CSE 103 Computational Models

Fall 2021 Prof. Fremont **HW 1**

Due October 6 at 11:59pm

(5 questions, 215 points total)

1. (25 pts.) Alphabets and Strings

Let $\Sigma = \{a, b, c\}$. Recall that we denote the empty string by ε.

(a) What is the set Σ^2?

(b) What is $|\Sigma^5|$?

(c) What is the set $\{w_4 w_6 \mid w \in \Sigma^8\}$?

(d) What is the set $\{w \in \Sigma^* \mid |w| = 0 \text{ or } |w| = 1\}$?

(e) What is the set $\{xx \mid x \in \Sigma\}$?

2. (30 pts.) Languages

Let $\Sigma = \{0, 1\}$. For each of the following pairs of languages over Σ, state whether they are equal ($=$), one is a proper subset of the other ($\subset$), or if they are incomparable (neither is a subset of the other). If $L_1 \subset L_2$, give an example of a string in L_2 that is not in L_1, or vice versa if $L_2 \subset L_1$; if the languages are incomparable, give an example string from each language that is not in the other.

(a) $L_1 = \Sigma^2 \cup \Sigma^4$
$L_2 = \Sigma^6$

(b) $L_1 = \{1\}^*$
$L_2 = \{x \in \Sigma^* \mid |x| \text{ is odd}\}$

(c) $L_1 = \{x \in \Sigma^8 \mid \exists y \in \Sigma^*.\ x = yyy\}$
$L_2 = \Sigma^3 \cap \Sigma^8$
(N.B. The dot in L_1 simply indicates the end of the quantifier $\exists y \in \Sigma^*$; you can read the condition as "there exists some $y \in \Sigma^*$ such that $x = yyy$".)

(d) $L_1 = \Sigma^*$
$L_2 = \cup_{k \geq 1} \Sigma^k$

(e) $L_1 = \{x \in \Sigma^* \mid x \text{ is a binary encoding of a prime number}\}$
$L_2 = \{10\} \cup \{x \in \Sigma^* \mid \text{the last symbol of } x \text{ is } 1\}$

3. (50 pts.) Working with a DFA

Consider the DFA $M = (Q, \Sigma, \delta, q_0, F)$ where:

- $Q = \{1, 2, 3, 4\}$
- $\Sigma = \{a, b\}$
- $\delta(q, s) = \begin{cases} 1 & q = 2 \text{ or } s = b \\ 3 & q = 1 \text{ and } s = a \\ 4 & \text{otherwise} \end{cases}$
- $q_0 = 2$
- $F = \{1, 4\}$

(a) *(20 pts.)* Draw M as a graph.

(b) *(18 pts.)* Which of the following strings does M accept: $\varepsilon, b, a, ba, aaa, baab$? Give an accepting path (the sequence of states traced out by the DFA as it executes) for each accepted string.

(c) *(12 pts.)* What is the language $L(M)$ of M?

4. (60 pts.) Designing DFAs

For each of the following languages, draw a DFA (as a graph) recognizing it:

(a) All strings over $\Sigma = \{a,b,c\}$ which either contain an *b* or contain *both* an *a* and a *c*.

(b) All strings over $\Sigma = \{a,b,c,\ldots,z\}$ containing the substring *aaa*.

(c) All strings over $\Sigma = \{a,b,c\}$ which are in alphabetical order (e.g. *aaac* and *bcc* but not *aba* or *ca*).

5. (50 pts.) Counting Strings

We argued in class that given an alphabet Σ with $m = |\Sigma|$ symbols, there are m^n strings of length n: $|\Sigma^n| = m^n$. Let's prove this formally for any $m \geq 1$ and $n \geq 0$ using induction.

(a) Which quantity should we do the induction on?

(b) Choose a base case and state the claim you need to prove for it.

(c) Prove the base case.

(d) State the claim you need to prove for the inductive case, and the inductive hypothesis you can assume while proving it.

(e) Prove the inductive case.

(f) Conclude your proof of the original statement.

N.B. Whenever doing a proof by induction, make sure to include all of the information above. You don't have to break up your presentation into as many pieces as we did here, but you need to clearly state what you're proving by induction, separate the base and inductive cases, etc. For example, I often use language like "We prove this for all $k \in \{0, \dots, n\}$ by induction on k in decreasing order. In the base case $k = n$, we have...", and then in a new paragraph for the inductive case, "Now suppose the hypothesis holds for $k > 0$; then it also holds for $k - 1$ because...".