Applications of Linear algebra in Analytics domain

- An overview of Image processing and recommender systems

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AGENDA

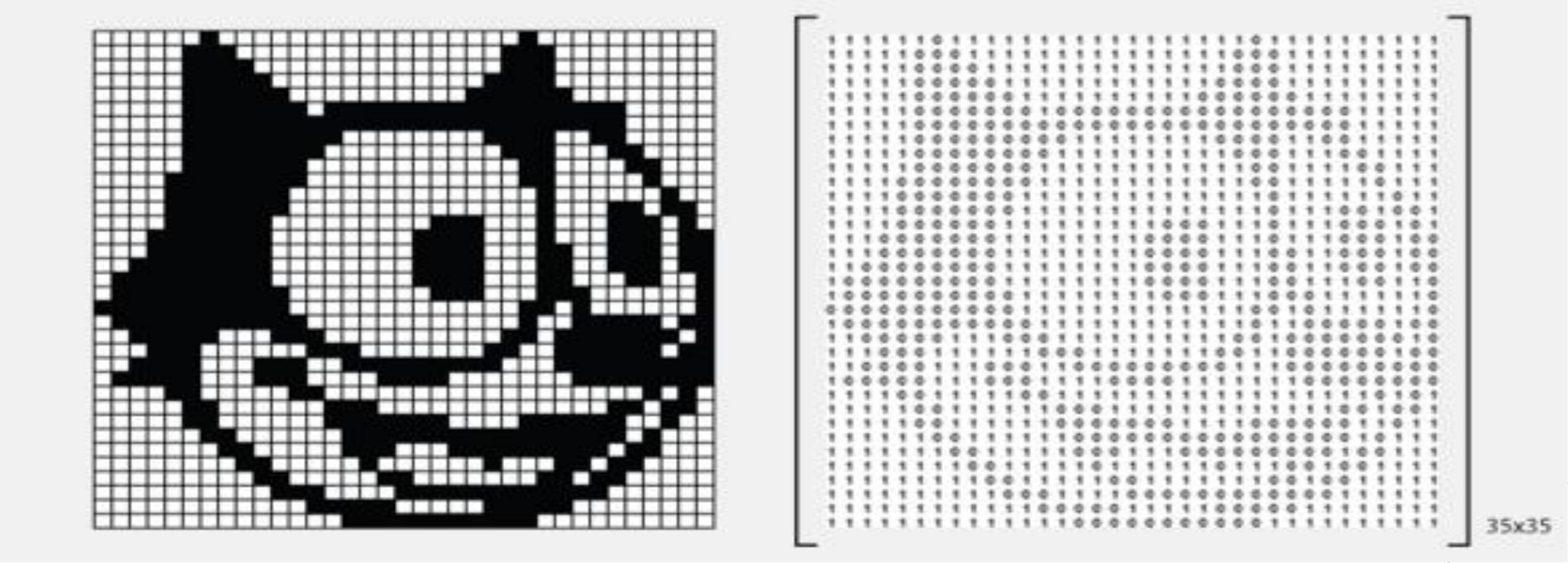
- Introduce Digital Image processing
- Science behind Image processing
- Case study – Image processing
- Introduce Recommender systems
- Science behind Recommender systems
- Case study- Recommendation engine
- Opportunities & Awareness - Discussion

Image processing & related techniques

-From the perspective of algebra application

How are digital images are read?

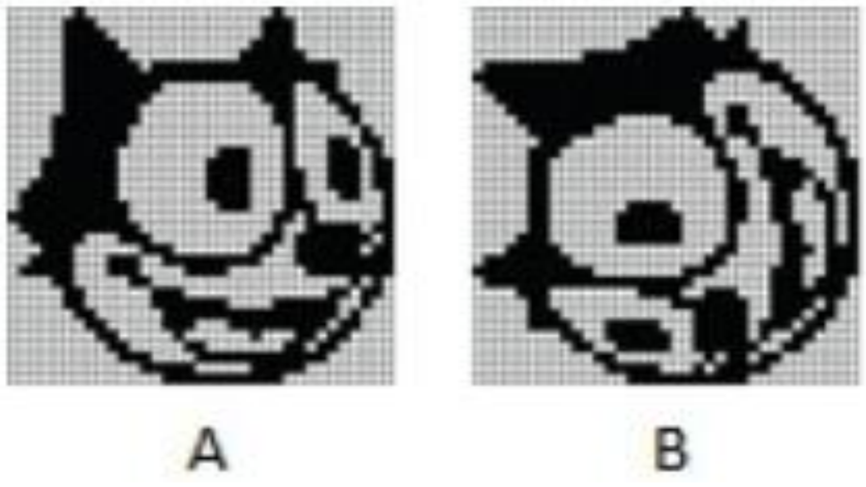
- Generally digital images are read represented as matrices.
- Each element of the matrix determines the intensity of the corresponding pixel



- Grayscale images can also be represented by matrices
- For our convenience, most of the gray digital images uses integer in the range (0-255),giving a total of $2^{8} = 256$ different levels of gray.
- Similarly Color images can also be represented by 3 matrices (3 Channels).
- RGB color system : Each matrix specifies the amount of red, green and blue that makes up the image
- In the RGB system, it is possible to represent 2^{24} different colors (2^{8} for each channel).

