PROBABILITY & STATISTICS (MATH F113)
COMPREHENSIVE EXAMINATION (CLOSED BOOK)

Date: 05.01.2014   Max. Marks: 120
Time: 3 hours   Weightage: 40 %

Answer Part A, B and C in separate answer books

$$F(-3.4) = 0.0003,\ F(-2.6) = 0.0047,\ F(-1) = 0.1587,\ z_{0.05} = 1.645,\ t_{29,0.025} = 2.045,\ \chi^2_{9,0.99} = 2.09$$

PART A

1. A control chart is used to monitor the average thread count produced by a machine making spandex cloth. Samples are taken periodically and each sample is classified into one of 5 categories. These are: in control but above average, in control and average, in control but below average, out of control and high, and out of control and low. In taking a series of 20 samples, in how many ways can we obtain a series in which there are exactly
   a) 5 samples in control but above average, 5 samples in control but below average, 5 samples in control and average, 3 samples out of control and high, and the rest out of control and low?
   b) 15 samples in control and 5 out of control?

2. A foundry is producing cast iron parts to be used in the automatic transmissions of trucks. There are two crucial dimensions to the part, A and B. Assume that if the part meets specifications on dimension A then there is a 98% chance that it will also meet specifications on dimension B. There is a 95% chance that it will meet specifications on dimension A and a 97% chance that it will meet specifications on dimension B. A part is randomly selected and inspected.
   a) What is the probability that it will meet specifications on both dimensions?
   b) If the part meets specification on dimension B what is the probability that it will meet specification on dimension A?

3. Production line workers assemble 15 automobiles per hour. During a given hour, four are produced with improperly fitted doors. Three automobiles are selected at random and inspected. Let X denote the number inspected that have improperly fitted doors.
   a) Find the density for X.
   b) Find the probability that atmost one will be found with improperly fitted doors.

4. A burr is a thin ridge or rough area that occurs when shaping a metal part. These must be removed by hand or means of some newer method such as water jets, thermal energy, or electrochemical processing before the part can be used. Assume that a part used in automatic transmissions typically averages two burrs each. What is the probability that the total number of burrs found on two randomly selected parts will be atmost four?

PTO

5. Let X denote the lag time in a printing queue at a particular computer center. That is, X denotes the difference between the time that a program is placed in the queue and the time at which printing begins. Assume that X is normally distributed with mean 15 minutes and variance 25. (10)
   a) Find the probability that a program will reach the printer with 2 minutes of arriving in the queue.
   b) What is the probability for a program to stay in the queue between 10 and 20 minutes?

6. The number of customers who visit a car dealer's showroom on a Saturday morning is a random variable with mean 18 and standard deviation 2.5. With what probability can we assert that there will be more than 8 but fewer than 28 customers? (8)

7. The joint density for (X, Y) is given by $f(x,y) = \dfrac{x^3 y^3}{16}$, $0 \le x \le 2, 0 \le y \le 2$ (12)
   a) Find the marginal densities for X and Y.
   b) Are X and Y independent?
   c) Find $P(X \le 1)$.
   d) Find $f_{X/y=1}$

8. A study of the computer market is conducted. Random samples are drawn from among the users of the two leading mainframes. The purpose of the study is to estimate the proportion of users in each population that either do use or would like to use the small office system built by the mainframe supplier. These data result: (10)

Type I	Type II
$n_1 = 200$	$n_2 = 190$
$x_1 = 62$	$x_2 = 76$

   a) Find the point estimates for $p_1, p_2, p_1 - p_2$
   b) Find a 90% confidence interval for $p_1 - p_2$.

9. A computer system currently has some terminals and uses a single printer. The average turnaround time for the system is 15 minutes. Some new terminals and a send printer are added to the system. We want to determine whether or not the mean turnaround time is affected. (8)
   a) Set up the appropriate null and alternative hypothesis.
   b) When the data are gathered, a sample of size 30 yields $\bar{x} = 14$ and $s = 3$. Test H_0 at 5% level of significance.

