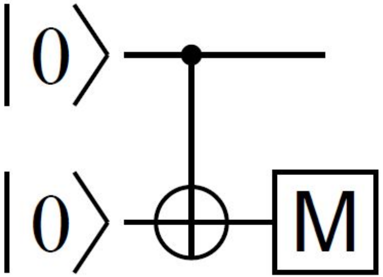
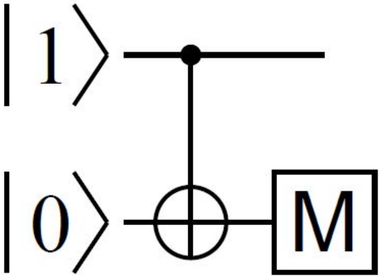
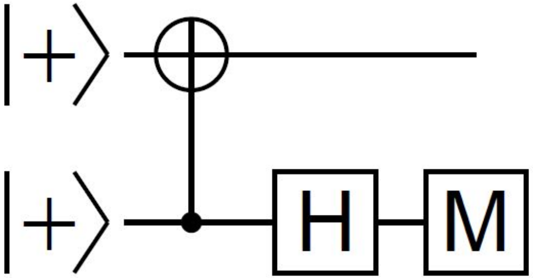
# Quantum error correction II:
## Detecting bit-flips and phase-flips

