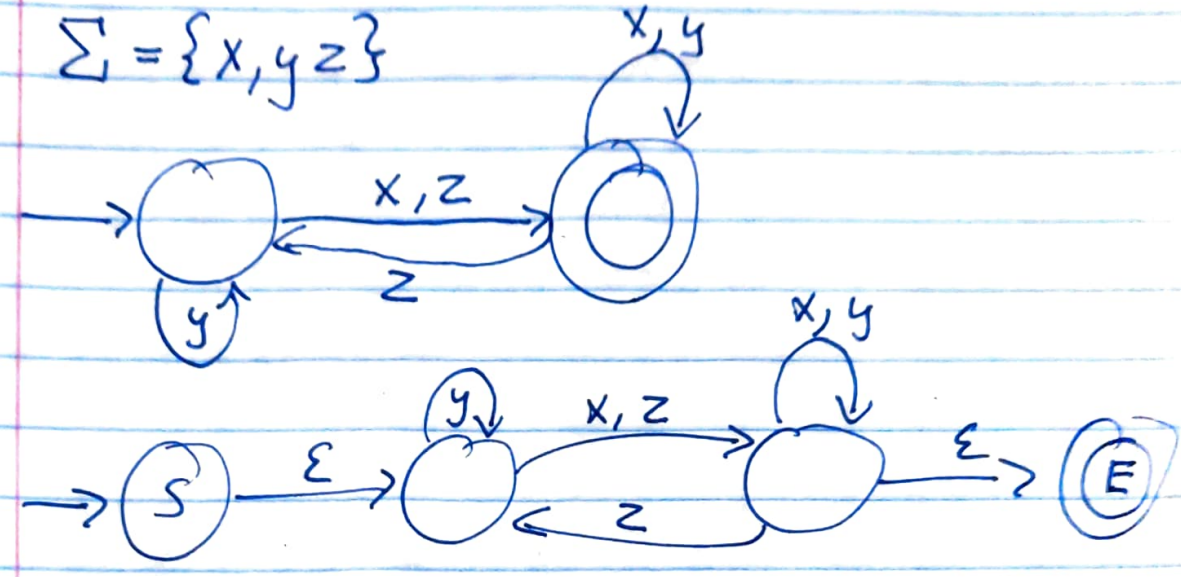
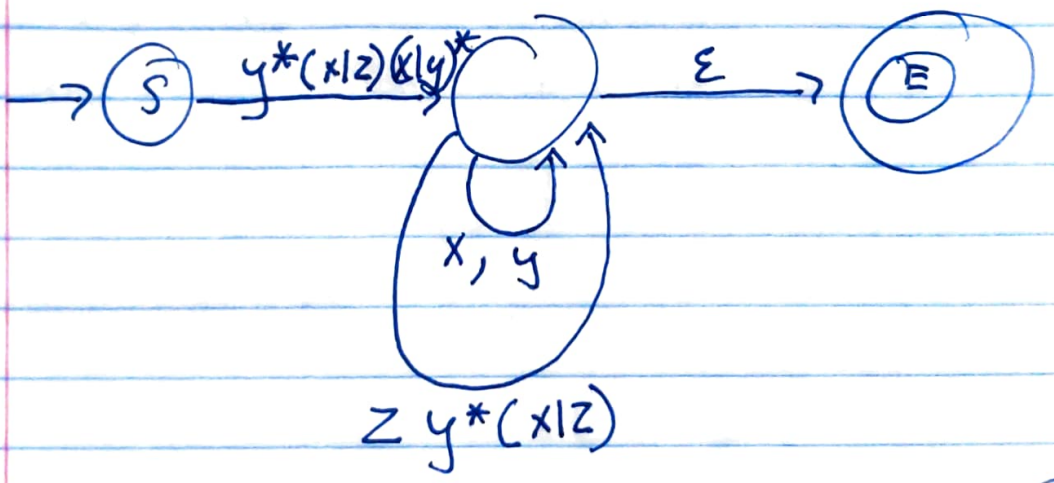
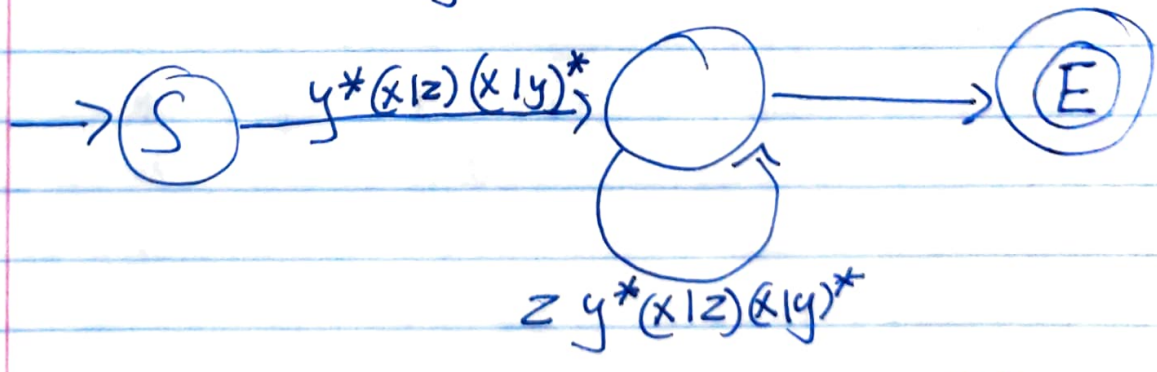
HW 4

