Mahindra Ecole Centrale

ME-305 Multiphysics Course Project

DEFORMATIONS DUE TO THERMOELASTICITY

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Team CE-8 Project 5

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0.1 Introduction

Stress, strain and temperature play a vital role in determining a components lifetime. Therefore, as an engineer it is very important for us to understand these quantities and how they interact with each other. We often encounter thermoelasticity in our daily life. In this project we have tried to understand this phenomenon and also showed how a component that experiences thermoelasticity behaves. An object is strongest when force is evenly distributed over its area, so a reduction in area, results in a localized increase in stress. The stress present at a particular location is termed as stress-concentration. Geometric discontinuities cause an object to experience a local increase in the intensity of a stress field. Cracks, sharp corners, holes and changes in the cross-sectional area of the object are the examples of shapes that cause these concentrations. The consideration of fatigue crack initiation mechanism under pure compressive cyclic stress is made with the association of stress concentration such as dents/notches that are inflicted in assembly period. This type of discontinuities is always present on the surface and introduce a non-uniform and complex stress field in the adjacent regions. This is the region where the cracks form and spread. Thus, the material fails. The effects of stress concentration, produced by holes/notches or other discontinuities under tension is of great importance for designing components. High local stresses can cause objects to fail more quickly. Therefore, one has to minimize the stress concentration at critical points.

0.2 Objective

The objective of this project is to quantify how stress concentration evolves as a function of temperature and specimen dimensions

0.3 Formulation

As thermoelasticity is a weakly coupled phenomenon between linear elastic deformation and thermal phenomenon the following equations govern the process.

$$\underline{\nabla}.\sigma + b = 0 \qquad\qquad \sigma = Stress\ tensor$$

b is taken to be because there are no body forces b = 0

$$\underline{\nabla}.\sigma = 0$$

$$\sigma = C : [\varepsilon - \alpha(T(x,t) - T_0)]$$

$$\rho C_p \frac{d(T(x,t))}{dt} = \underline{\nabla}.(k\underline{\nabla} T(x,t)) + Q$$

Assuming the specimen to be anisotropic we expanded the above equations by substituting each term as a matrix of appropriate dimensions. Here weve formulated the equations only for two dimensional problem. On expanding equation 2 and substituting in in equation 1 gives us 2 relations. Similarly on expanding equation 3 we obtain another relation.

$$e_a \frac{\partial^2 \underline{u}}{\partial t^2} + d_a \frac{\partial \underline{u}}{\partial t} - \underline{\nabla}.(c\underline{\nabla}\ \underline{u} + \alpha \underline{u} - \gamma) + \beta.\underline{\nabla}\ \underline{u} + a\underline{u} = f$$

The above mentioned equation is the general structure of the coefficient form pde in COMSOL. On expanding it and comparing the coefficients with the three relations obtained earlier we get the values of all the coefficients. After solving the equations and comparing the coefficients as shown in Primer we end up with.

$$C_{11}u_{1,11} + C_{12}u_{2,21} + C_{44}(u_{1,22} + u_{2,12}) - C_{11}\alpha\frac{\partial T}{\partial x_1} - C_{12}\alpha\frac{\partial T}{\partial x_1} - 2C_{44}\alpha\frac{\partial T}{\partial x_2} = 0$$

$$C_{11}u_{2,22} + C_{12}u_{1,12} + C_{44}(u_{1,21} + u_{2,11}) - C_{11}\alpha\frac{\partial T}{\partial x_2} - C_{12}\alpha\frac{\partial T}{\partial x_2} - 2C_{44}\alpha\frac{\partial T}{\partial x_1} = 0$$

$$k_1 u_{3,11} + k_2 u_{3,21} + k_3 u_{3,12} + k_4 u_{3,22} - \rho c_p \frac{\partial T}{\partial t} + \rho_f = 0$$

Here [u1,u2,u3] = [x,y,T],

$$\frac{\partial u_i}{\partial x_j} = u_{i,j} \quad and \quad \frac{\partial}{\partial x_k}(\frac{\partial u_i}{\partial x_j}) = u_{i,jk}$$

In the equations obtained earlier we do not have any second order derivative terms. Therefore, $e_a = 0$

There are no u_1, u_2, u_3 terms, so a $= 0$

On further simplification we obtain the values of d_a,c,α.

