# Software Requirements Specification for SSP: Slope Stability Analysis Program

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# 1 Revision History

| Date | Version | Notes |
|---|---|---|
| 09/24/18 | 1.0 | Removed RFEM |
| 09/25/18 | 1.1 | Traceability matrix work |
| 09/26/18 | 1.2 | Physical System Description expanded, Non-functional requirements itemized |
| 10/01/18 | 1.3 | Various improvements throughout |
| 10/02/18 | 1.4 | Initial revision of the solution characteristics specification |
| 10/03/18 | 1.5 | Completed revision of the solution characteristics specification and other sections |
| 10/04/18 | 1.6 | Minor fixes throughout |
| 10/12/18 | 1.7 | Minor fixes based on feedback |
| 10/17/18 | 1.8 | More fixes based on feedback |
| 12/05/18 | 1.9 | Completed major revisions |
| 12/09/18 | 1.10 | Further minor updates for final submission |
| 12/18/18 | 1.11 | Fixed a broken reference |

# 2 Reference Material

This section records information for easy reference.

## 2.1 Table of Units

The unit system used throughout is SI (Système International d'Unités). In addition to the basic units, several derived units are also used. For each unit, the table lists the symbol, a description and the SI name.

| Symbol | Unit | SI |
|---|---|---|
| N | force | newton |
| m | length | meter |
| $\mathrm{Pa} = \mathrm{N\,m^{-2}}$ | pressure | pascal |
| $^\circ$ | angle | degree |

## 2.2 Table of Symbols

The table that follows summarizes the symbols used in this document along with their units.

| Symbol | Unit | Description |
|---|---|---|
| $A$ | $\mathrm{m}^2$ | area on which a force acts |
| $b$ | m | width of the base of a slice in the $x$-direction |
| $\mathit{const\_f}$ | | boolean decision on which form of $f$ the user desires: constant if true, or a half-sine if false |
| $c'$ | Pa | effective cohesion |
| $C_{\text{num},i}$ | N | expression used to calculate the numerator of the interslice normal to shear force proportionality constant |
| $C_{\text{den},i}$ | N | expression used to calculate the denominator of the interslice normal to shear force proportionality constant |
| $F$ | N | force |
| $F_x$ | N | $x$-component of force |
| $F_y$ | N | $y$-component of force |
| $f$ | | variation of the interslice normal to shear force ratio as a function of distance in the $x$-direction |
| $F_{\text{S}}$ | | factor of safety |

| | | |
|---|---|---|
| $F_S^{Min}$ | | minimum factor of safety associated with the critical slip surface |
| $G$ | $\mathrm{N\,m^{-1}}$ | interslice normal force per meter in the $z$-direction |
| $H$ | $\mathrm{N\,m^{-1}}$ | interslice water force per meter in the $z$-direction |
| $h$ | m | height in the $y$-direction from the base of a slice to the slope surface, at the $x$-direction midpoint of the slice |
| $h_z$ | m | height in the $y$-direction from the base of a slice to the center of the slice |
| $h_{z,w}$ | m | height in the $y$-direction from the base of a slice halfway to the water table |
| $i$ | | index representing a single slice |
| $K_c$ | | horizontal seismic coefficient |
| $M$ | N m | moment |
| $N$ | $\mathrm{N\,m^{-1}}$ | normal force per meter in the $z$-direction |
| $N'$ | $\mathrm{N\,m^{-1}}$ | effective normal force per meter in the $z$-direction |
| $n$ | | the total number of slices |
| $P$ | $\mathrm{N\,m^{-1}}$ | resistive shear force per meter in the $z$-direction |
| $Q$ | $\mathrm{N\,m^{-1}}$ | imposed surface load or external force per meter in the $z$-direction |
| $R$ | $\mathrm{N\,m^{-1}}$ | resistive shear force without the influence of interslice forces per meter in the $z$-direction |
| $S$ | $\mathrm{N\,m^{-1}}$ | mobilized shear force per meter in the $z$-direction |
| $T$ | $\mathrm{N\,m^{-1}}$ | mobilized shear force without the influence of interslice forces per meter in the $z$-direction |
| $U_b$ | $\mathrm{N\,m^{-1}}$ | base hydrostatic force per meter in the $z$-direction |
| $U_t$ | $\mathrm{N\,m^{-1}}$ | surface hydrostatic force per meter in the $z$-direction |
| $u$ | Pa | pore pressure from water within the soil |

| | | |
|---|---|---|
| $W$ | $\mathrm{N\,m^{-1}}$ | self-weight per meter in the $z$-direction |
| $X$ | $\mathrm{N\,m^{-1}}$ | interslice shear force per meter in the $z$-direction |
| $x$ | m | $x$-ordinate in the Cartesian coordinate system |
| $x_{\text{cs}}$ | m | $x$-ordinate of a point on the critical slip surface |
| $x_{\text{slip}}$ | m | $x$-ordinate of a point on a slip surface |
| $x_{\text{slip}}^{\text{maxExt}}$ | m | maximum potential $x$-ordinate of the exit point of a slip surface |
| $x_{\text{slip}}^{\text{maxEtr}}$ | m | maximum potential $x$-ordinate of the entry point of a slip surface |
| $x_{\text{slip}}^{\text{minExt}}$ | m | minimum potential $x$-ordinate of the exit point of a slip surface |
| $x_{\text{slip}}^{\text{minEtr}}$ | m | minimum potential $x$-ordinate of the entry point of a slip surface |
| $x_{\text{slope}}$ | m | $x$-ordinate of a point on the slope |
| $x_{\text{wt}}$ | m | $x$-ordinate of a point on the water table |
| $y$ | m | $y$-ordinate in the Cartesian coordinate system |
| $y_{\text{cs}}$ | m | $y$-ordinate of a point on the critical slip surface |
| $y_{\text{slip}}$ | m | $y$-ordinate of a point on a slip surface |
| $y_{\text{slip}}^{\text{max}}$ | m | maximum potential $y$-ordinate of a point on a slip surface |
| $y_{\text{slip}}^{\text{min}}$ | m | minimum potential $y$-ordinate of a point on a slip surface |
| $y_{\text{slope}}$ | m | $y$-ordinate of a point on the slope |
| $y_{\text{wt}}$ | m | $y$-ordinate of a point on the water table |
| $z$ | m | $z$-ordinate in the Cartesian coordinate system |
| $\alpha$ | $^\circ$ | angle between the base of a slice and the horizontal |
| $\beta$ | $^\circ$ | angle between the surface of a slice and the horizontal |
| $\gamma$ | $\mathrm{N\,m^{-3}}$ | soil dry unit weight |
| $\gamma_{\text{Sat}}$ | $\mathrm{N\,m^{-3}}$ | soil saturated unit weight |

| | | |
|---|---|---|
| $\gamma_{\mathrm{w}}$ | $\mathrm{N\,m^{-3}}$ | unit weight of water |
| $\lambda$ | | proportionality constant for the interslice normal to shear force ratio |
| $\sigma$ | Pa | total stress on the soil mass |
| $\sigma'_N$ | Pa | effective normal stress |
| $\sigma'$ | Pa | effective stress provided by the soil skeleton |
| $\tau$ | Pa | shear strength |
| $\Upsilon$ | | generic minimization function or algorithm |
| $\varphi'$ | $^\circ$ | effective angle of friction |
| $\Phi$ | | first function used to convert shear without the influence of interslice forces to shear with the influence of interslice forces |
| $\Psi$ | | second function used to convert shear without the influence of interslice forces to shear with the influence of interslice forces |
| $\omega$ | $^\circ$ | angle between the imposed surface load and the vertical |
| $\ell_{\mathrm{b}}$ | m | base length of a slice in the direction parallel to the slope of the base |
| $\ell_{\mathrm{s}}$ | m | surface length of a slice in the direction parallel to the slope of the surface |

[The two symbols $h_{\mathrm{z}}$ and $h_{\mathrm{z,w}}$ were originally $z$ and $z_w$. You commented "Why use z?". The reason is because they are z in the literature. However, since z is also used for coordinates, I changed these symbols and reserved z for coordinates —BM]

## 2.3 Abbreviations and Acronyms

| Symbol | Description |
|---|---|
| 2D | Two-Dimensional |
| A | Assumption |
| DD | Data Definition |
| GD | General Definition |
| GS | Goal Statement |
| IM | Instance Model |
| LC | Likely Change |
| NFR | Non-Functional Requirement |
| PS | Physical System Description |
| R | Requirement |
| SRS | Software Requirements Specification |
| SSP | Slope Stability analysis Program |
| T | Theoretical Model |
| TU | Typical Uncertainty |
| UC | Unlikely Change |

# Contents

# 3 Introduction

A slope of geological mass, composed of soil and rock and sometimes water, is subject to the influence of gravity on the mass. This can cause instability in the form of soil or rock movement. The effects of soil or rock movement can range from inconvenient to seriously hazardous, resulting in significant life and economic losses. Slope stability is of interest both when analysing natural slopes, and when designing an excavated slope. Slope stability analysis is the assessment of the safety of a slope, identifying the surface most likely to experience slip and an index of its relative stability known as the factor of safety.

The following section provides an overview of the Software Requirements Specification (SRS) for a slope stability analysis problem. The developed program will be referred to as the Slope Stability analysis Program (SSP). This section explains the purpose of this document, the scope of the system, the characteristics of the intended readers, and the organization of the document.

## 3.1 Purpose of Document

The primary purpose of this document is to record the requirements of SSP and the models that will be used to meet those requirements. Goals, assumptions, theoretical models, definitions, and other model derivation information are specified, allowing the reader to fully understand and verify the purpose and scientific basis of SSP. With the exception of system constraints in Section 4.3, this SRS will remain abstract, describing *what* problem is being solved, but not *how* to solve it.

This document will be used as a starting point for subsequent development phases, including writing the design specification and the software verification and validation plan. The design document will show how the requirements are to be realized, including decisions on the numerical algorithms and programming environment. The verification and validation plan will show the steps that will be used to increase confidence in the software documentation and the implementation. Although the SRS fits in a series of documents that follow the so-called waterfall model, the actual development process is not constrained in any way. Even when the waterfall model is not followed, as Parnas and Clements (February 1986) point out, the most logical way to present the documentation is still to "fake" a rational design process.

## 3.2 Scope of Requirements

The scope of the requirements is stability analysis of a 2-Dimensional (2D) soil mass, composed of a single homogeneous layer with constant material properties. The soil mass is assumed to extend infinitely in the third dimension. The analysis will be at an instant in time; factors that may change the slope properties over time will not be considered.

## 3.3 Characteristics of Intended Reader

Reviewers of this documentation should have an understanding of undergraduate Level 4 physics and should have completed a second year or higher level undergraduate course in solid mechanics. A course specifically in soil mechanics would be an asset. The users of SSP can have a lower level of expertise, as explained in Section 4.2.

### 3.4 Organization of Document

The organization of this document follows the template for an SRS for scientific computing software proposed by Koothoor (2013) and Smith and Lai (2005). The presentation follows the standard pattern of presenting goals, theories, definitions, and assumptions. For readers that would like a more bottom up approach, they can start reading the instance models in Section 5.2.5 and trace back to find any additional information they require. The goal statements (Section 5.1.3) are refined to the theoretical models, and the theoretical models (Section 5.2.2) to the instance models (Section 5.2.5). The instance models provide the set of algebraic equations that must be solved.

## 4 General System Description

This section provides general information about the system. It identifies the interfaces between the system and its environment, describes the user characteristics, and lists the system constraints.

### 4.1 System Context

Figure 1 shows the system context. A circle represents an external entity outside the software. A rectangle represents the software system itself (SSP). Arrows are used to show the data flow between the system and its environment.

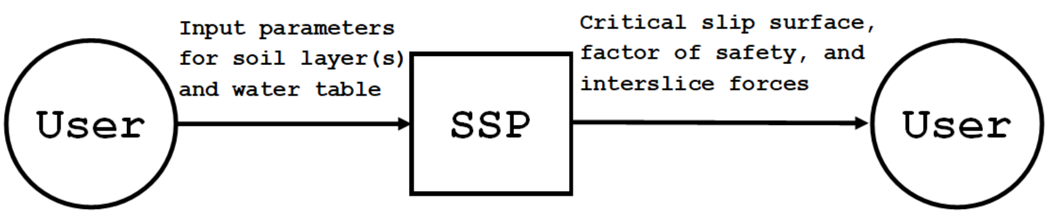


Figure 1: System context for SSP

The responsibilities of the user and the system are as follows:

- User Responsibilities:
  - Provide the input data related to the soil layer(s) and water table (if applicable), ensuring conformation to input data format required by SSP
  - Ensure that consistent units are used for input variables
  - Ensure required software assumptions (Section 5.2.1) are appropriate for the problem to which the user is applying the software
- SSP Responsibilities:
  - Detect data type mismatch, such as a string of characters input instead of a floating point number
  - Verify that the inputs satisfy the required physical constraints and other data constraints (Section 5.2.6)
  - Identify the critical slip surface within the possible input range

- Find the factor of safety for the slope
- Find the interslice normal and shear forces along the critical slip surface

## 4.2 User Characteristics

The end user of SSP should have an understanding of undergraduate Level 1 Calculus and Physics, and be familiar with soil and material properties, specifically cohesion, effective angle of friction, and unit weight.

