

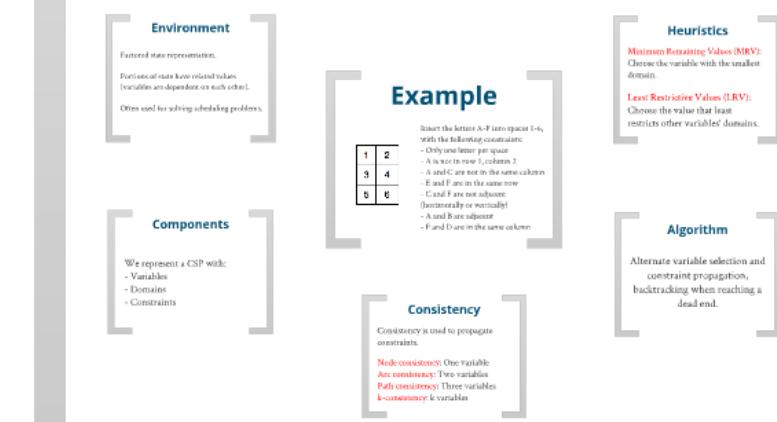
# Problem Solving

## Strategies for a Goal-Based Agent

### Search



### Constraint Satisfaction



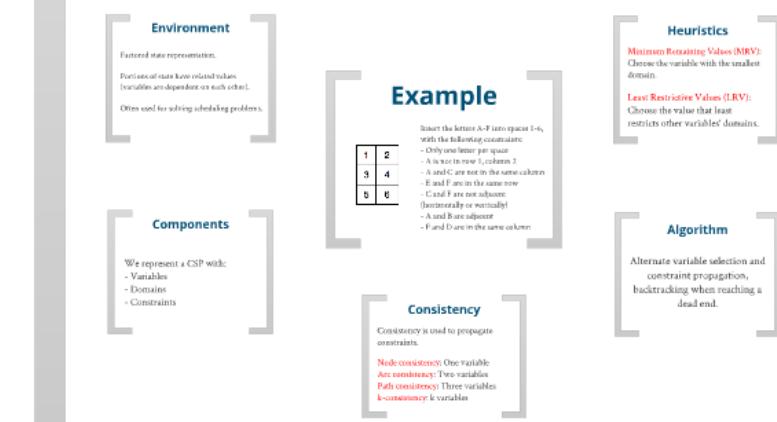
# Problem Solving

## Strategies for a Goal-Based Agent

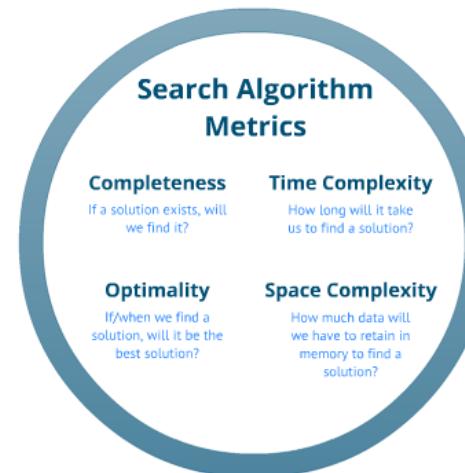
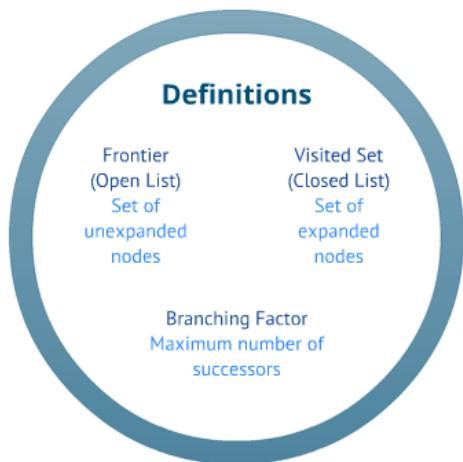
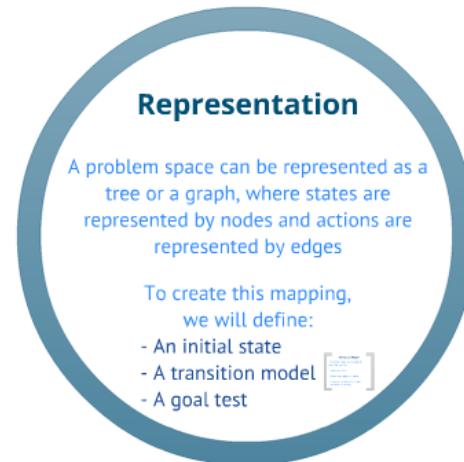
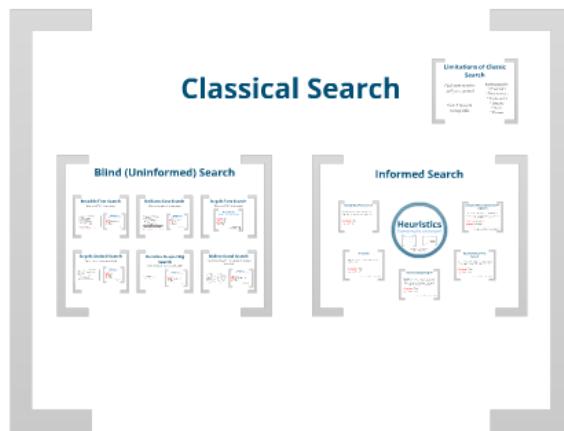
### Search



### Constraint Satisfaction



# Search



# Representation

A problem space can be represented as a tree or a graph, where states are represented by nodes and actions are represented by edges

To create this mapping,  
we will define:

- An initial state
- A transition model
- A goal test



# Transition Model

A transition model is a function of a state, that gives us:

- \* Available actions
- \* Cost of executing each action
- \* State resulting from each action  
(successors / children)

# Representation

A problem space can be represented as a tree or a graph, where states are represented by nodes and actions are represented by edges

To create this mapping,  
we will define:

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- A transition model
- A goal test



# Definitions

Frontier  
(Open List)

Set of  
unexpanded  
nodes

Visited Set  
(Closed List)

Set of  
expanded  
nodes

Branching Factor  
Maximum number of  
successors

# Search Algorithm Metrics

## Completeness

If a solution exists, will we find it?

## Time Complexity

How long will it take us to find a solution?

## Optimality

If/when we find a solution, will it be the best solution?

## Space Complexity

How much data will we have to retain in memory to find a solution?

# Completeness

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# Classical Search

## Limitations of Classic Search

Goal state must be easily recognized

Search space is manageable

- \* Observable
- \* Deterministic
- \* Single-agent
- \* Discrete
- \* Static
- \* Known

## Blind (Uninformed) Search



## Informed Search



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# Blind (Uninformed) Search

## Breadth-First Search

Maintain a FIFO frontier queue

Nodes are goal-tested as they are generated

### Attributes

Breadth-First Tree search

- Complete? Yes
- Optimal? Only if depth == cost
- Time?  $O(b^d)$
- Space?  $O(b^d)$

$b$  = branching factor  
 $d$  = depth of first solution

## Uniform-Cost Search

Maintain an ordered frontier queue

Nodes are goal-tested as they are selected for expansion

### Attributes

Uniform-Cost Tree Search

- Complete? Yes
- Optimal? Yes, assuming  $c > 0$
- Time?  $O(b^{1+\frac{c}{l}})$
- Space?  $O(b^{1+\frac{c}{l}})$

$b$  = branching factor  
 $c$  = cost of optimal solution  
 $l$  = maximum transition cost

## Depth-First Search

Maintain a LIFO frontier queue

Attributes  
Depth-First Tree Search

- Complete? No
- Optimal? No
- Time?  $O(b^m)$
- Space?  $O(bm)$

$b$  = branching factor  
 $m$  = maximum tree depth

## Depth-Limited Search

Depth-first with a maximum depth

Nodes are goal-tested as they are generated

### Attributes

Depth-Limited Tree search

- Complete? No
- Optimal? No
- Time?  $O(b^l)$
- Space?  $O(bl)$

$b$  = branching factor  
 $l$  = depth limit

## Iterative Deepening Search

Depth-limited with an increasing depth

Nodes are goal-tested as they are generated

### Attributes

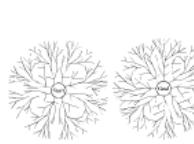
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## Bidirectional Search

Simultaneous breadth-first search from goal and initial state



### Attributes

Bi-directional Breadth-First Search

- Complete? Yes
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- Time?  $O(b^{d/2})$
- Space?  $O(b^{d/2})$

$b$  = branching factor  
 $d$  = depth of first solution

# Breadth-First Search

Maintain a FIFO frontier queue

```
function BREADTH-FIRST-SEARCH(problem) returns a solution, or failure
  node  $\leftarrow$  a node with STATE = problem.INITIAL-STATE, PATH-COST = 0
  if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
  frontier  $\leftarrow$  a FIFO queue with node as the only element
  explored  $\leftarrow$  an empty set
  loop do
    if EMPTY?(frontier) then return failure
    node  $\leftarrow$  POP(frontier) /* chooses the shallowest node in frontier */
    add node.STATE to explored
    for each action in problem.ACTIONS(node.STATE) do
      child  $\leftarrow$  CHILD-NODE(problem, node, action)
      if child.STATE is not in explored or frontier then
        if problem.GOAL-TEST(child.STATE) then return SOLUTION(child)
        frontier  $\leftarrow$  INSERT(child, frontier)
```

Figure 3.11 Breadth-first search on a graph.