$$d_a = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\rho c_p \end{bmatrix}$$

where ρ -density of the material

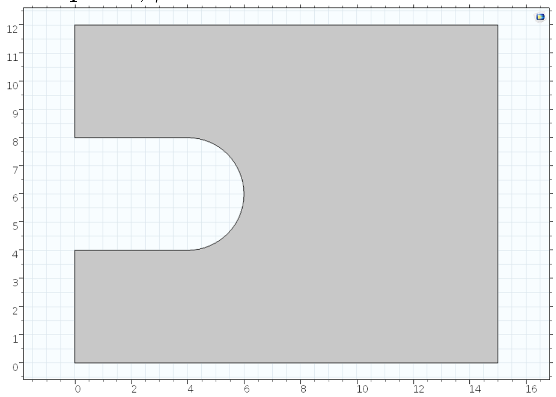
$$c = \begin{bmatrix} \begin{bmatrix} C_{11} & 0 \\ 0 & C_{44} \end{bmatrix} & \begin{bmatrix} 0 & C_{12} \\ C_{44} & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & C_{44} \\ C_{12} & 0 \end{bmatrix} & \begin{bmatrix} C_{44} & 0 \\ 0 & C_{11} \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} K_1 & K_2 \\ K_3 & K_4 \end{bmatrix} \end{bmatrix}$$

$$\alpha = \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \end{bmatrix} & \begin{bmatrix} -C_{11}\alpha - C_{12}\alpha \\ -2C_{44}\alpha \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \end{bmatrix} & \begin{bmatrix} -2C_{44}\alpha \\ C_{12}\alpha C_{11}\alpha \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{bmatrix}$$

0.4 Boundary conditions

$$\begin{aligned} &\hat{n}.\sigma = \underline{t} \\ &t = [150 * 10^6, 0, 0] \\ &\hat{n}.(c\underline{\nabla u} + \underline{\alpha u} - \gamma) = g - q\underline{u} \end{aligned}$$

Here q = 0,γ = 0



The above picture is the sample design of the model analyzed in the report

$$C_{11} = \frac{(1-\nu)}{(1+\nu)(1+2\nu)}E$$
$$C_{12} = \frac{\nu}{(1+\nu)(1+2\nu)}E$$
$$C_{44} = \frac{C_{11}-C_{12}}{2}$$

Study-1 The study involves fixing both the left ends and assigning it to a temperature at 298K. The right end is kept at the same temperature and a traction in the x-direction is applied. T1=298K and T2=298K. The traction in the first picture is in the negative x direction, whereas the traction applied in second picture is in positive direction.

Study-2 The study involves fixing the left end. The right end is kept at the sampe temperature and a traction in the x-direction is applied. T1=300K and T2=500K.The traction in the first picture is in the negative x direction, whereas the traction applied in second picture is in positive direction.

Study-3 The study involves fixing the left end.. The right end is kept at the sampe temperature and a traction in the x-direction is applied. T1=500K and T2=300K.The traction in the first picture is in the negative x direction, whereas the traction applied in second picture is in positive direction.

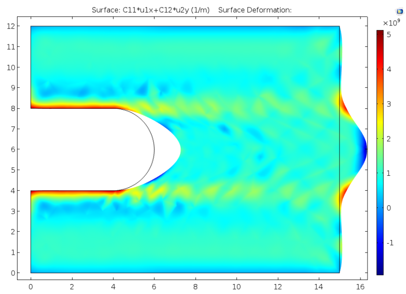


Figure 1: study 1.

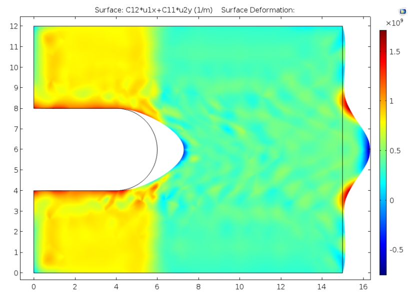


Figure 2: study 1

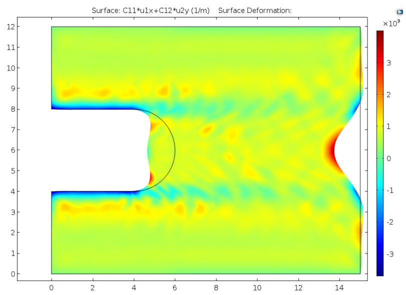


Figure 3: study 1.

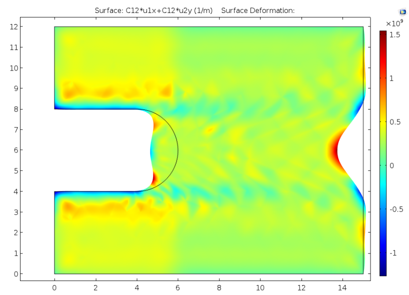


Figure 4: study 1

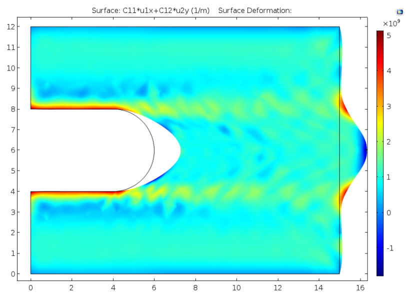


Figure 5: study 2.