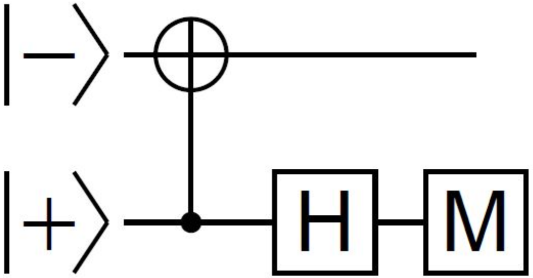
Austin Fowler

# What will the measurement results be?

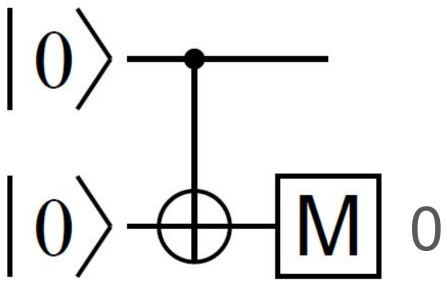


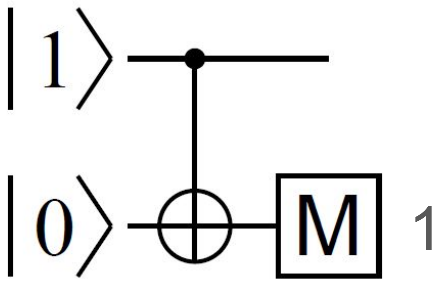



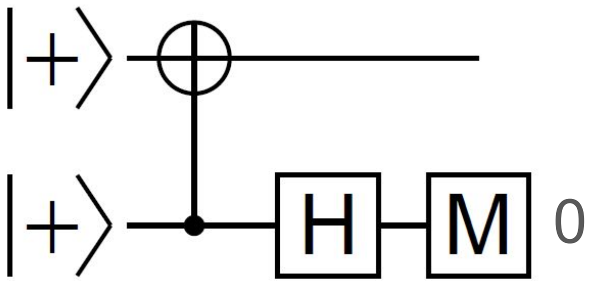



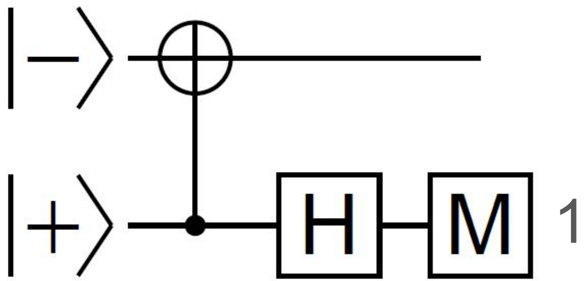



# Make sure you can derive these results

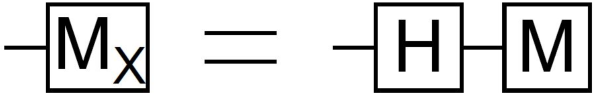


