Quantum states and circuits

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Classical states

- Bit strings: 0001100101
- Don't need to think about the physical implementation
- Just focus on the rules of manipulation
- Given two input registers A and B, and output C we can perform:
 - o C=A+B
 - C=A*B
 - C=A AND B
 - C=A XOR B
 - C=A>>B
 - o etc, etc, etc

Quantum states

$$|\Psi
angle = \sum_{i=0}^{2^n-1} c_i |i
angle, \quad c_i \in \mathbb{C}, \quad \sum_{i=0}^{2^n-1} |c_i|^2 = 1$$

quantum register or quantum state

Quantum states

$$|\Psi
angle = \sum_{i=0}^{2^n-1} c_i |i
angle, \quad c_i \in \mathbb{C}, \quad \sum_{i=0}^{2^n-1} |c_i|^2 = 1$$
 amplitude probability

Quantum gates

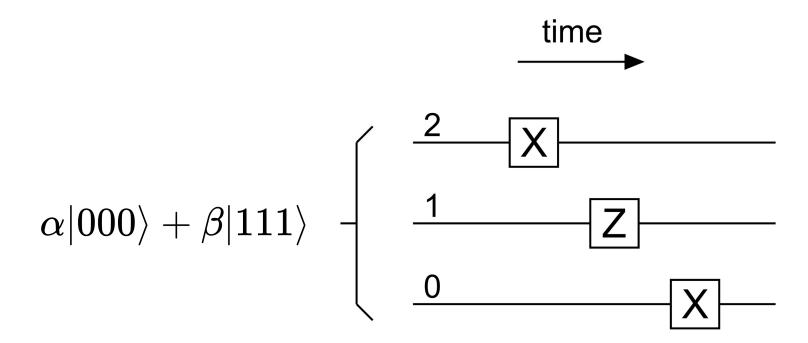
$$X|0
angle = |1
angle \ X|1
angle = |0
angle$$

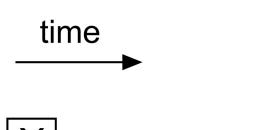
$$X_2(lpha|000
angle + eta|111
angle) = lpha|100
angle + eta|011
angle \ X_1(lpha|000
angle + eta|111
angle) = lpha|010
angle + eta|101
angle \ X_0(lpha|000
angle + eta|111
angle) = lpha|001
angle + eta|110
angle \ lpha_2 = lpha_1 = lpha_0$$

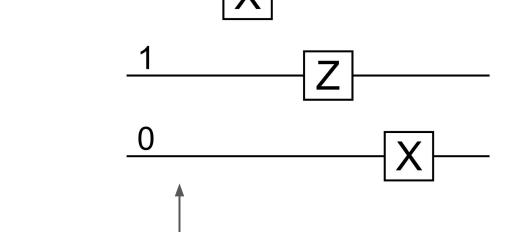
Quantum gates

$$egin{aligned} Z|0
angle &=|0
angle \ Z|1
angle &=-|1
angle \end{aligned}$$

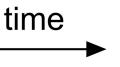
$$egin{aligned} Z_2(lpha|000
angle + eta|111
angle) &= lpha|000
angle - eta|111
angle \ Z_1(lpha|000
angle + eta|111
angle) &= lpha|000
angle - eta|111
angle \ Z_0(lpha|000
angle + eta|111
angle) &= lpha|000
angle - eta|111
angle \end{aligned}$$



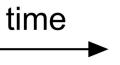


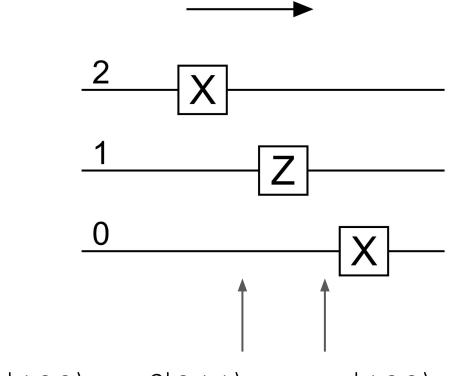


$$lpha|000
angle+eta|111
angle$$

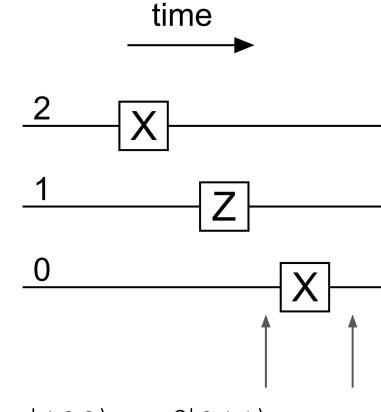


$$lpha|000
angle+eta|111
angle \qquad lpha|100
angle+eta|011
angle$$





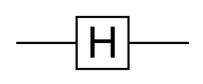
$$lpha|100
angle+eta|011
angle \qquad lpha|100
angle-eta|011
angle$$



$$lpha|100
angle-eta|011
angle \qquad lpha|101
angle-eta|010
angle$$

More quantum gates

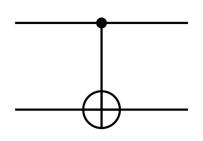




$$|\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = |+\rangle$$

$$egin{align} H|0
angle &=rac{1}{\sqrt{2}}(|0
angle+|1
angle)=|+
angle \ H|1
angle &=rac{1}{\sqrt{2}}(|0
angle-|1
angle)=|-
angle \ \end{gathered}$$