10. One random variable studied while designing the front-wheel-drive half-shaft of a new model automobile is the displacement (in millimeters) of the constant velocity joints. With the joint angle fixed at 12^0, 10 simulations are conducted, resulting in the following data: 6.2   1.9   4.4   4.9   3.5   4.6   4.2   1.1   1.3   4.8
Engineers designing the front-wheel-drive half-shaft claim that the standard deviation in the displacement of the CV shaft is less than 1.5 millimeters. Do these dates support the contention of the engineers? Test at 1% level of significance. (12)

PTO

11. A study is conducted to develop an equation by which the unit cost of producing a new drug (Y) can be predicted based on the number of units produced (X). Assume that these date are available:

Number of units produced (X)	5	5	10	10	15	15	20	20	25	25
Cost in hundreds of dollars (Y)	14.0	12.5	7.0	5.0	2.1	1.8	6.2	4.9	13.2	14.6

Find the linear regression of Y on X. (10)

12. In studying the effect of air quality on a lake, the experimenter takes observations on the pH of the water and the air quality as measured on an air quality index. The index goes from 0 to 100 with larger numbers representing high pollution. These data are obtained:

pH (X)	4.5	4.1	4.8	4.0	5.0	6.0	3.5	4.9	3.2	6.1
Air quality (Y)	40	50	30	60	20	10	70	30	85	15

Estimate the correlation coefficient. (10)

ALL THE BEST!

Date: 20.11.2013   Max. Marks: 60
Time: 50 minutes   Weightage: 20%

Answer all the questions

1. The joint probability distribution of X and Y is given by: (12)

		x		
		0	1	2
y	0	0.1	0.1	0.1
	1	0.1	0.2	0
	2	0.1	0.2	0.1

   (a) Find $P(X + Y \geq 2)$
   (b) Find $P(X \leq 1, Y > 0)$
   (c) Find $P(Y = 2)$
   (d) Find $P(X \geq 1)$
   (e) Find Cov (X, Y)

2. Given the joint density function

$$f_{XY}(x, y) = \frac{1}{6}, \quad 0 \leq x \leq 2, 0 \leq y \leq 3$$

$$0, \quad otherwise$$

   Find Cov (X, Y).   (8)

3. The joint density for (X, Y) is given by $f_{XY}(x, y) = cxy$ for $0 < x < 4$ and $1 < y < 5$.
   (a) Find C that makes $f_{XY}(x, y)$ a density?   (2)
   (b) Are X and Y independent? Explain on mathematical basis.   (4)
   (c) Find $P(X + Y < 3)$?   (6)

4. The joint density for (X, Y) is given by $f_{XY}(x, y) = \frac{e^x}{y}$ for $5 \leq x \leq y \leq 10$. Find $f_{X/Y=6}$,

   $f_{Y/X=8}$ (8)

5. Suppose that we observe a random variable having the binomial distribution. Let X be the number of successes in 12 trials.

   (a) Show that $\dfrac{\overline{X}}{12}$ is an unbiased estimator of the binomial parameter p. Also estimate p based on

   the following data: 3 5 2 3   4 1   4 3 3   3   (4)

   (b) Show that $\dfrac{\overline{X}+1}{14}$ is not an unbiased estimate of the binomial parameter p.   (4)

6. Consider the random variable X with density given by $f(x) = \theta e^{-\theta x}$, $x > 0$. Find the

   maximum likelihood estimator for θ based on the random sample of size n.   (12)

                                                                                                                                              All the best!

TEST – I (Closed Book)

Date: 09.10.2013
Time: 50 minutes

Max. Marks: 75
Weightage: 25%

Answer all the questions

1. A carton of 12 rechargeable batteries contains two that are defective. In how many ways can an inspector choose 3 of the batteries and
   (a) get the one that is defective.
   (b) not get the one that is defective. (10)

2. A consulting firm rents cars from three agencies, 20% from agency D, 20% from agency E, and 60% from agency F. If 10% of the cars from D, 12% of the cars from E and 4% of the cars from F have bad tyres, (15)
   (a) what is the probability that the firm will get a car with bad tyres?
   (b) What is the probability that the firm will get a car with bad tyres rented by the firm came from agency E? from agency F?

