sup>-2</sup> ).
Sources	
Ref. By	IM2

Number	DD8
Label	Impulse for collision response
Symbol	j
Units	Ns
Equation	$j = \frac{-(1 + C_{\mathrm{R}})\mathbf{v}_{\mathrm{i}}^{\mathrm{AB}} \cdot \mathbf{n}}{\left(\frac{1}{m_{\mathrm{A}}} + \frac{1}{m_{\mathrm{B}}}\right)  \mathbf{n}  ^{2} + \frac{  \mathbf{r}_{\mathrm{AP}} \times \mathbf{n}  ^{2}}{\mathbf{I}_{\mathrm{A}}} + \frac{  \mathbf{r}_{\mathrm{BP}} \times \mathbf{n}  ^{2}}{\mathbf{I}_{\mathrm{B}}}$
Description	j is the impulse (scalar) used to determine collision response (A5) between two rigid bodies (A1, A2) .
	$C_{\rm R}$ is the coefficient of restitution (DD12).
	<b>n</b> is the collision normal vector (m). Its signed direction is defined by (A4).
	$\mathbf{v}_{i}^{AB}$ is the relative velocity (DD11) between body A and body B (m s <sup>-1</sup> ).
	$m_{\rm A}$ and $m_{\rm B}$ are the masses of body A and B, respectively (kg).
	$\mathbf{r}_{AP}$ and $\mathbf{r}_{BP}$ are the displacement vectors between the centers of mass of body A and B, respectively, and the point of contact P (m).
	$I_A$ and $I_B$ are the moments of inertia (DD14) for body A and body B, respectively (kg m <sup>2</sup> ).
Sources	
Ref. By	IM4

#### Derivation for Impulse for Collision Response

Rearranging the equation for the coefficient of restitution (DD12), we get:

$$\mathbf{v}_{\mathrm{f}}^{\mathrm{AB}} \cdot \mathbf{n} = -C_{\mathrm{R}} \mathbf{v}_{\mathrm{i}}^{\mathrm{AB}} \cdot \mathbf{n}$$

Expanding the relative velocity (DD11) on the left:

$$(\mathbf{v}_{\mathrm{f}}^{\mathrm{AP}} - \mathbf{v}_{\mathrm{f}}^{\mathrm{BP}}) \cdot \mathbf{n} = -C_{\mathrm{R}} \mathbf{v}_{\mathrm{i}}^{\mathrm{AB}} \cdot \mathbf{n}$$

Applying Chasles' Theorem (DD9) and IM4 on the left-hand side: [This step does not make sense. IM3 comes from this DD. This makes this circular. The derivation should follow the game physics papers that I sent to you. —SS]

[Here it applies Chasles theorem although in the paper it does not specify Chasles theorem but it also uses equations generated from applying Center of Mass velocities and used that equation to calculate the velocities of an arbitrary point on rotation and translation, which I assume you are referring to rotation when you mentioned 'This makes it circular' above. Even though it does not exactly derive the equation as in the paper but the paper had derived some equation on page 14 which was not necessary and was implicitly stated but the collision response equation includes the term for angular velocity(in the paper) and thats the reason for Chasles law in the SRS. —OO]

$$\begin{split} & \left(\mathbf{v}_{\mathrm{f}}^{\mathrm{A}} + \boldsymbol{\omega}_{\mathrm{f}}^{\mathrm{A}} \times \mathbf{r}_{\mathrm{AP}} - \mathbf{v}_{\mathrm{f}}^{\mathrm{B}} - \boldsymbol{\omega}_{\mathrm{f}}^{\mathrm{B}} \times \mathbf{r}_{\mathrm{BP}}\right) \cdot \mathbf{n} \\ & \Longrightarrow \left(\mathbf{v}_{\mathrm{i}}^{\mathrm{A}} + \frac{j}{m_{\mathrm{A}}} \mathbf{n} + \left(\boldsymbol{\omega}_{\mathrm{i}}^{\mathrm{A}} + \frac{\mathbf{r}_{\mathrm{AP}} \times j \mathbf{n}}{\mathbf{I}_{\mathrm{A}}}\right) \times \mathbf{r}_{\mathrm{AP}} - \mathbf{v}_{\mathrm{i}}^{\mathrm{B}} + \frac{j}{m_{\mathrm{B}}} \mathbf{n} - \left(\boldsymbol{\omega}_{\mathrm{i}}^{\mathrm{B}} - \frac{\mathbf{r}_{\mathrm{BP}} \times j \mathbf{n}}{\mathbf{I}_{\mathrm{B}}}\right) \times \mathbf{r}_{\mathrm{BP}}\right) \cdot \mathbf{n} \end{split}$$