Nodes are goal-tested as they are generated

## Attributes

Breadth-First Tree search

Complete? Yes

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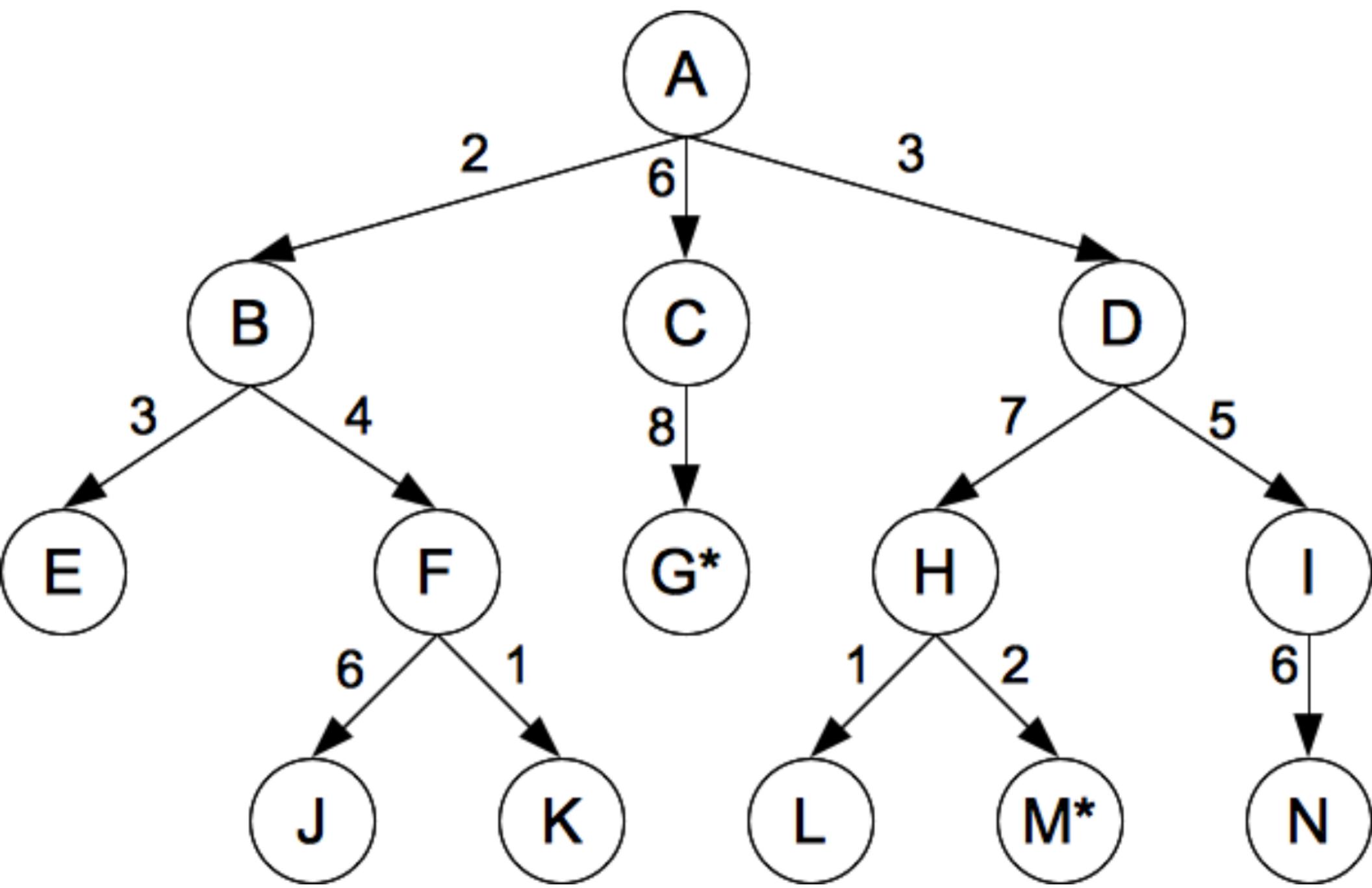
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**Figure 3.11** Breadth-first search on a graph.

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 $d$  = depth of first solution

# Uniform-Cost Search

Maintain an ordered frontier queue

```
function UNIFORM-COST-SEARCH(problem) returns a solution, or failure
  node  $\leftarrow$  a node with STATE = problem.INITIAL-STATE, PATH-COST = 0
  frontier  $\leftarrow$  a priority queue ordered by PATH-COST, with node as the only element
  explored  $\leftarrow$  an empty set
  loop do
    if EMPTY?(frontier) then return failure
    node  $\leftarrow$  POP(frontier) /* chooses the lowest-cost node in frontier */
    if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
    add node.STATE to explored
    for each action in problem.ACTIONS(node.STATE) do
      child  $\leftarrow$  CHILD-NODE(problem, node, action)
      if child.STATE is not in explored or frontier then
        frontier  $\leftarrow$  INSERT(child, frontier)
      else if child.STATE is in frontier with higher PATH-COST then
        replace that frontier node with child
```

**Figure 3.13** Uniform-cost search on a graph. The algorithm is identical to the general graph search algorithm in Figure ??, except for the use of a priority queue and the addition of an extra check in case a shorter path to a frontier state is discovered. The data structure for *frontier* needs to support efficient membership testing, so it should combine the capabilities of a priority queue and a hash table.

Nodes are goal-tested as they are selected for expansion

## Attributes

### Uniform-Cost Tree Search

Complete? Yes  
Optimal? Yes, assuming  $\varepsilon > 0$   
Time?  $O(b^{1+\lceil \frac{C^*}{\varepsilon} \rceil})$   
Space?  $O(b^{1+\lceil \frac{C^*}{\varepsilon} \rceil})$

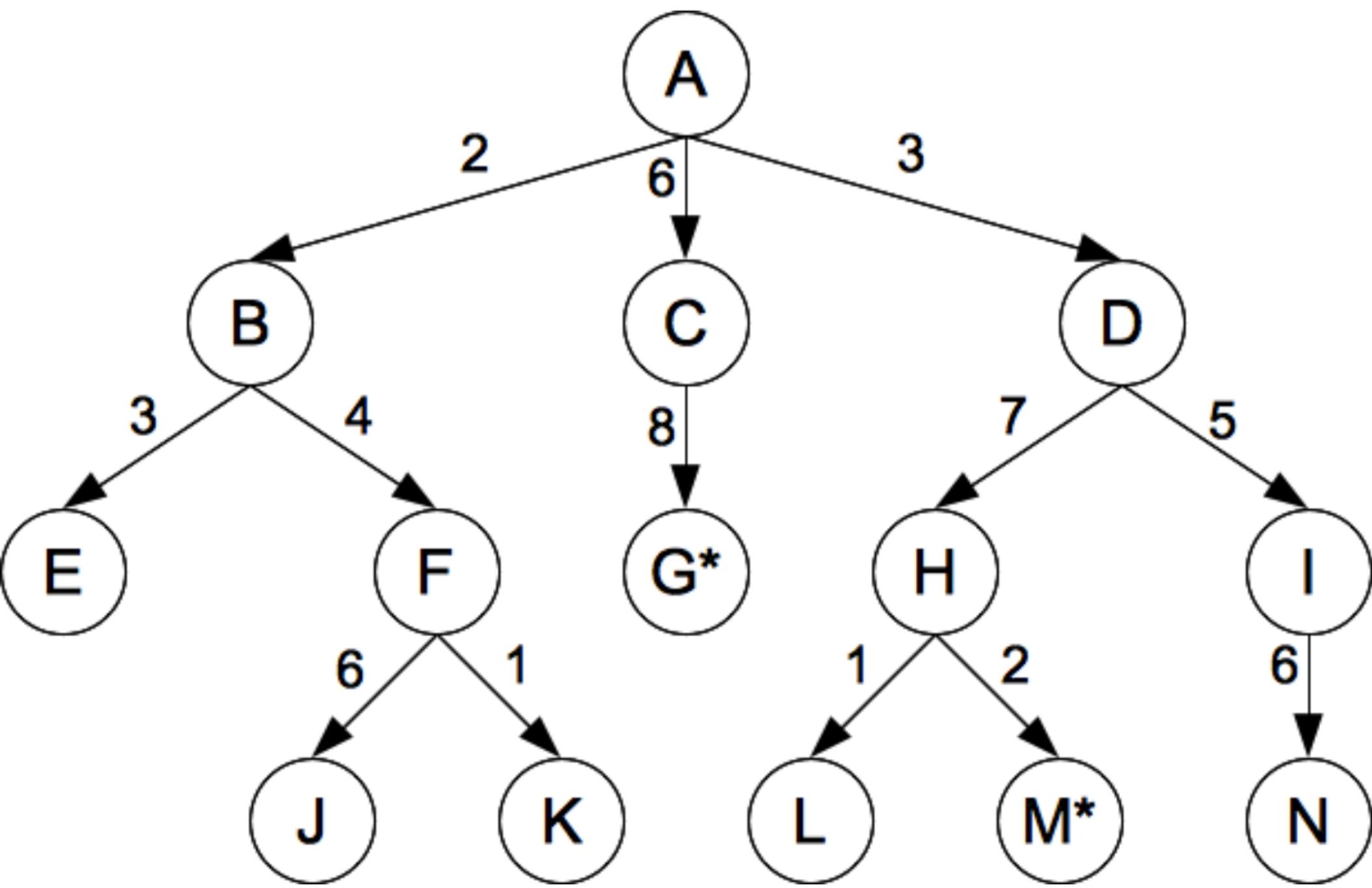
$b$  = branching factor  
 $C^*$  = cost of optimal solution  
 $\varepsilon$  = minimum transition cost

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# Attributes

## Uniform-Cost Tree Search

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Optimal? Yes, assuming  $\varepsilon > 0$