## 4.3 System Constraints

The Morgenstern-Price method (Morgenstern and Price, January 1965), which involves dividing the slope into vertical slices, will be used to derive the equations for analysing the slope.

# 5 Specific System Description

This section first presents the problem description, which gives a high-level view of the problem to be solved. This is followed by the solution characteristics specification, which presents the assumptions, theories, definitions and finally the instance models.

## 5.1 Problem Description

SSP is a computer program developed to evaluate the factors of safety for a slope's slip surfaces and identify the critical slip surface of the slope, as well as the interslice normal and shear forces along the critical slip surface. It is intended to be used as an educational tool for introducing slope stability issues, and to facilitate the analysis and design of a safe slope.

### 5.1.1 Terminology and Definitions

This subsection provides a list of terms that are used in the subsequent sections and their meaning, with the purpose of reducing ambiguity and making it easier to correctly understand the requirements.

- *Factor of safety:* The global stability metric of a slip surface of a slope, defined as the ratio of resistive shear force to mobile shear force
- *Slip surface:* A surface within a slope that has the potential to fail or displace due to load or other forces.
- *Critical slip surface:* Slip surface of the slope that has the lowest factor of safety, and is therefore most likely to experience failure.
- *Water table:* The upper boundary of a saturated zone in the ground.
- *Stress:* Force applied over an area.
- *Strain:* A measure of deformation of a body or plane under stress.
- *Normal force:* A force applied perpendicular to the plane of the material.

- *Shear force:* A force applied parallel to the plane of the material.
- *Resistive shear force:* Shear force in the direction opposite to the direction of potential motion, thus hindering motion along the plane.
- *Mobile shear force:* Shear force in the direction of potential motion, thus encouraging motion along the plane.
- *Effective forces and stresses*: The normal force or stress carried by the soil skeleton. The total normal force or stress is composed of the effective force or stress and the force or stress exerted by water.
- *Cohesion:* An attractive force between adjacent particles that holds the matter together.
- *Isotropic:* A condition where the value of a property is independent of the direction in which it is measured.
- *Plane strain:* A condition where the resultant stresses in one of the directions of a 3-dimensional body can be approximated as zero. Results when the length of one dimension of the body dominates the others, to the point where it can be assumed as infinite. Stresses in the direction of the dominant dimension can be approximated as zero.

### 5.1.2 Physical System Description

The Physical System (PS) of SSP, as shown in Figure 2, includes the following elements:

PS1: A slope comprised of one layer of soil.

PS2: A water table, which may or may not exist.

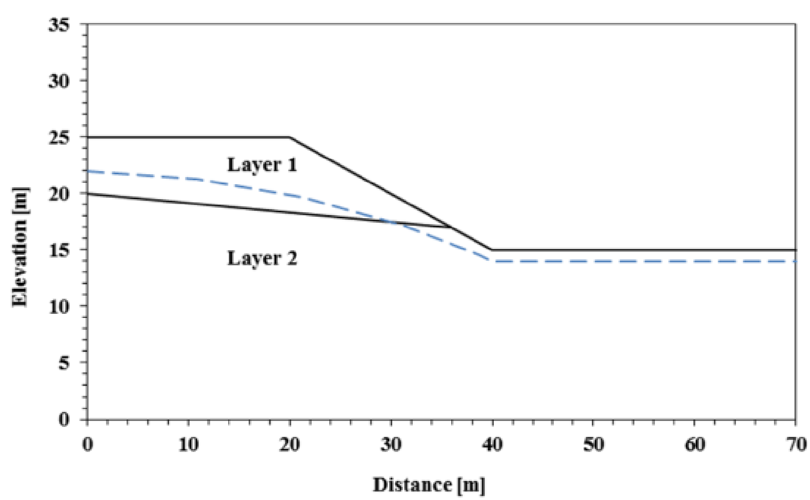


Figure 2: An example slope for analysis by SSP, where the dashed line represents the water table

Morgenstern-Price (Morgenstern and Price, January 1965) analysis of the slope involves representing the slope as a series of vertical slices. As shown in Figure 3, the index $i$ is used to denote a value for a single slice, and an interslice value at a given index $i$ refers to the value between slice $i$ and adjacent slice $i + 1$.

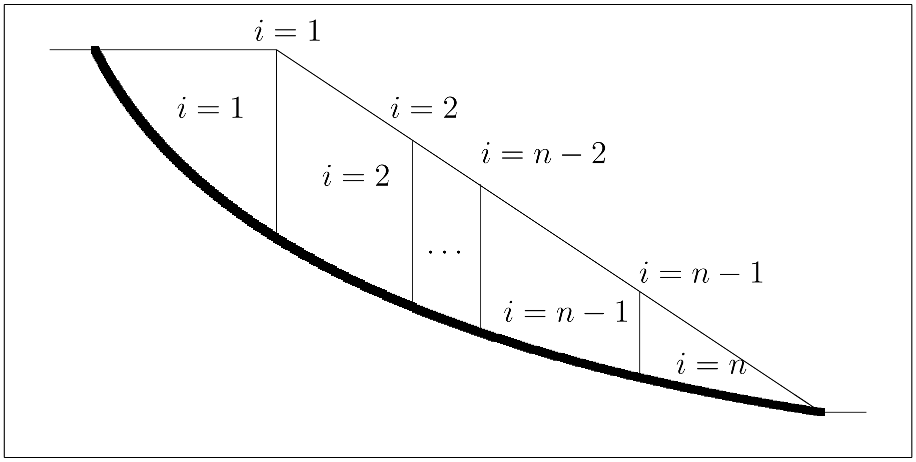


Figure 3: Index convention for slice and interslice values

A free body diagram of the forces acting on a slice is displayed in Figure 4. The specific forces and symbols will be discussed in detail in Sections 5.2.3 and 5.2.4.

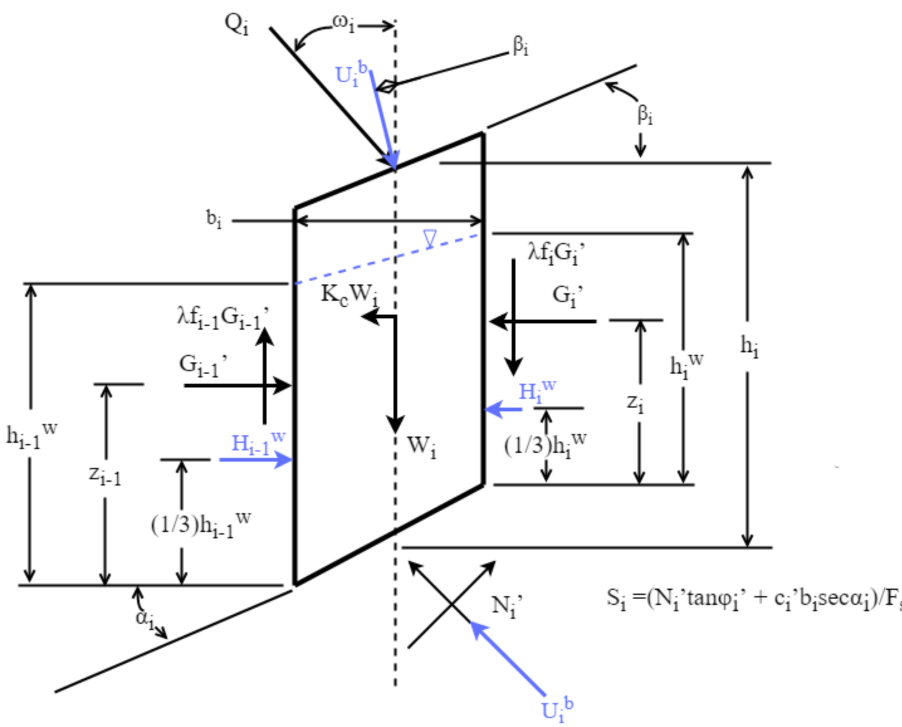


Figure 4: Free body diagram of forces acting on a slice

### 5.1.3 Goal statements

Given the shape of a soil mass, the location of a water table, and the material properties of the soil, the goal statements are to:

GS1: Identify the critical slip surface and the corresponding factor of safety.

GS2: Determine the interslice normal force between each pair of vertical slices of the slope.

GS3: Determine the interslice shear force between each pair of vertical slices of the slope.

## 5.2 Solution Characteristics Specification

The instance models that govern SSP are presented in Section 5.2.5. The information to understand the meaning of the instance models and their derivation is also presented, so that the instance models can be verified.

### 5.2.1 Assumptions

This section simplifies the original problem and helps in developing the theoretical models by filling in the missing information for the physical system. The numbers given in the square brackets refer to the theoretical model [T], general definition [GD], data definition [DD], instance model [IM], or likely change [LC], in which the respective assumption is used.

A1: The slip surface is concave with respect to the slope surface. The $(x_{\text{slip}}, y_{\text{slip}})$ coordinates of a slip surface follow a concave up function. [IM4]

A2: The factor of safety is assumed to be constant across the entire slip surface. [GD4]

A3: The soil mass is homogeneous, with consistent soil properties throughout. [GD3, DD1 LC1]

A4: The soil properties are independent of dry or saturated conditions, with the exception of unit weight. [GD3]

A5: The soil mass is treated as if the effective cohesion and effective angle of friction are isotropic properties. [GD3]

A6: Following the assumption of Morgenstern and Price (January 1965), interslice normal and shear forces have a proportional relationship, depending on a proportionality constant ($\lambda$) and a function ($f$) describing variation depending on $x$ position. [GD8, IM1, IM2]

A7: The slope and slip surface extends far into and out of the geometry ($z$-coordinate). This implies plane strain conditions, making 2D analysis appropriate. [T2, GD3, GD5]

A8: The effective normal stress is large enough that the resistive shear to effective normal stress relationship can be approximated as a linear relationship. [T3]

A9: The surface and base of a slice are approximated as straight lines [DD1, DD2, DD3, DD5, DD6, DD8, DD9, DD10].

A10: The interslice forces at the $0$th and $n$th interslice interfaces are zero. [IM1, IM2, IM3].

A11: There is no seismic force acting on the slope. [IM1, IM2, LC2]

A12: There is no imposed surface load, and therefore no external force, acting on the slope. [IM1, IM2, LC3]

### 5.2.2 Theoretical Models

This section focuses on the general equations and laws that SSP is based on.

| Number | T1 |
|---|---|
| Label | **Factor of Safety** |
| Equation | $F_{\text{S}} = \frac{P}{S}$ |
| Description | $F_{\text{S}}$ is the factor of safety, or stability metric of the slope.<br>$S$ is the mobile shear force per meter in the $z$-direction ($\text{N}\,\text{m}^{-1}$).<br>$P$ is the resistive shear force per meter in the $z$-direction ($\text{N}\,\text{m}^{-1}$). |
| Source | Fredlund and J.Krahn (4 April 1977) |
| Ref. By | GD4 |

| Number | T2 |
|---|---|
| Label | **Static Equilibrium** |
| Equation | $\sum F_{\text{x}} = \sum F_{\text{y}} = \sum M = 0$ |
| Description | For a body in static equilibrium the net forces and net moments acting on the body will cancel out. This equation assumes a 2D space (A7).<br>$F_x$ is the x-component of the net force (N).<br>$F_y$ is the y-component of the net force (N).<br>$M$ is the net moment ($\text{N}\,\text{m}$). |
| Source | Fredlund and J.Krahn (4 April 1977) |
| Ref. By | GD1, GD2, GD9 |

| | |
|---|---|
| Number | T3 |
| Label | **Mohr-Coulomb Shear Strength** |
| Equation | $\tau = \sigma'_N \cdot \tan(\varphi') + c'$ |
| Description | The $\tau$ versus $\sigma'_N$ relationship is not truly linear, but assuming the effective normal force is strong enough, it can be approximated with a linear fit (A8), where the cohesion $c'$ represents the $\tau$ intercept of the fitted line.<br>$\tau$ is the shear strength (Pa).<br>$\sigma'_N$ is the effective normal stress (Pa).<br>$\varphi'$ is the effective angle of friction (°).<br>$c'$ is the effective cohesion (Pa). |
| Source | Fredlund and J.Krahn (4 April 1977) |
| Ref. By | GD3 |

| | |
|---|---|
| Number | T4 |
| Label | **Effective Stress** |
| Equation | $\sigma' = \sigma - u$ |
| Description | $\sigma$ is the total stress on the soil mass (Pa), defined in DD**??**.<br>$\sigma'$ is the effective stress provided by the soil skeleton (Pa).<br>$u$ is the pore pressure from water within the soil (Pa). |
| Source | Fredlund and J.Krahn (4 April 1977) |
| Ref. By | GD5 |