Expanding and then collecting terms:

$$\begin{split} & \left[ \left( \mathbf{v}_{i}^{A} + \boldsymbol{\omega}_{i}^{A} \times \mathbf{r}_{AP} \right) - \left( \mathbf{v}_{i}^{B} + \boldsymbol{\omega}_{i}^{B} \times \mathbf{r}_{BP} \right) \right. \\ & + j \left( \frac{1}{m_{A}} + \frac{1}{m_{B}} \right) \mathbf{n} + j \left( \frac{\mathbf{r}_{AP} \times \mathbf{n} \times \mathbf{r}_{AP}}{\mathbf{I}_{A}} + \frac{\mathbf{r}_{BP} \times \mathbf{n} \times \mathbf{r}_{BP}}{\mathbf{I}_{B}} \right) \right] \cdot \mathbf{n} \\ & \Longrightarrow \left( \mathbf{v}_{i}^{AP} - \mathbf{v}_{i}^{BP} \right) \cdot \mathbf{n} + j \left[ \left( \frac{1}{m_{A}} + \frac{1}{m_{B}} \right) \mathbf{n} + \left( \frac{\mathbf{r}_{AP} \times \mathbf{n} \times \mathbf{r}_{AP}}{\mathbf{I}_{A}} + \frac{\mathbf{r}_{BP} \times \mathbf{n} \times \mathbf{r}_{BP}}{\mathbf{I}_{B}} \right) \right] \cdot \mathbf{n} \\ & \Longrightarrow \mathbf{v}_{i}^{AB} \cdot \mathbf{n} + j \left[ \left( \frac{1}{m_{A}} + \frac{1}{m_{B}} \right) \mathbf{n} \cdot \mathbf{n} + \left( \frac{\mathbf{r}_{AP} \times \mathbf{n} \times \mathbf{r}_{AP}}{\mathbf{I}_{A}} + \frac{\mathbf{r}_{BP} \times \mathbf{n} \times \mathbf{r}_{BP}}{\mathbf{I}_{B}} \right) \cdot \mathbf{n} \right] \\ & \Longrightarrow \mathbf{v}_{i}^{AB} \cdot \mathbf{n} + j \left[ \left( \frac{1}{m_{A}} + \frac{1}{m_{B}} \right) \mathbf{n} \cdot \mathbf{n} + \frac{(\mathbf{r}_{AP} \times \mathbf{n}) \cdot (\mathbf{r}_{AP} \times \mathbf{n})}{\mathbf{I}_{A}} + \frac{(\mathbf{r}_{BP} \times \mathbf{n}) \cdot (\mathbf{r}_{BP} \times \mathbf{n})}{\mathbf{I}_{B}} \right] \\ & \Longrightarrow \mathbf{v}_{i}^{AB} \cdot \mathbf{n} + j \left[ \left( \frac{1}{m_{A}} + \frac{1}{m_{B}} \right) ||\mathbf{n}||^{2} + \frac{||\mathbf{r}_{AP} \times \mathbf{n}||^{2}}{\mathbf{I}_{A}} + \frac{||\mathbf{r}_{BP} \times \mathbf{n}||^{2}}{\mathbf{I}_{B}} \right] \end{split}$$

Finally, equating the left and right-hand sides back together and rearranging for j, we obtain:

$$j = \frac{-(1 + C_{\mathrm{R}})\mathbf{v}_{\mathrm{i}}^{\mathrm{AB}} \cdot \mathbf{n}}{\left(\frac{1}{m_{\mathrm{A}}} + \frac{1}{m_{\mathrm{B}}}\right)||\mathbf{n}||^{2} + \frac{||\mathbf{r}_{\mathrm{AP}} \times \mathbf{n}||^{2}}{\mathbf{I}_{\mathrm{A}}} + \frac{||\mathbf{r}_{\mathrm{BP}} \times \mathbf{n}||^{2}}{\mathbf{I}_{\mathrm{B}}}}$$

Number	DD9
Label	Chasles' theorem [Please make this a DD; it isn't abstract, general or fundamental. —SS]
Symbol	v
Units	$\mathrm{m}\mathrm{s}^{-1}$
Equation	$\mathbf{v}_{\mathrm{B}} = \mathbf{v}_{\mathrm{O}} + (\boldsymbol{\omega} \times \mathbf{r}_{\mathrm{OB}})$
Description	The linear velocity $\mathbf{v}_{\mathrm{B}}$ (m s <sup>-1</sup> ) of a point $B$ in a rigid body (A1) is the sum of the body's linear velocity $\mathbf{v}_{\mathrm{O}}$ (m s <sup>-1</sup> ) at the origin (axis of rotation) and the resultant vector from the cross product of the body's angular velocity $\boldsymbol{\omega}$ (rad s <sup>-1</sup> ) and the vector between the origin and point $B$ , $\mathbf{r}_{\mathrm{OB}}$ (m).
Source	
Ref. By	IM2

[moved T4 in Theoretical model section to DD9 in Data Definition section —OO]

Number	DD10
Label	Impulse [Make this a data definition —SS]
Symbol	J
Units	Ns
Equation	$\mathbf{J} = \int \mathbf{F}  \mathrm{d}t = \Delta \mathbf{P} = m \Delta \mathbf{v}$
Description	An impulse $\bf J$ occurs when a force $\bf F$ acts over an interval of time.
	${f J}$ is the resultant impulse applied on the body (N s).
	<b>F</b> is the force applied on the body (N).
	$\Delta \mathbf{P}$ is the change in momentum of the body (N s).
	m is the mass of the body (kg).
	$\Delta \mathbf{v}$ is the change in velocity of the body (m s <sup>-1</sup> ).
Source	
Ref. By	GD1, DD8, IM4