Digital image processing and Matrices

- Once a digital image can be represented by matrices, we may ask how operations on their elements affect the corresponding image.
- A simple illustration is given below.
- If Binary Image A is represented as a matrix, say $A = (a_{i,j})$ then image B is the transpose of Image A (i.e) B = A^{T}.



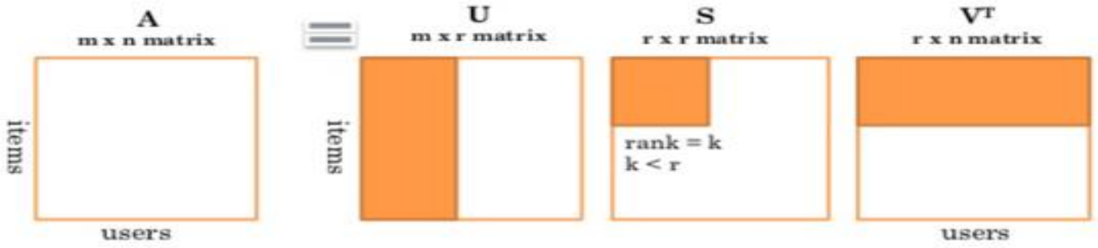
Practical usage of Linear Algebra concepts

- How does Images from Space and satellites gets transmitted through communication channel ?
- What is the mathematical technique generally we use for this ?

Singular Value Decomposition

- What is SVD ?
  - Any matrix A can be decomposed to three matrices U , Σ, and V such that A=UΣV, this is called singular value decomposition. The columns of U and V are orthonormal and Σ is diagonal.

$$A = U \times S \times V^T$$



$$A_k = U_k \times S_k \times V_k^T$$

- What does SVD yield in terms of dimensionality?

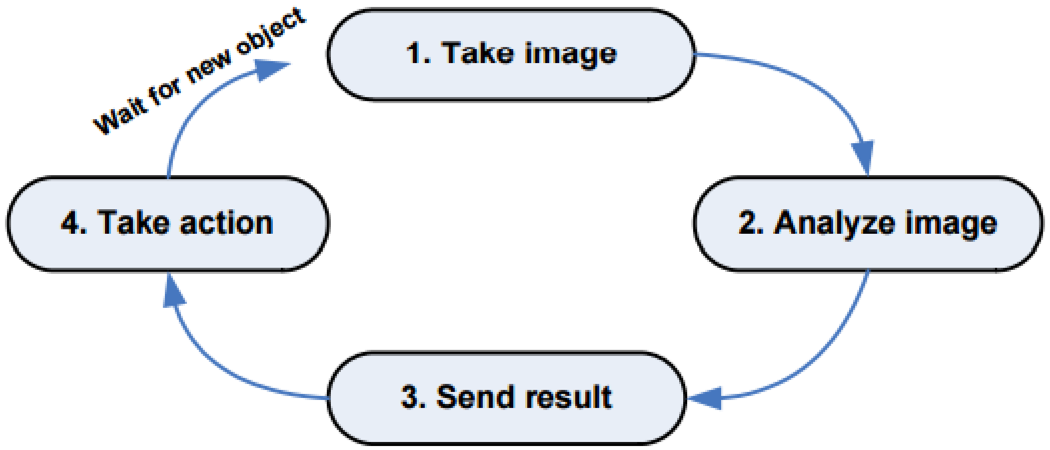
If A is $m \times n$ (m rows and n columns), U will be $m \times m$, Σ will be $m \times n$ and V will be $n \times n$. However, there are only $r = rank(A)$ non-zero values in Σ, i.e. $\sigma_1, \ldots, \sigma_r \neq 0$; $\sigma_{r+1}, \ldots, \sigma_n = 0$. Therefore columns of U beyond the r^{th} column and rows of V beyond the r^{th} row do not contribute to A and are usually omitted, leaving U an $m \times r$ matrix, Σ an $r \times r$ diagonal matrix and V an $r \times n$ matrix.

Image Compression using Python

Image
pression using Pyt

What is Machine Vision ?

- Machine vision is the technology to replace or complement manual inspections and measurements with digital cameras and image processing.
- Machine vision in operation can be described by a four-step flow:



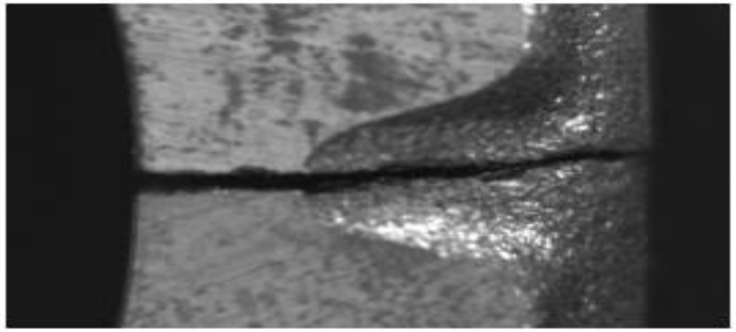
- From the application standpoint, Machine Vision can be divided in to 4 types:
  - Locate
  - Measure
  - Inspect
  - Identify
- Locate : In locating applications, the purpose of the vision system is to find the object and report its position and orientation.
- Measure : In measurement applications the purpose of the vision system is to measure physical dimensions of the object
- Inspect :In inspection applications the purpose of the vision system is to validate certain features, for example presence or absence of a correct label on a bottle, screws in an assembly, chocolates in a box, or defects.
- Identify :In an identification application the vision system reads various codes and alphanumeric characters (text and numbers).

