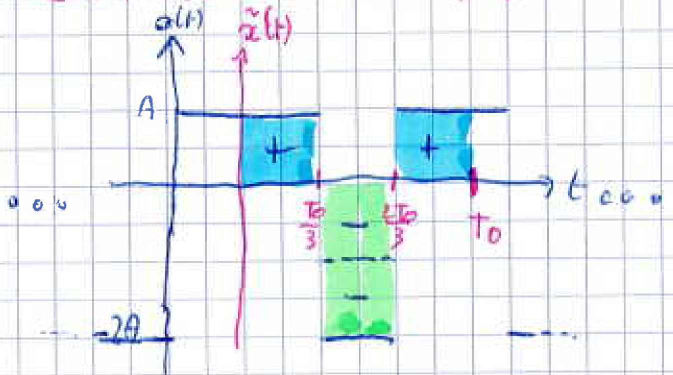
Exercice 3 : SMART ?



Existence

$\tilde{x}$ est $\mathcal{C}^0$ presque partout sauf en $\frac{T_0}{3}$ et $\frac{2T_0}{3}$ et bornée de période T_0

donc condi^e de Dirichlet OK ✓ elle admet une série.

$$a_0 = \frac{1}{T_0}\int_0^{T_0} \tilde{x}(t).dt = 2 \times \frac{A T_0}{3} + \frac{-2A . T_0}{3} = 0$$

$\tilde{x}(T_0 - t) = \tilde{x}(t) = \tilde{x}(-t)$ donc impaire $\Rightarrow b_n = 0 \quad \forall n \in \mathbb{N}^*$

$$\tilde{a}(n) = \frac{2}{T_0}\underbrace{\int_0^{T_0/3} A.\cos(2\pi n \tfrac{t}{T_0}).dt}_{I_1} + \frac{2}{T_0}\underbrace{\int_{T_0/3}^{2\frac{T_0}{3}} -2A \cos(2\pi n \tfrac{t}{T_0}).dt}_{I_2} + \frac{2}{T_0}\underbrace{\int_{2\frac{T_0}{3}}^{T_0} A \cos(2\pi n \tfrac{t}{T_0})}_{I_3}$$

$I_3 = I_1$ car $A.\cos(2\pi n \frac{t}{T_0})$ impaire et période T_0 ou chgt avec $x = T_0 - t$

$t = T_0 - x$

$t = T_0 \rightarrow 0 = x$

$t = \frac{2T_0}{3} \rightarrow x = \frac{T_0}{3}$

$dt = -dx$

$$-\int_0^{T_0/3} = \int_{T_0/3}^{0}$$

compense

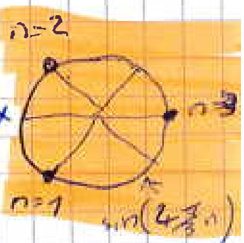
$$I_1 = \frac{2A}{T_0}\left[\frac{\sin(2\pi n\, t/T_0)}{2\pi n/T_0}\right]_0^{T_0/3} = \frac{2A}{2\pi n}\left(\sin(n\, 2\pi/3) - 0\right)$$

$$I_1 = \frac{2A}{2\pi n}\sin\left(n\frac{2\pi}{3}\right) = \begin{cases} \frac{A}{\cancel{2}\sqrt{2}\pi n} & k\%3 = 1 \\ -\frac{A}{\cancel{2}\sqrt{2}\pi n} & \text{si } k\%3 = 2 \\ 0 & \text{si } k = 3p \quad p \in \mathbb{N}. \end{cases}$$

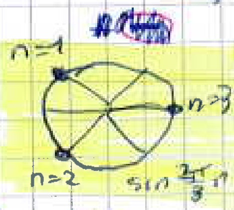
$$I_2 = -\frac{4A}{T_0}\left[\frac{\sin(2\pi n\, t/T_0)}{2\pi n/T_0}\right]_{T_0/3}^{2T_0/3} = -\frac{4A}{2\pi n}\left(\sin(2 . 2\pi/3\, n) - \sin(2\pi/3\, n)\right)$$

$$= \frac{4A}{2\pi n}\sin(2\pi/3\, n) - \frac{4A}{2\pi n}\sin\left(\frac{4\pi}{3}.n\right)$$

$-1\times$



$2\times$



$$\tilde{a}(n) = 2I_1 + I_2$$

$$= \frac{2A}{2\pi n}\sin\left(n\frac{2\pi}{3}\right) + \frac{4A}{2\pi n}\sin\left(n\frac{2\pi}{3}\right) - \frac{4A}{2\pi n}\sin\left(\frac{4\pi}{3}n\right)$$

$$= \frac{4A}{2\pi n}\sin\left(n\frac{2\pi}{3}\right) - \frac{2A}{\cancel{2}\pi n}\sin\left(n\frac{4\pi}{3}\right) = \frac{2A}{\pi n}\left(2\sin\left(n\frac{2\pi}{3}\right) - \sin\left(n\frac{4\pi}{3}\right)\right)$$

$$\boxed{\tilde{a}(n) = \frac{2A}{\pi n}\begin{cases} 3/\sqrt{2} & \text{si } n\%3 = 1 \\ -3/\sqrt{2} & \text{si } n\%3 = 2 \\ 0 & \text{si } n\%3 = 0 \Leftrightarrow n = 3p,\ p \in \mathbb{N} \end{cases} = \frac{6A}{\pi n}\sin\left(n\frac{2\pi}{3}\right)}$$

Vérifications :

$\tilde{a}(n) \xrightarrow[n\to+\infty]{} 0$ sinon Σ DV

$\tilde{a}(n) = A_0 \cdots$ car $\times 2$ le sig $\Leftrightarrow \times 2$ les composantes