3. Suppose a researcher goes to a engineering college consisting of 200 faculty, 12 of which have blood type O-negative. He obtains a simple random sample of 20 of the faculty. Let the random variable X represent the number of faculty in the sample of size that have blood type O-negative.
   (a) Find the density function and also find the probability that 3 of the faculty have blood type O-negative?
   (b) What is the probability that at least one of the faculty has blood type O-negative?
   (c) What is the average number of randomly selected faculty that will have blood type O-negative? (14)

4. An underground mine has 5 pumps installed for pumping out storm water, the probability of any one of the pumps failing during the storm is 1/8. What is the probability that
   (a) atleast 2 pumps will be working?
   (b) all the pumps will be working during particular storm?
   (c) what is moment generation function? (11)

5. If the probability is 0.05 that a certain kind of measuring device will show excessive drift, what is the probability that the sixth measuring device tested will be the first to show excessive drift. Find the mean and variance. (10)

6. If the probability density of a random variable is given by

$$f(x) = \begin{cases} k\,x^3, & 0 < x < 1 \\ 0, & otherwise \end{cases}$$

   (a) Find k.
   (b) P(0.6 < X< 1.2)
   (c) P(2 ≤ X≤ 2.5)
   (d) Find cdf F(x) (15)

All the best!

BITS, Pilani - Dubai Campus
Dubai International Academic City, Dubai
First year – I Semester 2013 – 2014
Probability and Statistics (MATH F113)

A

Quiz - 2		05.12.2013
Time: 20 Minutes	**Max Marks: 21**	**Weightage: 7%**
Name:	**ID:**	**Faculty's Name:**

Answer all the questions. Each question carries 3 marks.

$$\chi^2_{19,0.025} = 8.91, \quad \chi^2_{19,0.975} = 32.9, \quad t^2_{9,0.005} = 3.250, \quad z_{0.05} = 1.645, \quad z_{0.01} = 2.33,$$

1. If probability of type II error is 0.623, then the power of the test is _____.

2. Researchers are experimenting with a new compound used to bond Teflon to steel. The compounds currently in use require an average drying time of 3 minutes. It is though that the new compound dries in a shorter length of time. Set up the null and alternative hypothesis:
_____.

3. The standard deviation of the lifetimes of a sample of 20 electric light bulbs was computed to be 100 hours. The 95% confidence limit for the standard deviation of all such electric light bulbs is _____.

4. A sample of 10 measurements of the diameter of a sphere gave a mean 4.38 inches and a sample standard deviation of 0.06 inch. The 99% confidence limit for mean of the actual diameter is _____.

5. The minimum required sample size for estimating the average number of designer shirts sold per day to within 10 units with 90% confidence if the standard deviation of the number of shirts sold per day is about 50 is _____.

6. Let $H_0 : \mu \le 0, \quad H_1 : \mu > 0$. A sample of 25 measurement yields sample mean 0.03 millimeter and sample standard deviation 0.1. The value of test statistics is _____.

7. Let $H_0 : \mu \ge 2.3, \quad H_1 : \mu < 2.3$. The critical point/ value for $\alpha = 0.01$, if σ known, is _____.

All the best!

A

Quiz - 2   05.12.2013

Time: 20 Minutes   **Max Marks: 21**   **Weightage: 7%**

Name: **ID:**   **Faculty's Name:**

Answer all the questions. Each question carries 3 marks.

$$\chi^2_{19,0.025} = 8.91, \quad \chi^2_{19,0.975} = 32.9, \quad t^2_{9,0.005} = 3.250, \quad z_{0.05} = 1.645, \quad z_{0.01} = 2.33,$$

1. If probability of type II error is 0.632, then the power of the test is _____.

2. Researchers are experimenting with a new compound used to bond Teflon to steel. The compounds currently in use require an average drying time of 2 minutes. It is though that the new compound dries in a shorter length of time. Set up the null and alternative hypothesis:

   _____.

3. The standard deviation of the lifetimes of a sample of 20 electric light bulbs was computed to be 90 hours. The 95% confidence limit for the standard deviation of all such electric light bulbs is _____.