Processing and Analysis of Image

- Once the Image has been grabbed, we can do analysis based on our application/necessity.
- Some common analysis include Region of Interest (ROI), Pixel counting, Edge detection, Thresholding, Median Filter etc.
- More sophisticated analysis include Blob Analysis, Object detection, Pattern matching, Optical character Recognition etc.
- How does ROI and Pixel counting are used in real time ?.

Region of interest and Pixel counting

- ROI
  - A ROI (Region of Interest) is a selected area of concern within an image.
  - The purpose of ROIs is to restrict area of analysis.
- Pixel counting
  - The algorithm finds the number of pixels within a ROI that have intensities within a certain gray level interval.
  - Pixel counting is used to measure area and to find deviances from a normal appearance of an object, for example missing pieces, spots, or cracks.



Automotive part with crack.

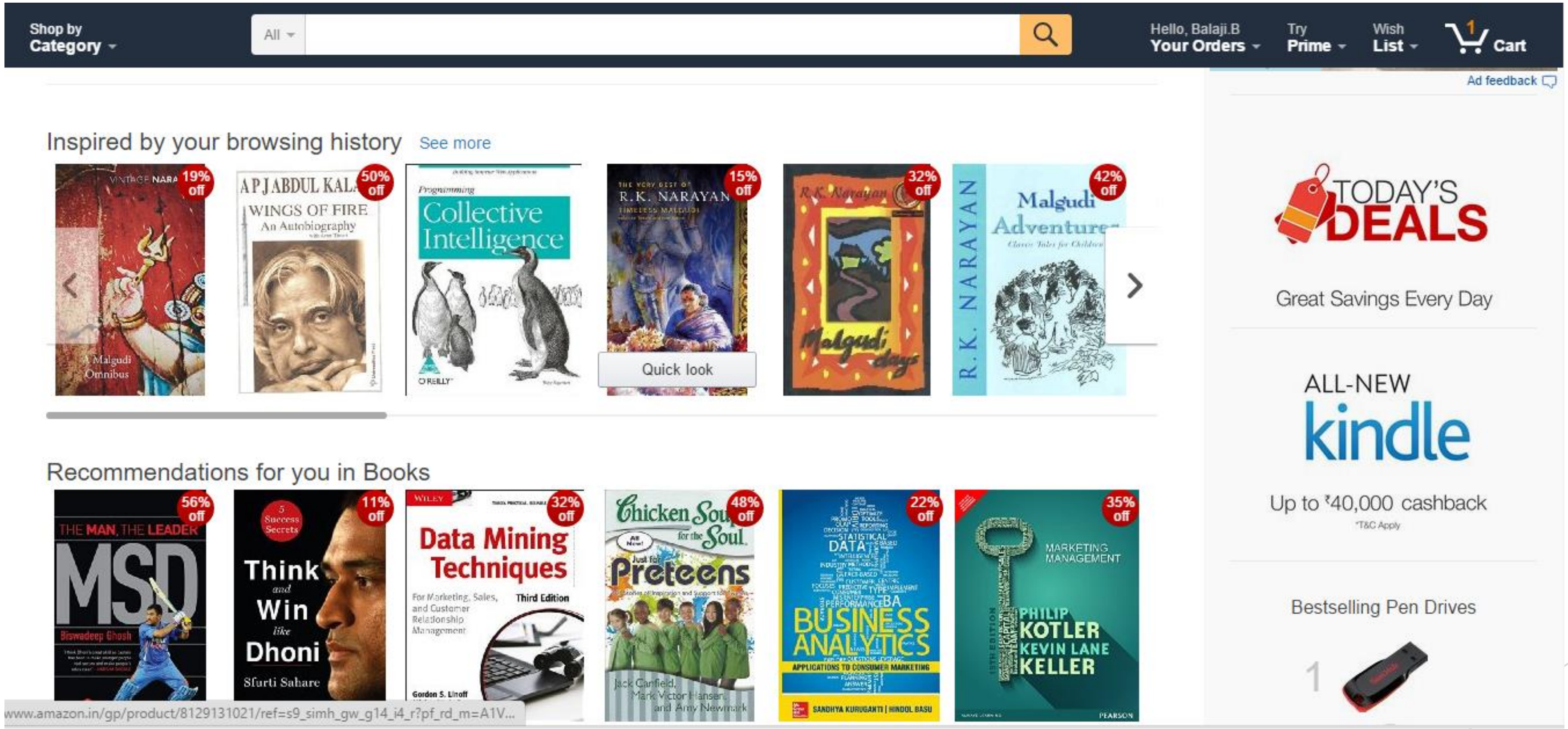
→

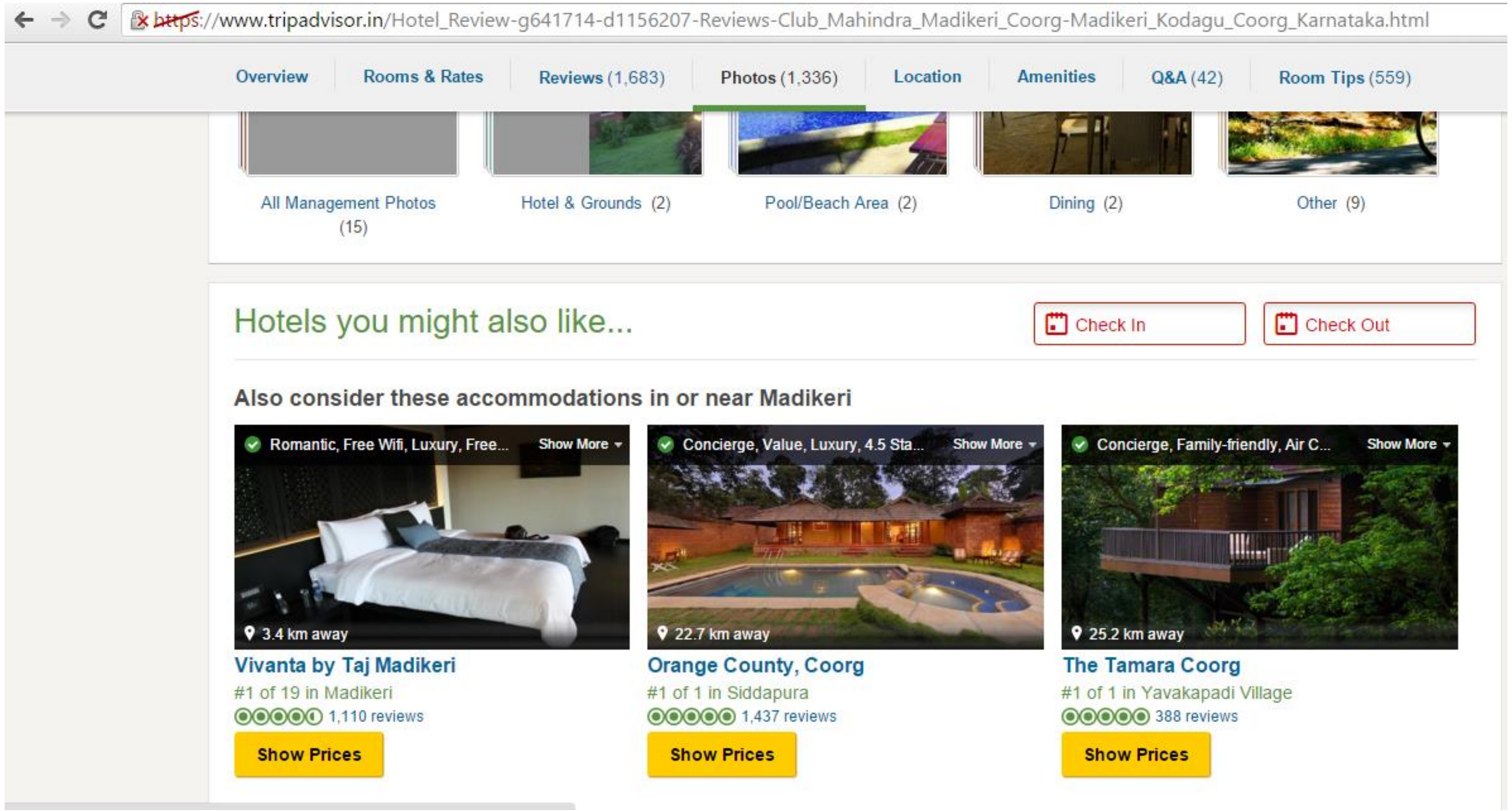
The crack is found using a darkfield illumination and by counting the dark pixels inside the ROI.

