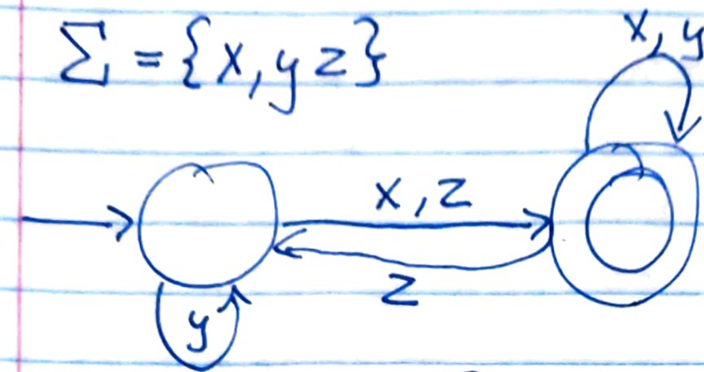
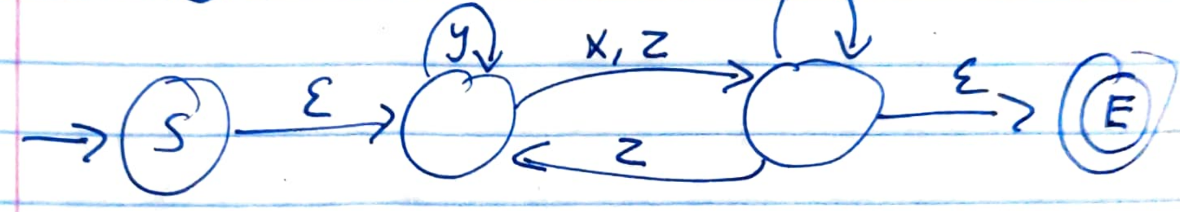
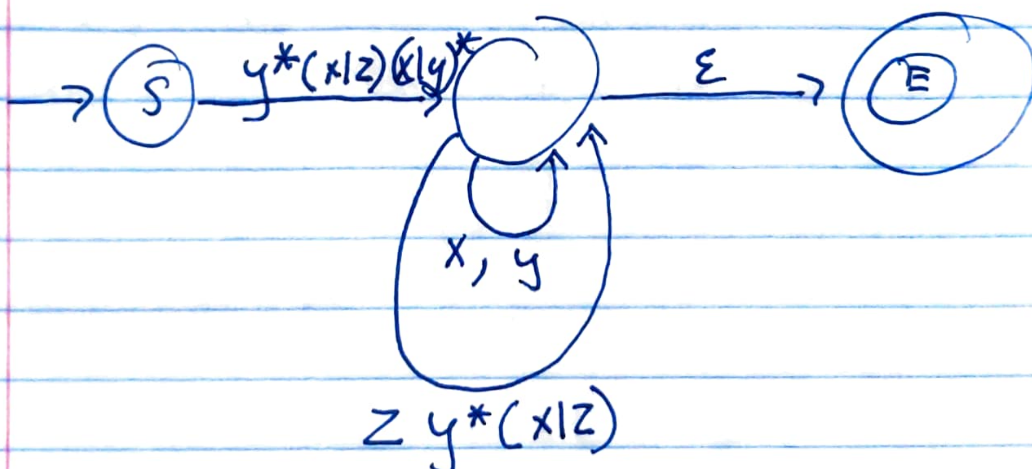
# HW 4