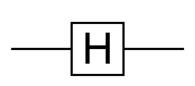
$$CNOT(1,0)|00
angle = |00
angle \ CNOT(1,0)|01
angle = |01
angle$$

$$CNOT(1,0)|10\rangle = |11\rangle$$

$$CNOT(1,0)|11
angle=|10
angle$$

More quantum gates

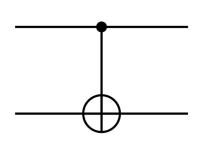
Hadamard



$$egin{align} H|0
angle &=rac{1}{\sqrt{2}}(|0
angle+|1
angle)=|+
angle \ H|1
angle &=rac{1}{\sqrt{2}}(|0
angle-|1
angle)=|-
angle \ \end{gathered}$$

$$|1\rangle=rac{1}{\sqrt{2}}(|0
angle-|1
angle)=|-
angle$$

Controlled-X

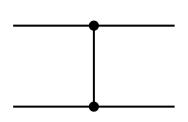


$$CX(1,0)|00
angle=|00
angle$$

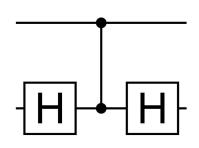
$$egin{aligned} CX(1,0)|01
angle &=|01
angle \ CX(1,0)|10
angle &=|11
angle \ CX(1,0)|11
angle &=|10
angle \end{aligned}$$

More quantum gates

Controlled-Z



$$egin{aligned} CZ(1,0)|00
angle = |00
angle \ CZ(1,0)|01
angle = |01
angle \ CZ(1,0)|10
angle = |10
angle \ CZ(1,0)|11
angle = -|11
angle \end{aligned}$$



$$egin{array}{c} |00
angle
ightarrow\ |01
angle
ightarrow\ |10
angle
ightarrow\ |11
angle
ig$$

$$|10\rangle \rightarrow |1\rangle \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$= \frac{1}{\sqrt{2}}(|1\rangle|0\rangle + |1\rangle|1\rangle)$$

$$= \frac{1}{\sqrt{2}}(|10\rangle + |11\rangle)$$

$$= \frac{1}{\sqrt{2}}(|10\rangle + |11\rangle)$$

$$|00\rangle \rightarrow ?$$

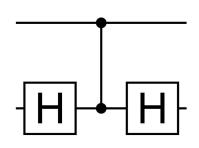
$$|01\rangle \rightarrow ?$$

$$|10\rangle \rightarrow ?$$

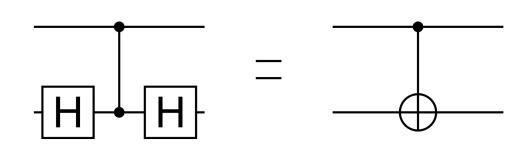
$$|10\rangle \rightarrow ?$$

$$|11\rangle \rightarrow ?$$

$$= |11\rangle$$

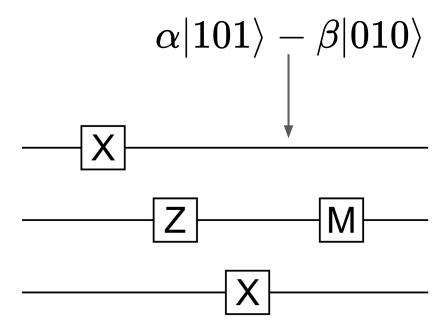


$$egin{array}{ll} |00
angle
ightarrow & |00
angle \ |01
angle
ightarrow & |01
angle \ |10
angle
ightarrow & |11
angle \ |11
angle
ightarrow & |10
angle \end{array}$$

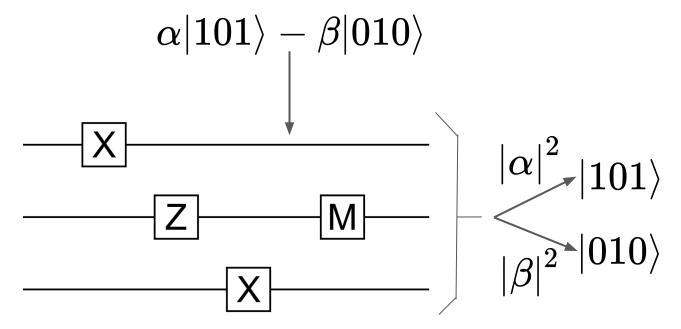


$$egin{array}{l} \ket{00}
ightarrow \ket{00} \ \ket{01}
ightarrow \ket{01} \ \ket{10}
ightarrow \ket{11} \ \ket{11}
ightarrow \ket{10} \end{array}$$

Measurement



Measurement



Entire system collapses to one state or the other with probabilities $|\alpha|^2$, $|\beta|^2$