Recommender Systems

-From the perspective of algebra application

Recommender system




https://www.tripadvisor.in/Hotel_Review-g641714-d1156207-Reviews-Club_Mahindra_Madikeri_Coorg-Madikeri_Kodagu_Coorg_Karnataka.html
Overview
Rooms & Rates
Reviews (1,683)
Photos (1,336)
Location
Amenities
Q&A (42)
Room Tips (559)
All Management Photos (15)
Hotel & Grounds (2)
Pool/Beach Area (2)
Dining (2)
Other (9)
Hotels you might also like...
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Also consider these accommodations in or near Madikeri
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3.4 km away
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#1 of 19 in Madikeri
1,110 reviews
Show Prices
Concierge, Value, Luxury, 4.5 Sta...
Show More
22.7 km away
Orange County, Coorg
#1 of 1 in Siddapura
1,437 reviews
Show Prices
Concierge, Family-friendly, Air C...
Show More
25.2 km away
The Tamara Coorg
#1 of 1 in Yavakapadi Village
388 reviews
Show Prices

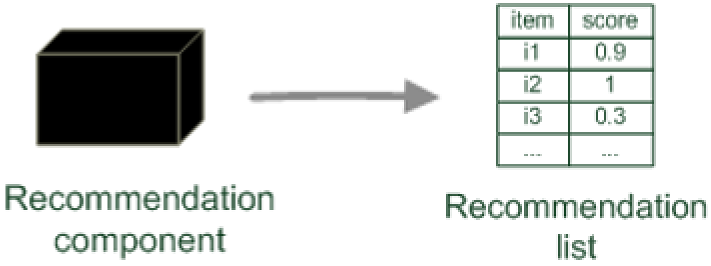
What is Recommender system ?