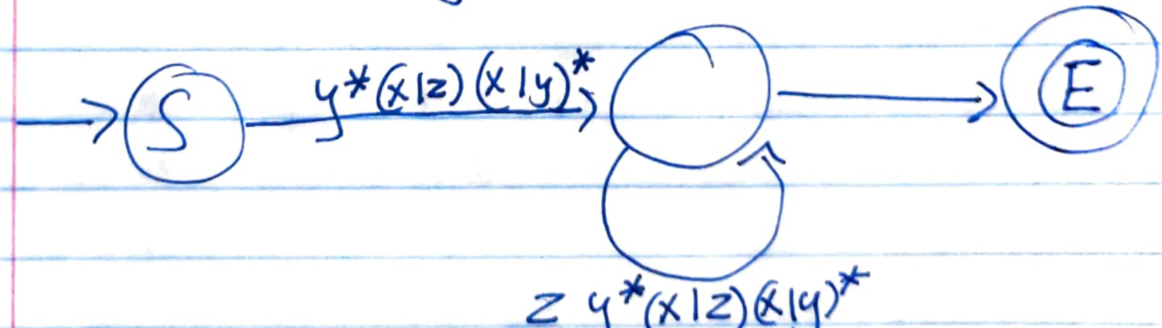
1) Convert DFA → Reg Exp

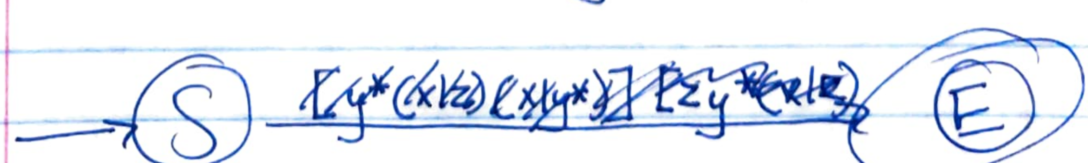
$\Sigma = \{x, y z\}$




Regex: [y*(x|z)(x|y)*] [zy*(x|z)(x|y)*]

Prove languag is not Regular.

2a $L = \{\text{properly nested parenthesis}\}$

let $w = (())$. ~~∈L~~ $w \in L$ ✓

if $w = xyz$

$x = ($
$y = ($
$z = ))$

$y$ consists of "("
$y \neq \varepsilon$ so there is atleast 1

$xyz$ is valid however $xyyz$ is not since $xy^2z$ will be imbalanced in terms of parenthesis. There fore $xy^iz \notin L$ $\forall i \geq 2$ and L is not Regular, as a resut.

2b $L = \{xx^R \mid x \in \Sigma^*\}$
$\Sigma = \{0,1\}$

Let $w \in L$ and $w = \emptyset^p 1 1 \emptyset^p$

$\boxed{\emptyset = \text{zero}}$

$w = xyz$
$x = \varepsilon$
$y = \emptyset^p$
$z = 11\emptyset^p$

~~$|w| \geq p \geq 1$~~

$|w| = 2p + 2 \geq p$
$|xy| \leq n$

"p" is given by pumping lemma.

$y$ is only $\emptyset$'s. Thus, $xyyz \notin L$ as then $x \neq x^R$ resulting in the Conclusion that L is not regular.

Attempt 3 brain hurty....

3 ~~A~~ ~~This is the official answer.~~

If $L$ is recognized by a DFA w/ $n$ states then Prove $|L| \leq |\Sigma^{\leq n-1}|$

In a nutshell, a DFA w/ $n$ states means that there are $n-1$ transitions requiring input from word $w \in L$. Therefore word $w \in L$ has a max length of $n-1$. If there exists an accepting string of length greater than $n-1$ that means that there is a loop within the DFA introducing the addition of the Kleene star. This means that the language is now infinite due to the loop. This then means that $|L| \leq |\Sigma^{\leq n-1}|$ is true since the language (finite) ~~can~~ contains words of max length $n-1$.

① $n-1$ since the start state is an extra state that has an ~~empty~~ empty enter transition.

3 L is ~~[illegible]~~ recognized by a DFA w/n states. if L is finite then $|L| \leq |\Sigma^{\leq n-1}|$

☆ In a nutshell, ~~A~~ a DFA w/ nstates means that any word w in L is

① of max length $\underline{n-1}$. If $|w|>n$, then there exists a loop within the DFA. The Existence of a loop results in the language of the DFA being infinite. This is because the Kleene star for the loop means that ∅ or more of that symbol is required to be a valid string and since there is no limit that means that the language is infinite due to the loop. Therefore $|L| \leq |\Sigma^{\leq n-1}|$. ✓

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~~Cont.~~

This is main answer

In terms of pumping lemma:

$w \in L$

$K = \max_{w \in L} |w|$

This is 2nd attempt

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Then $p = k+1 \Rightarrow$ Pumping Length min.

