II TFD

$e^{i2\pi f t}$ avec $t = kT_e$ et $f = n\Delta f$

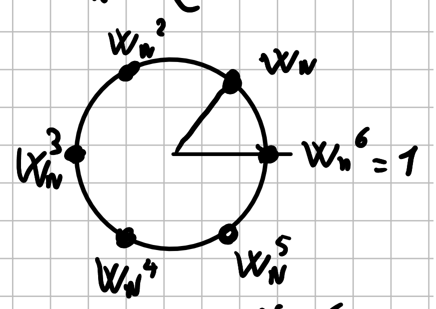
N points de $T_e \Rightarrow$ période $NT_e \Rightarrow \Delta f = \frac{1}{NT_e} = \frac{F_e}{N}$

$$e^{i2\pi n \Delta f k T_e} = e^{i2\pi \frac{F_e}{N} T_e n k} = e^{\frac{i2\pi}{N} n k} = W_N^{nk}$$

$$\hat{S}[n] = \langle\langle s, k \mapsto e^{\frac{i2\pi}{N}kn} \rangle\rangle_p = \sum_{k=0}^{N-1} s[k] \cdot W_N^{nk}$$

$$\boxed{\hat{S}[n] = \sum_{k=0}^{N-1} s[k] \cdot e^{\frac{i2\pi}{N} n \cdot k}}$$

$$W_N = e^{\frac{i2\pi}{N}}$$



pour $N = 6$

W_N racine Nème de 1

$\hat{\delta_0}$:
$$\hat{\delta_0}[n] = \sum_{k=0}^{N-1} \underbrace{\delta_0[k]}_{=0 \text{ sauf pour } k=0} \cdot \overline{W_N^{nk}}$$
$$= \delta_0[0] \cdot W_N^{n.0} = 1.1 = 1$$

$\hat{\delta_0} : n \mapsto 1$ [V] Verifs : ✓ $\hat{\delta_0}[-n] = \overline{\hat{\delta_0}[n]}$ car $\bar{1} = 1$

spectre de signal réel.

Vérif laplace : δ_0 (impulsion unité (discret)) $\longleftrightarrow$ δ_0 (impulsion dirac (continu))

$\mathcal{L}[\delta_0] : p \mapsto 1$

$\hat{\delta_\ell}$:
$$\hat{\delta_\ell}[n] = \sum_{k=0}^{N-1} \underbrace{\delta_\ell[k]}_{=0 \text{ sauf pour } k=\ell} \cdot \overline{W_N^{nk}}$$
$$= \underbrace{\delta_\ell[\ell]}_{1} \cdot \underbrace{W_N^{-n.\ell}}_{e^{-\frac{i2\pi}{N} n.\ell}}$$

$$\hat{\delta_\ell}[n] = e^{-\frac{i2\pi}{N} \ell . n}$$

$\hat{\delta_\ell} : n \mapsto e^{-\frac{i2\pi}{N}\ell . n}$ [V]

✓ Vérif spectre Hilbert : $\hat{\delta_\ell}[-n] = \overline{\hat{\delta_\ell}[n]}$
$$e^{\frac{i2\pi}{N} n \ell} = \overline{e^{-\frac{i2\pi}{N} n \ell}}$$

✓ $e^{-i2\pi \ell \frac{n}{N}}$ [1] sans dimention [1]

✓ Verif Laplace :

δ_ℓ (impulsion unité retardée) $\longleftrightarrow$ δ_ℓ (impulsion de dirac retardée de $\ell . T_e$)

Th. retard : $\mathcal{L}[\delta_\ell] = e^{-\ell p} \cdot \mathcal{L}[\delta_0]$

$e^{-\tau p} \leftrightarrow$ retard de $\tau = \ell T_e$

$$\mathcal{L}[\delta_\ell] = e^{-\ell \underbrace{T_e p}_{[s]\,[\frac{1}{s}]}}$$

[1] sans dimention

On remplace p [1/s] par $i2\pi f = i2\pi n \Delta f = \frac{i2\pi}{N} n F_e$
$$\Rightarrow \hat{\delta_\ell}[n] = e^{\underbrace{-\ell \cancel{T_e} i2\pi n \frac{\cancel{F_e}}{N}}_{[1]}} = e^{-\frac{i2\pi}{N}\ell n}$$