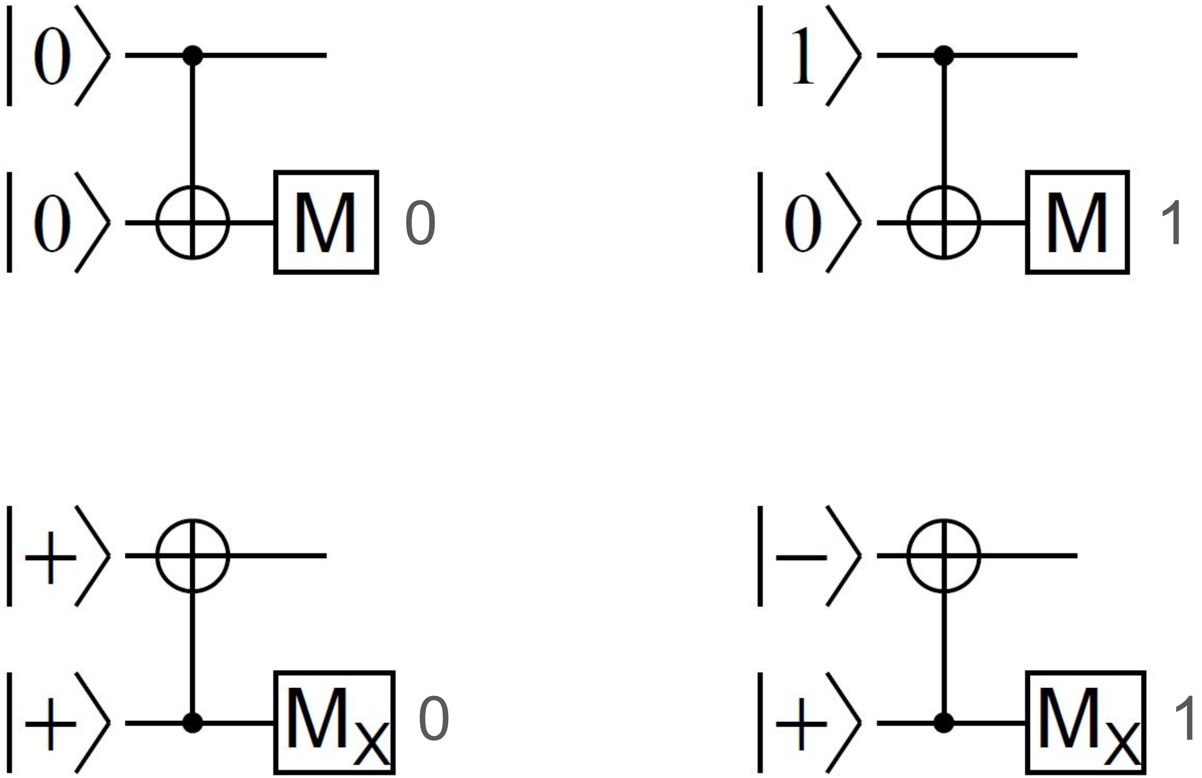



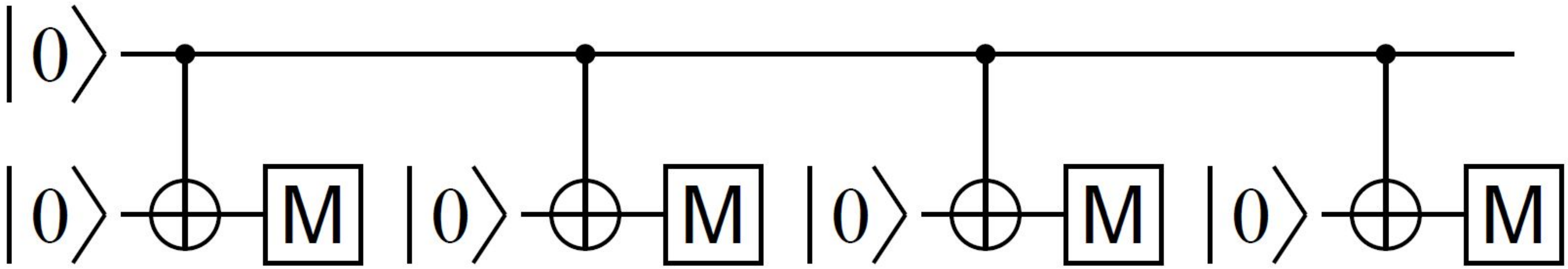



A useful definition



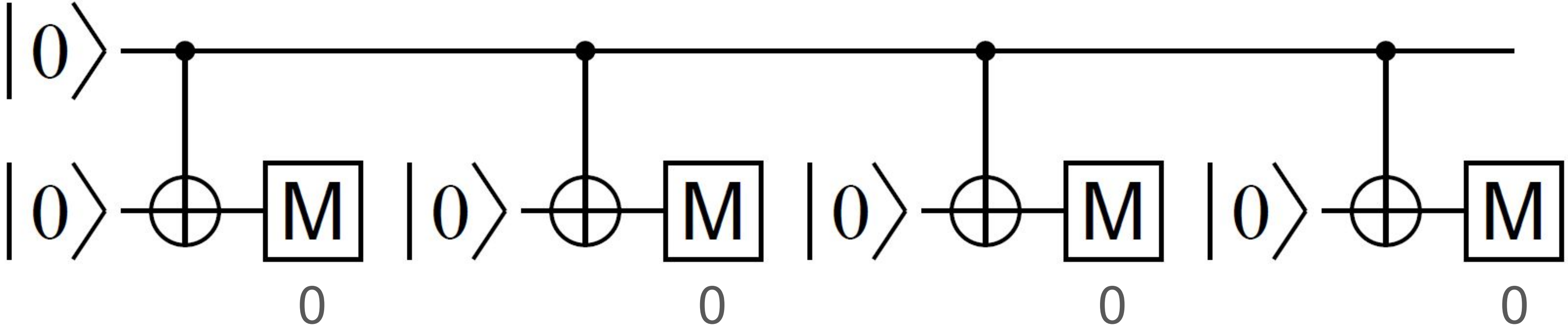



# Repeated detection



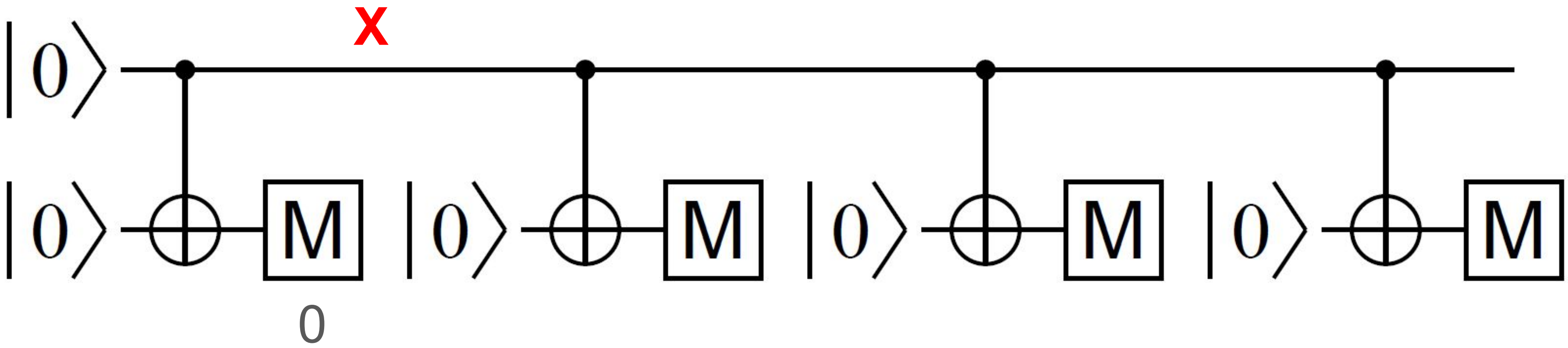

Consider a data qubit (top) being checked by a measure qubit (bottom)