- Recommender system/Engine are a subclass of information filtering system that seek to predict the “rating” or “Preference” that a user would give to an item.
- Popular Areas of usage : Movies, music, news, books, research articles, search queries, social tags, and products in general.
- Basic flavors of recommendation methods:
  - Content-based filtering
  - Collaborative filtering
  - Hybrid approach
  - Knowledge-based

Why use Recommender systems?

- Value for the customer
  - Find things that are interesting
  - Narrow down the set of choices
  - Help me explore the space of options
  - Discover new things
  - Entertainment

Recommender systems reduce information overload by estimating relevance



- Value for the provider
  - Additional and probably unique personalized service for the customer
  - Increase trust and customer loyalty
  - Increase sales, click through rates, conversion etc.
  - Opportunities for promotion, persuasion
  - Obtain more knowledge about customers

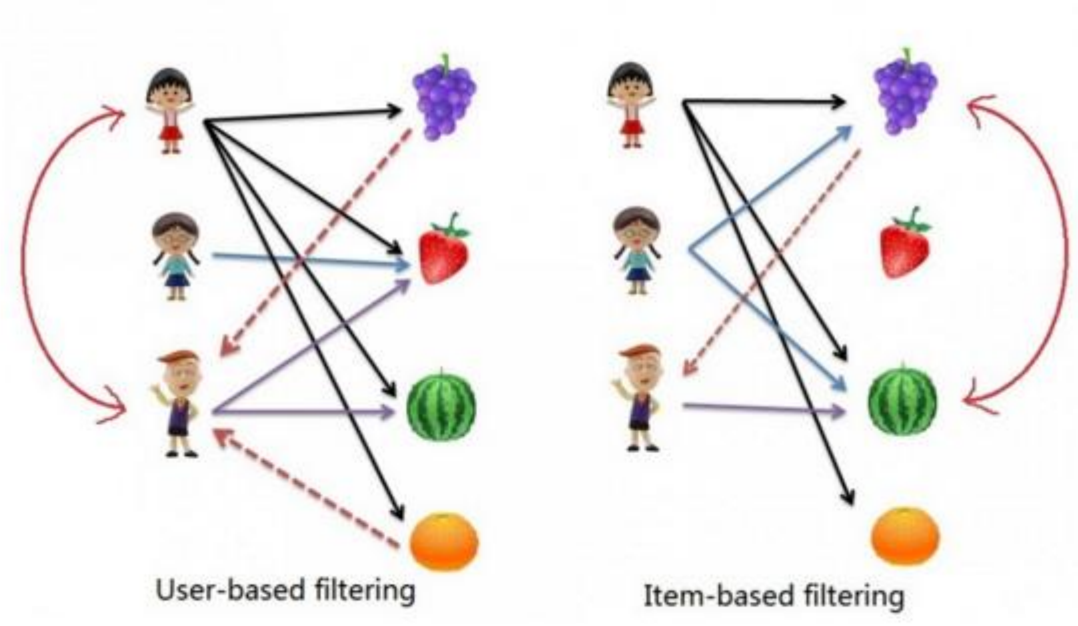
Flavors of Recommender system

- A visual overview of the methods used

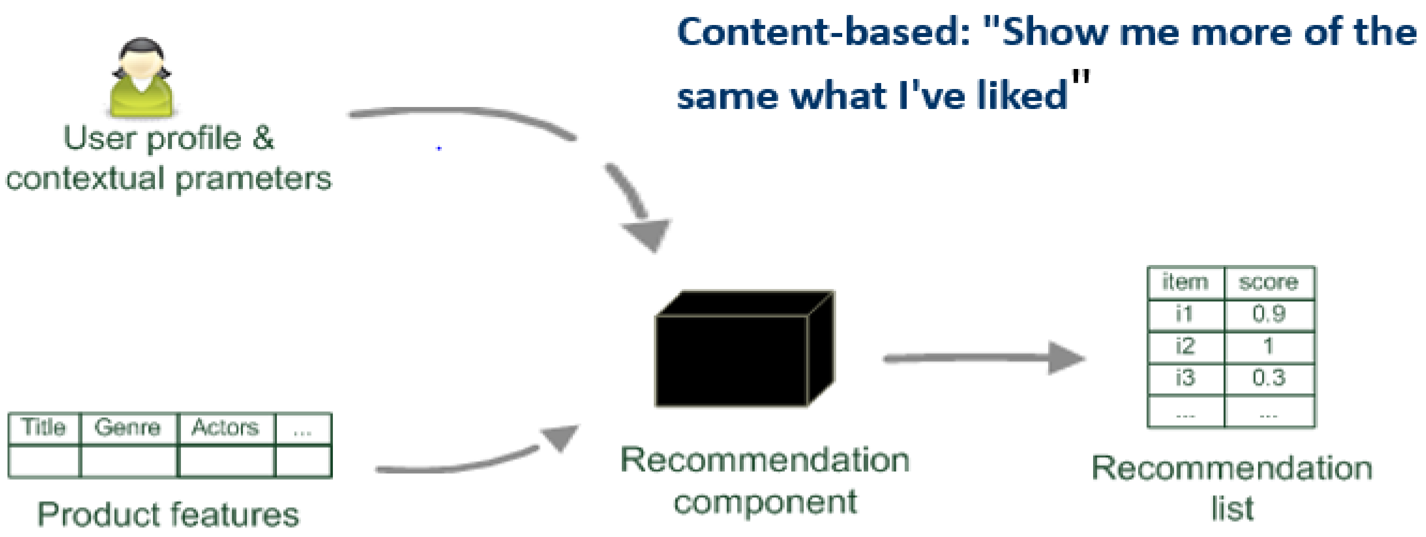
Collaborative filtering

Types of collaborative filtering

- **Item Based Collaborative Filtering** takes the similarities between items' consumption history.
- **User Based Collaborative Filtering** considers similarities between user consumption history.