Time?  $O(b^{1+\frac{C^*}{\varepsilon}})$

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$b$  = branching factor

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# Depth-First Search

Maintain a LIFO frontier queue

## Attributes

### Depth-First Tree Search

Complete? No

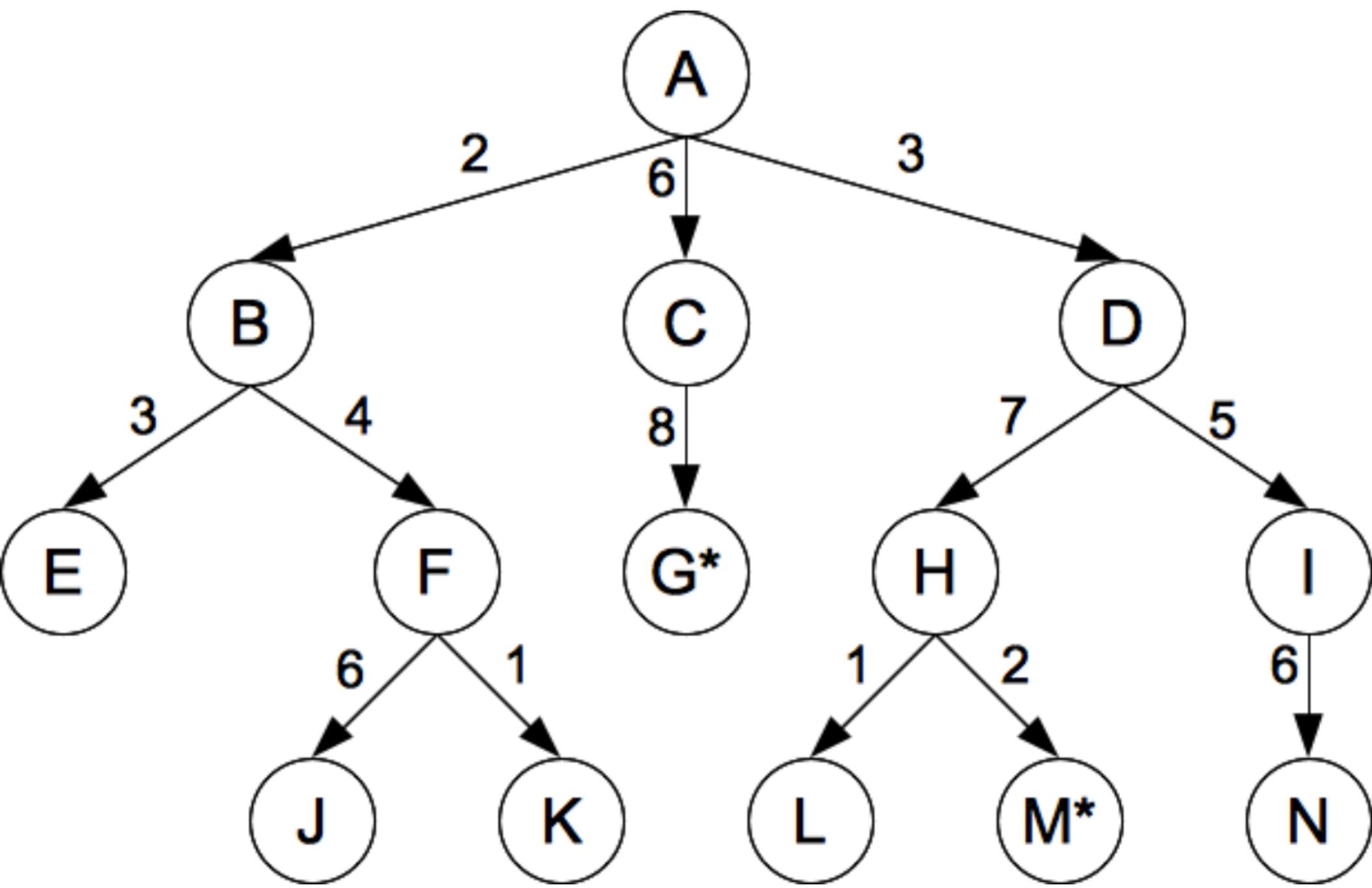
Optimal? No

Time?  $O(b^m)$

Space?  $O(bm)$

$b$  = branching factor

$m$  = maximum tree depth



# Attributes

## Depth-First Tree Search

Complete? No

Optimal? No

Time?  $O(b^m)$

Space?  $O(bm)$

$b$  = branching factor

$m$  = maximum tree depth

# Depth-Limited Search

Depth-first with a maximum depth

```
function DEPTH-LIMITED-SEARCH(problem, limit) returns a solution, or failure/cutoff
  return RECURSIVE-DLS(MAKE-NODE(problem.INITIAL-STATE), problem, limit)
 
function RECURSIVE-DLS(node, problem, limit) returns a solution, or failure/cutoff
  if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
  else if limit = 0 then return cutoff
  else
    cutoff_occurred? ← false
    for each action in problem.ACTIONS(node.STATE) do
      child ← CHILD-NODE(problem, node, action)
      result ← RECURSIVE-DLS(child, problem, limit - 1)
      if result = cutoff then cutoff_occurred? ← true
      else if result ≠ failure then return result
    if cutoff_occurred? then return cutoff else return failure
```

Figure 3.16 A recursive implementation of depth-limited tree search.

## Attributes

Depth-Limited Tree search

Complete? No  
Optimal? No  
Time?  $O(b^l)$   
Space?  $O(bl)$

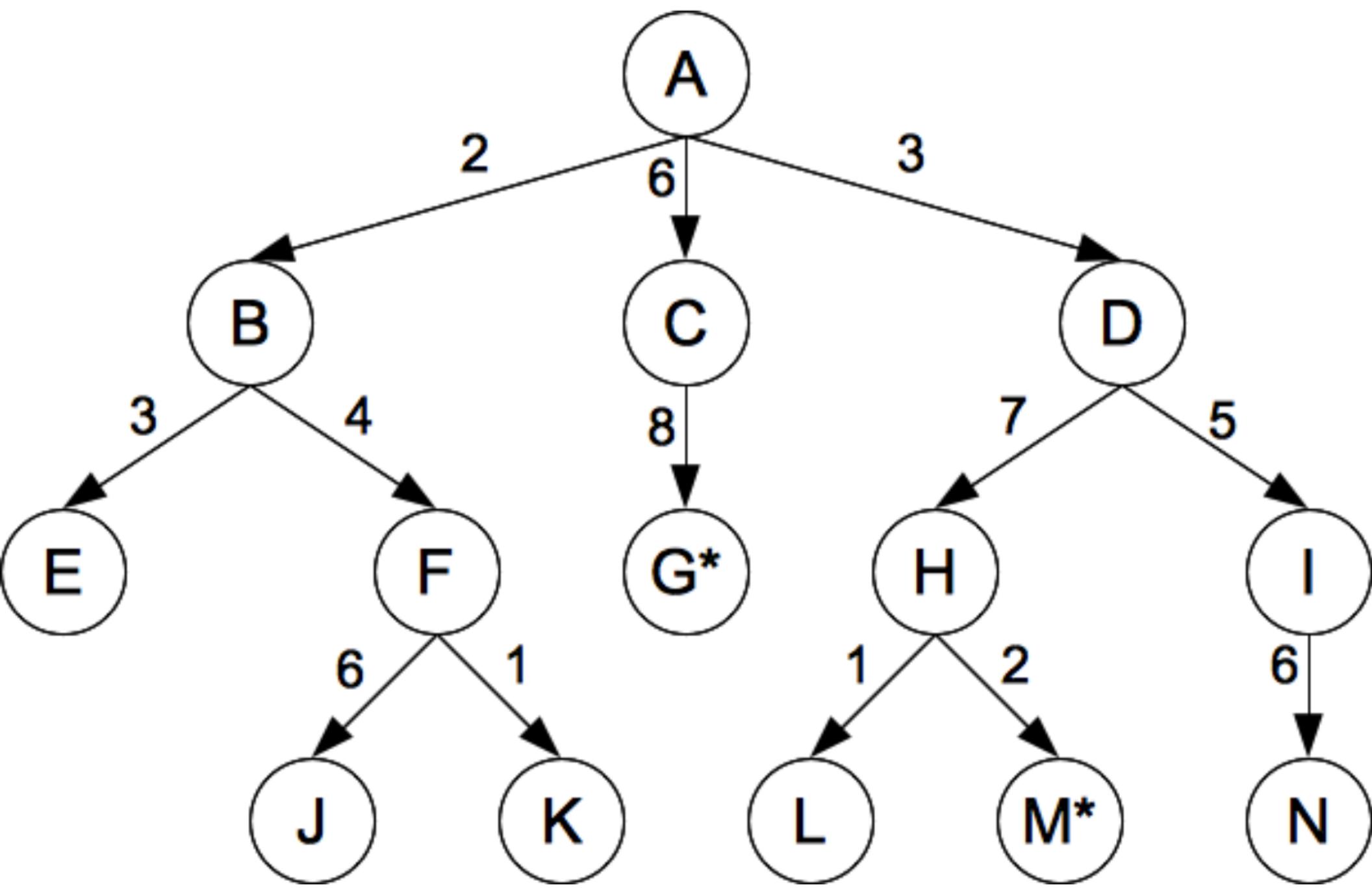
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      if result = cutoff then cutoff-occurred?  $\leftarrow$  true
      else if result  $\neq$  failure then return result
    if cutoff-occurred? then return cutoff else return failure
  
```

**Figure 3.16** A recursive implementation of depth-limited tree search.



# Attributes

Depth-Limited Tree search

Complete? No

Optimal? No

Time?  $O(b^l)$

Space?  $O(bl)$

$b$  = branching factor

$l$  = depth limit

# Iterative Deepening Search

Depth-limited with an increasing depth

