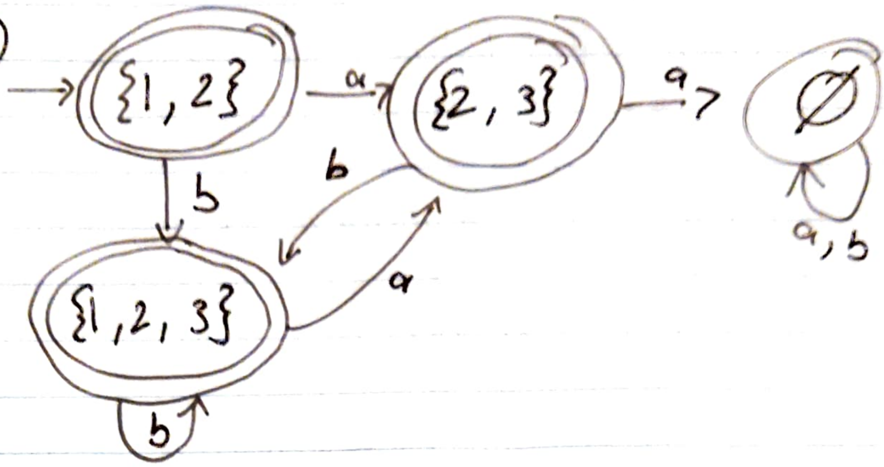
Hw3 - CSE 103

1a)



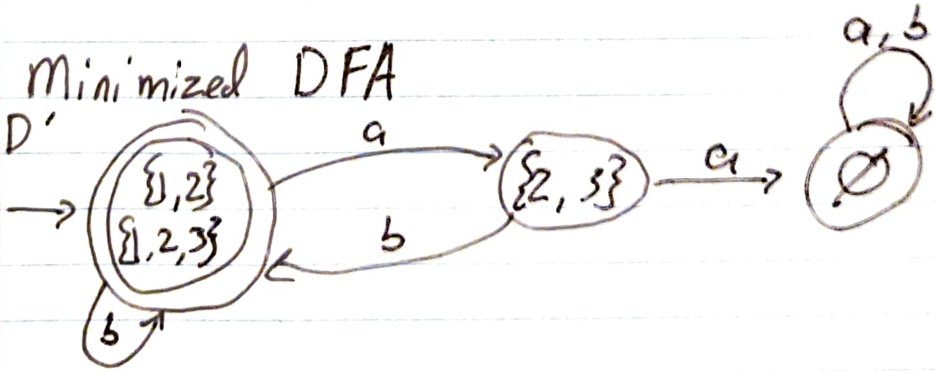
1b) $\boxed{\text{DFA } D}$

	a	b
{1,2}	{2,3}	{1,2,3}
{2,3}	∅	{1,2,3}
{1,2,3}	{2,3}	{1,2,3}
∅	∅	∅

$\boxed{\text{DFA } D'}$

	a	b
∅	∅	∅
{1,2} {1,2,3}	{2,3}	{1,2} {1,2,3}
{2,3}	∅	{1,2} {1,2,3}

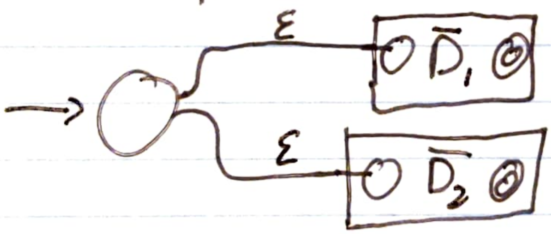
Minimized DFA



2a Intersection of 2 DFA's can be constructed by:

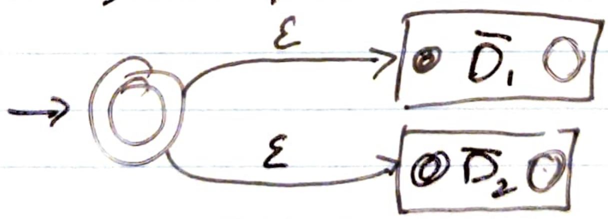
1) getting the complement of L_1 & L_2
  - $\overline{D}_1$: n states
  - $\overline{D}_2$: m states
  - Complement of DFA is turning accepting states to non-accepting and non-accepting states to accepting

2) Union $\overline{D}_1$ and $\overline{D}_2$ by using an "epsilon fork" as follows:



Total States: $n+m+1$

3) Complement the new NFA:



Total States: $n+m+1$

4) Convert NFA → DFA using subset construction which results in a DFA w/

Total state : $2^{(n+m+1)}$

2b Language L_1 and L_2 are regular since there exists D_1 and D_2 DFAs respectively.

Therefore $\hat{\delta}_1$ and $\hat{\delta}_2$ exist for the finite languages L_1 and L_2.

Due to this we can say that there are valid paths within D_1 and D_2 and by product construction D is constructed of states consisting of pairs of states from D_1 and D_2 where p is from D_1 and q is from D_2.