### 5.2.3 General Definitions

This section collects the laws and equations that will be used to build the instance models.

| Number | GD1 |
|---|---|
| Label | **Normal Force Equilibrium** |
| SI Units | $\mathrm{N\,m^{-1}}$ |
| Equation | $N_i = [W_i - X_{i-1} + X_i + U_{\mathrm{t},i} \cos(\beta_i) + Q_i \cos(\omega_i)] \cos(\alpha_i) + [-K_\mathrm{c}\, W_i - G_i + G_{i-1} - H_i + H_{i-1} + U_{\mathrm{t},i} \sin(\beta_i) + Q_i \sin(\omega_i)] \sin(\alpha_i)$ |
| Description | This equation satisfies T2 in the normal direction. Force equilibrium is derived from the free body diagram of Figure 4 in Section 5.1.2.<br>$i$ is the index representing a single slice.<br>$N$ is the normal force per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$).<br>$W$ is the weight per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$), defined in DD1.<br>$X$ is the interslice shear force per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$).<br>$U_\mathrm{t}$ is the surface hydrostatic force per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$), defined in DD3.<br>$\beta$ is the angle between the surface of a slice and the horizontal (°), defined in DD6.<br>$Q$ is the external force per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$).<br>$\omega$ is the angle between the imposed surface load acting into the surface and the vertical (°).<br>$\alpha$ is the angle between the base of a slice and the horizontal (°), defined in DD5.<br>$K_\mathrm{c}$ is the seismic coefficient.<br>$G$ is the interslice normal force per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$).<br>$H$ is the interslice water force per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$), defined in DD4. |
| Source | Zhu et al. (19 February 2005) |
| Ref. By | IM1 |

| Number | GD2 |
|---|---|
| Label | **Base Shear Force Equilibrium** |
| SI Units | $\mathrm{N\,m^{-1}}$ |
| Equation | $S_i = \begin{array}{l}[W_i - X_{i-1} + X_i + U_{\mathrm{t},i} \cos(\beta_i) + Q_i \cos(\omega_i)] \sin(\alpha_i) \\ -[-K_{\mathrm{c}} W_i - G_i + G_{i-1} - H_i + H_{i-1} + U_{\mathrm{t},i} \sin(\beta_i) + Q_i \cos(\omega_i)] \cos(\alpha_i)\end{array}$ |
| Description | This equation satisfies T2 in the shear direction. Force equilibrium is derived from the free body diagram of Figure 4 in Section 5.1.2.<br><br>$i$ is the index representing a single slice.<br><br>$S$ is the mobile shear force per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$).<br><br>$W$ is the weight per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$), defined in DD1.<br><br>$X$ is the interslice shear force per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$).<br><br>$U_{\mathrm{t}}$ is the surface hydrostatic force per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$), defined in DD3.<br><br>$\beta$ is the angle between the surface of a slice and the horizontal ($^\circ$), defined in DD6.<br><br>$Q$ is the external force per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$).<br><br>$\omega$ is the angle between the imposed surface load acting into the surface and the vertical ($^\circ$).<br><br>$\alpha$ is the angle between the base of a slice and the horizontal ($^\circ$), defined in DD5.<br><br>$K_{\mathrm{c}}$ is the seismic coefficient.<br><br>$G$ is the interslice normal force per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$).<br><br>$H$ is the interslice water force per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$), defined in DD4. |
| Source | Zhu et al. (19 February 2005) |
| Ref. By | IM1 |

| | |
|---|---|
| Number | GD3 |
| Label | **Resistive Shear Force** |
| SI Units | $\mathrm{N\,m^{-1}}$ |
| Equation | $P_i = N'_i \cdot \tan(\varphi') + c' \cdot \ell_{\mathrm{b},i}$ |
| Description | Derived by substituting DD11 into the Mohr-Coulomb resistive shear strength, T3, and multiplying both sides of the equation by the area of the slice in the shear-$z$ plane. Since the slope is assumed to extend infinitely in the $z$-direction (A7), the resulting forces are expressed per meter in the $z$-direction. The effective angle of friction $\varphi'$ and the effective cohesion $c'$ are not indexed by $i$ because they are assumed to be isotropic (A5) and the soil is assumed to be homogeneous, with constant soil properties throughout (A3).<br><br>$i$ is the index representing a single slice.<br><br>$P$ is the resistive shear force per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$).<br><br>$N'$ is the effective normal force per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$).<br><br>$\varphi'$ is the effective angle of friction ($^\circ$).<br><br>$c'$ is the effective cohesion (Pa).<br><br>$\ell_{\mathrm{b}}$ is the width of the base of a slice in the $x$ direction (m), defined in DD8. |
| Source | Zhu et al. (19 February 2005) |
| Ref. By | GD4 |

| | |
|---|---|
| Number | GD4 |
| Label | **Mobile Shear Force** |
| SI Units | $\mathrm{N\,m^{-1}}$ |
| Equation | $S_i = \frac{P_i}{F_\mathrm{S}} = \frac{N_i' \cdot \tan(\varphi') + c' \cdot \ell_{\mathrm{b},i}}{F_\mathrm{S}}$ |
| Description | Mobile shear force as derived from the definition of the factor of safety in T1, and the definition of $P$ in GD3. The factor of safety $F_\mathrm{S}$ is not indexed by $i$ because it is assumed to be constant for the entire slip surface (A2).<br><br>$i$ is the index representing a single slice.<br><br>$S$ is the mobile shear force per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$).<br><br>$P$ is the resistive shear force per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$).<br><br>$N'$ is the effective normal force per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$).<br><br>$\varphi'$ is the effective angle of friction ($°$).<br><br>$c'$ is the effective cohesion (Pa).<br><br>$\ell_\mathrm{b}$ is the width of the base of a slice in the $x$ direction (m), defined in DD8.<br><br>$F_\mathrm{S}$ is the factor of safety. |
| Source | Zhu et al. (19 February 2005) |
| Ref. By | IM1 |

| Number | GD5 |
|---|---|
| Label | **Effective Normal Force** |
| SI Units | $\mathrm{N\,m^{-1}}$ |
| Equation | $N'_i = N_i - U_{\mathrm{b},i}$ |
| Description | Derived by substituting DD11 into T4 and multiplying both sides of the equation by the area of the slice in the shear-$z$ plane. Since the slope is assumed to extend infinitely in the $z$-direction (A7), the resulting forces are expressed per meter in the $z$-direction.<br><br>$i$ is the index representing a single slice.<br><br>$N'$ is the effective normal force per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$).<br><br>$N$ is the normal force per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$).<br><br>$U_{\mathrm{b}}$ is the base hydrostatic force per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$), defined in DD2. |
| Source | Zhu et al. (19 February 2005) |
| Ref. By | IM1 |

| | |
|---|---|
| Number | GD6 |
| Label | **Resistive Shear, Without Interslice Normal and Shear Forces** |
| Equation | $R_i = \left(\begin{array}{l}[W_i + U_{\text{t},i} \cos(\beta_i)] \cos(\alpha_i) \\ + [-H_i + H_{i-1} + U_{\text{t},i} \sin(\beta_i)] \sin(\alpha_i) - U_{\text{b,i}}\end{array}\right) \cdot (\tan(\varphi') + c' \cdot \ell_{\text{b},i})$ |
| Description | This equation for $R$ arises as part of the derivation for IM1, so that derivation should be consulted for information relating to the derivation of $R$.<br><br>$i$ is the index representing a single slice.<br><br>$R$ is the resistive shear force without the influence of interslice forces per meter in the $z$-direction ($\text{N}\,\text{m}^{-1}$).<br><br>$W$ is the weight per meter in the $z$-direction ($\text{N}\,\text{m}^{-1}$), defined in DD1.<br><br>$U_{\text{t}}$ is the surface hydrostatic force per meter in the $z$-direction ($\text{N}\,\text{m}^{-1}$), defined in DD3.<br><br>$\beta$ is the angle between the surface of a slice and the horizontal ($^\circ$), defined in DD6.<br><br>$\alpha$ is the angle between the base of a slice and the horizontal ($^\circ$), defined in DD5.<br><br>$H$ is the interslice water force per meter in the $z$-direction ($\text{N}\,\text{m}^{-1}$), defined in DD4.<br><br>$U_{\text{b}}$ is the base hydrostatic force per meter in the $z$-direction ($\text{N}\,\text{m}^{-1}$), defined in DD2.<br><br>$\varphi'$ is the effective angle of friction ($^\circ$).<br><br>$c'$ is the effective cohesion (Pa).<br><br>$\ell_{\text{b}}$ is the base length of a slice in the direction parallel to the slope of the base (m), defined in DD8. |
| Sources | Zhu et al. (19 February 2005), Karchewski et al. (2012) |
| Ref. By | IM1, IM3 |

[You had a comment that the symbol for R should be $R'$ because it is an effective force value. However, even though the effective normal force is used in this equation, does that mean that R itself is effective? Our two primary sources (Brandon's paper and the Zhu et al. paper) do not refer to R or T as effective. If we do decide to define R as the effective resistive shear, then we should also define T as effective mobile shear, right? —BM]

| Number | GD7 |
|---|---|
| Label | **Mobile Shear, Without Interslice Normal and Shear Forces** |
| Equation | $T_i = (W_i + U_{t,i} \cos(\beta_i)) \sin(\alpha_i) - (-H_i + H_{i-1} + U_{t,i} \sin(\beta_i)) \cos(\alpha_i)$ |
| Description | This equation for $T$ arises as part of the derivation for IM1, so that derivation should be consulted for information relating to the derivation of $T$.<br><br>$i$ is the index representing a single slice.<br><br>$T$ is the mobilized shear force without the influence of interslice forces per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$).<br><br>$W$ is the weight per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$), defined in DD1.<br><br>$U_t$ is the surface hydrostatic force per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$), defined in DD3.<br><br>$\beta$ is the angle between the surface of a slice and the horizontal (°), defined in DD6.<br><br>$\alpha$ is the angle between the base of a slice and the horizontal (°), defined in DD5.<br><br>$H$ is the interslice water force per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$), defined in DD4. |
| Sources | Zhu et al. (19 February 2005), Karchewski et al. (2012) |
| Ref. By | IM1, IM3 |

| Number | GD8 |
|---|---|
| Label | **Interslice Normal and Shear Force Proportionality** |
| Equation | $X = \lambda \cdot f \cdot G$ |
| Description | Mathematical representation of the primary assumption for the Morgenstern-Price method (A6).<br><br>$X$ is the interslice shear force per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$).<br><br>$G$ is the interslice normal force per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$).<br><br>$\lambda$ is the proportionality constant.<br><br>$f$ is the variation of the interslice normal to shear force ratio as a function of distance in the $x$-direction, defined in DD12. |
| Source | Zhu et al. (19 February 2005) |
| Ref. By | IM1, IM2 |

| Number | GD9 |
|---|---|
| Label | **Moment Equilibrium** |
| Equation | $$0 = \begin{aligned} & -G_i\left[h_{\mathrm{z},i} + \frac{b_i}{2}\tan(\alpha_i)\right] + G_{i-1}\left[h_{\mathrm{z},i-1} - \frac{b_i}{2}\tan(\alpha_i)\right] - H_i\left[h_{\mathrm{z,w},i} + \frac{b_i}{2}\tan(\alpha_i)\right] \\ & +H_{i-1}\left[h_{\mathrm{z,w},i-1} - \frac{b_i}{2}\tan(\alpha_i)\right] + \frac{b_i}{2}(X_i + X_{i-1}) - K_\mathrm{c} W_i \frac{h_i}{2} + U_{\mathrm{t},i}\sin(\beta_i)\,h_i \\ & +Q_i\sin(\omega_i)h_i \end{aligned}$$ |
| Description | This equation satisfies T2 for the net moment. Force equilibrium is derived from the free body diagram of Figure 4 in Section 5.1.2.<br><br>$i$ is the index representing a single slice.<br><br>$G$ is the interslice normal force per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$).<br><br>$h_\mathrm{z}$ is the height in the $y$-direction from the base of a slice to the center of the slice (m).<br><br>$b$ is the width of the base of a slice in the $x$-direction (m), defined in DD7.<br><br>$\alpha$ is the angle between the base of a slice and the horizontal (°), defined in DD5.<br><br>$H$ is the interslice water force per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$), defined in DD4.<br><br>$h_\mathrm{z,w}$ is the height in the $y$-direction from the base of a slice halfway to the water table (m).<br><br>$X$ is the interslice shear force per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$).<br><br>$K_\mathrm{c}$ is the seismic coefficient.<br><br>$W$ is the weight per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$), defined in DD1.<br><br>$h$ is the height in the $y$-direction from the base of a slice to the slope surface, at the $x$-direction midpoint of the slice (m), defined in DD10.<br><br>$U_\mathrm{t}$ is the surface hydrostatic force per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$), defined in DD3.<br><br>$\beta$ is the angle between the surface of a slice and the horizontal (°), defined in DD6.<br><br>$Q$ is the external force per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$).<br><br>$\omega$ is the angle between the imposed surface load acting into the surface and the vertical (°). |
| Source | Zhu et al. (19 February 2005) |
| Ref. By | IM2 |