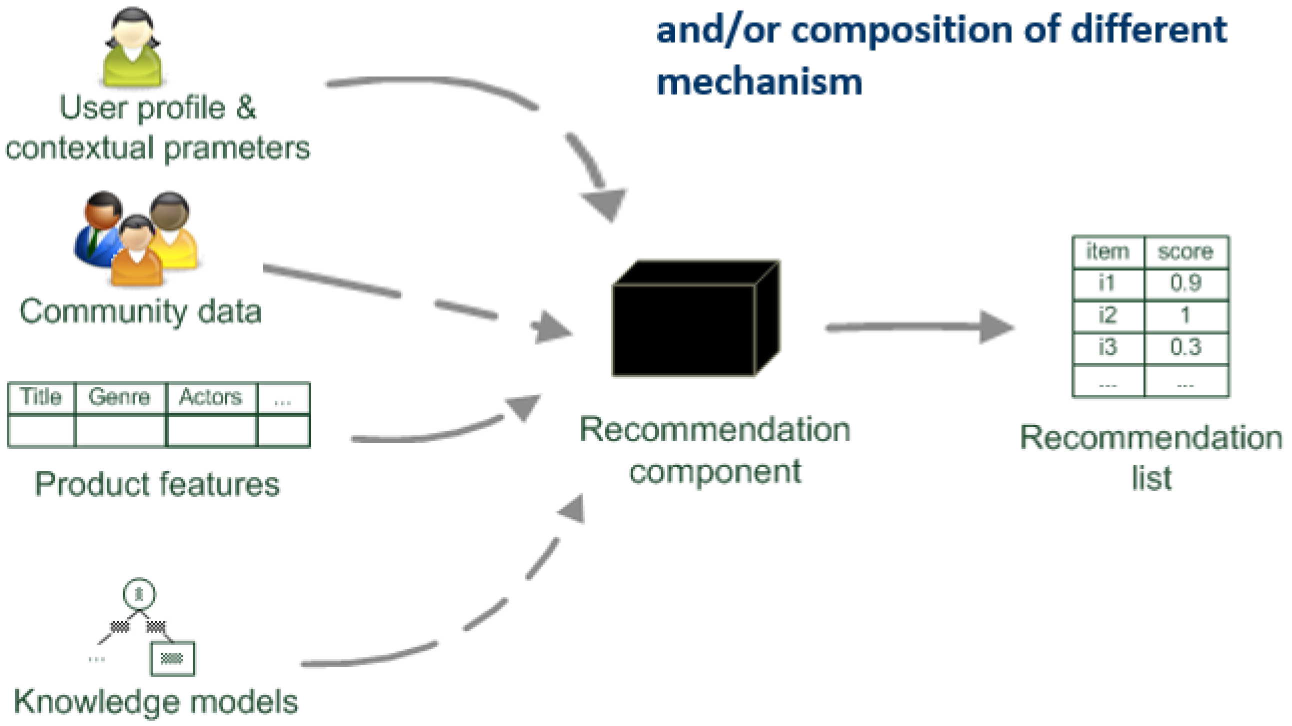


Content based filtering

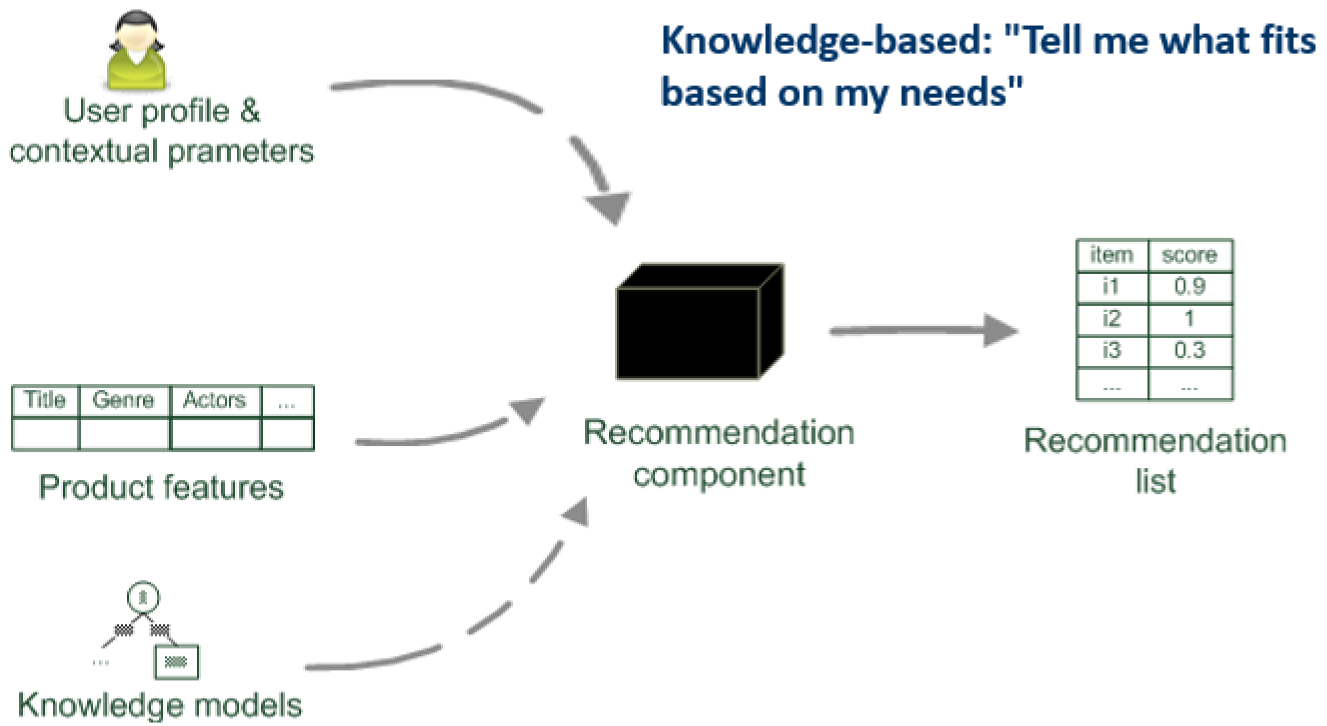


Hybrid approach

Hybrid: combinations of various inputs and/or composition of different mechanism



Knowledge based approach



Pros and Cons of methods

	Pros	Cons
Collaborative	No knowledge-engineering effort, serendipity of results, learns market segments	Requires some form of rating feedback, cold start for new users and new items
Content-based	No community required, comparison between items possible	Content descriptions necessary, cold start for new users, no surprises
Knowledge-based	Deterministic recommendations, assured quality, no cold-start, can resemble sales dialogue	Knowledge engineering effort to bootstrap, basically static, does not react to short-term trends

Understanding Recommender system

- To understand how recommendation engine works, let's slice the data into a sample set of five smartphones with two major features "Battery and Display". The five smartphones have following properties:
  - S1 has good battery life but poor display
  - S2 has an amazing battery performance but very rough display
  - S3's battery is one of the best but display lacks quality
  - S4 & S5 are good in terms of display but poor in terms of battery performance.
- Imagine a Item- Feature matrix created from the above definition of smartphone (S1-S5) and the features Battery and Display.

Item – Feature Matrix

Smartphone	Battery	Display
S1	0.9	0.1
S2	1	0
S3	0.99	0.01
S4	0	1
S5	0.1	0.9

- Our sample set also consist of four active users with their preferences.
  - **Aman**: He prefers battery over display as an ideal smartphone feature.