# Repeated detection: results without errors



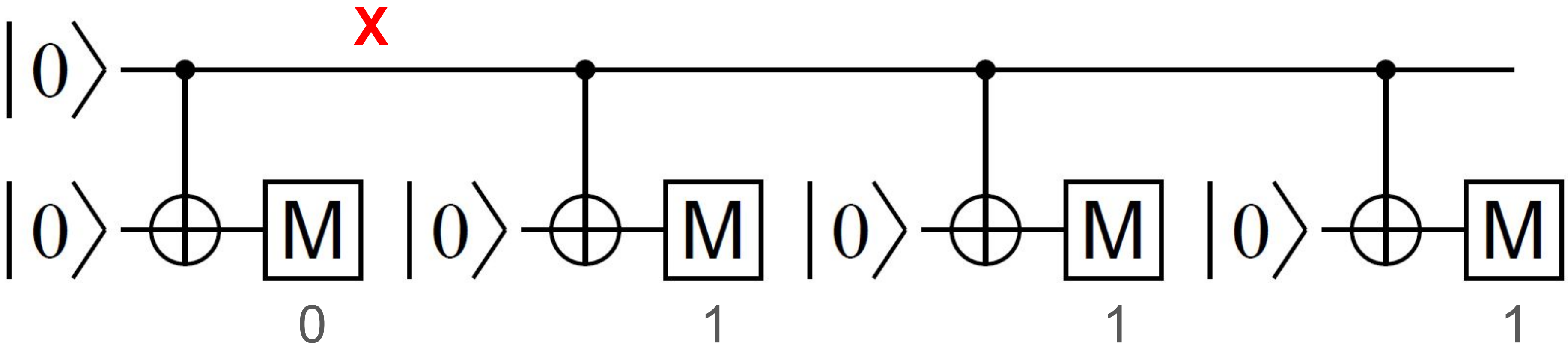

In the absence of errors, each measurement will report zero

# Repeated detection: results with a data error?



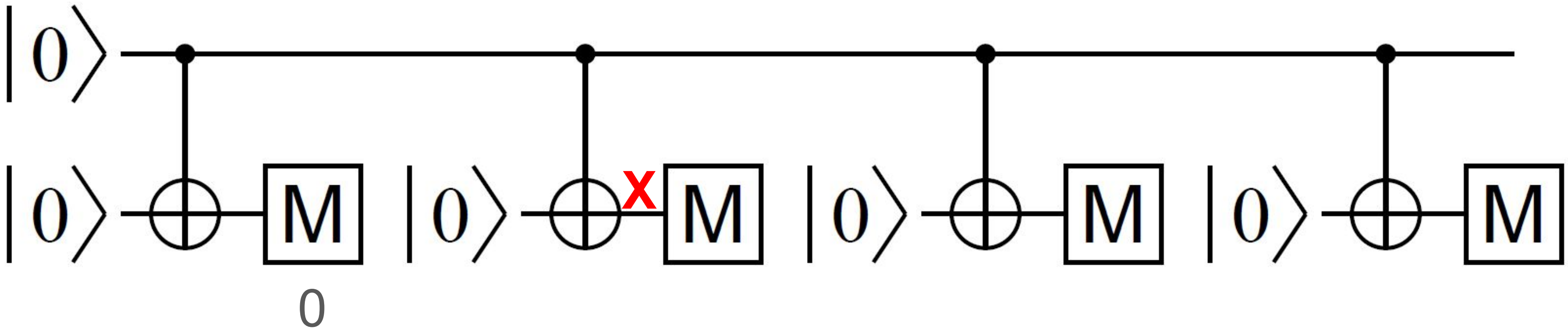

Now let's consider the case of a bit flip error occurring on the data qubit