4. A sample of 10 measurements of the diameter of a sphere gave a mean 4.5 inches and a sample standard deviation of 0.03 inch. The 99% confidence limit for mean of the actual diameter is _____.

5. The minimum required sample size for estimating the average number of designer shirts sold per day to within 10 units with 90% confidence if the standard deviation of the number of shirts sold per day is about 40 is _____.

6. Let $H_0 : \mu \le 0, \quad H_1 : \mu > 0$. A sample of 25 measurement yields sample mean 0.03 millimeter and sample standard deviation 0.2. The value of test statistics is _____.

7. Let $H_0 : \mu \le 2.3, \quad H_1 : \mu > 2.3$. The critical point/ value for $\alpha = 0.01$, if σ known, is _____.

All the best!

B

Quiz - 2   05.12.2013

Time: 20 Minutes   Max Marks: 21   **Weightage: 7%**

Name: ID: **Faculty's Name:**

Answer all the questions. Each question carries 3 marks.

$$\chi^2_{24,0.025} = 12.4, \; \chi^2_{24,0.975} = 39.4, \; t_{24,0.005} = 2.797, \; z_{0.025} = 1.96, \; z_{0.05} = 1.645,$$

1. If probability of type II error is 0.564, then the power of the test is _____.

2. Researchers are experimenting with a new compound used to bond Teflon to steel. The compounds currently in use require an average drying time of 2 minutes. It is though that the new compound dries in _____. Set up the null and alternative hypothesis:

   _____.

3. The standard deviation of the lifetimes of a sample of 25 electric light bulbs was computed to be 90 hours. The 95% confidence limit for the standard deviation of all such electric light bulbs is _____.

4. A sample of 25 measurements of the diameter of a sphere gave a mean 4.5 inches and a sample standard deviation 0.20 inch. The 99% confidence limit for mean of the actual diameter is _____.

5. The minimum required sample size for estimating the average number of designer shirts sold per day to within 10 units with 95% confidence if the standard deviation of the number of shirts sold per day is about 40 is _____.

6. Let $H_0 : \mu \le 0, \quad H_1 : \mu > 0$. A sample of 36 measurement yields sample mean 0.04 millimeter and sample standard deviation 0.2. The value of test statistics is _____.

7. Let $H_0 : \mu \ge 2.3, \quad H_1 : \mu < 2.3$. The critical point/ value for $\alpha = 0.05$, if σ known, is _____.

All the best!

B

Quiz - 2   05.12.2013

Time: 20 Minutes   Max Marks: 21   Weightage: 7%

Name: ID: Faculty's Name:

Answer all the questions. Each question carries 3 marks.

$$\chi^2_{24, 0.025} = 12.4, \quad \chi^2_{24, 0.975} = 39.4, \quad t^2_{24, 0.005} = 2.797, \quad z_{0.025} = 1.96, \quad z_{0.05} = 1.645,$$

1. If probability of type II error is 0.546, then the power of the test is _____.

2. Researchers are experimenting with a new compound used to bond Teflon to steel. The compounds currently in use require an average drying time of 3 minutes. It is though that the new compound dries in _____. Set up the null and alternative hypothesis:
_____.

3. The standard deviation of the lifetimes of a sample of 25 electric light bulbs was computed to be 100 hours. The 95% confidence limit for the standard deviation of all such electric light bulbs is _____.

4. A sample of 25 measurements of the diameter of a sphere gave a mean 4.38 inches and a sample standard deviation of 0.06 inch. The 99% confidence limit for mean of the actual diameter is _____.