```
function ITERATIVE-DEEPENING-SEARCH(problem) returns a solution, or failure
  for depth = 0 to  $\infty$  do
    result  $\leftarrow$  DEPTH-LIMITED-SEARCH(problem, depth)
    if result  $\neq$  cutoff then return result
```

Figure 3.17 The iterative deepening search algorithm, which repeatedly applies depth-limited search with increasing limits. It terminates when a solution is found or if the depth-limited search returns *failure*, meaning that no solution exists.

## Attributes

Iterative Deepening Tree Search

Complete? Yes

Optimal? Yes, if  $\text{depth} == \text{cost}$

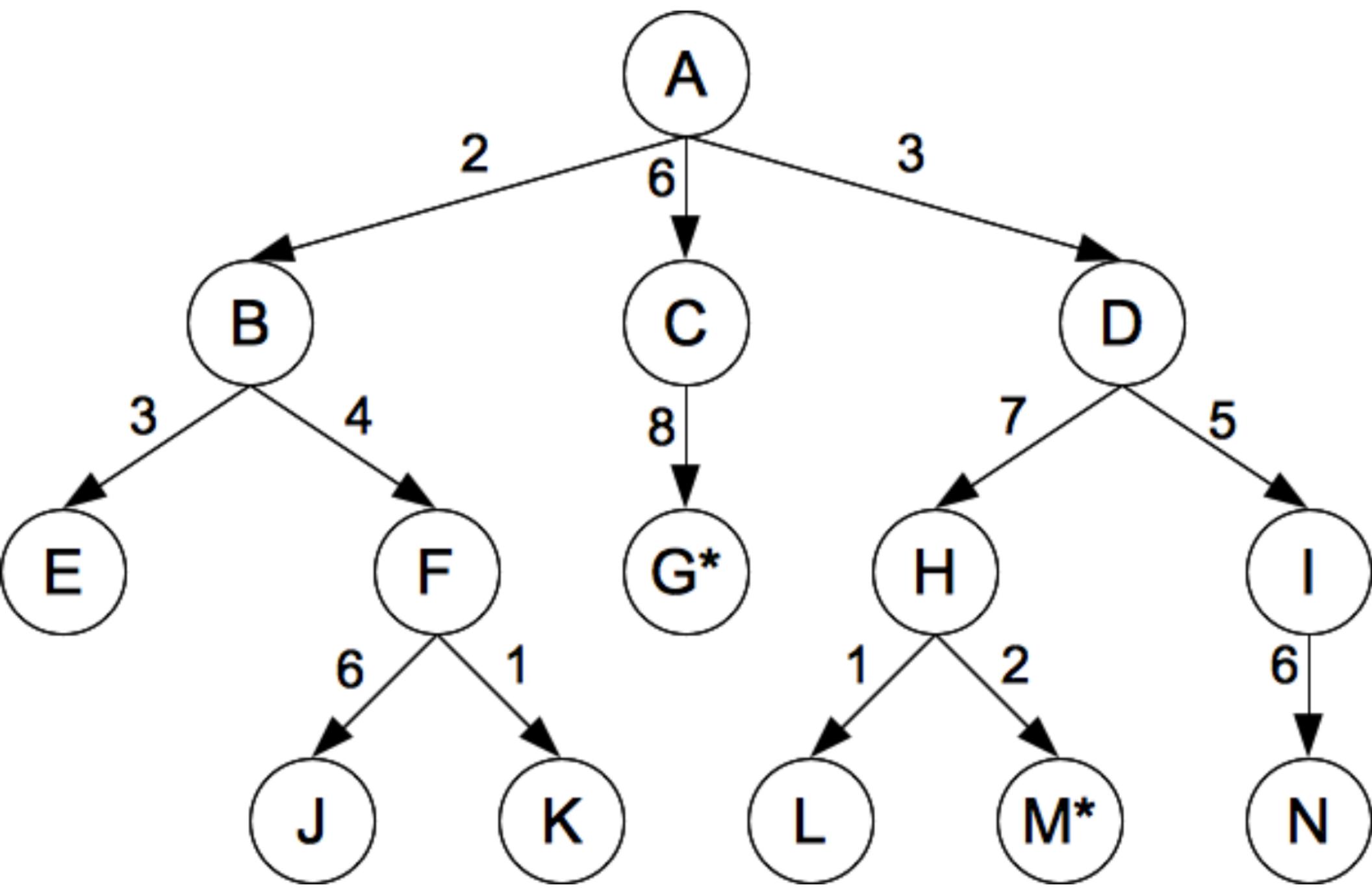
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# Attributes

Iterative Deepening Tree Search

Complete? Yes

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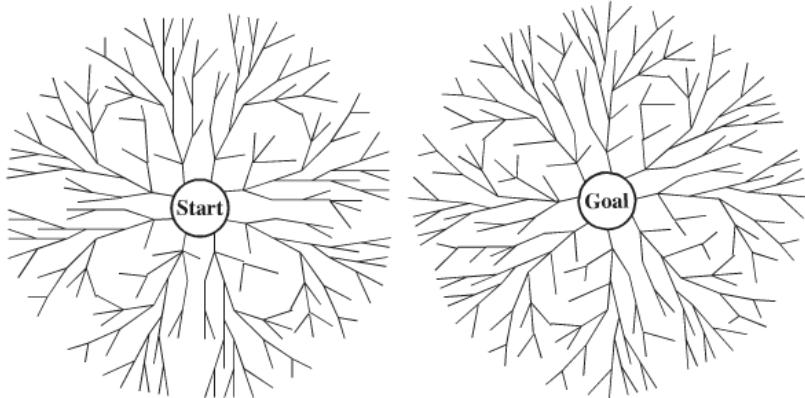
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# Bidirectional Search

Simultaneous breadth-first search from goal and initial state



## Attributes

Bi-directional Breadth-First Search

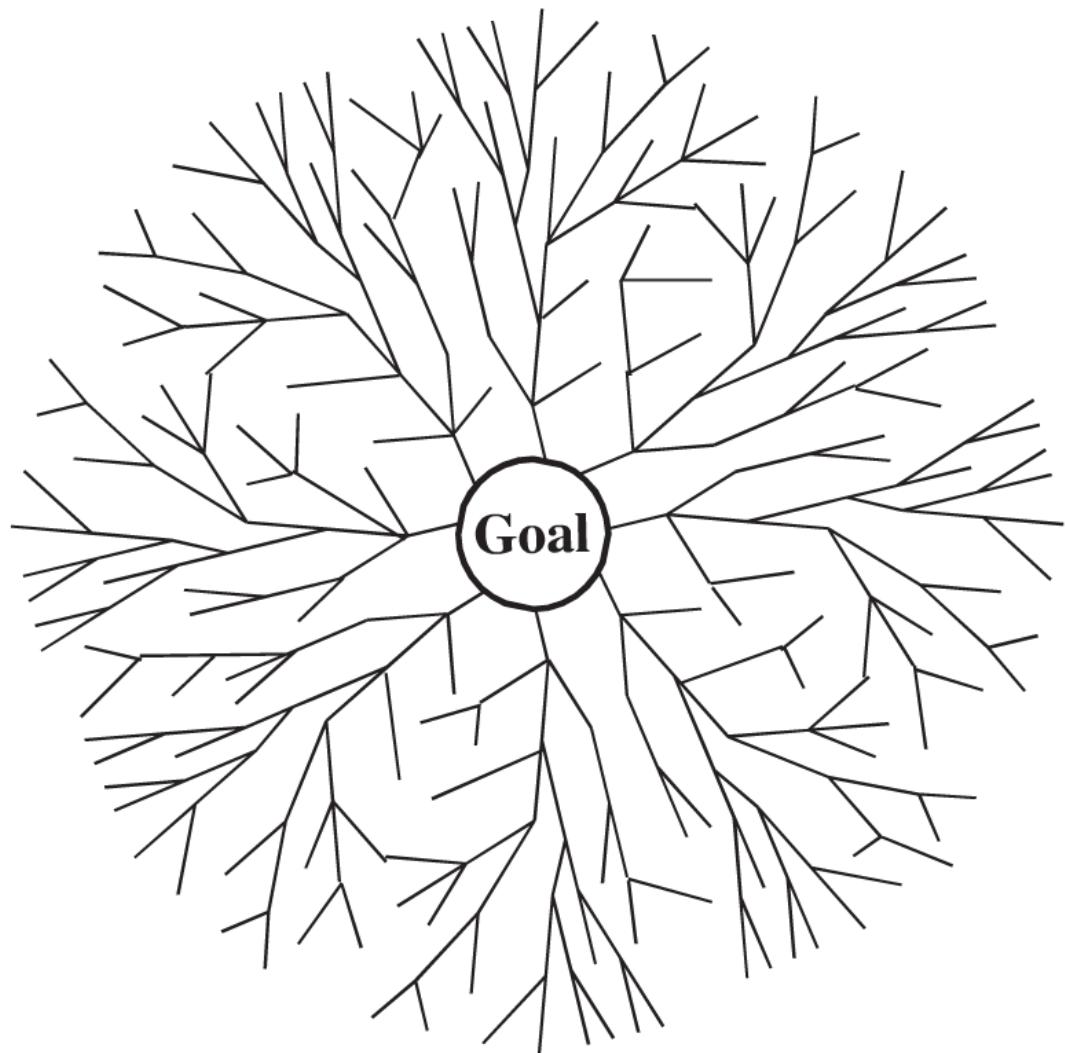
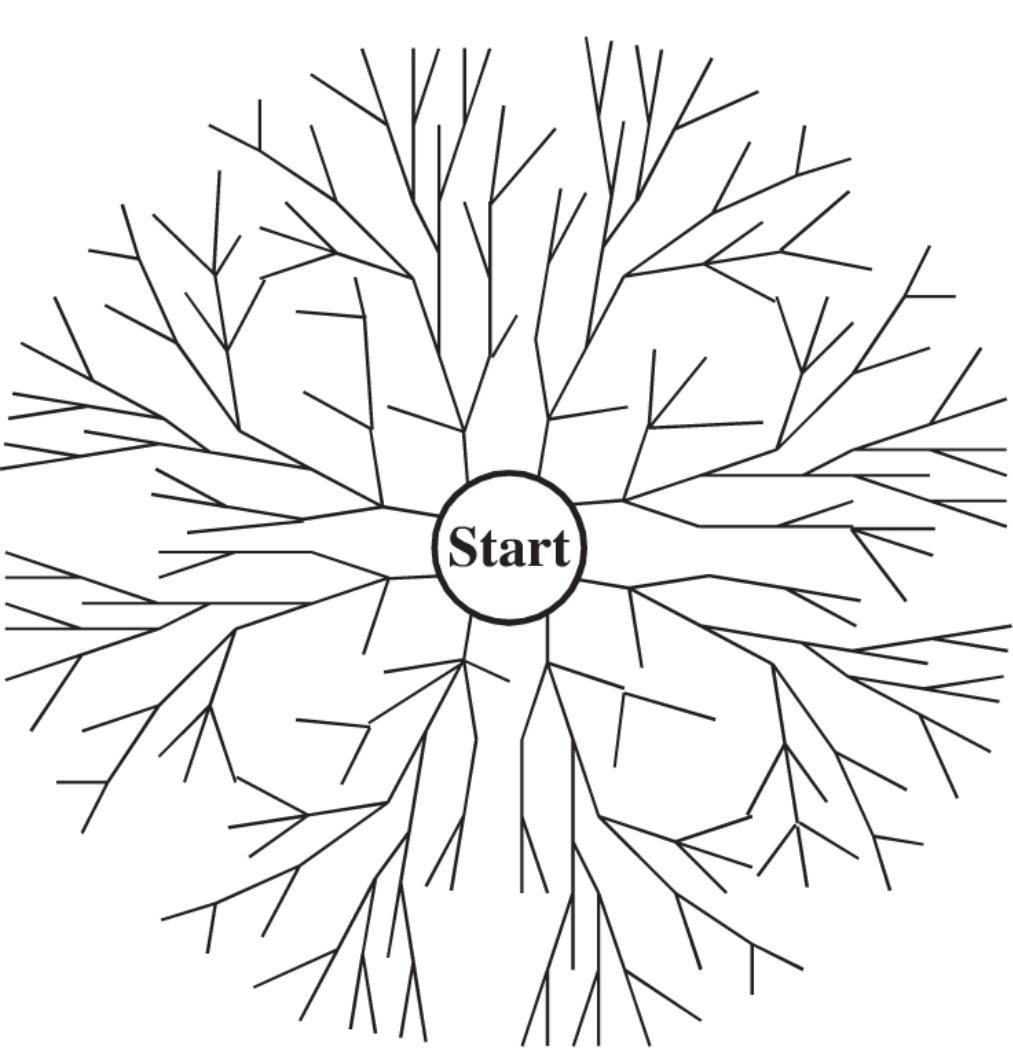
Complete? Yes

Optimal? Yes, if depth == cost

Time?  $O(b^{d/2})$

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# Attributes

Bi-directional Breadth-First Search

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# Classical Search

## Limitations of Classic Search

Goal state must be easily recognized

Search space is manageable

- \* Observable
- \* Deterministic
- \* Single-agent
- \* Discrete
- \* Static
- \* Known

## Blind (Uninformed) Search



## Informed Search



# Informed Search

## Greedy Best-First Search

Maintain frontier as a list sorted by heuristic.  
Expand the node with the lowest heuristic first.

Complete? No  
Optimal? No

## A\* Search

Take the action with the lowest total of cost plus heuristic value.

Complete? Yes\*  
Optimal? Yes\*

\*if our heuristic is admissible and consistent

# Heuristics

Estimating the cost to goal