[moved GD1 to DD10 —OO]

# **Derivation of Impulse**

Newton's second law of motion  $(T_1)$  states:

$$\mathbf{F} = m\mathbf{a} = m\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}$$

Rearranging:

$$\int_{t_1}^{t_2} \mathbf{F} \, \mathrm{d}t = m \int_{v_1}^{v_2} \, \mathrm{d}\mathbf{v}$$

Integrating the right hand side:

$$\int_{t_1}^{t_2} \mathbf{F} \, \mathrm{d}t = m\mathbf{v_2} - m\mathbf{v_1} = m\Delta\mathbf{v}$$

Number	DD11
Label	Relative velocity in collisions [Make this a data definition — SS][updated to DD —OO]
Units	$\mathrm{m}\mathrm{s}^{-1}$
Equation	$\mathbf{v}^{\mathrm{AB}} = \mathbf{v}^{\mathrm{AP}} - \mathbf{v}^{\mathrm{BP}}$
Description	In a collision, the velocity of a rigid body A colliding with another body B relative to that body, $\mathbf{v}^{AB}$ , is the difference between the velocities of A and B at point P.
	$\mathbf{v}^{\mathrm{AB}}$ is the velocity of A relative to B (m s <sup>-1</sup> ).
	P is the common collision point on both bodies (m).
	$\mathbf{v}^{\mathrm{AP}}$ is the velocity of point P in body A (m s <sup>-1</sup> ).
	$\mathbf{v}^{\mathrm{BP}}$ is the velocity of point P in body B (m s <sup>-1</sup> ).
Source	
Ref. By	DD12, DD8

Number	DD12						
Label	Coefficient of restitution [Make this a data definition — SS][updated —OO]						
Equation	$C_{ m R} = -rac{{f v}_{ m f}^{ m AB}\cdot{f n}}{{f v}_{ m i}^{ m AB}\cdot{f n}}$						
Description	The coefficient of restitution $C_{\rm R}$ is a unitless, dimensionless quantity that determines the elasticity of a collision between two bodies. $C_{\rm R}=1$ results in an elastic collision, while $C_{\rm R}<1$ results in an inelastic collision, and $C_{\rm R}=0$ results in a totally inelastic collision.						
	$C_{\rm R}$ is the coefficient of restitution (unitless).						
	<b>n</b> is the collision normal vector (m). Its signed direction is defined by (A4).						
	$\mathbf{v}_{i}^{AB}$ is the initial relative velocity (DD11) of body A with respect to body B before collision (m s <sup>-1</sup> ).						
	$\mathbf{v}_{\mathrm{f}}^{\mathrm{AB}}$ is the final relative velocity (DD11) of body A with respect to body B after collision (m s <sup>-1</sup> ).						
Source							
Ref. By	DD8						

Number	DD13					
Label	Torque [Make this a data definition —SS][updated —OO]					
Units	N m					
Equation	$oldsymbol{ au} = \mathbf{r}  imes \mathbf{F}$					
Description	The torque $\tau$ on a body measures the tendency of a force to rotate the body around an axis or pivot.					
	au is the torque on the body (N m).					
	<b>F</b> is the force applied to the lever arm (N).					
	<b>r</b> is a position vector of the point where the force is applied, measured from the axis of rotation (m).					
Source						
Ref. By	T5, IM4					

Number	DD14						
Label	Moment of inertia [Make this a data definition —SS][updated — OO]						
Units	$ m kgm^2$						
Equation	$\mathbf{I} = \sum_{i=0}^{n} m_i r_{\mathbf{p}_i}^2$						
Description	The moment of inertia I of a body measures how much torque is needed for the body to achieve an angular acceleration about axis of rotation.						
	I is the moment of inertia $(kg m^2)$ .						
	n is number of particles of the body.						
	$m_i$ is the mass of the <i>i</i> -th particle (kg).						
	$r_{\mathbf{p}_i}$ is the distance between the <i>i</i> -th particle and the axis of rotation (m).						
Source							
Ref. By	T5, DD8, IM4						

Number	DD15						
Label	Kinetic Energy						
Units	J						
Equation	$\mathbf{KE} = mv^2/2$						
Description	The kinetic energy <b>KE</b> of a body is the measure of the energy a body possesses due to its motion. "It is defined as the work needed to accelerate a body of a given mass from rest to its stated velocity".						
	<b>KE</b> is the kinetic energy (J).						
	m is the mass of the body (kg).						
	${f v}$ is the velocity of the body. (m s <sup>-1</sup> ).						
Source	https://www.omnicalculator.com/physics/kinetic-energy#kinetic-energy-definition						
Ref. By	T5, DD8, IM4 [fix ref by —OO]						

Number	DD16						
Label	Potential Energy						
Units	J						
Equation	$\mathbf{PE} = mgh$						
Description	The potential energy <b>PE</b> of a body is the measure of the force of gravity exerted on relative to how far it has to fall.						
	<b>PE</b> is the potential energy (J).						
	m is the mass of the body (kg).						
	$\mathbf{g}$ is the gravitational force acting the body. (m s <sup>-1</sup> ).						
	h is the distance through which a body is falling. (m).						
Source	https://www.omnicalculator.com/physics/kinetic-energy#kinetic-energy-definition						
Ref. By	T5, DD8, IM4 [fix ref by —OO]						

# 5.2.5 Instance Models

This section transforms the problem defined in Section 5.1 into one expressed in mathematical terms. It uses concrete symbols defined in Section 5.2.4 to replace the abstract symbols in the models identified in Sections 5.2.2 and 5.2.3.