5. The minimum required sample size for estimating the average number of designer shirts sold per day to within 10 units with 95% confidence if the standard deviation of the number of shirts sold per day is about 50 is _____.

6. Let $H_0 : \mu \le 0$, $H_1 : \mu > 0$. A sample of 36 measurement yields sample mean 0.03 millimeter and sample standard deviation 0.1. The value of test statistics is _____.

7. Let $H_0 : \mu \le 2.3$, $H_1 : \mu > 2.3$. The critical point/ value for $\alpha = 0.05$, if σ known, is _____.

All the best!

BITS, Pilani - Dubai Campus
Dubai International Academic City, Dubai
First year – I Semester 2013 – 2014
Probability and Statistics (MATH F113)

A

Quiz - 1   30.10.2013

Time: 20 Minutes   Max Marks: 24   Weightage: 8%

Name: ID: Faculty's Name:

Answer all the questions. Each question carries 3 marks except the last question.

F(1.5)=0.9332, F(-1.58)=0.0571

1. If the mean and variance of a gamma distribution is12 and 48 respectively then the parameters α = _____ and β = _____

2. If on average three trucks arrived per hour to be unloaded at a warehouse. Assuming that the Poisson distribution is in operation, the probability that the time between the arrival of successive trunks will be exactly 30 minutes is _____.

3. On a statistics examination, the student's grade follows a normal distribution mean 78 and standard deviation 10. The standard value z, whose grade is 93, is _____.

4. If X is normally distributed with mean 5 and standard deviation2, then $P(X > 8)$ is _____

5. The number of customers who visit a car dealer's showroom on a particular morning is a random variable with mean 15 and standard deviation 2.5. Using Chebyshev's inequality, the probability that there will be between 5 to 25 customers is atleast _____.

6. A machine produces bolts which are 10% defective. Using normal approximation, the probability that in a random sample of 400 bolts produced by this machine atmost 30 of the bolts will be defective is _____.

7. The following table defines the joint distribution of (X, Y). Then   (6)

Y \ X	0	1	2
1	0.2	0.02	0.1
2	0.04	0.05	0.1
3	0.25	0.2	c

a) c = _____
b) $P(X \geq 2 \text{ and } Y \leq 1)$ = _____.

All the best!

BITS, Pilani - Dubai Campus
Dubai International Academic City, Dubai
First year – I Semester 2013 – 2014
Probability and Statistics (MATH F113)

B

Quiz - 1   30.10.2013

Time: 20 Minutes   Max Marks: 24   Weightage: 8%

Name: ID: Faculty's Name:

Answer all the questions. Each question carries equal marks except the last question.

F(1.5)=0.9332, F(-1.75)=0.0401

1. If the mean and variance of a gamma distribution is12 and 36 respectively then the parameters α = _____ and β = _____

2. If on average three trucks arrived per hour to be unloaded at a warehouse. Assuming that the Poisson distribution is in operation, the probability that the time between the arrival of successive trunks will be exactly 45 minutes is _____.

3. On a statistics examination, the student's grade follows a normal distribution mean 78 and standard deviation 10. The standard value z, whose grade is 90, is _____.

4. If X is normally distributed with mean 5 and standard deviation2, then $P(X < 8)$ is _____.

5. The number of customers who visit a car dealer's showroom on a particular morning is a random variable with mean 20 and standard deviation 2.5. Using Chebyshev's inequality, the probability that there will be between 5 to 25 customers is atleast _____.

6. A machine produces bolts which are 10% defective. Using normal approximation, the probability that in a random sample of 400 bolts produced by this machine atleast 30 of the bolts will be defective is _____.

7. The following table defines the joint distribution of (X, Y). Then   (6)

Y \ X	0	1	2
1	c	0.02	0.1
2	0.04	0.06	0.1
3	0.25	0.2	0.03

a) c = _____
b) $P(X \le 2 \text{ and } Y \ge 1)$ = _____.

All the best!