### 5.2.4 Data Definition

This section collects and defines all the data needed to support the general definitions of 5.2.3 or build the instance models of 5.2.5. The dimension of each quantity is also given.

| Number | DD1 |
|---|---|
| Label | **Weight** |
| Symbol | $W$ |
| SI Units | $\mathrm{N\,m^{-1}}$ |
| Equation | $W_{i,1} = b_i \begin{cases} \left(y_{\text{slope},i} - y_{\text{slip},i}\right)\gamma_{\text{Sat}} & y_{\text{wt},i} \geq y_{\text{slope},i} \\ \left(y_{\text{slope},i} - y_{\text{wt},i}\right)\gamma + \left(y_{\text{wt},i} - y_{\text{slip},i}\right)\gamma_{\text{Sat}} & y_{\text{slope},i} > y_{\text{wt},i} > y_{\text{slip},i} \\ \left(y_{\text{slope},i} - y_{\text{slip},i}\right)\gamma & y_{\text{wt},i} \leq y_{\text{slip},i} \end{cases}$ <br> $W_{i,2} = b_i \begin{cases} \left(y_{\text{slope},i-1} - y_{\text{slip},i-1}\right)\gamma_{\text{Sat}} & y_{\text{wt},i-1} \geq y_{\text{slope},i-1} \\ \left(\begin{array}{l} \left(y_{\text{slope},i-1} - y_{\text{wt},i-1}\right)\gamma \\ + \left(y_{\text{wt},i-1} - y_{\text{slip},i-1}\right)\gamma_{\text{Sat}} \end{array}\right) & y_{\text{slope},i-1} > y_{\text{wt},i-1} > y_{\text{slip},i-1} \\ \left(y_{\text{slope},i-1} - y_{\text{slip},i-1}\right)\gamma & y_{\text{wt},i-1} \leq y_{\text{slip},i-1} \end{cases}$ <br> $W_i = 0.5(W_{i,1} + W_{i,2})$ |
| Description | This equation is based on the assumption that the surface and base of a slice are straight lines (A9). The soil dry unit weight $\gamma$ and the soil saturated unit weight $\gamma_{\text{Sat}}$ are not indexed by $i$ because the soil is assumed to be homogeneous, with constant soil properties throughout (A3). <br><br> $i$ is the index representing a single slice. <br><br> $W$ is the weight per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$). <br><br> $b$ is the width of the base of a slice in the $x$-direction (m), defined in DD7. <br><br> $y_{\text{slope}}$ is the $y$-ordinate of a point on the slope (m). <br><br> $y_{\text{slip}}$ is the $y$-ordinate of a point on a slip surface (m). <br><br> $\gamma_{\text{Sat}}$ is the soil saturated unit weight ($\mathrm{N\,m^{-3}}$). <br><br> $y_{\text{wt}}$ is the $y$-ordinate of a point on the water table (m). <br><br> $\gamma$ is the soil dry unit weight ($\mathrm{N\,m^{-3}}$). |
| Sources | Fredlund and J.Krahn (4 April 1977) |
| Ref. By | GD1, GD2, GD6, GD7, GD9 |

| Number | DD2 |
|---|---|
| Label | **Base Water Force** |
| Symbol | $U_{\text{b}}$ |
| SI Units | $\text{N}\,\text{m}^{-1}$ |
| Equation | $U_{\text{b},i,1} = \ell_{\text{b},i} \begin{cases} (y_{\text{wt},i} - y_{\text{slip},i})\,\gamma_{\text{w}} & y_{\text{wt},i} > y_{\text{slip},i} \\ 0 & y_{\text{wt},i} \leq y_{\text{slip},i} \end{cases}$ <br> $U_{\text{b},i,2} = \ell_{\text{b},i} \begin{cases} (y_{\text{wt},i-1} - y_{\text{slip},i-1})\,\gamma_{\text{w}} & y_{\text{wt},i-1} > y_{\text{slip},i-1} \\ 0 & y_{\text{wt},i-1} \leq y_{\text{slip},i-1} \end{cases}$ <br> $U_{\text{b},i} = 0.5(U_{\text{b},i,1} + U_{\text{b},i,2})$ |
| Description | This equation is based on the assumption that the base of a slice is a straight line (A9). <br> $i$ is the index representing a single slice. <br> $U_{\text{b}}$ is the base hydrostatic force per meter in the $z$-direction ($\text{N}\,\text{m}^{-1}$). <br> $\ell_{\text{b}}$ is the base length of a slice in the direction parallel to the slope of the base (m), defined in DD8. <br> $y_{\text{wt}}$ is the $y$-ordinate of a point on the water table (m). <br> $y_{\text{slip}}$ is the $y$-ordinate of a point on a slip surface (m). <br> $\gamma_{\text{w}}$ is the unit weight of water ($\text{N}\,\text{m}^{-3}$). |
| Sources | Fredlund and J.Krahn (4 April 1977) |
| Ref. By | GD5, GD6 |

| | |
|---|---|
| Number | DD3 |
| Label | **Surface Hydrostatic Force** |
| Symbol | $U_{\text{t}}$ |
| SI Units | $\text{N}\,\text{m}^{-1}$ |
| Equation | $U_{\text{t},i,1} = \ell_{\text{s},i} \begin{cases} (y_{\text{wt},i} - y_{\text{slope},i})\,\gamma_{\text{w}} & y_{\text{wt},i} > y_{\text{slope},i} \\ 0 & y_{\text{wt},i} \leq y_{\text{slope},i} \end{cases}$ <br> $U_{\text{t},i,2} = \ell_{\text{s},i} \begin{cases} (y_{\text{wt},i-1} - y_{\text{slope},i-1})\,\gamma_{\text{w}} & y_{\text{wt},i-1} > y_{\text{slope},i-1} \\ 0 & y_{\text{wt},i-1} \leq y_{\text{slope},i-1} \end{cases}$ <br> $U_{\text{t},i} = 0.5(U_{\text{t},i,1} + U_{\text{t},i,2})$ |
| Description | This equation is based on the assumption that the surface of a slice is a straight line (A9). <br> $i$ is the index representing a single slice. <br> $U_{\text{t}}$ is the surface hydrostatic force per meter in the $z$-direction ($\text{N}\,\text{m}^{-1}$). <br> $\ell_{\text{s}}$ is the surface length of a slice in the direction parallel to the slope of the surface (m), defined in DD9. <br> $y_{\text{wt}}$ is the $y$-ordinate of a point on the water table (m). <br> $y_{\text{slope}}$ is the $y$-ordinate of a point on the slope (m). <br> $\gamma_{\text{w}}$ is the unit weight of water ($\text{N}\,\text{m}^{-3}$). |
| Sources | Fredlund and J.Krahn (4 April 1977) |
| Ref. By | GD1, GD2, GD6, GD7, GD9, IM2 |

[I think DD2 and DD3 should be multiplied by $b$, not by $\ell$, however this matches the original code by Brandon so I'm keeping it for now. But do you agree that it makes no sense to multiply by $\ell$ here and not $b$? —BM]

| Number | DD4 |
| --- | --- |
| Label | **Interslice Water Force** |
| Symbol | $H$ |
| SI Units | $\mathrm{N\,m^{-1}}$ |
| Equation | $H_i = \begin{cases} \frac{\left[y_{\text{slope},i} - y_{\text{slip},i}\right]^2}{2}\gamma_{\text{w}} + \left[y_{\text{wt},i} - y_{\text{slope},i}\right]^2 \gamma_{\text{w}} & y_{\text{wt},i} \geq y_{\text{slope},i} \\ \frac{\left[y_{\text{wt},i} - y_{\text{slip},i}\right]^2}{2}\gamma_{\text{w}} & y_{\text{slope},i} > y_{\text{wt},i} > y_{\text{slip},i} \\ 0 & y_{\text{wt},i} \leq y_{\text{slip},i} \end{cases}$ |
| Description | $i$ is the index representing a single slice.<br>$H$ is the interslice water force per meter in the $z$-direction ($\mathrm{N\,m^{-1}}$).<br>$y_{\text{slope}}$ is the $y$-ordinate of a point on the slope (m).<br>$y_{\text{slip}}$ is the $y$-ordinate of a point on a slip surface (m).<br>$\gamma_{\text{w}}$ is the unit weight of water. ($\mathrm{N\,m^{-3}}$).<br>$y_{\text{wt}}$ is the $y$-ordinate of a point on the water table (m). |
| Sources | Fredlund and J.Krahn (4 April 1977) |
| Ref. By | GD1, GD2, GD6, GD7, GD9, IM2 |

| Number | DD5 |
| --- | --- |
| Label | **Base Angle** |
| Symbol | $\alpha$ |
| SI Units | $^\circ$ |
| Equation | $\alpha_i = \arctan\left(\frac{y_{\text{slip},i} - y_{\text{slip},i-1}}{x_{\text{slip},i} - x_{\text{slip},i-1}}\right)$ |
| Description | This equation is based on the assumption that the base of a slice is a straight line (A9).<br>$i$ is the index representing a single slice.<br>$\alpha$ is the angle between the base of a slice and the horizontal ($^\circ$).<br>$y_{\text{slip}}$ is the $y$-ordinate of a point on a slip surface (m).<br>$x_{\text{slip}}$ is the $x$-ordinate of a point on a slip surface (m). |
| Sources | Fredlund and J.Krahn (4 April 1977) |
| Ref. By | GD1, GD2, GD6, GD7, GD9, DD8, IM2 |

| | |
|---|---|
| Number | DD6 |
| Label | **Surface Angle** |
| Symbol | $\beta$ |
| SI Units | $^{\circ}$ |
| Equation | $\beta_i = \arctan\left(\frac{y_{\text{slope},i} - y_{\text{slope},i-1}}{x_{\text{slope},i} - x_{\text{slope},i-1}}\right)$ |
| Description | This equation is based on the assumption that the surface of a slice is a straight line (A9).<br><br>$i$ is the index representing a single slice.<br><br>$\beta$ is the angle between the surface of a slice and the horizontal ($^{\circ}$).<br><br>$y_{\text{slope}}$ is the $y$-ordinate of a point on the slope (m).<br><br>$x_{\text{slope}}$ is the $x$-ordinate of a point on the slope (m). |
| Sources | Fredlund and J.Krahn (4 April 1977) |
| Ref. By | GD1, GD2, GD6, GD7, GD9, DD9, IM2 |

| | |
|---|---|
| Number | DD7 |
| Label | **Base $x$-Direction Width of a Slice** |
| Symbol | $b$ |
| SI Units | m |
| Equation | $b_i = x_{\text{slip},i} - x_{\text{slip},i-1}$ |
| Description | $i$ is the index representing a single slice.<br><br>$b$ is the width of the base of a slice in the $x$-direction (m).<br><br>$x_{\text{slip}}$ is the $x$-ordinate of a point on a slip surface (m). |
| Sources | Fredlund and J.Krahn (4 April 1977) |
| Ref. By | GD9, DD1, DD8, DD9, IM2 |

| | |
|---|---|
| Number | DD8 |
| Label | **Total Base Length of a Slice** |
| Symbol | $\ell_{\text{b}}$ |
| SI Units | m |
| Equation | $\ell_{\text{b},i} = b_i \sec(\alpha_i)$ |
| Description | This equation is based on the assumption that the base of a slice is a straight line (A9).<br><br>$i$ is the index representing a single slice.<br><br>$\ell_{\text{b}}$ is the base length of a slice in the direction parallel to the slope of the base (m).<br><br>$b$ is the width of the base of a slice in the $x$-direction (m), defined in DD7.<br><br>$\alpha$ is the angle between the base of a slice and the horizontal (°), defined in DD5. |
| Sources | Fredlund and J.Krahn (4 April 1977) |
| Ref. By | GD3, GD4, GD6, DD2 |