Number	IM1						
Label	Force on the translational motion of a set of 2D rigid bodies						
Input	$m_i, \mathbf{g}, \mathbf{p}_i(t_0), \mathbf{v}_i(t_0), \mathbf{F}_i(t_0)$						
Output	$\mathbf{p}_i(t), \mathbf{v}_i(t)$ , such that the following ODE is satisfied:						
	$\mathbf{a}_i(t) = rac{\mathrm{d}\mathbf{v}_i(t)}{\mathrm{d}t} = \mathbf{g} + rac{\mathbf{F}_i(t)}{m_i}$						
Description	The above equation expresses the total acceleration of the rigid body (A2) $i$ as the sum of gravitational acceleration (GD2) and acceleration of to applied force $\mathbf{F}_i(t)$ (T1). The resultant outputs are then obtained from this equation using DD2, DD3 and DD4. It is currently assumed that the is no damping (A6) or constraints (A7) involved.						
	$m_i$ is the mass of the <i>i</i> -th rigid body (kg).						
	$\mathbf{g}$ is the acceleration due to gravity (m s <sup>-2</sup> ).						
	$t$ is a point in time and $t_0$ denotes the initial time (s).						
	$\mathbf{p}_i(t)$ is the <i>i</i> -th body's position (specifically, the position of its center of mass, $\mathbf{p}_{\mathrm{CM}}(t)$ (DD1)) at time $t$ (m).						
	$\mathbf{a}_i(t)$ is the <i>i</i> -th body's acceleration at time $t$ (m s <sup>-2</sup> ).						
	$\mathbf{v}(t)$ is the <i>i</i> -th body's velocity at time $t$ (m s <sup>-1</sup> ).						
	$\mathbf{F}(t)$ is the force applied to the <i>i</i> -th body at time $t$ (N).						
Sources							
Ref. By	GS1, R2, R5						

Number	IM2						
Label	Force on the rotational motion of a set of 2D rigid body						
Input	$m_i, \mathbf{g}, oldsymbol{\phi}_i(t_0), oldsymbol{\omega}_i(t_0), oldsymbol{ au}_i(t_0), \mathbf{I}_i$						
Output	$\phi_i(t), \boldsymbol{\omega}_i(t)$ , such that the following ODEs is satisfied:						
	$oldsymbol{lpha}_i(t) = rac{\mathrm{d} oldsymbol{\omega}_i(t)}{\mathrm{d} t} = rac{oldsymbol{ au}_i(t)}{\mathbf{I}_i}$						
Description	The above equation for the total angular acceleration of the rigid body (A1, A2) $i$ is derived from T5, and the resultant outputs are then obtained from this equation using DD5, DD6 and DD7. It is currently assumed that there is no damping (A6) or constraints (A7) involved.						
	$m_i$ is the mass of the <i>i</i> -th rigid body (kg).						
	$\mathbf{g}$ is the acceleration due to gravity (m s <sup>-2</sup> ).						
	$t$ is a point in time and $t_0$ denotes the initial time (s).						
	$\phi_i(t)$ is the <i>i</i> -th body's orientation at time $t$ (rad).						
	$\omega_i(t)$ is the <i>i</i> -th body's angular velocity at time $t$ (rad s <sup>-1</sup> ).						
	$\alpha_i(t)$ is the <i>i</i> -th body's angular acceleration at time $t$ (rad s <sup>-2</sup> ).						
	$\tau_i(t)$ is the torque applied to the <i>i</i> -th body at time $t$ (N m). Signed direction of torque is defined by (A4).						
	$\mathbf{I_i}$ is the moment of inertia of the <i>i</i> -th body (kg m <sup>2</sup> ).						
Sources							
Ref. By	GS2, R6						