# Repeated detection: results with a data error?



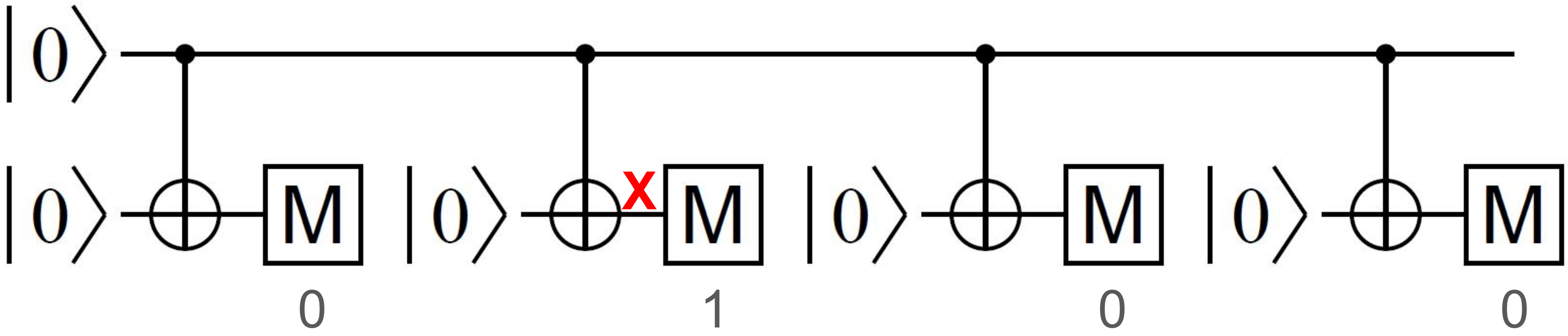

A single data qubit bit flip changes all future measurements

# Repeated detection: what about a measurement error?



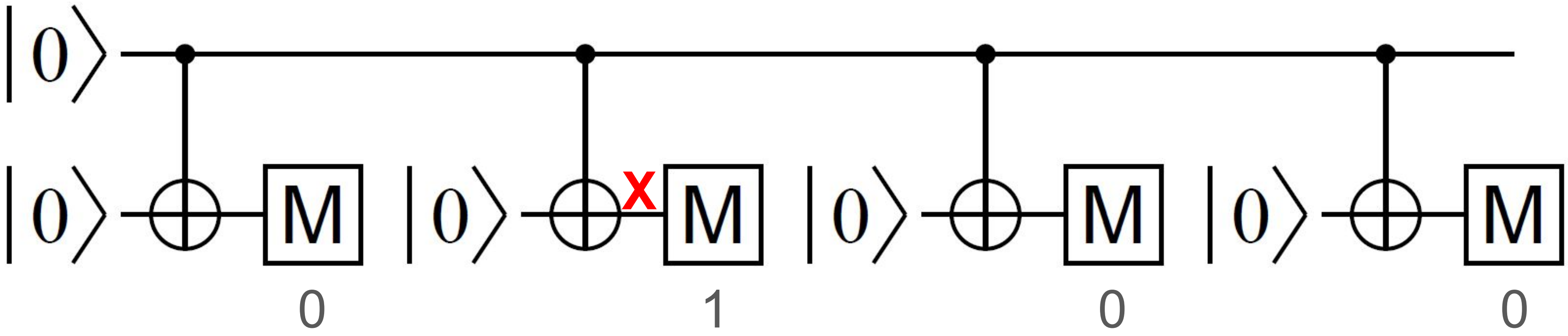

A measurement error can be modeled as a bit flip before the measurement gate