| | |
|---|---|
| Number | DD9 |
| Label | **Total Surface Length of a Slice** |
| Symbol | $\ell_{\text{s}}$ |
| SI Units | m |
| Equation | $\ell_{\text{s},i} = b_i \sec(\beta_i)$ |
| Description | This equation is based on the assumption that the surface of a slice is a straight line (A9).<br><br>$i$ is the index representing a single slice.<br><br>$\ell_{\text{s}}$ is the surface length of a slice in the direction parallel to the slope of the surface (m).<br><br>$b$ is the width of the base of a slice in the $x$-direction (m), defined in DD7.<br><br>$\beta$ is the angle between the surface of a slice and the horizontal (°), defined in DD6. |
| Sources | Fredlund and J.Krahn (4 April 1977) |
| Ref. By | DD3 |

| Number | DD10 |
|---|---|
| Label | **$y$-Direction Height of a Slice** |
| Symbol | $h$ |
| SI Units | m |
| Equation | $h_i = 0.5((y_{\text{slope},i} - y_{\text{slip},i}) + (y_{\text{slope},i-1} - y_{\text{slip},i-1}))$ |
| Description | This equation is based on the assumption that the surface and base of a slice are straight lines (A9).<br><br>$i$ is the index representing a single slice.<br><br>$h$ is the height in the $y$-direction from the base of a slice to the slope surface, at the $x$-direction midpoint of the slice (m).<br><br>$y_{\text{slope}}$ is the $y$-ordinate of a point on the slope (m).<br><br>$y_{\text{slip}}$ is the $y$-ordinate of a point on a slip surface (m). |
| Sources | Fredlund and J.Krahn (4 April 1977) |
| Ref. By | GD9, IM2 |

| Number | DD11 |
|---|---|
| Label | **Stress** |
| Symbol | $\sigma$ |
| SI Units | Pa |
| Equation | $\sigma = \frac{F}{A}$ |
| Description | $\sigma$ is the total stress on the soil mass (Pa).<br><br>$F$ is the force (N).<br><br>$A$ is the area on which a force acts ($m^2$). |
| Sources | Huston and Josephs (2008) |
| Ref. By | GD3, GD5 |

| Number | DD12 |
|---|---|
| Label | **Interslice Normal to Shear Force Ratio Variation Function** |
| Symbol | $f$ |
| SI Units | unitless |
| Equation | $f_i = \begin{cases} 1 & const\_f \\ \sin\left(\pi \frac{x_{\text{slip},i} - x_{\text{slip},0}}{x_{\text{slip},n} - x_{\text{slip},0}}\right) & \neg const\_f \end{cases}$ |
| Description | $i$ is the index representing a single slice.<br><br>$f$ is the variation of the interslice normal to shear force ratio as a function of distance in the $x$-direction.<br><br>$const\_f$ is a boolean decision on which form of $f$ the user desires: constant if true, or half-sine if false.<br><br>$x_{\text{slip}}$ is the $x$-ordinate of a point on a slip surface (m). |
| Sources | Fredlund and J.Krahn (4 April 1977) |
| Ref. By | GD8, DD13, DD14, IM2 |

| | |
|---|---|
| Number | DD13 |
| Label | **First Function for Incorporating Interslice Forces into Shear Force** |
| Symbol | $\Phi$ |
| SI Units | unitless |
| Equation | $\Phi_i = [\lambda \cdot f_i \cos(\alpha_i) - \sin(\alpha_i)] [\tan(\varphi')] - [\lambda \cdot f_i \sin(\alpha_i) + \cos(\alpha_i)] (F_S)$ |
| Description | The equation for $\Phi$ arises as part of the derivation for IM1, so that derivation should be consulted for information relating to the derivation of $\Phi$.<br><br>$i$ is the index representing a single slice.<br><br>$\Phi$ is the first function used to convert shear without the influence of interslice forces to shear with the influence of interslice forces.<br><br>$\lambda$ is the proportionality constant for the interslice normal to shear force ratio.<br><br>$f$ is the variation of the interslice normal to shear force ratio as a function of distance in the $x$-direction.<br><br>$\alpha$ is the angle between the base of a slice and the horizontal (°).<br><br>$\varphi'$ is the effective angle of friction (°).<br><br>$F_S$ is the factor of safety. |
| Sources | Zhu et al. (19 February 2005), Karchewski et al. (2012) |
| Ref. By | IM1, IM3 |

| Number | DD14 |
|---|---|
| Label | **Second Function for Incorporating Interslice Forces into Shear Force** |
| Symbol | $\Psi$ |
| SI Units | unitless |
| Equation | $\Psi_{i-1} = \frac{[\lambda \cdot f_{i-1} \cos(\alpha_i) - \sin(\alpha_i)][\tan(\varphi')] - [\lambda \cdot f_{i-1} \sin(\alpha_i) + \cos(\alpha_i)](F_{\text{S}})}{\Phi_{i-1}}$ |
| Description | The equation for $\Psi$ arises as part of the derivation for IM1, so that derivation should be consulted for information relating to the derivation of $\Psi$.<br><br>$i$ is the index representing a single slice.<br><br>$\Psi$ is the second function used to convert shear without the influence of interslice forces to shear with the influence of interslice forces.<br><br>$\lambda$ is the proportionality constant for the interslice normal to shear force ratio.<br><br>$f$ is the variation of the interslice normal to shear force ratio as a function of distance in the $x$-direction.<br><br>$\alpha$ is the angle between the base of a slice and the horizontal (°).<br><br>$\varphi'$ is the effective angle of friction (°).<br><br>$F_{\text{S}}$ is the factor of safety.<br><br>$\Phi$ is the first function used to convert shear without the influence of interslice forces to shear with the influence of interslice forces. |
| Sources | Zhu et al. (19 February 2005), Karchewski et al. (2012) |
| Ref. By | IM1, IM3 |

### 5.2.5 Instance Models

This section transforms the problem defined in the Section 5.1 into one which is expressed in mathematical terms. It uses concrete symbols defined in Section 5.2.4 to replace the abstract symbols in the models identified in the Sections 5.2.2 and 5.2.3.

The goals GS1, GS2, and GS3 are met by the simultaneous solution of IM1, IM2, and IM3. The goal GS1 is also contributed to by IM4.

The Morgenstern-Price Method is a vertical slice, limit equilibrium slope stability analysis method. Analysis is performed by breaking the assumed slip surface into a series of vertical slices of mass. Static equilibrium analysis is performed, using two force equations and one moment equation as in T2. The problem is statically indeterminate with only these 3 equations and one constitutive equation (the Mohr-Coulomb shear strength of T3), so the assumption A6 and corresponding equation GD8 are used. The force equilibrium equations can be modified to be expressed only in terms of known physical values, as done in GD6 and GD7.

| Number | IM1 |
|---|---|
| Label | **Factor of Safety** |
| Input | $\{(x_{\text{slope}}, y_{\text{slope}})\}$, $\{(x_{\text{wt}}, y_{\text{wt}})\}$, $c'$, $\varphi'$, $\gamma$, $\gamma_{\text{Sat}}$, $\gamma_{\text{w}}$, $\{(x_{\text{slip}}, y_{\text{slip}})\}$, *const_f* |
| Output | $F_{\text{S}} = \dfrac{\sum_{i=1}^{n-1}\left[R_i \prod_{c=i}^{n-1} \Psi_c\right] + R_n}{\sum_{i=1}^{n-1}\left[T_i \prod_{c=i}^{n-1} \Psi_c\right] + T_n}$ |
| Description | $i$ is the index representing a single slice.<br>$n$ is the total number of slices.<br>$F_{\text{S}}$ is the factor of safety.<br>$R$ is the resistive shear force without the influence of interslice forces per meter in the $z$-direction ($\text{N}\,\text{m}^{-1}$), defined in GD6.<br>$\Psi$ is the second function used to convert shear without the influence of interslice forces to shear with the influence of interslice forces, defined in GD14.<br>$T$ is the mobile shear force without the influence of interslice forces per meter in the $z$-direction ($\text{N}\,\text{m}^{-1}$), defined in GD7. |
| Sources | Zhu et al. (19 February 2005), Karchewski et al. (2012) |
| Ref. By | IM2, IM3 |

### Factor of Safety Derivation

The mobile shear force defined in GD2 can be substituted into the definition of mobile shear force based on the factor of safety, from GD4, yielding Equation 1 below.

$$\left(\begin{array}{l}[W_i - X_{i-1} + X_i + U_{\text{t},i}\cos(\beta_i) + Q_i\cos(\omega_i)]\sin(\alpha_i)\\ -[-K_{\text{c}}\,W_i - G_i + G_{i-1} - H_i + H_{i-1} + U_{\text{t},i}\sin(\beta_i) + Q_i\cos(\omega_i)]\cos(\alpha_i)\end{array}\right) = \frac{N'_i \cdot \tan(\varphi') + c' \cdot \ell_{b,i}}{F_{\text{S}}} \tag{1}$$

An expression for the effective normal force, $N'_i$, can be derived by substituting the normal force equilibrium from GD1 into the definition for effective normal force from GD5. This results in Equation 2.

$$\begin{aligned} N'_i = \; & [W_i - X_{i-1} + X_i + U_{\text{t},i}\cos(\beta_i) + Q_i\cos(\omega_i)]\cos(\alpha_i) \\ & + [-K_{\text{c}}\,W_i - G_i + G_{i-1} - H_i + H_{i-1} + U_{\text{t},i}\sin(\beta_i) + Q_i\sin(\omega_i)]\sin(\alpha_i) \\ & - U_{\text{b,i}} \end{aligned} \tag{2}$$

Substituting Equation 2 into Equation 1 gives

$$\frac{\begin{pmatrix} [W_i - X_{i-1} + X_i + U_{\mathrm{t},i} \cos(\beta_i) + Q_i \cos(\omega_i)] \sin(\alpha_i) \\ -[-K_{\mathrm{c}}\, W_i - G_i + G_{i-1} - H_i + H_{i-1} + U_{\mathrm{t},i} \sin(\beta_i) + Q_i \cos(\omega_i)] \cos(\alpha_i) \end{pmatrix} = \begin{pmatrix} [W_i - X_{i-1} + X_i + U_{\mathrm{t},i} \cos(\beta_i) + Q_i \cos(\omega_i)] \cos(\alpha_i) \\ +[-K_{\mathrm{c}}\, W_i - G_i + G_{i-1} - H_i + H_{i-1} + U_{\mathrm{t},i} \sin(\beta_i) + Q_i \sin(\omega_i)] \sin(\alpha_i) - U_{\mathrm{b,i}} \end{pmatrix} \cdot \tan(\varphi') + c' \cdot \ell_{b,i}}{F_{\mathrm{S}}}$$

Since the interslice shear force $X$ and interslice normal force $G$ are unknown, they are separated from the other terms as follows:

$$\frac{\begin{pmatrix} [W_i + U_{\mathrm{t},i} \cos(\beta_i) + Q_i \cos(\omega_i)] \sin(\alpha_i) + (-X_{i-1} + X_i) \sin(\alpha_i) \\ -[-K_{\mathrm{c}}\, W_i - H_i + H_{i-1} + U_{\mathrm{t},i} \sin(\beta_i) + Q_i \cos(\omega_i)] \cos(\alpha_i) - (-G_i + G_{i-1}) \cos(\alpha_i) \end{pmatrix} = \begin{pmatrix} [W_i + U_{\mathrm{t},i} \cos(\beta_i) + Q_i \cos(\omega_i)] \cos(\alpha_i) + (-X_{i-1} + X_i) \cos(\alpha_i) \\ +[-K_{\mathrm{c}}\, W_i - H_i + H_{i-1} + U_{\mathrm{t},i} \sin(\beta_i) + Q_i \sin(\omega_i)] \sin(\alpha_i) + (-G_i + G_{i-1}) \sin(\alpha_i) \\ -U_{\mathrm{b,i}} \end{pmatrix} \cdot \tan(\varphi') + c' \cdot \ell_{b,i}}{F_{\mathrm{S}}}$$

Applying assumptions A11 and A12, which state that the seismic coefficient and the external force, respectively, are zero, allows for further simplification as shown below.

$$\frac{\begin{pmatrix} [W_i + U_{\mathrm{t},i} \cos(\beta_i)] \sin(\alpha_i) + (-X_{i-1} + X_i) \sin(\alpha_i) \\ -[-H_i + H_{i-1} + U_{\mathrm{t},i} \sin(\beta_i)] \cos(\alpha_i) - (-G_i + G_{i-1}) \cos(\alpha_i) \end{pmatrix} = \begin{pmatrix} [W_i + U_{\mathrm{t},i} \cos(\beta_i)] \cos(\alpha_i) + (-X_{i-1} + X_i) \cos(\alpha_i) \\ +[-H_i + H_{i-1} + U_{\mathrm{t},i} \sin(\beta_i)] \sin(\alpha_i) + (-G_i + G_{i-1}) \sin(\alpha_i) - U_{\mathrm{b,i}} \end{pmatrix} \cdot \tan(\varphi') + c' \cdot \ell_{b,i}}{F_{\mathrm{S}}}$$