Number	IM3								
Label	Collisions on 2D rigid bodies								
Input	$m_k, \mathbf{p}_k(t_0), \mathbf{v}_k(t_0), \boldsymbol{\phi}_k(t_0), \boldsymbol{\omega}_k(t_0), C_{\mathrm{R}}$								
Output	$\mathbf{v}_k(t), \mathbf{p}_k(t), \boldsymbol{\phi}_k(t), \boldsymbol{\omega}_k(t)$ such that momentum is conserved:								
	$\sum_{k=0}^{n} m_k \mathbf{v}_{i_k} = \sum_{k=0}^{n} m_k \mathbf{v}_{f_k} $ (GD1)								
	and that for any colliding pair of rigid bodies $A$ and $B$ , the following equations are satisfied:								
	$\mathbf{v}_{\mathrm{A}}(t_c) = \mathbf{v}_{\mathrm{A}}(t) + rac{j}{m_{\mathrm{A}}}\mathbf{n}$								
	$\mathbf{v}_{\mathrm{B}}(t_c) = \mathbf{v}_{\mathrm{B}}(t) - rac{j}{m_{\mathrm{B}}}\mathbf{n}$								
	$oldsymbol{\omega}_{ m A}(t_c) = oldsymbol{\omega}_{ m A}(t) + rac{{f r}_{ m AP} imes j{f n}}{{f I}_{ m A}}$								
	$oldsymbol{\omega}_{ m B}(t_c) = oldsymbol{\omega}_{ m B}(t) - rac{{f r}_{ m BP} imes j{f n}}{{f I}_{ m B}}$								
Description	This instance model is based on our assumptions regarding rigid body (A1, A2) collisions (A5). Again, this does not take damping (A6) or constraints (A7) into account.								
	$m_k$ is the mass of the k-th rigid body (kg).								
	$\mathbf{I}_k$ is the moment of inertia of the k-th rigid body (kg m <sup>2</sup> ).								
	$t$ is a point in time, $t_0$ denotes the initial time, and $t_c$ denotes the time at collision (s).								
	$\mathbf{p}_k(t)$ is the k-th body's position (specifically, the position of its center of mass, $\mathbf{p}_{\mathrm{CM}}(t)$ (DD1)) at time $t$ (m).								
	$\mathbf{v}_k(t)$ is the k-th body's velocity at time $t$ (m s <sup>-1</sup> ).								
	$\phi_k(t)$ is the k-th body's orientation at time t (rad).								
	$\omega_k(t)$ is the k-th body's angular velocity at time $t$ (rad s <sup>-1</sup> ).								
	<b>n</b> is the collision normal vector (m). Its signed direction is determined by (A4).								
	j is the collision impulse (DD8) (Ns).								
	P is the point of collision (m).								
	$\mathbf{r}_{kP}$ is the displacement vector between the center of mass of the $k$ -th body and point $P$ (m).								
Sources									
Ref. By	GS1, GS2, DD8, R3, R8 [removed GS4 as it no longer exists and replaced with GS1 & 2 and GS1 and 2 now covers collision —OO]								

Number	IM4						
Label	Collision detection of rectangles - Axis Aligned Bounding Box						
Input	$x_1, y_1, x_2, y_2, w_1, h_1, w_2, h_2, rect_1, rect_2$						
Output	True(collision is detected) if the below statements are satisfied: $rect_1.x_1 < rect_2.x_2 + rect_2.w_2$ and $rect_1.x_1 + rect_1.w_1 > rect_2.x_2$ and $rect_1.y_1 < rect_2.y_2 + rect_2.h_2$ and $rect_1.y_1 + rect_1.h_1 > rect_2.y_2$						
Description	This instance model is based on the assumptions regarding rigid body (AA2) and collisions (A5) and is defined using the Axis Aligned Bounding B method for collision detection.						
	$rect_1$ and $rect_2$ are 2 rectangular shapes referred to as the bounding boxes of rigid bodies.						
	$\mathbf{x}_1$ is x coordinate position of $rect_1$ in space						
	$\mathbf{y}_1$ is y coordinate position of $rect_1$ in space						
	$\mathbf{x}_2$ is x coordinate position of $rect_2$ in space						
	$\mathbf{y}_2$ is y coordinate position of $rect_2$ in space						
	** The x, y coordinates are the position of the center of mass						
	$\mathbf{w}_1$ is the width of $rect_1$ (m)						
	$\mathbf{w}_2$ is the width of $rect_2$ (m)						
	$\mathbf{h}_1$ is the width of $rect_1$ (m)						
	$\mathbf{h}_2$ is the width of $rect_2$ (m)						
Ref. By	GS1, R3, R8, R7						

## Collision Diagram

This section presents an image of a typical collision between two 2D rigid bodies labeled A and B, showing the position of the two objects, the collision normal vector  $\mathbf{n}$  and the vectors from the approximate center of mass of each object to the point of collision P,  $\mathbf{r}_{AP}$  and  $\mathbf{r}_{BP}$ . Note that this figure only presents vertex-to-edge collisions, as per our assumptions (A5).

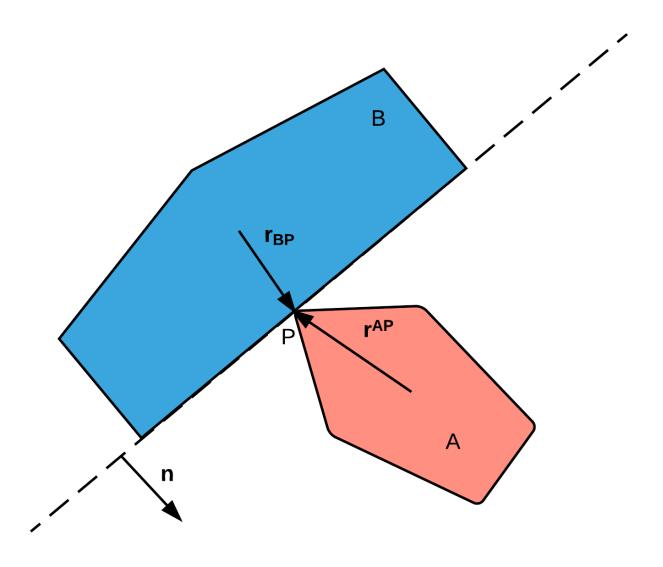


Figure 2: Collision between two rigid bodies

#### 5.2.6 Data Constraints

Table 1 and 2 show the data constraints on the input and output variables, respectively. The "Physical Constraints" column gives the physical limitations on the range of values that can be taken by the variable. The constraints are conservative, to give the user of the model the flexibility to experiment with unusual situations. The column of typical values is intended to provide a feel for a common scenario.