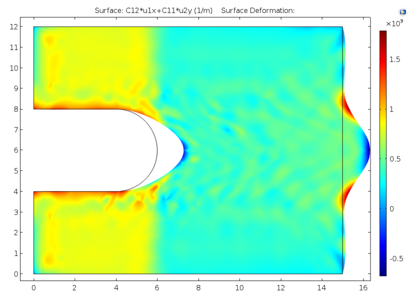


Figure 6: study 2

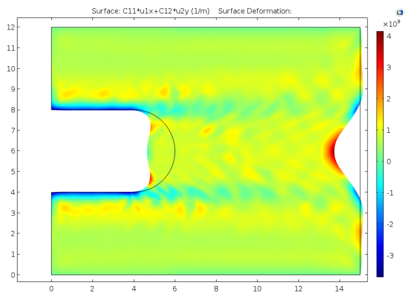


Figure 7: study 2.

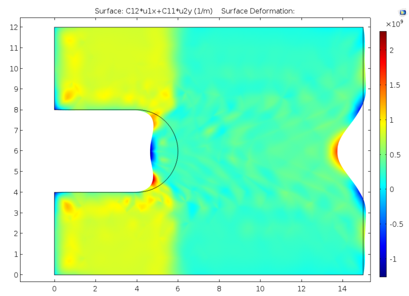


Figure 8: study 2

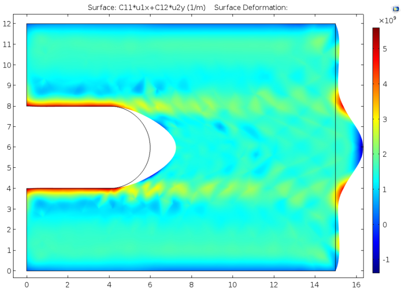


Figure 9: study 1.

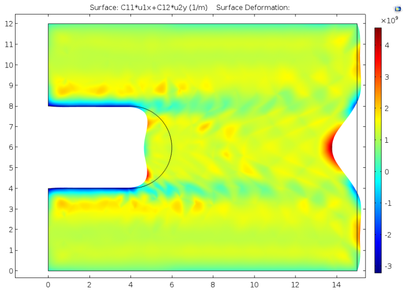


Figure 10: study 1

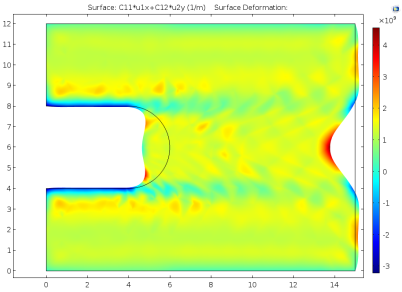


Figure 11: study 1.

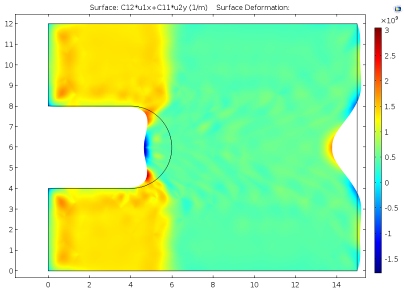


Figure 12: study 1

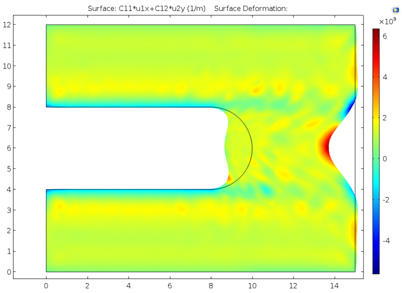


Figure 13: change in w/d ratio.

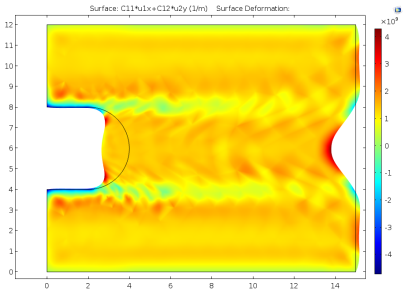


Figure 14: change in w/d ratio.

0.5 conclusion

As the w/d ratio inceases, the stress concentration increases. Here, d- depth of the noth, and w- width of the piston. This has a practical implementation in the piston of the four strokes engine, where the effect of the temperature on the notch is determined. The higher the depth of the notch, the less will be the stress.

0.6 References

engineeringtoolbox-

The above webiste provoided the needed information.