Heuristic  $h_1$  **dominates** heuristic  $h_2$  if  
for any node  $n$ ,  $h_1(n) \geq h_2(n)$



## Iterative-Deepening A\*

Use cost+heuristic calculation of A\*, but perform a depth-first search, on each iteration increasing limit to include node with lowest value that was excluded on the previous iteration.

Complete? Yes\*  
Optimal? Yes\*

\*if our heuristic is admissible and consistent

## (Simple) Memory-Bounded A\* ((S)MA\*)

A\*, but limits the number of nodes retained in memory (forgets those with worst cost + heuristic value), with RBFS-style backup.

Complete? Usually\*  
Optimal? Usually\*

\* if our heuristic is admissible and consistent, and the depth of the optimal goal node is less than the memory size.

## Recursive Best-First Search

Similar to depth-first, but keep track of best alternate path available from ancestors of the current node.

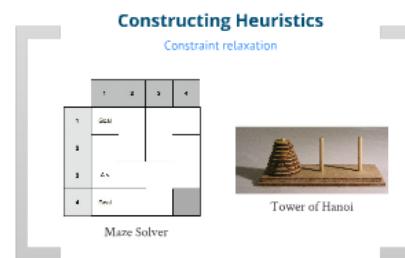
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# Heuristics

## Estimating the cost to goal

Desirable Properties of Heuristics	
<b>Accurate</b>	Strong correlation between heuristic and actual result
<b>Efficient (time)</b>	
<b>Efficient (memory)</b>	
<b>Admissible (Optimistic)</b>	Never overestimates costs
<b>Consistent</b>	Differences in heuristics should also be optimistic



Heuristic  $h_1$  **dominates** heuristic  $h_2$  if  
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# Desirable Properties of Heuristics

Accurate

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Efficient (time)

Efficient (memory)

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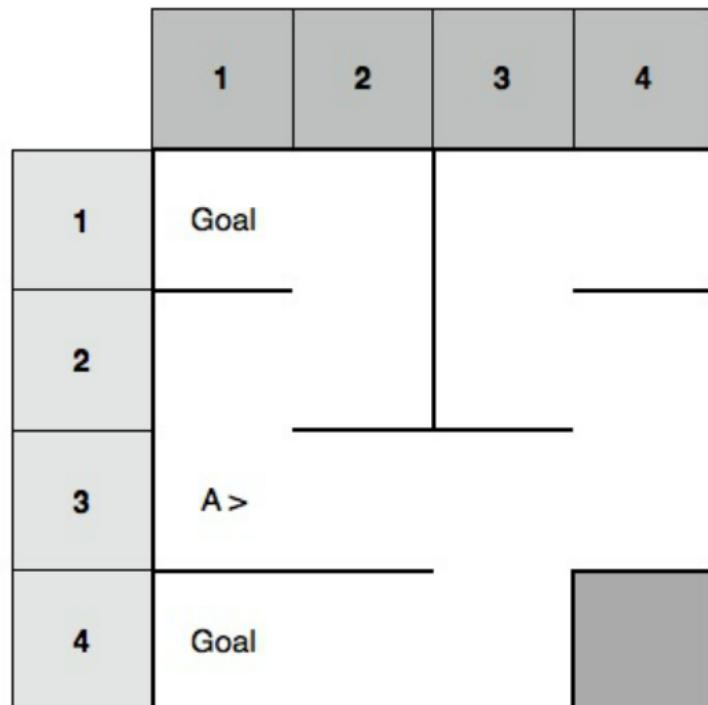
costs

Consistent

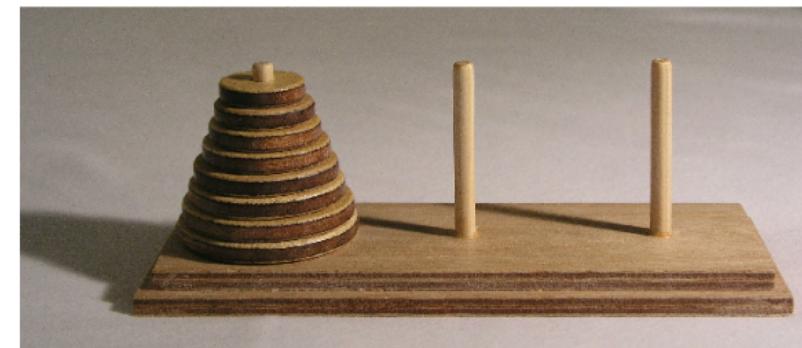
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# Constructing Heuristics

Constraint relaxation



Maze Solver



Tower of Hanoi



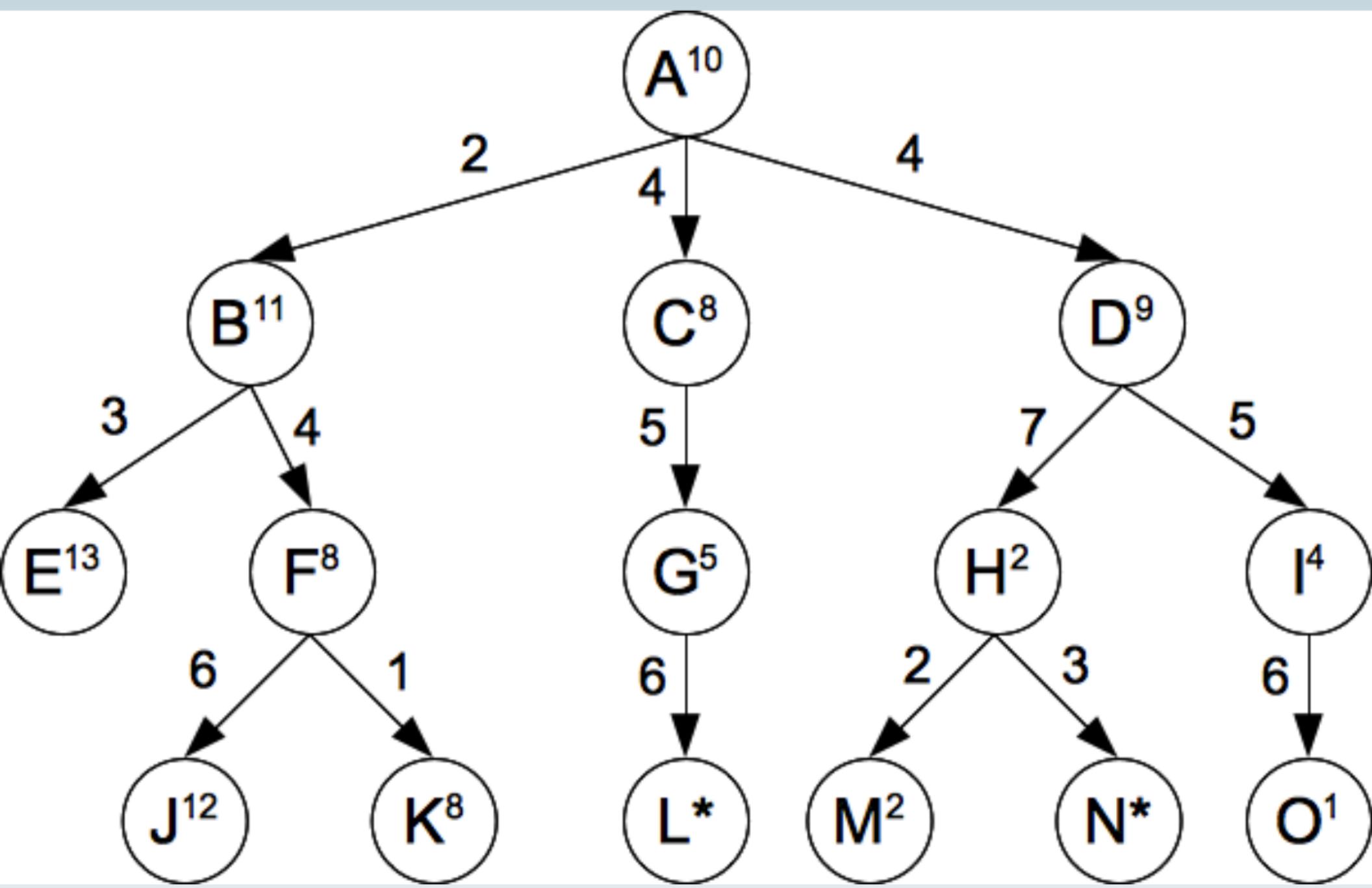
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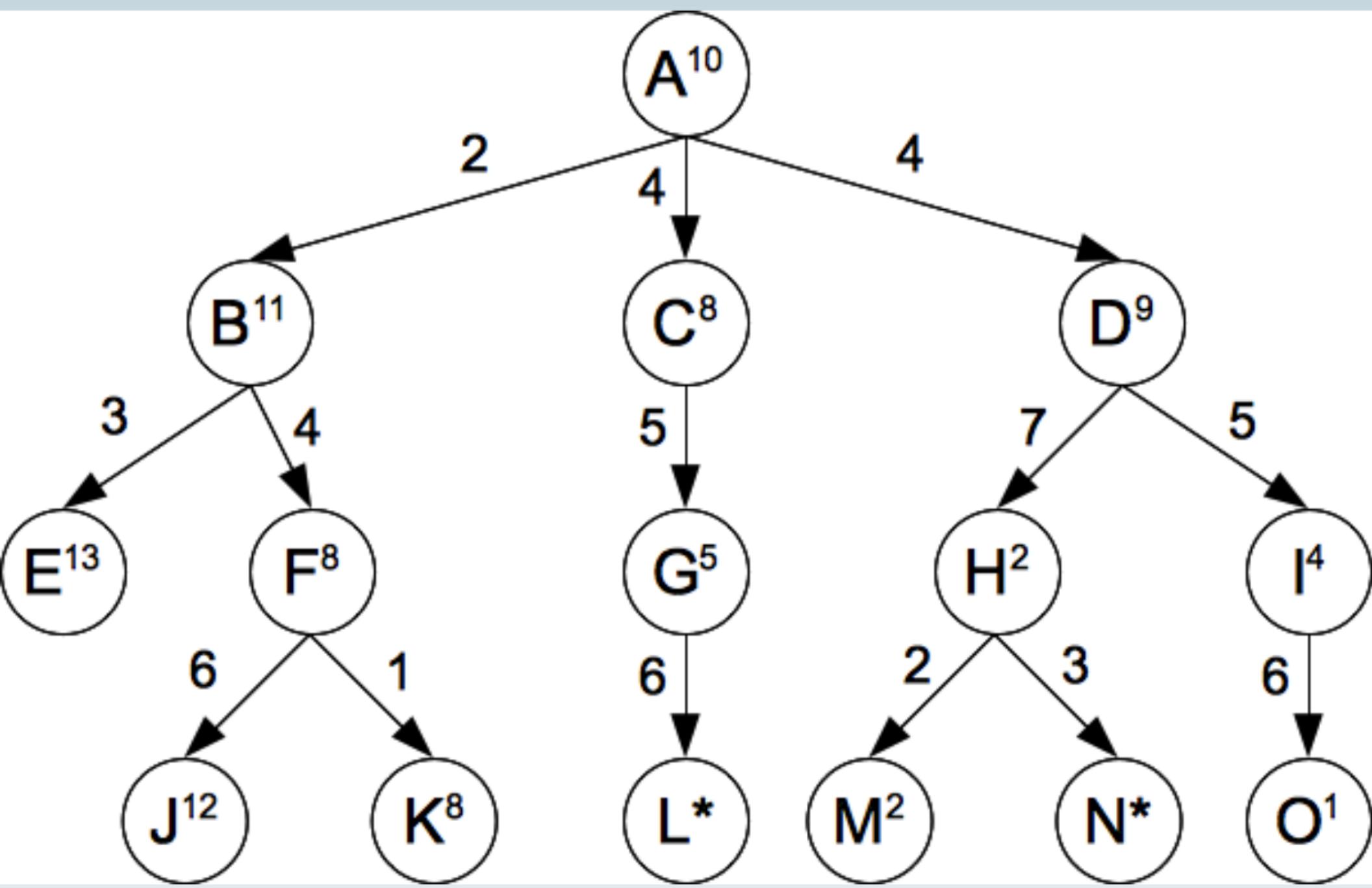