Var	Physical Constraints	Typical Value
$C_{\mathrm{R}}$	$0 \le C_{\mathrm{R}} \le 1$	0.8
${f F}$	None	98.1 N
g	None	$9.8   \mathrm{m  s^{-2}}$
Ι	$I \ge 0$	$74.5 \text{ kg m}^2$
L	$L \ge 0$	44.2 m
m	$m \ge 0$	56.2  kg
$\mathbf{p}$	None	(0.412, 0.502)  m
$\mathbf{v}$	None	$2.51~{\rm ms^{-1}}$
au	None	$200~\mathrm{Nm}$
$\omega$	None	$2.1~\mathrm{rads^{-1}}$
$\phi$	$0 \le \phi < 2\pi$	$\frac{\pi}{2}$ rad

Table 1: Input Variables

Var	Physical Constraints				
p	None				
$\mathbf{v}$	None				
$oldsymbol{\phi}$	$0 \le \phi < 2\pi$				
$\omega$	None				

Table 2: Output Variables

#### 5.2.7 Properties of a Correct Solution

A correct solution must exhibit conservation of energy. The sum of the potential energies and kinetic energy of all bodies should remain constant or decrease. Energy can decrease if the  $C_R$  is less than 1.0. Momentum is conserved.

[No, this is just stating (in another way) that the software should follow its requirements. This is a given for any SRS. Instead, you should state that energy is conserved. The sum of the potential energies and kinetic energy of all bodies should remain constant or decrease. (Energy can decrease of the CR is less than 1.0.) You will have to add the definitions for kinetic and potential energy. —SS]

[updated properties of a correct solution —OO]

# 6 Requirements

This section provides the functional requirements: the business tasks that the software is expected to complete, and the nonfunctional requirements: the qualities that the software is expected to exhibit.

## 6.1 Functional Requirements

- R1: The software shall create a space for all of the rigid bodies in the physical simulation to interact in.
- R2: The software shall receive the input, initial mass, velocities, positions, orientations, angular velocities of, and forces applied on rigid bodies (IM1, IM2, R4).
- R3: The software shall receive the surface properties of the bodies, such as [friction should not be included because we do not have damping—SS] elasticity which is the coefficient of restitution. [is this the coefficient of restitution? These are rigid bodies, so they do not deform.—SS] (IM4, R4),
- R4: The software shall verify that the inputs satisfy the required physical constraints (Section 5.2.6).
- R5: The software shall determine the position and velocities over a period of time of the 2D rigid bodies acted upon by a force (IM1).
- R6: The software shall determine the orientation and angular velocities over a period of time of the 2D rigid bodies (IM2).
- R7: The software shall determine if any of the rigid bodies in the space have collided (R1).
- R8: The software shall determine the position and velocities over a period of time of 2D rigid bodies that have undergone a collision (IM4, R7).

[reworded functional requirement items—OO]

[There is information missing. From these requirements. How is the shape of the rigid bodies defined? How are collisions defined? You should decide how to represent the shapes. For now, I suggest that we make the shapes all rectangles. (A degenerate rectangle could give us a triangle.) Also, please add a theoretical model for detecting collision. A collision will occur between two bodies when any point on both bodies occupies the same location in space. You could then add a corresponding IM that defines what this means for rectangular bodies. —SS][Added theoretical model —OO] [Added IM4 —OO]

# 6.2 Nonfunctional Requirements

Games are resource-intensive, so performance is a high priority. Other non-functional requirements that are a priority are: correctness, understandability, portability, reliability, and maintainability.

- 1. Performance: The execution time for collision detection and collision resolution shall be comparable to an existing 2D physics library on the market (e.g. Pymunk). [This is ambiguous. How would you make this unambiguous and something that can be measured? —SS] [updated "Performance", is this less abstract than it should be? —OO] [You are on the right track with your revised NFR. You want this requirement to be less abstract, so that you can make it unambiguous and verifiable. To accomplish this, we need to be specific about the existing 2D physics library and we need to be specific about the test case or test cases. My advice is to refer to your System Verification and Validation plan. Ideally you could reference specific tests and say that your requirement is to describe the percent relative difference between the execution times. You can use the same idea for correctness comparison. —SS]
- 2. Correctness: The output of simulation results shall be compared to an existing implementation like Pymunk.
- 3. Usability: Software shall be easy to learn and use. Usability shall be measured by how long it takes a user to learn how to use the library to create a small program to simulate the movement of 2 bodies over time in space. Creating a program should take no less than 30 to 60 minutes for an intermediate to experienced programmer. Please refer Usability NFR test in section 5.2.3 of System VnV Plan located at https://github.com/smiths/caseStudies/blob/gamephy\_finaldoc/CaseStudies/gamephys/docs/VnVPlan/SystVnVPlan/SystVnVPlan.pdf [ambiguous—SS] [updated usability, hope this makes sense—OO] [This is a great start. As mentioned for performance, I think you could make this usability test part of your VnV plan. You could point to the VnV plan here.—SS] [Updated usability—OO]
- 4. Understandability: Users of Tamias2D shall be able to learn the software with ease. Users shall be able to easily create a small program using the library. Creating a small program to simulate the movement of 2 bodies in space should take no less than 60 minutes. [imposing modularization is actually a design constraint. It is a good idea,