# Repeated detection: what about a measurement error?



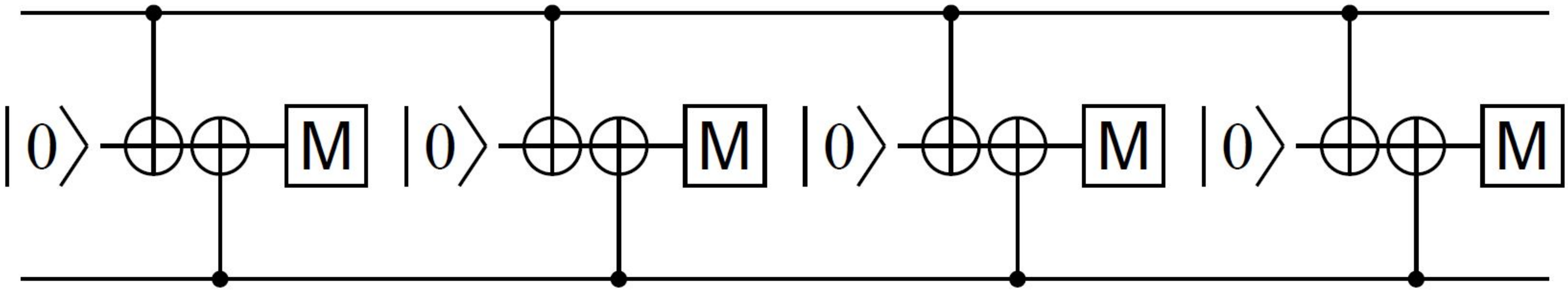

A measurement error flips one result

# Repeated detection: summary



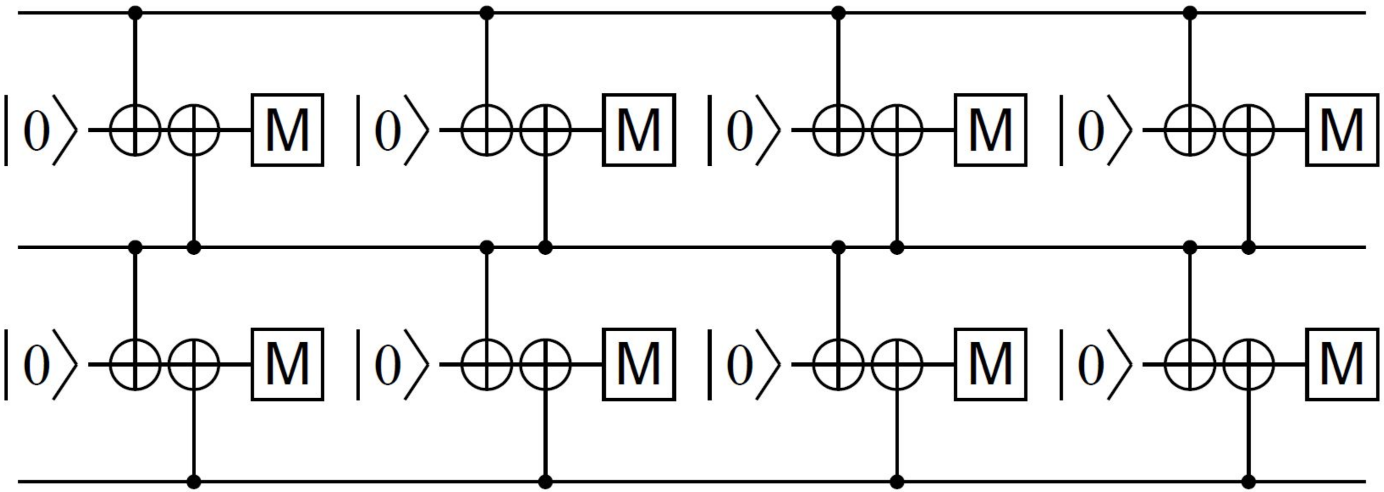

- A single data error permanently flips the measurement stream
- A single measurement error just flips a single bit