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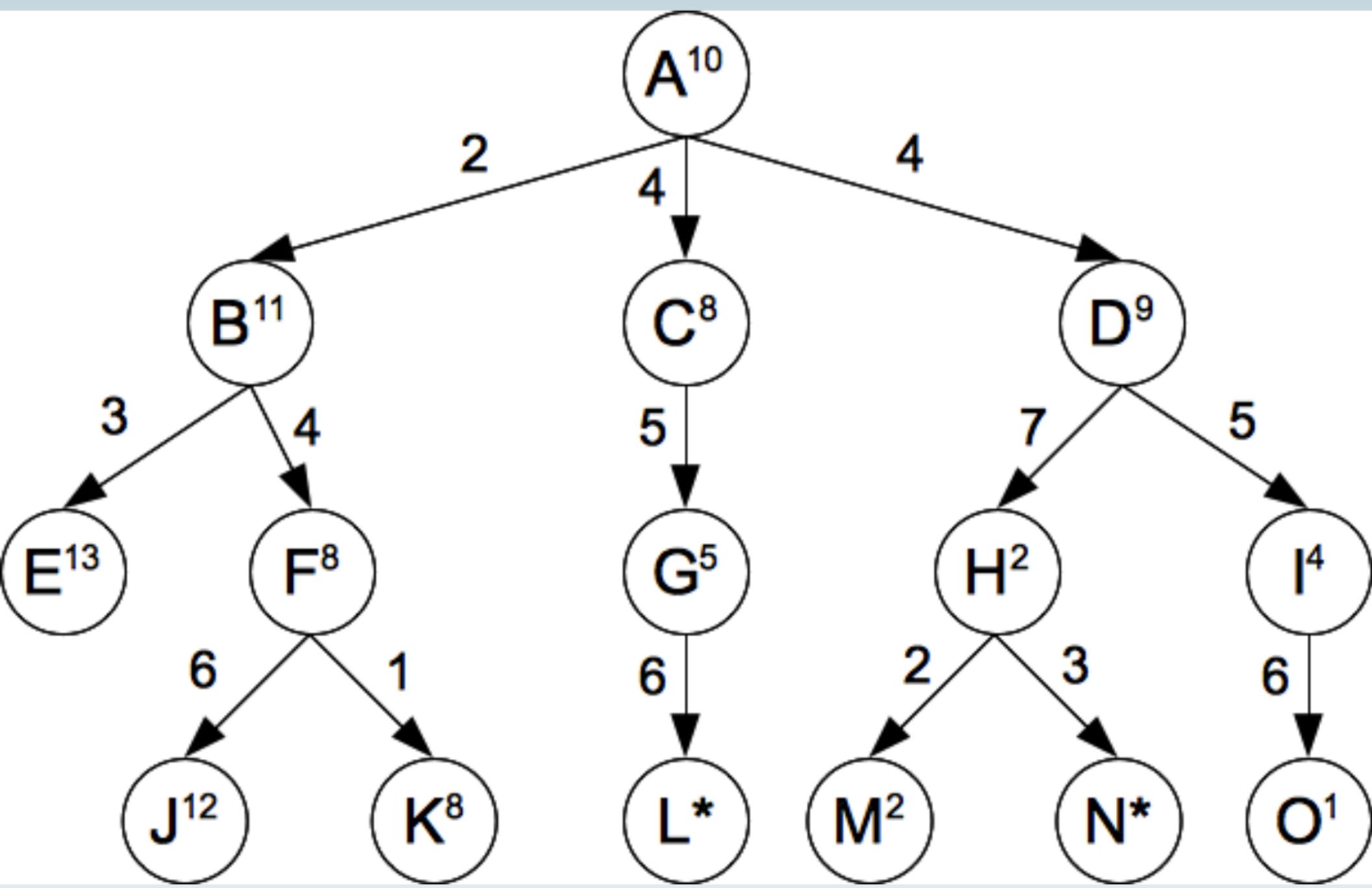
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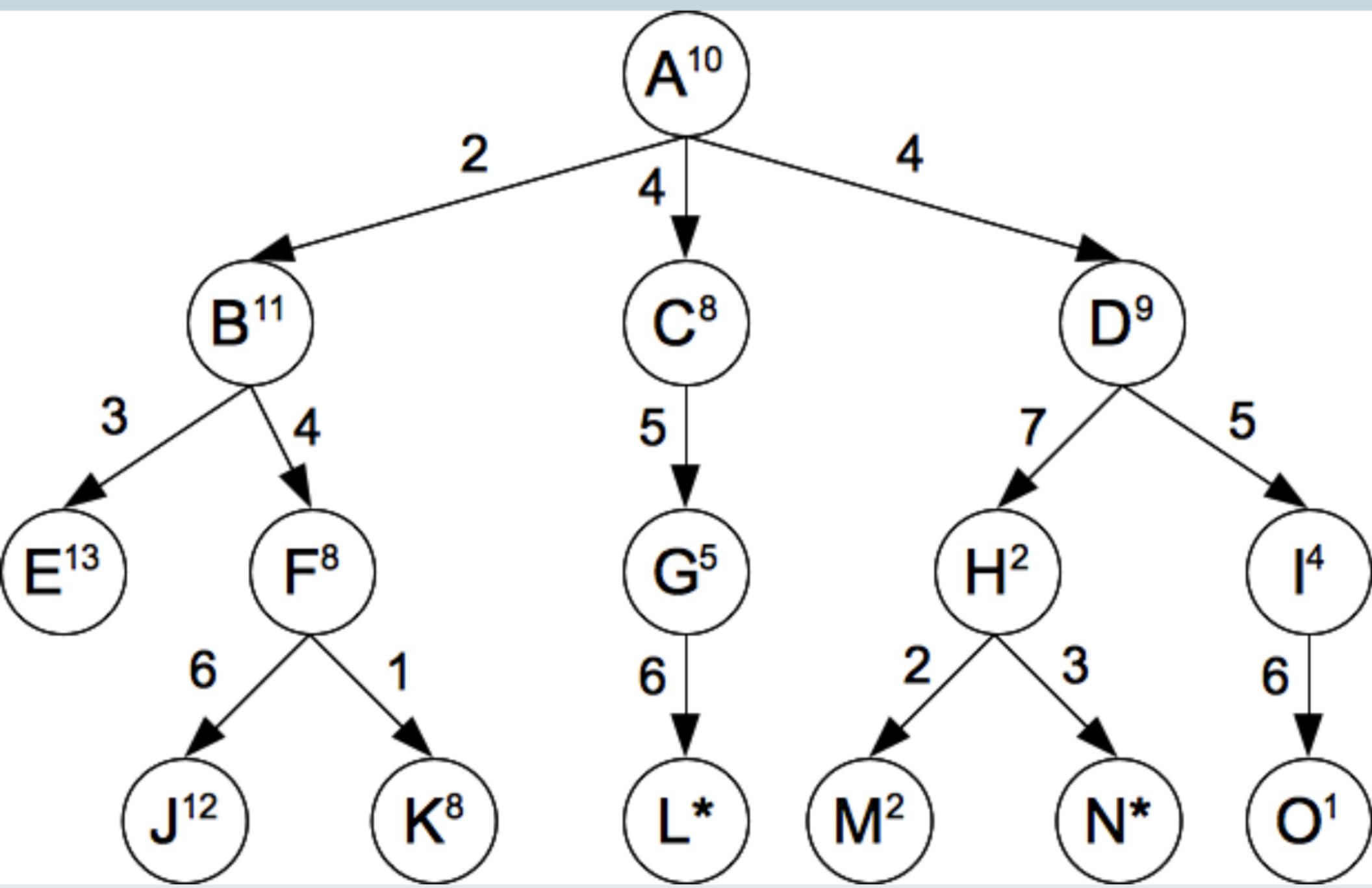
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Search space is manageable

Environment is:

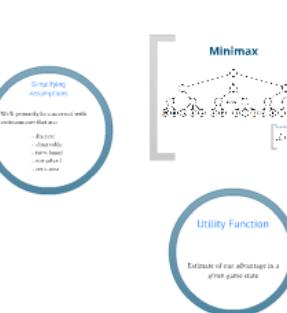
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# Alternate Search Strategies

## Optimization Problems



## Multi-Agent Environments

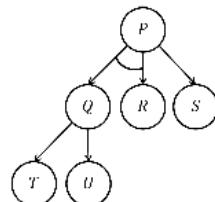


## Partially Observable Environments



Incremental belief state search allows us to consider multiple possible worlds simultaneously

## Stochastic Environments



An and-or tree represents multiple possible outcomes using connected branches.

## Continuous Environments

Two approaches:

1. Treat the search space as a discrete environment by partitioning it.
2. Examine slope of change of evaluation function and use to guide increments.

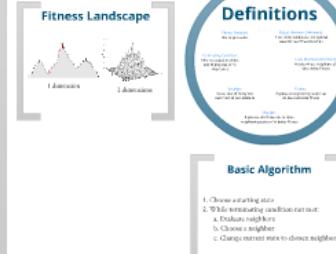
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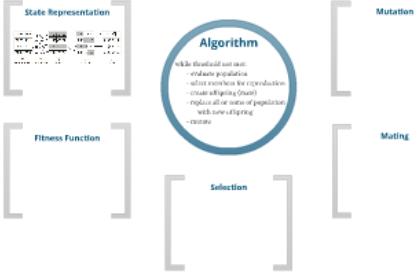
## Characteristics

- Goal state is not clearly defined
- Search space is vast, potentially infinite
- Path to reach solution is generally unimportant

### Local Search



### Genetic Algorithms





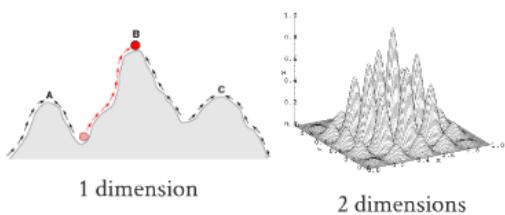
You Tube

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# Local Search

## Fitness Landscape



1 dimension

2 dimensions

## Definitions

**Fitness Function:**  
Our target metric

**Global Maximum (Minimum):**  
In our entire landscape, the optimal value for our fitness function

**Terminating Condition:**  
When to stop (threshold, lack of progress, time, steps, etc.)

**Local Maximum (Minimum):**  
A state whose neighbors all have worse fitness

**Neighbor:**  
States that differ by one increment of one attribute

**Plateau:**  
A group of neighboring states that all have identical fitness