Since there are no strings of length $K+1$ or more in the language then every word $w \in L$ satisfies all 3 conditions of the pumping lemma.

Therefore if ~~[illegible]~~ $k+1>n$ then all words are in $\Sigma^{\leq n-1}$

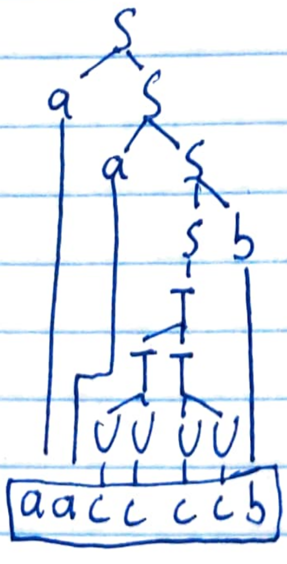
4 CFG G

$\Sigma = \{a, b, c\}$
$V = \{S, T, U\}$
$P = \text{rules} = \begin{cases} S \to aS \mid Sb \mid T \\ T \to TT \mid UU \\ U \to c \end{cases}$

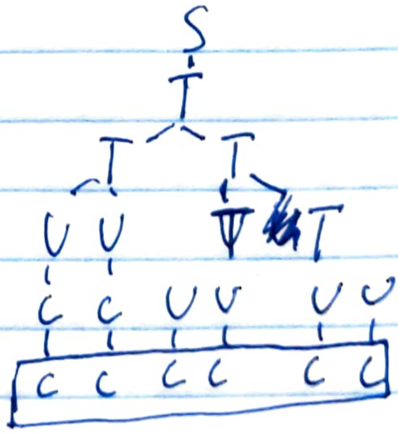
a) $\varepsilon$ → not derivable

b) c → not derivable

c) aacccb




f) cccccc




d ab not deriveable

e abcc not derivable

5 Design CFG

a) $\Sigma = \{x, y, z, +, =\}$

$L = \begin{cases} x = x \\ y + y = x + z \end{cases}$

$V = \{S, E, \cancel{V} T\}$

$P = \text{rules} = \left\{ \begin{array}{l} S \Rightarrow E = E \\ E \Rightarrow E + V \\ E \Rightarrow x \\ E \Rightarrow V + V \\ V \Rightarrow y \mid z \end{array} \right\}$

$q_0 = S$

$F = \{ l \in L \}$

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b $\Sigma = \{ f, g, x, y, (, ), , \}$

$L = \begin{cases} x \\ f(f(y)) \\ g(x, g(f(x), f(y))) \end{cases}$

$P = \text{rules} = \quad S \to V \mid f(E) \mid g(E, E)$

$E \to f(V) \mid V \mid g(V, V)$

$V \to x \mid y \mid g(E, E)$

$\text{Variables} = \{S, E, V\}$

$q_0 = S$

$\Sigma = \{a, b\}$  $V = \{S, T\}$

6 a) $S \to TT \mid a$ unambiguous

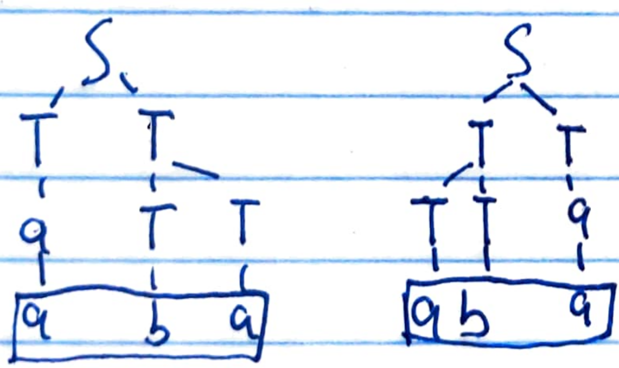
$T \to a \mid b$

b) $S \to \cancel{TT}\ ST \mid Sb$ unambigous

$T \to a \mid b$ (does not terminate)

c) $S \to TT \mid \varepsilon$ ambiguous

$T \to TT \mid a \mid b$ $w = aba$




2 Derivations