Assignment B3: State estimation

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23rd January 2025

1 State Space modeling

State Equation

The state equation represents the evolution of the system's state $\mathbf{x}_t$ over time. Considering the system to be linear, the general form is given by kinematics law:

$$\mathbf{x}_{t+1} = \mathbf{x}_t + T_s \mathbf{v}_t$$

$$\mathbf{v}_{t+1} = \mathbf{v}_t + \mathbf{w}_t$$

For this system:

$$\mathbf{x}_{t+1} = \begin{bmatrix} 1 & T_s \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{x}_t \\ \mathbf{v}_t \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} W_t$$

where:

- $\begin{bmatrix} \mathbf{x}_t \\ \mathbf{v}_t \end{bmatrix}$: State vector, where $\mathbf{x}_t$ is the position and $\mathbf{v}_t$ is the velocity.
- T_s: Sampling time.
- W_t: Process noise affecting the velocity.

Measurement Equation

The measurement equation represents the relationship between the observed variable z_t and the system's state $\mathbf{x}_t$. The general form is:

$$z_t = H\mathbf{x}_t + v_t$$

For this system:

$$z_t = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x}_t + v_t$$

where:

- $H = \begin{bmatrix} 1 & 0 \end{bmatrix}$: Measurement matrix.
- v_t: Measurement noise.

This model captures the system's dynamics and measurements, suitable for use in state estimation techniques like the Extended Kalman Filter (EKF).