**Shoulder:**  
A plateau which has one or more neighboring states with better fitness

## Flavors of Local Search

**Steepest-ascent:**  
Always choose the neighbor with the highest fitness

**Random restart (shotgun):**  
Keep trying

**Stochastic:**  
Most likely choose the neighbor with the highest fitness

**Simulated Annealing:**  
Stochastic with allowed negative progress, threshold decreases over time

**First Choice:**  
Always choose the first neighbor with a better fitness

**Beam Search:**  
Track multiple results for greater coverage

## Basic Algorithm

1. Choose a starting state
2. While terminating condition not met:
  - a. Evaluate neighbors
  - b. Choose a neighbor
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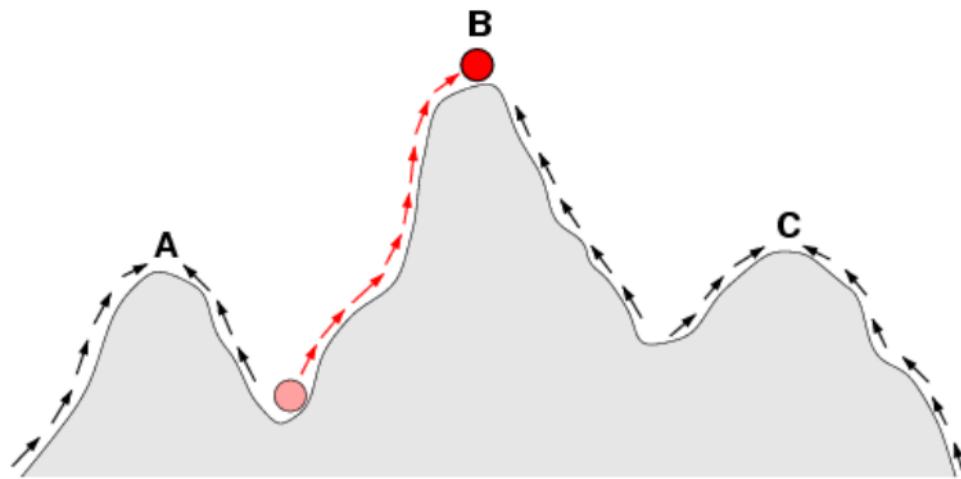
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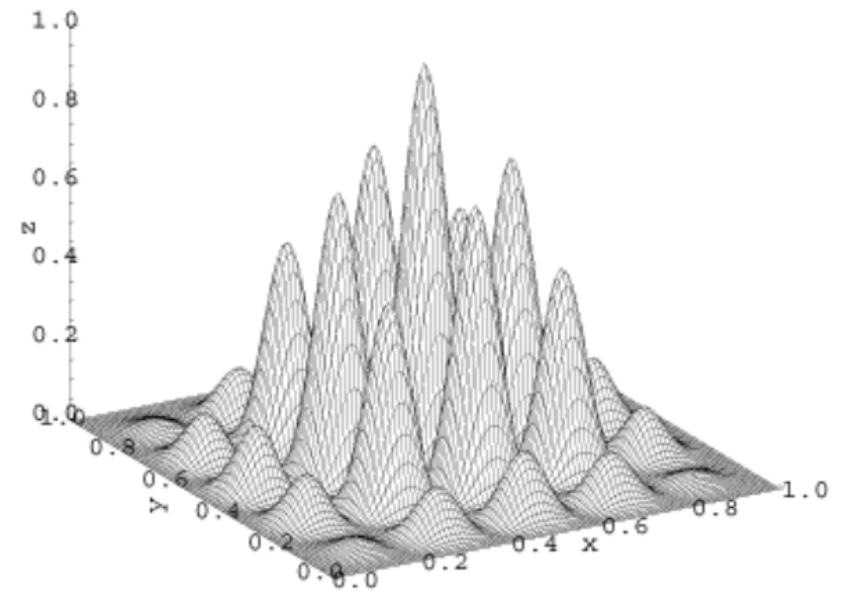


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# Fitness Landscape



1 dimension



2 dimensions

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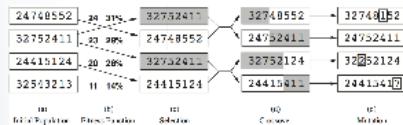
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# Genetic Algorithms

## State Representation



## Algorithm

while threshold not met:

- evaluate population
- select members for reproduction
- create offspring (mate)
- replace all or some of population with new offspring
- mutate

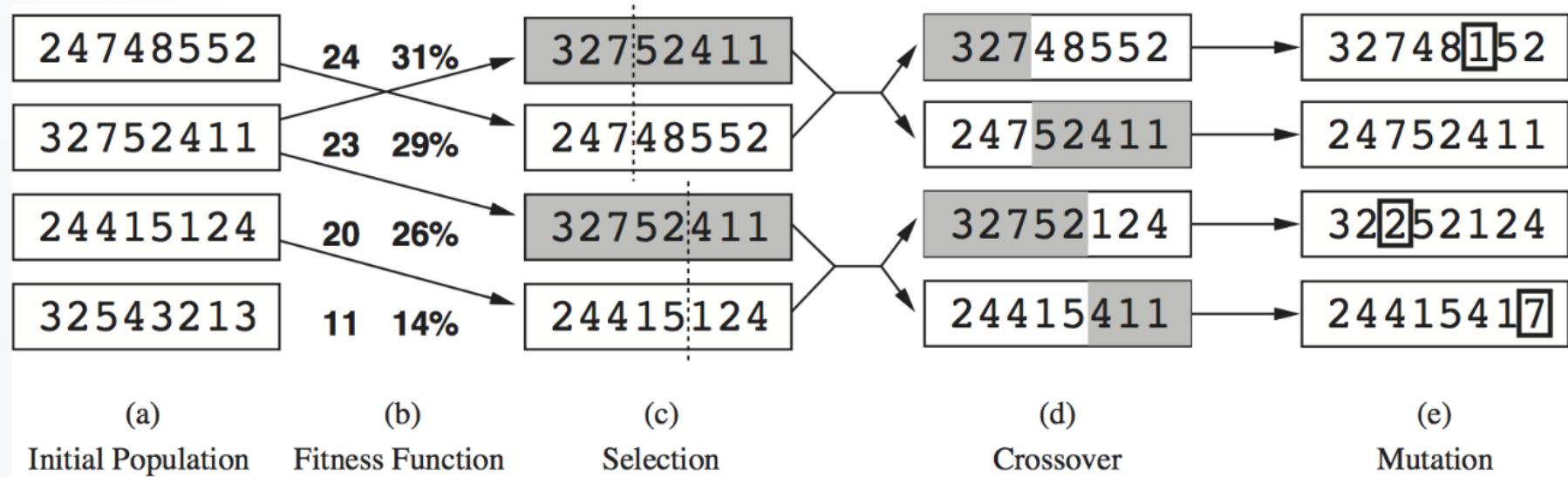
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## Mutation

## Selection

## Mating

# State Representation

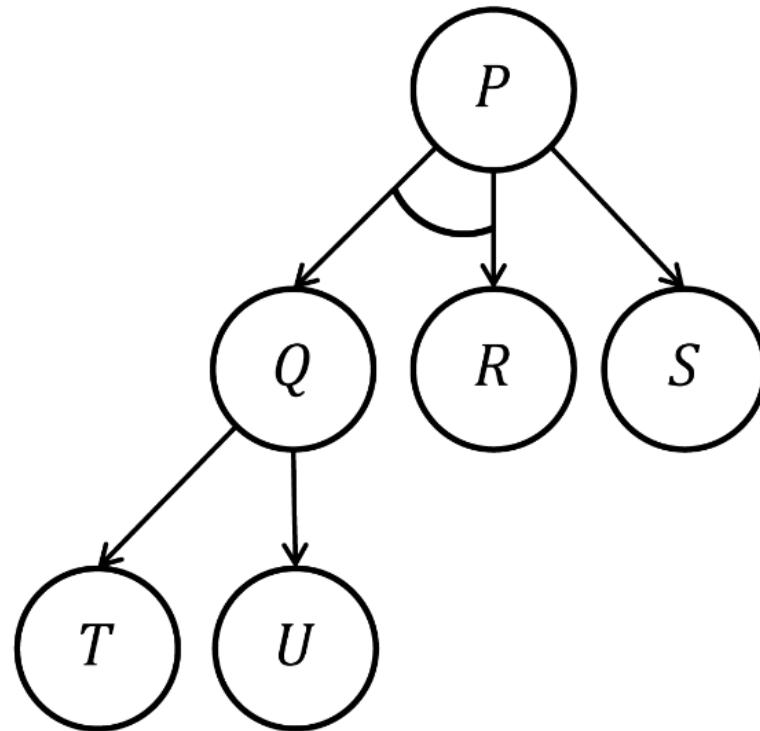


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# Stochastic Environments



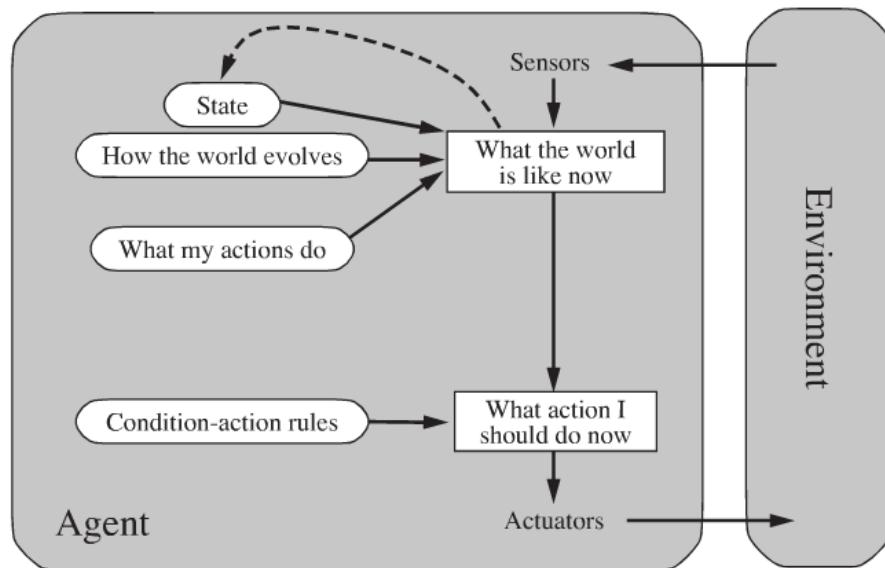
An and-or tree represents multiple possible outcomes using connected branches.

# Continuous Environments

Two approaches:

1. Treat the search space as a discrete environment by partitioning it.
2. Examine slope of change of evaluation function and use to guide increments.