The definitions of GD6 and GD7 are present in this equation, and can thus be replaced by $R_i$ and $T_i$, respectively.

$$\frac{(T_i + (-X_{i-1} + X_i) \sin(\alpha_i) - (-G_i + G_{i-1}) \cos(\alpha_i)) = R_i + ((-X_{i-1} + X_i) \cos(\alpha_i) + (-G_i + G_{i-1}) \sin(\alpha_i)) \cdot \tan(\varphi')}{F_{\mathrm{S}}}$$

The interslice shear force $X$ can be expressed in terms of the interslice normal force $G$ using GD8, resulting in

$$\frac{(T_i + (-\lambda f_{i-1} G_{i-1} + \lambda f_i G_i) \sin(\alpha_i) - (-G_i + G_{i-1}) \cos(\alpha_i)) = R_i + ((-\lambda f_{i-1} G_{i-1} + \lambda f_i G_i) \cos(\alpha_i) + (-G_i + G_{i-1}) \sin(\alpha_i)) \cdot \tan(\varphi')}{F_{\mathrm{S}}}$$

Rearranging yields the following:

$$G_i \begin{bmatrix} [\lambda \cdot f_i \cos(\alpha_i) - \sin(\alpha_i)] \tan({\varphi'}_i) \\ -[\lambda \cdot f_i \sin(\alpha_i) + \cos(\alpha_i)] (F_{\mathrm{S}}) \end{bmatrix} = G_{i-1} \begin{bmatrix} [\lambda \cdot f_{i-1} \cos(\alpha_i) - \sin(\alpha_i)] \tan({\varphi'}_i) \\ -[\lambda \cdot f_{i-1} \sin(\alpha_i) + \cos(\alpha_i)] (F_{\mathrm{S}}) \end{bmatrix} + (F_{\mathrm{S}}) \cdot T_i - R_i$$

The definitions for $\Phi$ and $\Psi$ from DD13 and DD14 simplify the above to Equation 3.

$$G_i \Phi_i = \Psi_{i-1} G_{i-1} \Phi_{i-1} + F_{\mathrm{S}} T_i - R_i \tag{3}$$

Versions of Equation 3 instantiated for slices 1 to $n$ are shown below.

$$G_1\Phi_1 = \Psi_0 G_0 \Phi_0 + F_{\mathrm{S}} T_1 - R_1$$

$$G_2\Phi_2 = \Psi_1 G_1 \Phi_1 + F_{\mathrm{S}} T_2 - R_2 \tag{4}$$

$$G_3\Phi_3 = \Psi_2 G_2 \Phi_2 + F_{\mathrm{S}} T_3 - R_3 \tag{5}$$

$$\ldots$$

$$G_{\text{n-2}}\Phi_{\text{n-2}} = \Psi_{\text{n-3}} G_{\text{n-3}} \Phi_{\text{n-3}} + F_{\mathrm{S}} T_{\text{n-2}} - R_{\text{n-2}} \tag{6}$$

$$G_{\text{n-1}}\Phi_{\text{n-1}} = \Psi_{\text{n-2}} G_{\text{n-2}} \Phi_{\text{n-2}} + F_{\mathrm{S}} T_{\text{n-1}} - R_{\text{n-1}} \tag{7}$$

$$G_{\text{n}}\Phi_{\text{n}} = \Psi_{\text{n-1}} G_{\text{n-1}} \Phi_{\text{n-1}} + F_{\mathrm{S}} T_{\text{n}} - R_{\text{n}}$$

Applying A10, which says that $G_0$ and $G_n$ are zero, results in the following special cases: Equation 8 for the first slice and Equation 9 for the $n$th slice.

$$G_1\Phi_1 = F_{\mathrm{S}} T_1 - R_1 \tag{8}$$

$$-\frac{F_{\mathrm{S}} T_{\text{n}} - R_{\text{n}}}{\Psi_{\text{n-1}}} = G_{\text{n-1}}\Phi_{\text{n-1}} \tag{9}$$

Substituting Equation 8 into Equation 4 yields Equation 10, which can be substituted into Equation 5 to get Equation 11, and so on until Equation 12 is obtained from Equation 7.

$$G_2\Phi_2 = \Psi_1 \left(F_{\mathrm{S}} T_1 - R_1\right) + F_{\mathrm{S}} T_2 - R_2 \tag{10}$$

$$G_3\Phi_3 = \Psi_2 \left(\Psi_1 \left(F_{\mathrm{S}} T_1 - R_1\right) + F_{\mathrm{S}} T_2 - R_2\right) + F_{\mathrm{S}} T_3 - R_3 \tag{11}$$

$$\ldots$$

$$\begin{split} G_{\text{n-1}}\Phi_{\text{n-1}} =& \Psi_{\text{n-2}} \left(\Psi_{\text{n-3}} \left(\ldots \left(\Psi_1 \left(F_{\mathrm{S}} T_1 - R_1\right) + F_{\mathrm{S}} T_2 - R_2\right) \ldots\right) + F_{\mathrm{S}} T_{\text{n-2}} - R_{\text{n-2}}\right) \\ &+ F_{\mathrm{S}} T_{\text{n-1}} - R_{\text{n-1}} \end{split} \tag{12}$$

Equation 9 can then be substituted into the left-hand side of Equation 12, resulting in:

$$\begin{split} -\frac{F_{\mathrm{S}} T_{\text{n}} - R_{\text{n}}}{\Psi_{\text{n-1}}} =& \Psi_{\text{n-2}} \left(\Psi_{\text{n-3}} \left(\ldots \left(\Psi_1 \left(F_{\mathrm{S}} T_1 - R_1\right) + F_{\mathrm{S}} T_2 - R_2\right) \ldots\right) + F_{\mathrm{S}} T_{\text{n-2}} - R_{\text{n-2}}\right) \\ &+ F_{\mathrm{S}} T_{\text{n-1}} - R_{\text{n-1}} \end{split}$$

This can be rearranged by multiplying both sides by $\Psi_{\text{n-1}}$ and then distributing the multiplication of each $\Psi$ over addition to obtain:

$$-\left(F_{\mathrm{S}} T_{\text{n}} - R_{\text{n}}\right) = \Psi_{\text{n-1}} \Psi_{\text{n-2}} \ldots \Psi_1 \left(F_{\mathrm{S}} T_1 - R_1\right) + \Psi_{\text{n-1}} \Psi_{\text{n-2}} \ldots \Psi_2 \left(F_{\mathrm{S}} T_2 - R_2\right) + \cdots + \Psi_{\text{n-1}} \left(F_{\mathrm{S}} T_{\text{n-1}} - R_{\text{n-1}}\right)$$

The multiplication of the $\Psi$ terms can be further distributed over the subtractions, resulting in the equation having terms that each either contain an $R$ or a $T$. The equation can then be rearranged so terms containing an $R$ are on one side of the equality, and terms containing a $T$ are on the other. The multiplication by the factor of safety is common to all of the $T$ terms, and thus can be factored out, resulting in:

$$F_{\mathrm{S}}\left(\Psi_{\text{n-1}}\Psi_{\text{n-2}}\ldots\Psi_1 T_1+\Psi_{\text{n-1}}\Psi_{\text{n-2}}\ldots\Psi_2 T_2+\ldots\Psi_{\text{n-1}}T_{\text{n-1}}+T_{\text{n}}\right)=$$
$$\Psi_{\text{n-1}}\Psi_{\text{n-2}}\ldots\Psi_1 R_1+\Psi_{\text{n-1}}\Psi_{\text{n-2}}\ldots\Psi_2 R_2+\cdots+\Psi_{\text{n-1}}R_{\text{n-1}}+R_{\text{n}}$$

Isolating the factor of safety on the left-hand side and using compact notation for the products and sums yields Equation 13, which can also be seen in IM1. $F_{\mathrm{S}}$ depends on the unknowns $\lambda$ (IM2) and $G$ (IM3).

$$F_{\mathrm{S}}=\frac{\sum_{i=1}^{n-1}\left[R_i\prod_{c=i}^{n-1}\Psi_c\right]+R_n}{\sum_{i=1}^{n-1}\left[T_i\prod_{c=i}^{n-1}\Psi_c\right]+T_n} \tag{13}$$

| Number | IM2 |
|---|---|
| Label | **Normal and Shear Force Proportionality Constant** |
| Input | $\{(x_{\text{slope}}, y_{\text{slope}})\}$, $\{(x_{\text{wt}}, y_{\text{wt}})\}$, $\gamma_{\text{w}}$, $\{(x_{\text{slip}}, y_{\text{slip}})\}$, *const_f* |
| Output | $C_{\text{num},i} = \begin{cases} b_1 \left[G_1 + H_1\right] \tan\left(\alpha_1\right) & i = 1 \\ b_i \left[\left(G_i + G_{i-1}\right) + \left(H_i + H_{i-1}\right)\right] \tan\left(\alpha_i\right) + h_i \left(-2\, U_{\text{t},i} \sin\left(\beta_i\right)\right) & 2 \leq i \leq n-1 \\ b_n \left[G_{n-1} + H_{n-1}\right] \tan\left(\alpha_{n-1}\right) & i = n \end{cases}$<br>$C_{\text{den},i} = \begin{cases} b_1 G_1 f_1 & i = 1 \\ b_i \left(f_i G_i + f_{i-1} G_{i-1}\right) & 2 \leq i \leq n-1 \\ b_n G_{n-1} f_{n-1} & i = n \end{cases}$<br>$\lambda = \frac{\sum_{i=1}^{n} C_{\text{num},i}}{\sum_{i=1}^{n} C_{\text{den},i}}$ |
| Description | $i$ is the index representing a single slice.<br>$n$ is the total number of slices.<br>$b$ is the width of the base of a slice in the $x$-direction (m), defined in DD7.<br>$G$ is the interslice normal force per meter in the $z$-direction ($\text{N}\,\text{m}^{-1}$).<br>$H$ is the interslice water force per meter in the $z$-direction ($\text{N}\,\text{m}^{-1}$), defined in DD4.<br>$\alpha$ is the angle between the base of a slice and the horizontal (°), defined in DD5.<br>$h$ is the height in the $y$-direction from the base of a slice to the slope surface, at the $x$-direction midpoint of the slice (m), defined in DD10.<br>$U_{\text{t}}$ is the surface hydrostatic force per meter in the $z$-direction ($\text{N}\,\text{m}^{-1}$), defined in DD3.<br>$\beta$ is the angle between the surface of a slice and the horizontal (°), defined in DD6.<br>$f$ is the variation of the interslice normal to shear force ratio as a function of distance in the $x$-direction, defined in DD12.<br>$\lambda$ is the proportionality constant. |
| Sources | Zhu et al. (19 February 2005) |
| Ref. By | IM1, IM3 |

**Normal/Shear Force Ratio Derivation**

From the moment equilibrium of GD9, with the primary assumption for the Morgenstern-Price method of A6 and associated definition GD8, Equation (14) can be derived.

$$0 = \begin{array}{l} -G_i\left[h_{\mathrm{z},i} - \frac{b_i}{2}\tan\left(\alpha_i\right)\right] + G_{i-1}\left[h_{\mathrm{z},i-1} + \frac{b_i}{2}\tan\left(\alpha_i\right)\right] - H_i\left[h_{\mathrm{z,w},i} - \frac{b_i}{2}\tan\left(\alpha_i\right)\right] \\ +H_{i-1}\left[h_{\mathrm{z,w},i-1} + \frac{b_i}{2}\tan\left(\alpha_i\right)\right] - \lambda\frac{b_i}{2}\left(G_i f_i + G_{i-1} f_{i-1}\right) + K_{\mathrm{c}} W_i \frac{h_i}{2} - U_{\mathrm{t},i}\sin\left(\beta_i\right) h_i - Q_i \sin\left(\omega_i\right) h_i \end{array} \tag{14}$$

Rearranging the equation in terms of $\lambda$ leads to Equation (15).

$$\lambda = \frac{\begin{array}{l} -G_i\left[h_{\mathrm{z},i} - \frac{b_i}{2}\tan\left(\alpha_i\right)\right] + G_{i-1}\left[h_{\mathrm{z},i-1} + \frac{b_i}{2}\tan\left(\alpha_i\right)\right] - H_i\left[h_{\mathrm{z,w},i} - \frac{b_i}{2}\tan\left(\alpha_i\right)\right] \\ +H_{i-1}\left[h_{\mathrm{z,w},i-1} + \frac{b_i}{2}\tan\left(\alpha_i\right)\right] + K_{\mathrm{c}} W_i \frac{h_i}{2} - U_{\mathrm{t},i}\sin\left(\beta_i\right) h_i - Q_i \sin\left(\omega_i\right) h_i \end{array}}{\frac{b_i}{2}\left[G_i f_i + G_{i-1} f_{i-1}\right]} \tag{15}$$