but we don't know for sure if this is required, or the best idea. Is there a way you can specify understandability without imposing a design solution? —SS] [it might be a bit hard to evaluate, also understandability might be relative to humans, can I take this out from the list of NFR? Understandability can also be measured by the level of documentation maybe, how well the software artifacts are documented? —OO] [I don't think we need to take this out. You could describe a usability experiment along the lines of what you described for usability. I don't think understandability is important enough to add a VnV test. We can keep this requirement a little ambiguous and not flesh out all of the details. —SS] [updated understandability —OO]

5. Maintainability: The development time for any of the likely changes should not exceed 10% percent of the original development time.

[ambiguous—SS] [updated "Maintainability"—OO] [Again, this is not abstract. This isn't an easy requirement to phrase. My suggestion is to characterize maintainability in terms of the likely changes. You can say that the development time for any of the likely changes should not exceed RE-DEV-PERCENT percent of the original development time. RE-DEV-PERCENT can be a symbolic constant that you define as something like 10%. This isn't an ideal requirement, but it is the best idea I have at this time. :-)—SS]

[updated NonFunctional requirements section —OO]

# 7 Likely Changes

This section lists the likely changes to be made to the physics library. [fixed sentence—OO]

LC1: The library may be expanded to deal with edge-to-edge and vertex-to-vertex collisions (A5).

LC2: The library may be expanded to include motion with damping (A6).

LC3: The library may be expanded to include joints and constraints (A7).

# 8 Unlikely Changes

[This section does not exist in blank template, should I keep it in here? —OO]

UC1: The goal of the system is to simulate the interactions of rigid bodies.

UC2: There will always be a sourse of input data external to the software.

UC3: A Cartesian coordinate system is used.

UC4: All objects are rigid bodies.

# 9 Traceability Matrices and Graphs

The purpose of traceability matrices is to provide easy references on what has to be additionally modified if a certain component is changed. Every time a component is changed, the items in the column of that component that are marked with an "X" should be modified as well. Table 3 shows the dependencies of goal statements, requirements, instance models and data constraints with each other. Table 4 shows the dependencies of theoretical models, general definitions, data definitions and instance models on the assumptions. Finally, Table 5 shows the dependencies of the theoretical models, general definitions, data definitions and instance models on each other.

[As per discussion with Dr. Smith, traceability graphs will be fixed automatically —OO] [updated traceability matrix for merged GS3 and 4 —OO]

	IM1	IM2	IM4	R1	R4	R7	Data Constraints (5.2.6)
GS1	X		X			X	
GS2		X	X			X	
R1							
R2	X	X			X		
R3			X		X		
R4							X
R5	X						
R6		X					
R7				X			
R8			X			X	

Table 3: Traceability Matrix showing the connections between Goal Statements, Requirements, Data Constraints and Instance Models

	A1	A2	A3	A4	A5	A6	A7
T1							
T2							
T3							
T??	X						
T5							
GD??							
GD1							
GD2		X	X				
GD??							
GD??							
GD??							
GD??							
DD1	X	X					
DD2	X	X				X	
DD3	X	X				X	
DD4	X	X				X	
$DD_{5}$	X	X				X	
DD6	X	X				X	
DD7	X	X				X	
DD8	X	X		X	X		
IM1	X	X				X	X
IM2	X	X		X		X	X
IM4	X	X			X	X	X
LC??							
LC1					X		
LC2						X	
LC3							X

Table 4: Traceability Matrix showing the connections between Assumptions and other items