# Partially Observable Environments



Incremental belief state search allows us to consider multiple possible worlds simultaneously

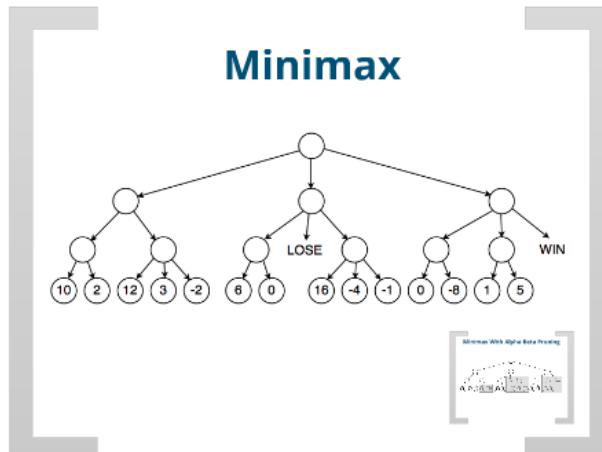
# Multi-Agent Environments

## Simplifying Assumptions

We'll primarily be concerned with environments that are:

- discrete
- observable
- turn-based
- not solved
- zero-sum

## Minimax



## Weaknesses

We may spend time exploring branches that are unlikely to be productive.

Cutoff test

We may stop immediately before an important event occurs.

Horizon effect

Quiescence

## Utility Function

Estimate of our advantage in a given game state

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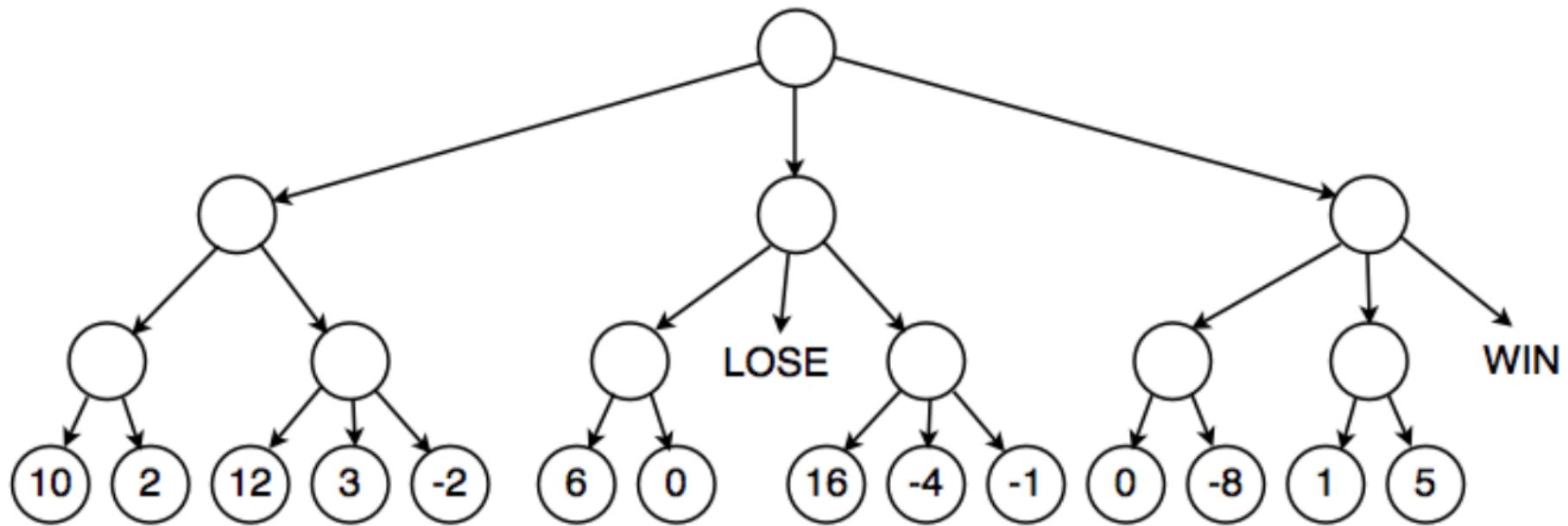
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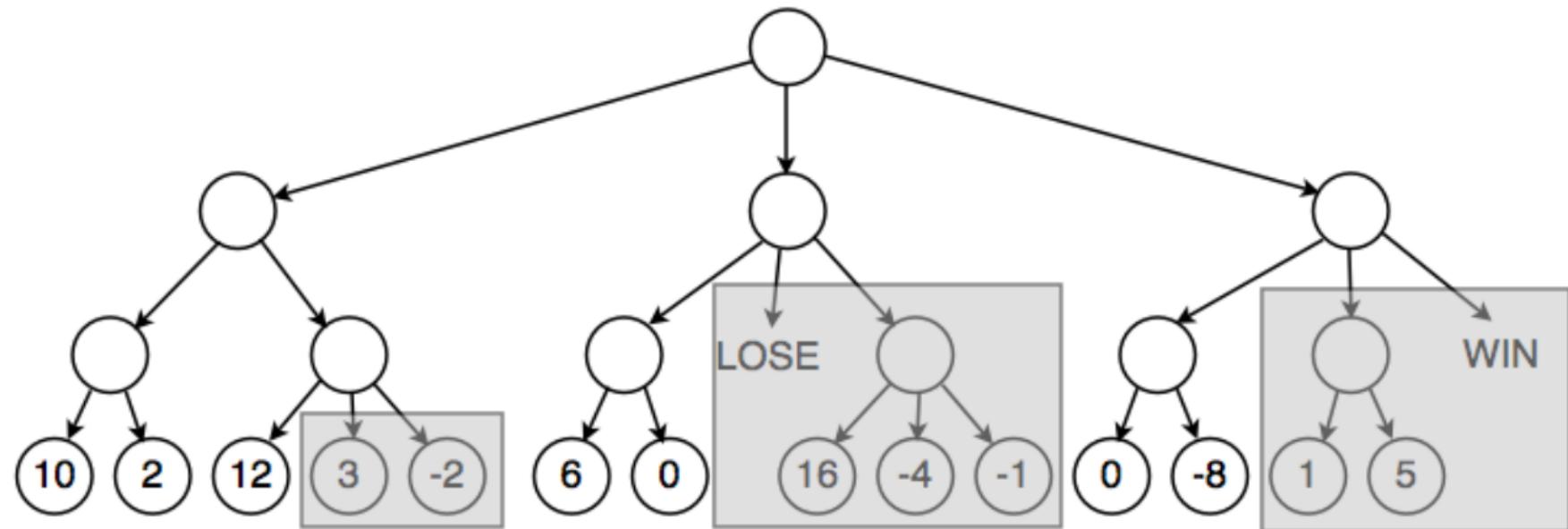
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# Constraint Satisfaction

## Environment

Factored state representation.

Portions of state have related values  
(variables are dependent on each other).

Often used for solving scheduling problems.

## Components

We represent a CSP with:

- Variables
- Domains
- Constraints

## Example

Insert the letters A-F into spaces 1-6,  
with the following constraints:

- Only one letter per space
- A is not in row 1, column 2
- A and C are not in the same column
- E and F are in the same row
- C and F are not adjacent  
(horizontally or vertically)
- A and B are adjacent
- F and D are in the same column

1	2
3	4
5	6

## Consistency

Consistency is used to propagate  
constraints.

**Node consistency:** One variable

**Arc consistency:** Two variables

**Path consistency:** Three variables

**k-consistency:** k variables

## Heuristics

**Minimum Remaining Values (MRV):**  
Choose the variable with the smallest  
domain.

**Least Restrictive Values (LRV):**  
Choose the value that least  
restricts other variables' domains.

## Algorithm

Alternate variable selection and  
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