$\tilde{a}(n)$ indep de T_0 car $\times 2$ T_0 $\Rightarrow$ composantes 2x lentes mais de m^ amplitude

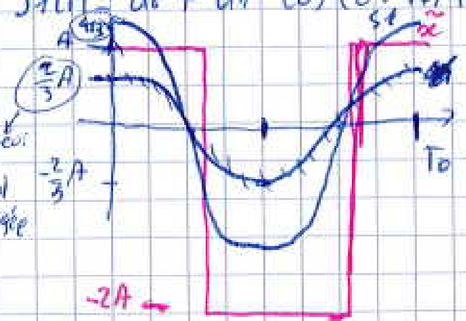
Donc $\sum_{n\in\mathbb{N}^*} \frac{3A}{\pi n} \sin\left(n\frac{2\pi}{3}\right) \cdot \cos\left(n 2\pi \frac{t}{T_0}\right)$ CV

et CV presque partout vers $\tilde{x}(t)$.

$\tilde{a}(0) = 0$

$\tilde{a}(1) = \frac{3A}{\sqrt{2}\pi} \approx \frac{2A}{3}$

$S_1(t) = a_0 + a_1 \cos(2\pi t/T_0) \approx \tilde{x}(t)$



En tout cas pas erreur de signe !

Passage au $\tilde{c}(n)$:

$\tilde{c}(n) = \frac{\tilde{a}(n) - i\,\tilde{b}(n)}{2}$ or $\tilde{b}(n) = 0$

$\tilde{c}(n) = \frac{\tilde{a}(n)}{2} = \frac{3A}{2\pi n} \sin\left(n\frac{2\pi}{3}\right)$

$= 2A \operatorname{sinc}\left(n\frac{2\pi}{3}\right)$

sinc : $x \mapsto \frac{\sin(x)}{x}$

$a(n).\cos(n\omega_0 t) + b(n).\sin(n\omega_0 t)$

$\| \quad$ Euler ou phaseur

$\mathcal{R}\left[a(n).e^{in\omega_0 t}\right] + \mathcal{R}\left[b(n).e^{-i\frac{\pi}{2}} e^{in\omega_0 t}\right]$

↳ sin en retard de $\frac{\pi}{2}$ sur cos.

$\|$

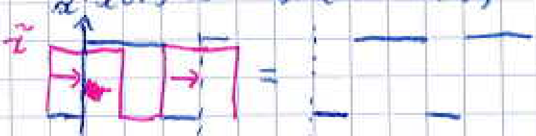
$\mathcal{R}\left[(a(n) - i\,b(n)).e^{in\omega_0 t}\right]$

$\| \qquad z + \bar{z} = 2\,\mathcal{R}(z)$

$\underbrace{\frac{a(n) - i\,b(n)}{2}}_{c(n)} e^{in\omega_0 t} + \underbrace{\frac{a(n) + i\,b(n)}{2}}_{c(-n) = \bar{c}(n)} . e^{-in\omega_0 t}$

Retour à $x(t)$

$x(t) = \tilde{x}(t - T_0/3)$



x = retard de $\frac{T_0}{3}$ de $\tilde{x}$

$c(n) = e^{-\tau p}.\tilde{c}(n) = e^{-in\frac{2\pi}{3}}.\tilde{c}(n)$

$p = in\omega_0 = i2\pi n/T_0$

$\tau = \frac{T_0}{3}$

$\tilde{x}(t) = \sum_{n=-\infty}^{+\infty} \tilde{c}(n).e^{in\omega_0 t}$

$\tilde{x}(t-\tau) = \sum_{n=-\infty}^{+\infty} \tilde{c}(n).e^{in\omega_0(t-\tau)}$

$\tilde{x}(t-\tau) = \sum_{n=-\infty}^{+\infty} \underbrace{\tilde{c}(n).e^{-in\omega_0\tau}}_{c(n)} . e^{in\omega_0 t}$

$\| $

$x(t) = \sum_{n\in\mathbb{Z}} c(n).e^{in\omega_0 t}$

$c(n) = \frac{3A}{2\pi n} \sin\left(n\frac{2\pi}{3}\right) . e^{-in2\pi/3}$

↓ Euler

$= \frac{3A}{2\pi n}\left[\frac{e^{in2\pi/3} - e^{-in2\pi/3}}{2i} . e^{-in\frac{2\pi}{3}}\right] = \frac{3A}{2\pi n} . \frac{+i}{2} . \left(e^{in\frac{4\pi}{3}} - e^{0}\right)$

↓ Euler

$= \frac{3A}{4\pi n}\left[\left(-1 + \cos\left(n\frac{4\pi}{3}\right)\right).i - \sin\left(n\frac{4\pi}{3}\right)\right]$

$c(n) = \underbrace{\frac{3A}{4\pi n} \sin\left(n\frac{4\pi}{3}\right)}_{\frac{a(n)}{2}} + i . \underbrace{\frac{3A}{4\pi n}\left(-1 + \cos\left(n\frac{4\pi}{3}\right)\right)}_{-\frac{b(n)}{2}}$

$a(n) = \frac{3A}{2\pi n}\left(\sin \frac{4\pi}{3} n\right) = \begin{cases} 0 & \text{si } n = 3p \in \mathbb{N} \\ -\frac{3A}{8\sqrt{2}\pi n} & \text{si } n = 3p+1 \in \mathbb{N} \\ \frac{3A}{8\sqrt{2}\pi n} & \text{si } n = 3p+2 \in \mathbb{N} \end{cases}$

$b(n) = \frac{3A}{2\pi n}\left(\cos\left(n\frac{4\pi}{3}\right) - 1\right) = \begin{cases} 0 & \text{si } n = 3p \\ -\frac{9A}{2\pi n} & \text{sinon} \end{cases}$