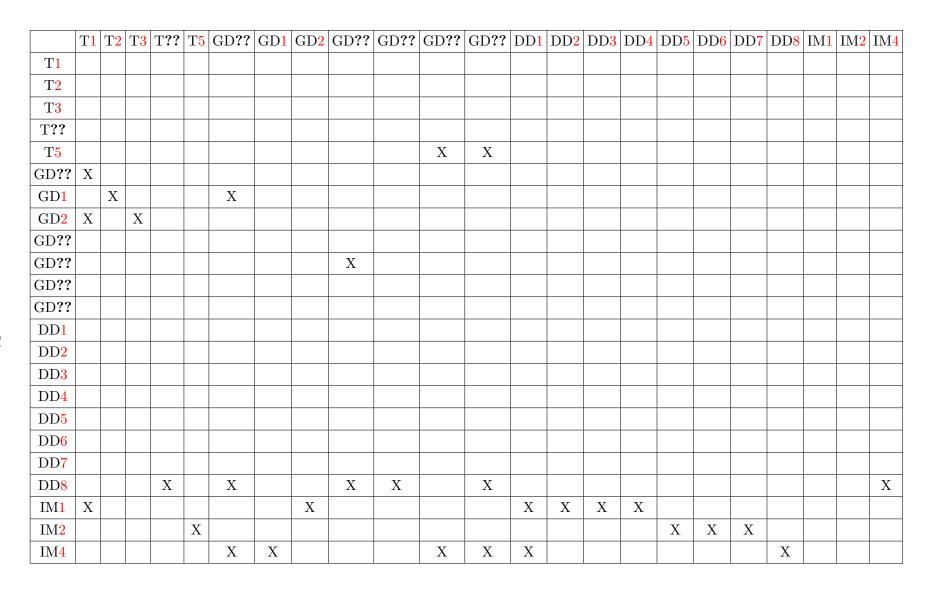


Table 5: Traceability Matrix showing the connections between items of different sections

The purpose of the traceability graphs is also to provide easy references on what has to be additionally modified if a certain component is changed. The arrows in the graphs represent dependencies. The component at the tail of an arrow is depended on by the component at the head of that arrow. Therefore, if a component is changed, the components that it points to should also be changed. Figure 3 shows the dependencies of goal statements, requirements, instance models and data constraints with each other. Figure 4 shows the dependencies of theoretical models, general definitions, data definitions and instance models on the assumptions. Finally, Figure 5 shows the dependencies of the theoretical models, general definitions, data definitions and instance models on each other. Building a tool to automatically generate the graphical representation of the matrix by scanning the label and reference can be future work.

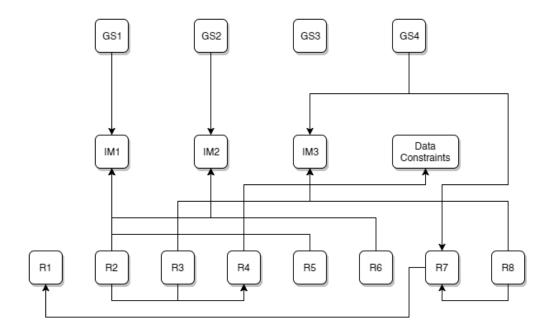


Figure 3: Traceability Graph showing the connections between Goal Statements, Requirements, Data Constraints and Instance Models

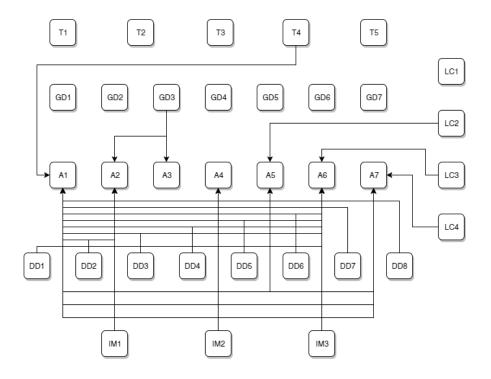


Figure 4: Traceability Graph showing the connections between Assumptions and other items

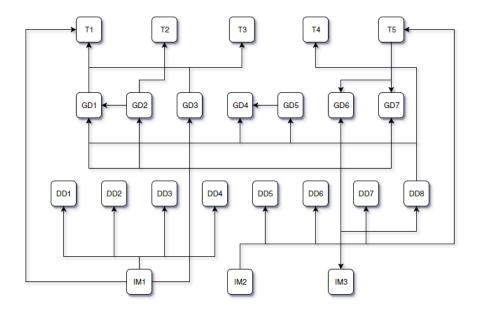


Figure 5: Traceability Graph showing the connections between items of different sections

## 10 Off the Shelf Solutions

[Add Chipmunk to this section. —SS] As mentioned in section 5.1, there already exist free open source game physics libraries. Similar 2D physics libraries are:

- Box2D http://box2d.org/
- Nape Physics Engine http://napephys.com/
- Chipmunk2D http://chipmunk-physics.net/
- Pymunk www.pymunk.org/

Free open source 3D game physics libraries include:

- Bullet http://bulletphysics.org/
- Open Dynamics Engine http://www.ode.org/
- Newton Game Dynamics http://newtondynamics.com/

[Added to Off the Shelf Solutions —OO]

# References

- [1] Nirmitha Koothoor. A document drive approach to certifying scientific computing software. Master's thesis, McMaster University, Hamilton, Ontario, Canada, 2013.
- [2] W. Spencer Smith and Lei Lai. A new requirements template for scientific computing. In J. Ralyté, P. Agerfalk, and N. Kraiem, editors, Proceedings of the First International Workshop on Situational Requirements Engineering Processes Methods, Techniques and Tools to Support Situation-Specific Requirements Engineering Processes, SREP'05, pages 107–121, Paris, France, 2005. In conjunction with 13th IEEE International Requirements Engineering Conference.

#### [References are okay —OO]

[What references are not showing? I see two cite commands in the document and two references in the generated file. This looks right to me? Not everything in the bib file should show up. The only entries in the references section should be those that were actually cited.—SS]