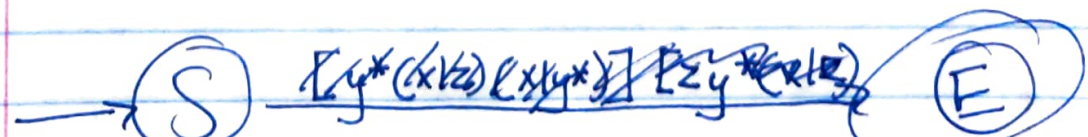
1) Convert DFA → Reg Exp

$\Sigma = \{x, y, z\}$









Regex: $[y^*(x|z)(x|y)^*][zy^*(x|z)(x|y)^*]$

Prove language is not Regular.

2a

$L = \{$ properly nested parenthesis$\}$

let $w = (())$. $w \in L$ ✓

if $w = xyz$

$x = ($
$y = ($
$z =))$

} y consists of "(" $y \neq \varepsilon$ so there is atleast 1

xyz is valid however $xyyz$ is not since xy^2z will be imbalanced in terms of parenthesis. Therefore $xy^iz \notin L$ $\forall i \geq 2$ and L is not Regular, as a result.

2b $L = \{ xx^R \mid x \in \Sigma^* \}$

$\Sigma = \{0, 1\}$

Let $w \in L$ and $w = 0^p 1 1 0^p$

$\emptyset$ = zero

"p" is given by pumping lemma.

$w = xyz$
$x = \varepsilon$
$y = 0^p$
$z = 11 0^p$

$|w| = 2p + 2 \geq p$
$|xy| \leq n$

y is only 0's. Thus, $xyyz \notin L$ as then $x \neq x^R$ resulting in the conclusion that L is not regular.

3 L is recognized by a DFA w/ n states.
if L is finite then $|L| \leq |\Sigma^{\leq n-1}|$

In a nutshell, a DFA w/ n states means that any word w in L is of max length n. If $|w| > n$, then there exists a loop within the DFA. The Existence of a loop results in the language of the DFA being infinite. This is because the kleene star for the loop means that 0 or more of that symbol is required to be a valid string and since there is no limit that means that the language is infinite due to the loop. Therefore $|L| \leq |\Sigma^{\leq n-1}|$. ✓

This is main answer

This is 2nd attempt

In terms of pumping lemma:

$w \in L$

$k = \max_{w \in L} |w|$

Then $p = k+1$ => Pumping Length min.
Since there are no strings of length $k+1$ or more in the language then every word $w \in L$ satisfies all 3 conditions of the pumping lemma.