# Repeated detection



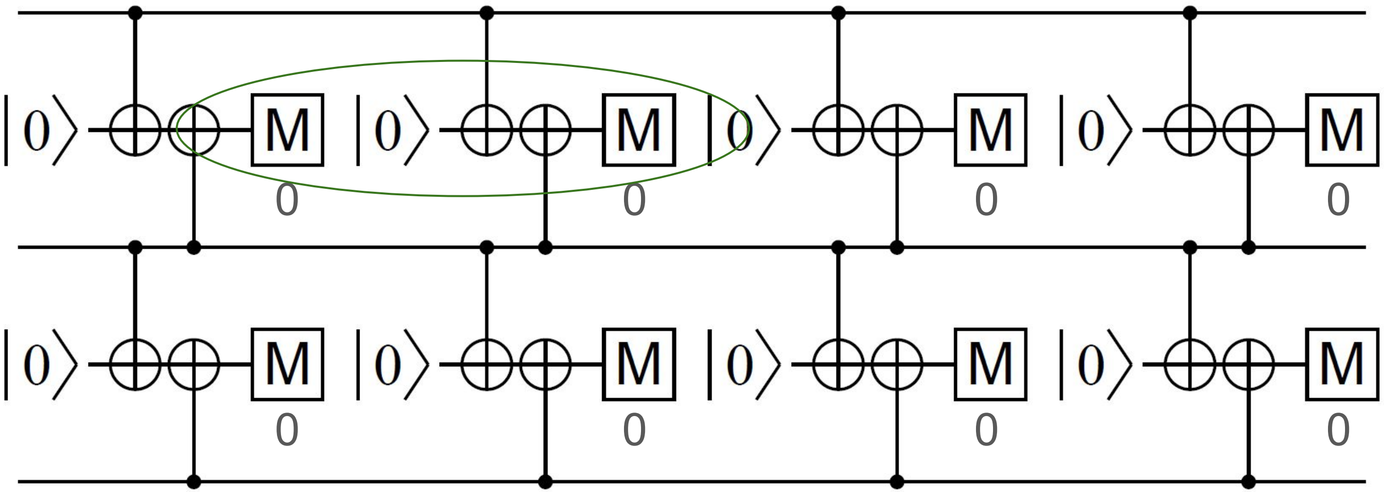

- Circuit detects if the top and bottom qubits are the same or different, it calculates the parity as was done in the classical repetition code
- Can use it to find errors in states $\alpha|00\rangle + \beta|11\rangle$ without collapse
- Not enough to identify which qubit suffered an error