  - **Bob**: He likes a long lasting battery.
  - **Chandan**: For C, display should be decent, battery should be normal.
  - **David**: For D, Display is extremely important but not the battery.
- From the users interest we can create User-Feature matrix as shown below.

User	Battery	Display
Aman	0.9	0.1
Bob	0.8	0.2
Chandan	0.1	0.9
David	0.01	0.99

- Now we have 2 matrices – Item-Feature matrix & User-Feature matrix

Applying Content based Recommendations

- Content based systems, recommends item based on a similarity comparison between the content of the items and a user's profile. Basically it understands the user's previous liking to recommend a new item to him/her.
- The feature of items are mapped with feature of users in order to obtain user – item similarity.
- The top matched pairs are given as recommendations.
- Representing every user by a feature vector:

	User	Feature Vector
U_1	Aman	[0.9 0.1]
U_2	Bob	[0.8 0.2]
U_3	Chandan	[0.1 0.9]
U_4	David	[0.01 0.99]

- Also, every item representation as a feature vector:

Smartphone	Feature Vector
S_1	[0.9 0.1]
S_2	[1 0]

- Content Based ***Item - User Mapping Recommendations*** are given by the equation:

$$MAX\ (\ U_{(j)}^{T}\ .\ I_{(i)}\ )$$

- For User U1 (Aman), Smartphone recommendation is:
  - MAX($U1^{T}S1$, $U1^{T}S2$, $U1^{T}S3$, $U1^{T}S4$, $U1^{T}S5$)
  - MAX($[0.9\ 0.1]^{T}[0.9\ \ 0.1]$, $[0.9\ \ 0.1]^{T}[1\ \ 0]$, $[0.9\ \ 0.1]^{T}[0.99\ 0.01]$, $[0.9\ \ 0.1]^{T}[0.1\ \ 0.9]$,$[0.9\ \ 0.1]^{T}[0.01\ \ 0.99]$)
  - MAX(0.82 , 0.9 , 0.89 , 0.18 , 0.10)
  - = S2(0.9), S3(0.89) & S1(0.82)

- Smartphones S2, S3 and S1 has the highest recommendation scores, Hence S2, S3 and S1 are recommended to Aman.