$(p,q) \to (p',q')$ iff there exists

a transition from $p \to p'$ and $q \to q'$. There fore, $\hat{\delta}$ for DFA D exist as there will be a valid path given 2 valid paths from D_1 and D_2.

$\boxed{\text{Proof 2:}}$

$L_1 \cap L_2 = \overline{\overline{L_1} \cup \overline{L_2}}$

↳ regular languges are closed under Union and complement

↳ $\overline{L_1}$ and $\overline{L_2}$ are reg

↳ $\overline{L_1} \cup \overline{L_2}$ is reg

↳ $\overline{\overline{L_1} \cup \overline{L_2}}$ is reg.

3 a) $R_1 = a^* b^*$ $\rightarrow$ 0 or more a's followed by 0 or more b's

$R_2 = (a^* b^*)^*$ $\rightarrow$ 0 or more (0 or more a's followed by 0 or more b's)

$\underline{R_1 \subset R_2}$ $\quad$ $\underline{R_2 = abab \notin R_1}$

b) $R_1 = (a|b|c)^*$

$R_2 = (a^* b^* c^*)^*$

$\underline{R_1 = R_2}$

c) $R_1 = c^* c$

$R_2 = (cc)^* \mid c(cc)^*$

$\underline{R_1 \subset R_2}$ $\quad$ $\underline{R_1 = \varepsilon \notin R_2}$

d) $R_1 = c^*(a|b|\varepsilon)\, c^*$

$R_2 = c^* a c^* \mid c^* b c^*$

$\underline{R_2 \subset R_1}$ $\quad$ $\underline{R_1 = c\,\varepsilon\, c \notin R_2}$

e) $R_1 = \varnothing b \mid aa^*$

$R_2 = a^*$

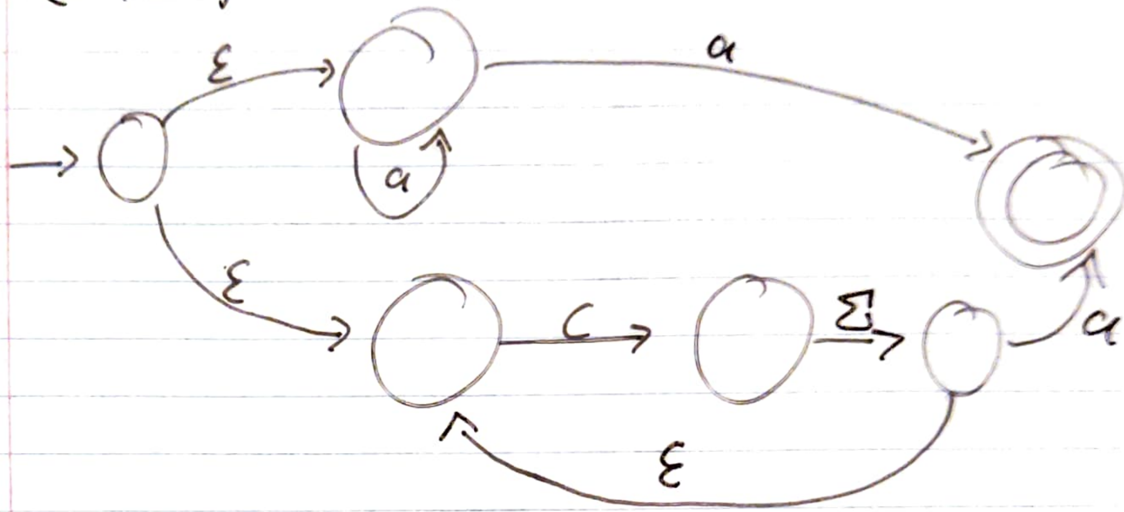
$R_1 \subset R_2$ $\quad$ $R_1 = \varepsilon \notin R_2$

4 a) $\Sigma^*(dog)\Sigma^*(cat\,|\,rat)\Sigma^*$

b) $x(\Sigma\Sigma\,|\,\Sigma\,|\,\varepsilon)y$

c) $(zz)^*$

5 $(a \mid c\Sigma)^* \rightarrow$ NFA



6a Proof:

- let A be a finite language w/ finite # of strings
  $\rightarrow \{a_0, a_1 \cdots a_n\}$
- Language Containing $\{a_i\}$ containing a singular string $\rightarrow$ regular.
- Union of finite language is regular (from class)
  ↳ $A = \{a_0\} \cup \{a_1\} \cup \cdots \{a_n\}$

b) Proof:

- Concatenation of 2 regular languages is regular
  ↳ $\boxed{x \in I \rightarrow y \in T}$
- Complement of regular language is regular
  ↳ $\boxed{x \notin I \rightarrow y \in E}$
- Union of regular languages are regular
  ↳ $\boxed{x \in I \rightarrow y \in T \text{ and } x \in I \rightarrow y \in E}$

c) Proof:

→ Since R is regular then the kleene star of R (R^*) is also regular. Same for any reg lang.

→ $(L^* | R^*)^*$ is regular as union of 2 regular languages is regular.