This equation can be simplified by applying assumptions A11 and A12, which state that the seismic and external forces, respectively are zero.

$$\lambda = \frac{\begin{array}{l} -G_i\left[h_{\mathrm{z},i} - \frac{b_i}{2}\tan\left(\alpha_i\right)\right] + G_{i-1}\left[h_{\mathrm{z},i-1} + \frac{b_i}{2}\tan\left(\alpha_i\right)\right] - H_i\left[h_{\mathrm{z,w},i} - \frac{b_i}{2}\tan\left(\alpha_i\right)\right] \\ +H_{i-1}\left[h_{\mathrm{z,w},i-1} + \frac{b_i}{2}\tan\left(\alpha_i\right)\right] - U_{\mathrm{t},i}\sin\left(\beta_i\right) h_i \end{array}}{\frac{b_i}{2}\left[G_i f_i + G_{i-1} f_{i-1}\right]}$$

Taking the summation of all slices, and applying A10 to set $G_0$, $G_n$, $H_0$, and $H_n$ equal to zero, a general equation for the constant $\lambda$ is developed in Equation (16), also found in IM2.

$$\lambda = \frac{\sum_{i=1}^{n} b_i\left[\left(G_i + G_{i-1}\right) + \left(H_i + H_{i-1}\right)\right]\tan\left(\alpha_i\right) + h_i\ \left[-2\ U_{\mathrm{t},i}\sin\left(\beta_i\right)\right]}{\sum_{i=1}^{n} b_i\left[f_i G_i + f_{i-1} G_{i-1}\right]} \tag{16}$$

Equation (16) for $\lambda$ is a function of the unknown interslice normal force, $G$ (IM3), which itself depends on the unknown factor of safety, $F_{\mathrm{S}}$ (IM1).

| Number | IM3 |
|---|---|
| Label | **Interslice Normal Forces** |
| Input | $\{(x_{\text{slope}}, y_{\text{slope}})\}$, $\{(x_{\text{wt}}, y_{\text{wt}})\}$, $c'$, $\varphi'$, $\gamma$, $\gamma_{\text{Sat}}$, $\gamma_{\text{w}}$, $\{(x_{\text{slip}}, y_{\text{slip}})\}$, *const_f* |
| Output | $G_i = \begin{cases} \frac{(F_{\text{S}})T_1 - R_1}{\Phi_i} & i = 1 \\ \frac{\Psi_{i-1} \cdot G_{i-1} + (F_{\text{S}}) \cdot T_i - R_i}{\Phi_i} & 2 \leq i \leq n-1 \\ 0 & i = 0 \lor i = n \end{cases}$ |
| Description | $i$ is the index representing a single slice.<br><br>$n$ is the total number of slices.<br><br>$G$ is the interslice normal force per meter in the $z$-direction ($\text{N}\,\text{m}^{-1}$).<br><br>$F_{\text{S}}$ is the factor of safety.<br><br>$T$ is the mobile shear force without the influence of interslice forces per meter in the $z$-direction ($\text{N}\,\text{m}^{-1}$), defined in DD7.<br><br>$R$ is the resistive shear force without the influence of interslice forces per meter in the $z$-direction ($\text{N}\,\text{m}^{-1}$), defined in DD6.<br><br>$\Phi$ is the first function used to convert shear without the influence of interslice forces to shear with the influence of interslice forces, defined in DD13.<br><br>$\Psi$ is the second function used to convert shear without the influence of interslice forces to shear with the influence of interslice forces, defined in DD14. |
| Sources | Zhu et al. (19 February 2005) |
| Ref. By | IM1, IM2 |

**Interslice Force Derivation**

This derivation is identical to the derivation for IM1 up until Equation 3, shown again below.

$$G_i \Phi_i = \Psi_{i-1} G_{i-1} \Phi_{i-1} + F_{\text{S}} T_i - R_i$$

A simple rearrangement of Equation 3 leads to Equation 17, also seen in IM3.

$$G_i = \frac{\Psi_{i-1}\, G_{i-1} + F_{\text{S}} T_i - R_i}{\Phi_i} \tag{17}$$

The cases shown in IM3 for when $i = 0$, $i = 1$, or $i = n$ are derived by applying A10, which says that $G_0$ and $G_n$ are zero, to Equation 17. $G$ depends on the unknowns $F_{\text{S}}$ (IM1) and $\lambda$ (IM2).

| Number | IM4 |
|---|---|
| Label | **Critical Slip Surface Identification** |
| Input | $\{(x_{\text{slope}}, y_{\text{slope}})\}$, $\{(x_{\text{wt}}, y_{\text{wt}})\}$, $c'$, $\varphi'$, $\gamma$, $\gamma_{\text{Sat}}$, $\gamma_{\text{w}}$, *const_f* |
| Output | $\left(F_{\text{S}}^{\text{Min}}, \{(x_{\text{cs}}, y_{\text{cs}})\}\right) =$ <br> $\Upsilon\left(\{(x_{\text{slope}}, y_{\text{slope}})\}, \{(x_{\text{wt}}, y_{\text{wt}})\}, c', \varphi', \gamma, \gamma_{\text{Sat}}, \gamma_{\text{w}}, \mathit{const\_f}\right)$ |
| Description | The minimization algorithm must enforce the constraints on the critical slip surface expressed in A1 and Table 2. <br> $F_{\text{S}}^{\text{Min}}$ is the minimum factor of safety associated with the critical slip surface. <br> $x_{\text{cs}}$ is the $x$-ordinate of a point on the critical slip surface (m). <br> $x_{\text{cs}}$ is the $y$-ordinate of a point on the critical slip surface (m). <br> $\Upsilon$ is a minimization algorithm or function. <br> $x_{\text{slope}}$ is the $x$-ordinate of a point on the slope (m). <br> $y_{\text{slope}}$ is the $y$-ordinate of a point on the slope (m). <br> $x_{\text{wt}}$ is the $x$-ordinate of a point on the water table (m). <br> $y_{\text{wt}}$ is the $y$-ordinate of a point on the water table (m). <br> $c'$ is the effective cohesion (Pa). <br> $\varphi'$ is the effective angle of friction (°). <br> $\gamma$ is the soil dry unit weight ($\text{N}\,\text{m}^{-3}$). <br> $\gamma_{\text{Sat}}$ is the soil saturated unit weight ($\text{N}\,\text{m}^{-3}$). <br> $\gamma_{\text{w}}$ is the unit weight of water ($\text{N}\,\text{m}^{-3}$). <br> *const_f* is a boolean decision on which form of $f$ the user desires: constant if true, or half-sine if false. |
| Sources | Li et al. (25 June 2010) |

[Should this IM exist? It doesn't arise from any T —BM]

[We need something to explain that we pick the slip surface with the minimum factor of safety. I'll give this some further thought on whether this is the best way to say it. —SS]

#### 5.2.6 Data Constraints

Tables 1 and 2 show the data constraints on the input and output variables, respectively. The column for physical constraints gives the physical limitations on the range of values that can be taken by the variable. The uncertainty column provides an estimate of the confidence with which the physical quantities can be measured. This information would be part of the input if one were performing an uncertainty quantification exercise. The constraints are conservative, to give the user of the model the flexibility to experiment with unusual situations. The column of typical values is intended to provide a feel for a common scenario.

| Var | Physical Constraints | Typical Value | Uncertainty |
|---|---|---|---|
| $\{(x_{\text{wt}}, y_{\text{wt}})\}$ (*) | At least two ordered pairs must be specified. First and last $x$ values must be the same as the soil mass. $x$ values must be monotonically increasing. | N/A | 10% |
| $\{(x_{\text{slope}}, y_{\text{slope}})\}$ | At least two ordered pairs must be specified. $x$ values must be monotonically increasing. | N/A | 10% |
| $x_{\text{slip}}^{\text{maxExt}}$, $x_{\text{slip}}^{\text{minExt}}$, $x_{\text{slip}}^{\text{maxEtr}}$, $x_{\text{slip}}^{\text{minEtr}}$ | must be between or equal to the minimum and maximum $x_{\text{slope}}$ values. | N/A | 10% |
| $y_{\text{slip}}^{\text{min}}$ | Cannot be above the maximum $y_{\text{slope}}$ value. | N/A | 10% |
| $y_{\text{slip}}^{\text{max}}$ | Cannot be below the minimum $y_{\text{slope}}$ value. | N/A | 10% |
| $c'$ | $c > 0$ | 10000 | 10% |
| $\varphi'$ | $0 < \varphi < 90$ | 25 | 10% |
| $\gamma$ | $\gamma > 0$ | 20000 | 10% |
| $\gamma_{\text{Sat}}$ | $\gamma_{\text{Sat}} > 0$ | 20000 | 10% |
| $\gamma_{\text{w}}$ | $\gamma_{\text{w}} > 0$ | 9800 | 10% |

Table 1: Input variables for SSP

**(*)** Optional input.

#### 5.2.7 Properties of a Correct Solution

Not applicable for SSP.

| Var | Physical Constraints |
|---|---|
| $F_S$ | $F_S > 0$ |
| $\{(x_{cs}, y_{cs})\}$ | All $x$ values must be between $x_{slip}^{minEtr}$ and $x_{slip}^{maxExt}$. $y$ values must not be below $y_{slip}^{min}$. For any given vertex, the $y_{cs}$ value must not exceed the $y_{slope}$ value corresponding to the same $x_{cs}$ value. The first and last vertices must each be equal to one of the vertices in $\{(x_{slope}, y_{slope})\}$. The slope between consecutive vertices must be always increasing as $x$ increases. The internal angle between consecutive vertices should not be below 110 degrees. |

Table 2: Output variables for SSP

# 6 Requirements

This section provides the functional requirements, the business tasks that the software is expected to complete, and the nonfunctional requirements, the qualities that the software is expected to exhibit.

## 6.1 Functional Requirements

R1: Read the inputs, shown in the table below, and store the data.

| Symbol | Unit | Description |
|---|---|---|
| $(x, y)$ | m | $x$ and $y$-coordinates for vertices of the soil mass, for the water table if one exists, and for potential entry and exit points of a slip surface. |
| $c'$ | Pa | Cohesion for each slope layer. |
| $\varphi'$ | $^\circ$ | Effective angle of friction for each slope layer. |
| $\gamma$ | $\mathrm{N\,m^{-3}}$ | Unit weight of dry soil for each slope layer. |
| $\gamma_{Sat}$ | $\mathrm{N\,m^{-3}}$ | Unit weight of saturated soil for each slope layer. |
| $\gamma_w$ | $\mathrm{N\,m^{-3}}$ | Unit weight of water. |
| *const_f* | N/A | Boolean decision on which form of $f$ the user desires: constant if true, or half-sine if false. |

R2: Verify that the input data lies within physical constraints shown in Table 1.

R3: Generate potential critical slip surfaces for the input slope (using IM4).

R4: Calculate the factors of safety for each of the potential critical slip surfaces (using IM1, IM2, and IM3).

R5: Compare the factor of safety for each potential critical slip surface to determine the minimum factor of safety, corresponding to the critical slip surface (using IM4).

[R3-R5 have never sat well with me because they read too much like an algorithm. I always end up leaving them because I like how they separate the calculation of factor of safety from the minimization of the factor of safety. I still wonder if I should replace them with one all-encompassing requirement like "Determine the critical slip surface corresponding to the minimum factor of safety". What do you think? —BM]

R6: Verify that the factor of safety and critical slip surface satisfy the physical constraints shown in Table 2.

R7: Display as output the user-supplied inputs listed in the table below:

| Symbol | Description |
|---|---|
| $x_{\text{slip}}^{\text{maxExt}}$ | Maximum potential $x$-ordinate of the exit point of a slip surface |
| $x_{\text{slip}}^{\text{minExt}}$ | Minimum potential $x$-ordinate of the exit point of a slip surface |
| $x_{\text{slip}}^{\text{maxEtr}}$ | Maximum potential $x$-ordinate of the entry point of a slip surface |
| $x_{\text{slip}}^{\text{minEtr}}$ | Minimum potential $x$-ordinate of the entry point of a slip surface |
| $y_{\text{slip}}^{\text{max}}$ | Maximum potential $y$-ordinate of a point on a slip surface |
| $y_{\text{slip}}^{\text{min}}$ | Minimum potential $y$-ordinate of a point on a slip surface |
| $const\_f$ | Boolean decision on which form of $f$ the user desires: constant if true, or half-sine if false. |

R8: Display the critical slip surface of the 2D slope, as determined from IM4, graphically.