Therefore if $k+1 > n$ then all words are in $\Sigma^{\leq n-1}$

4 CFG G

$\Sigma = \{a, b, c\}$

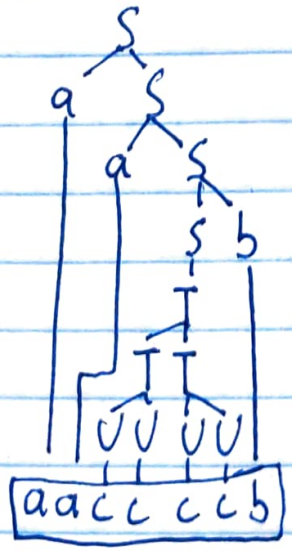
$V = \{S, T, U\}$

$P = \text{rules} = \begin{cases} S \to aS \mid Sb \mid T \\ T \to TT \mid UU \\ U \to C \end{cases}$

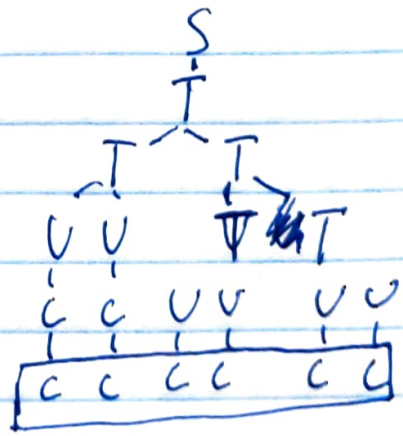
a) ε → not derivable

b) C → not derivable

c) aaccccb



f) CCCCCC



d ab not derivable

e abcc not derivable

5. Design CFG

a) $\Sigma = \{x, y, z, +, =\}$

$L = \{ x = x,\ y + y = x + z \}$

$V = \{S, E, T\}$

$P = \text{rules} = \{ S \Rightarrow E = E,\ E \Rightarrow E + V,\ E \Rightarrow x,\ E \Rightarrow V + V,\ V \Rightarrow y \mid z \}$

$q_0 = S$

$F = \{ l \in L \}$

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b) $\Sigma = \{ f, g, x, y, (,), , \}$

$L = \{ x,\ f(f(y)),\ g(x, g(f(x), f(y))) \}$

$P = \text{rules} =$

$S \rightarrow V \mid f(E) \mid g(E, E)$

$E \rightarrow f(V) \mid V \mid g(V, V)$

$V \rightarrow x \mid y \mid g(E, E)$

Variables = $\{S, E, V\}$

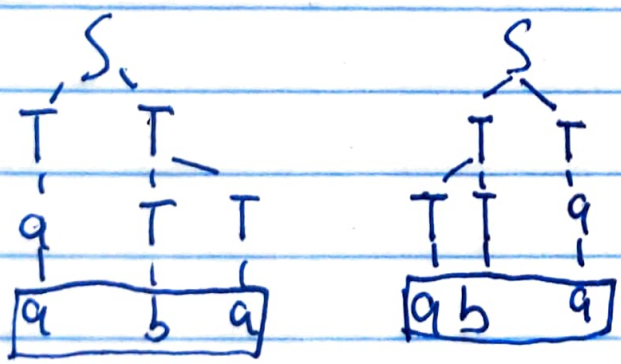
$q_0 = S$

$\Sigma = \{a, b\}$ $V = \{S, T\}$

6 a) $S \to TT \mid a$ unambiguous
$T \to a \mid b$

b) $S \to \cancel{TT}\ ST \mid Sb$ unambiguous (does not terminate)
$T \to a \mid b$

c) $S \to TT \mid \varepsilon$ ambiguous
$T \to TT \mid a \mid b$ $w = aba$



2 Derivations