Applying Collaborative filtering

- Collaborative filtering uses user behavior for recommending items.
- It considers the peer groups of each user for recommendation (i.e) it uses transaction history, ratings, interest, selection etc. for the process.
- We build a User-Feature matrix as given below.

User	Battery	Display	Feature Vector
Aman	0.9	0.1	[0.9 0.1]
Bob	0.8	0.2	[0.8 0.2]
Chandan	0.1	0.9	[0.1 0.9]
David	0.3	0.7	[0.01 0.99]

- Here we don’t have the information regarding the item but we do information on the user behavior like how they bought or rated the items they purchased.

User- Behaviour Matrix

Smartphone	Aman	Bob	Chandan	David
S1	5	4.5	?	?
S2	5	?	0.5	?
S3	?	4	0.5	?
S4	?	?	5	4
S5	?	?	5	4.5

- where values of the behavior matrix can be described as:
  - $B_{i,j}$ = {r , if Uj has given "r" rating to a Si
  - ?, if no rating is given}

- S1 is rated 5 by U1,S1 is rated 4.5 by U2, S1 rating by U3 & U4 are not known
  - Using this information Feature Vector of S1 can be assumed as:
  - S1 : [x1 x2] and the equations are:
    - $U1^T S1 = 5$
    - $U2^T S1 = 4.5$
    - $[0.9 \quad 0.1]^T [x1 \quad x2] = 5$
    - $[0.8 \quad 0.2]^T [x1 \quad x2] = 4.5$
    - $0.9 * x1 + 0.1 * x2 = 5$
    - $0.8 * x1 + 0.1 * x2 = 4.5$
    - solving these equations, gives x1 = 5.5 and x2 = 0.5

- S1 = [5.5 0.5]

- Similarly,
  - S2 = [5.5 0]
  - S3 = [5 0]
  - S4 = [0.5 5.5]
  - S5 = [2.7 5.25]
- Now all the feature vectors are known. Hence the recommendations will be mappings of User Feature Vectors and Item Feature Vectors.
- Thus for Aman, based on his preferences and behaviors, recommendation will be:

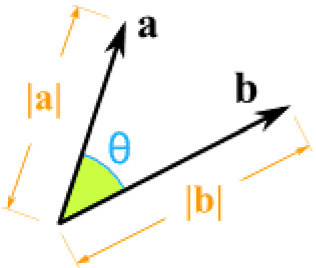
- $MAX(U1^TS1, U1^TS2, U1^TS3, U1^TS4, U1^TS5)$
- MAX([0.9 0.1]^T [5.5 0.5] , [0.9 0.1]^T [5.5 0], [0.9 0.1]^T [5   0], [0.9 0.1]^T
- [0.5 5.5],[0.9   0.1]T [2.7   5.25])
- MAX(5, 4.99, 4.95, 1, 2.9)

- So S1, S2 and S3 are the items which are coming as output here.
- Since S1 and S2 are already rated by Aman, So we will recommend him a new smartphone S3.
- We assumed the user had rated just 2 primary features. In real time we may end up with many users rating the features. For example, if we had data for all the N number of users who rated S1, then feature vector look like:
  - S1: [x1 x2 x3 x4 x5 …]

The science behind Recommender system

Why transpose of matrix in this calculation?

- As matrix multiplication rules have not been defined for 2 column vectors, we transpose one of the matrices.
- Matrix multiplication rule No of rows in the first matrix should match No of columns in the second matrix.
- Actually the dot product of any 2 vectors is same as the matrix transpose multiplication given previously.



$\cdot\ \mathbf{b} = |\mathbf{a}| \times |\mathbf{b}| \times \cos(\theta)$

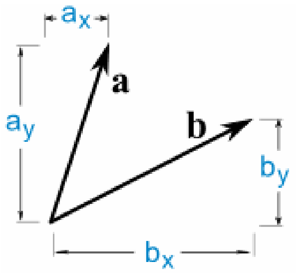
Where:
$|\mathbf{a}|$ is the magnitude (length) of vector **a**
$|\mathbf{b}|$ is the magnitude (length) of vector **b**
θ is the angle between **a** and **b**

How does dot product and matrix transpose equate ?

- The transposition of vectors are critical for providing the properties of sizes and angles. Once the notions of sizes and angles are obtained, similarity can be provided.

If we multiply $\mathbf{x}^T$ (a $1 \times n$ matrix) with any n-dimensional vector $\mathbf{y}$ (viewed as an $n \times 1$ matrix), we end up with a matrix multiplication equivalent to the familiar dot product of $\mathbf{x} \cdot \mathbf{y}$:

$$\mathbf{x}^T\mathbf{y} = \begin{bmatrix} x_1 & x_2 & x_3 & \cdots & x_n \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{bmatrix} = x_1y_1 + x_2y_2 + x_3y_3 + \ldots + x_ny_n = \mathbf{x}$$



$$\mathbf{a} \cdot \mathbf{b} = a_x \times b_x + a_y \times b_y$$

- Thus linear algebra is useful in the form that *vectors are the* ***simplest*** *mathematical objects for which notions of sizes and angles, and thus* ***similarity****, can be provided.*

QUESTIONS?

THANKS FOR THE OPPORTUNTITY

Reference

- https://www.youtube.com/watch?v=ZLHs1Fs4b2o
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