R9: Display the value of the factor of safety for the critical slip surface, as determined from IM1, IM2, and IM3.

R10: Using IM1, IM2, and IM3, calculate and graphically display the interslice normal forces.

R11: Using IM1, IM2, and IM3, calculate and graphically display the interslice shear forces.

## 6.2 Nonfunctional Requirements

SSP is intended to be an educational tool, therefore accuracy and performance speed are secondary program priorities. Instead, the following non-functional requirements are prioritized:

NFR1: Correctness, achieved if the outputs of the code have the properties described in 5.2.7.

NFR2: Understandability, achieved if the code is modularized with complete module guide and module interface specification.

NFR3: Reusability, achieved if the code is modularized.

NFR4: Maintainability, achieved if the traceability between requirements, assumptions, theoretical models, general definitions, data definitions, instance models, likely changes, and modules is completely recorded in traceability matrices in the SRS and module guide.

# 7 Likely Changes

LC1: The system currently assumes the soil mass is homogeneous (A3). In the future, implementation can be added for inconsistent soil properties throughout.

LC2: The system currently assumes no seismic force (A11). In the future, implementation can be added for the presence of seismic force.

LC3: The system currently assumes no external force (A12). In the future, implementation can be added for an imposed surface load on the slope.

# 8 Unlikely Changes

If changes were to be made with regard to the following, a different algorithm would be needed.

UC1: Changes related to A6 are not possible due to the dependency of the calculations on the proportional relationship between interslice normal and shear forces.

UC2: A7 allows for 2D analysis with these models only because stress along $z$-direction is zero. These models do not take into account stress in the $z$-direction, and therefore cannot be used without manipulation to attempt 3-dimensional analysis.

[This section is not on the template, not sure if it should be kept —BM]

[I'm going to think about adding this section to the template. It is a way to show that some of the assumptions are critical to the identity of the problem. —SS]

# 9 Traceability Matrices and Graphs

The purpose of the traceability matrices is to provide easy references on what has to be additionally modified if a certain component is changed. Every time a component is changed, the items in the column of that component that are marked with an "X" may have to be modified as well. Tables 5 and 6 show the dependencies of theoretical models, general definitions, data definitions, and instance models with each other. Table 4 shows the dependencies of instance models, requirements, and data constraints on each other. Table 3 shows the dependencies of theoretical models, general definitions, data definitions, instance models, and likely changes on the assumptions.

The purpose of the traceability graphs is also to provide easy references on what has to be additionally modified if a certain component is changed. The arrows in the graphs represent dependencies. The component at the tail of an arrow is depended on by the component at the head of that arrow. Therefore, if a component is changed, the components that it points to should also be changed. Figure 5 shows the dependencies of theoretical models, general definitions, data definitions, instance models, likely changes, and assumptions on each other. Figure 6 shows the dependencies of instance models, requirements, and data constraints on each other.

| | A1 | A2 | A3 | A4 | A5 | A6 | A7 | A8 | A9 | A10 | A11 | A12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T1 | | | | | | | | | | | | |
| T2 | | | | | | | X | | | | | |
| T3 | | | | | | | | X | | | | |
| T4 | | | | | | | | | | | | |
| GD1 | | | | | | | | | | | | |
| GD2 | | | | | | | | | | | | |
| GD3 | | | X | X | X | | | | | | | |
| GD4 | | X | X | X | X | | | | | | | |
| GD5 | | | | | | | | X | | | | |
| GD8 | | | | | | X | | | | | | |
| GD9 | | | | | | | | | | | | |
| DD1 | | | | | | | | | | | | |
| DD2 | | | | | | | | | X | | | |
| DD3 | | | | | | | | | X | | | |
| DD4 | | | | | | | | | | | | |
| DD5 | | | | | | | | | X | | | |
| DD6 | | | | | | | | | X | | | |
| DD7 | | | | | | | | | | | | |
| DD8 | | | | | | | | | X | | | |
| DD9 | | | | | | | | | X | | | |
| GD6 | | | X | X | X | | | | | | X | X |
| GD7 | | | X | X | X | | | | | | X | X |
| IM1 | | X | | | | X | | | | X | X | X |
| IM2 | | | | | | X | | | | X | X | X |
| IM3 | | X | | | | X | | | | X | X | X |
| IM4 | X | | | | | | | | | | | |
| LC1 | | | X | | | | | | | | | |
| LC2 | | | | | | | | | | | X | |
| LC3 | | | | | | | | | | | | X |

Table 3: Traceability matrix showing the connections between assumptions and other items

| | IM1 | IM2 | IM3 | IM4 | 5.2.6 | R1 |
|---|---|---|---|---|---|---|
| IM1 | | | | | | X |
| IM2 | | | | | | X |
| IM3 | | | | | | X |
| IM4 | | | | | | X |
| R1 | | | | | | |
| R2 | | | | | X | |
| R3 | | | | X | | |
| R4 | X | X | X | | | |
| R5 | | | | X | | |
| R6 | | | | | X | |
| R7 | | | | | | X |
| R8 | | | | | X | |
| R9 | X | X | X | | | |
| R10 | X | X | X | | | |
| R11 | X | X | X | | | |

Table 4: Traceability matrix showing the connections between requirements and instance models

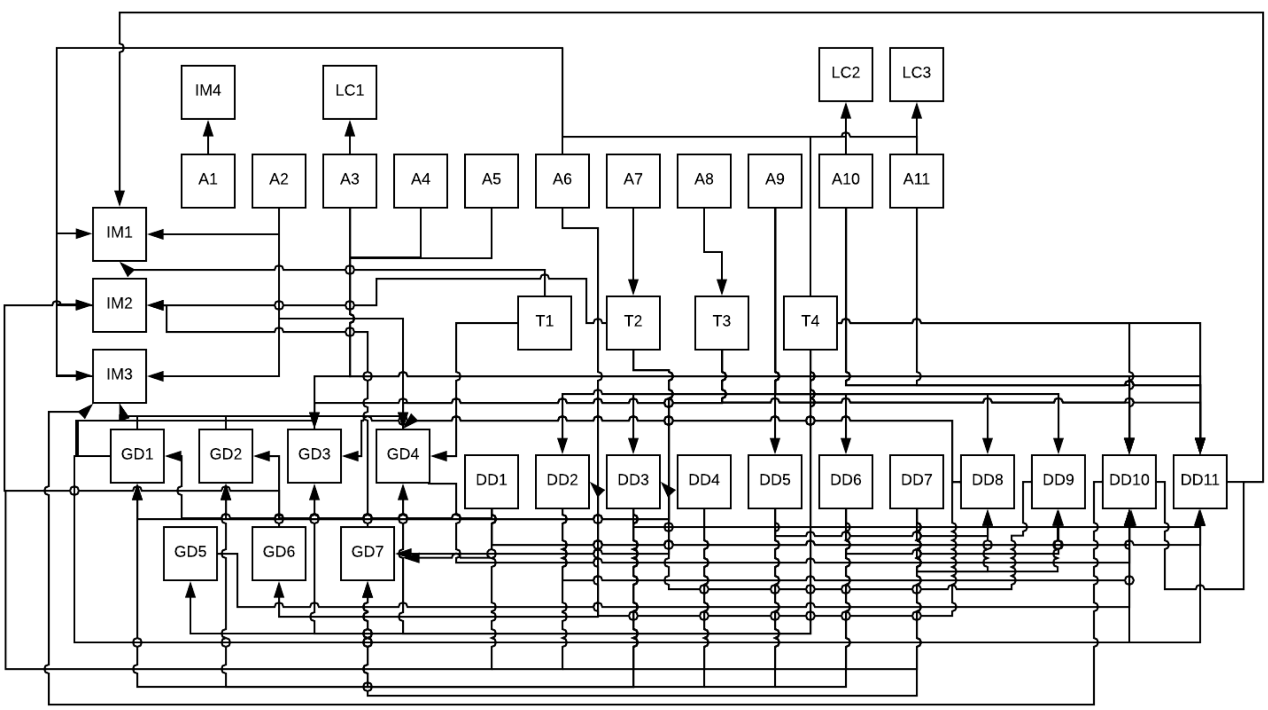


Figure 5: Traceability matrix showing the connections between items of different sections

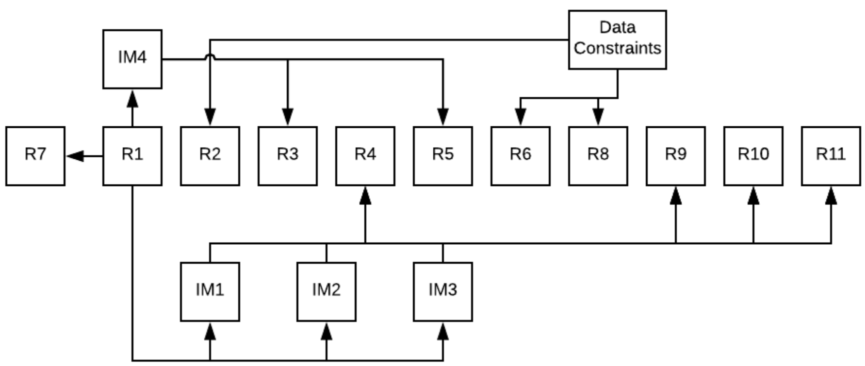


Figure 6: Traceability matrix showing the connections between requirements, instance models, and data constraints

| | T1 | T2 | T3 | T4 | GD1 | GD2 | GD3 | GD4 | GD5 | GD8 | GD9 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| T1 | | | | | | | | | | | |
| T2 | | | | | | | | | | | |
| T3 | | | | | | | | | | | |
| T4 | | | | | | | | | | | |
| GD1 | | X | | | | | | | | | |
| GD2 | | X | | | | | | | | | |
| GD3 | | | X | X | | | | | | | |
| GD4 | X | | X | X | | | X | | | | |
| GD5 | | | | X | | | | | | | |
| GD8 | | | | | | | | | | | |
| GD9 | | X | | | | | | | | | |
| DD1 | | | | | | | | | | | |
| DD2 | | | | | | | | | | | |
| DD3 | | | | | | | | | | | |
| DD4 | | | | | | | | | | | |
| DD5 | | | | | | | | | | | |
| DD6 | | | | | | | | | | | |
| DD7 | | | | | | | | | | | |
| DD8 | | | | | | | | | | | |
| DD9 | | | | | | | | | | | |
| GD6 | | | X | X | X | X | X | X | X | | |
| GD7 | | | X | X | X | X | X | | | | |
| IM1 | X | | | X | | | | | | X | |
| IM2 | | X | | X | | | | | | X | X |
| IM3 | | | | X | X | X | | X | | X | |
| IM4 | | | | | | | | | | | |

Table 5: Traceability matrix showing the connections between items of different sections with theory models and general definitions

| | DD1 | DD2 | DD3 | DD4 | DD5 | DD6 | DD7 | DD8 | DD9 | GD6 | GD7 | IM1 | IM2 | IM3 | IM4 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T1 | | | | | | | | | | | | | | | |
| T2 | | | | | | | | | | | | | | | |
| T3 | | | | | | | | | | | | | | | |
| T4 | | | | | | | | | | | | | | | |
| GD1 | X | | X | X | X | X | | | | | | | | | |
| GD2 | X | | X | X | X | X | | | | | | | | | |
| GD3 | | | | | | | | X | | | | | | | |
| GD4 | | | | | | | | X | | | | | | | |
| GD5 | | | | | | | | | | | | | | | |
| GD8 | | | | | | | | | | | | | | | |
| GD9 | X | | X | X | X | X | X | | | | | | | | |
| DD1 | | | | | | | | | | | | | | | |
| DD2 | | | | | | | | X | | | | | | | |
| DD3 | | | | | | | | | X | | | | | | |
| DD4 | | | | | | | | | | | | | | | |
| DD5 | | | | | | | | | | | | | | | |
| DD6 | | | | | | | | | | | | | | | |
| DD7 | | | | | | | | | | | | | | | |
| DD8 | | | | | X | | X | | | | | | | | |
| DD9 | | | | | | X | X | | | | | | | | |
| GD6 | X | X | X | X | X | X | X | | | | | | | | |
| GD7 | X | | X | X | X | X | X | | | | | | | | |
| IM1 | X | X | X | X | X | X | X | | | X | X | | X | X | |
| IM2 | X | X | X | X | X | X | X | | | | | X | | X | |
| IM3 | X | X | X | X | X | X | X | | | X | | X | X | | |
| IM4 | | | | | | | | | | | | | | | |

Table 6: Traceability matrix showing the connections between items of different sections with data definitions and instance models

# 10 References

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# 11 Appendix

## 11.1 Symbolic Parameters

There are no symbolic parameters.