# The quantum repetition code



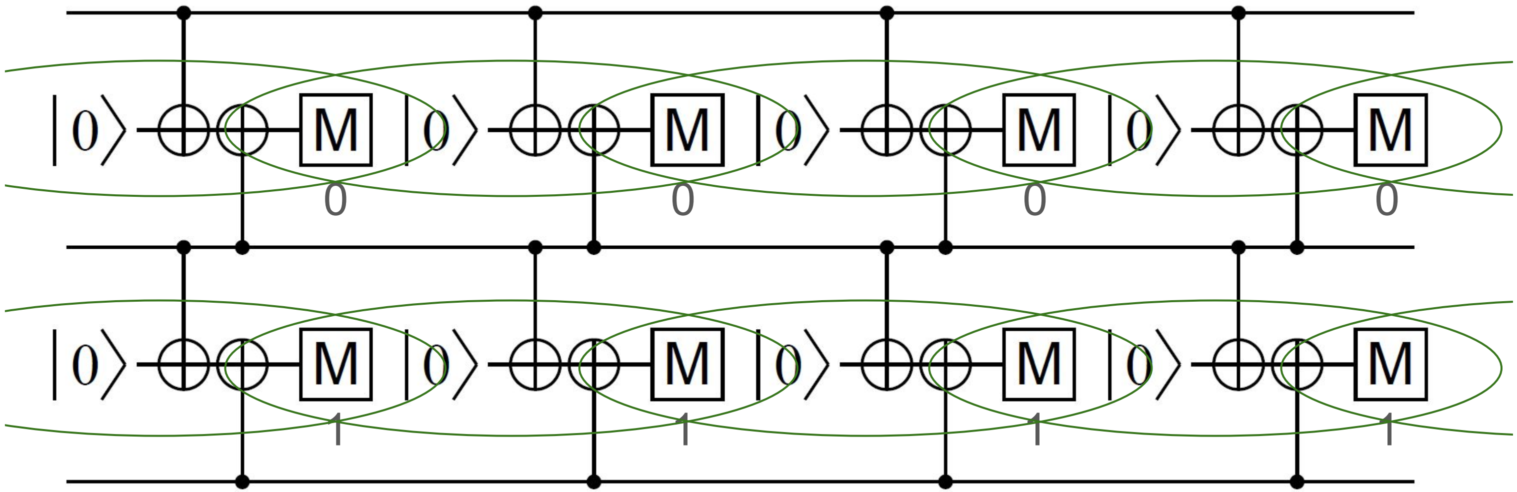

- The above circuit can monitor states of the form $\alpha|000\rangle + \beta|111\rangle$
- Let’s discuss how the above can reliably find any single bit flip

# The quantum repetition code: no errors



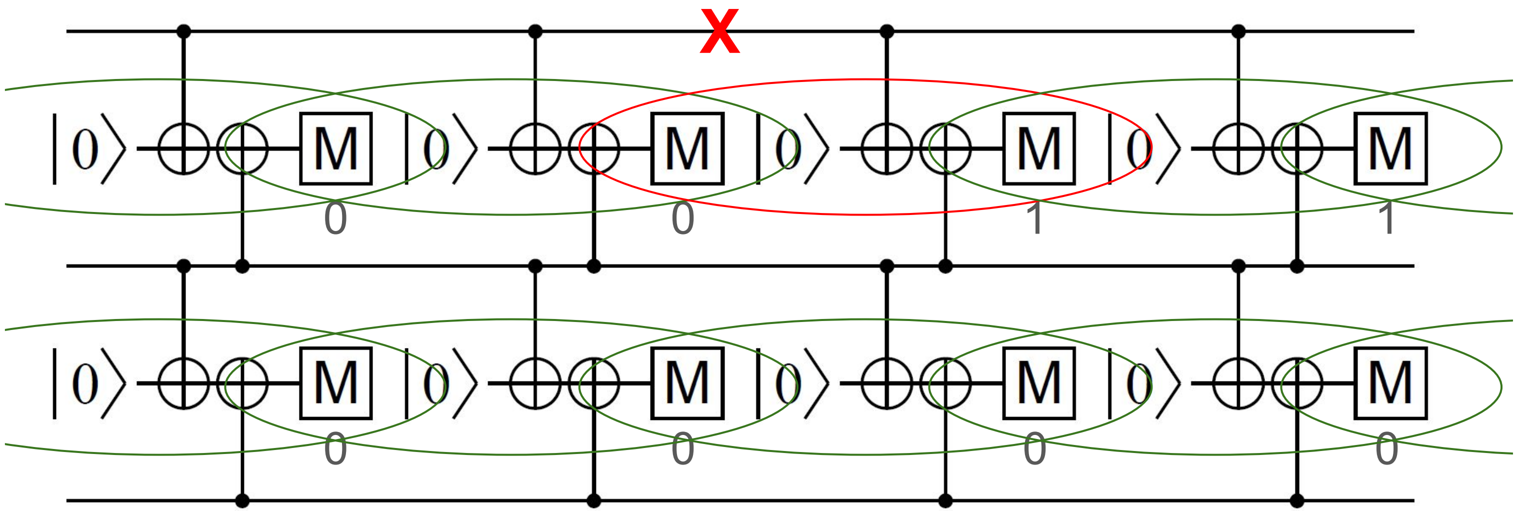

- In the absence of errors, each new measurement will be the same as the previous measurement
- A detector is a set of measurements with an expected parity in the absence of errors

# The quantum repetition code: no errors



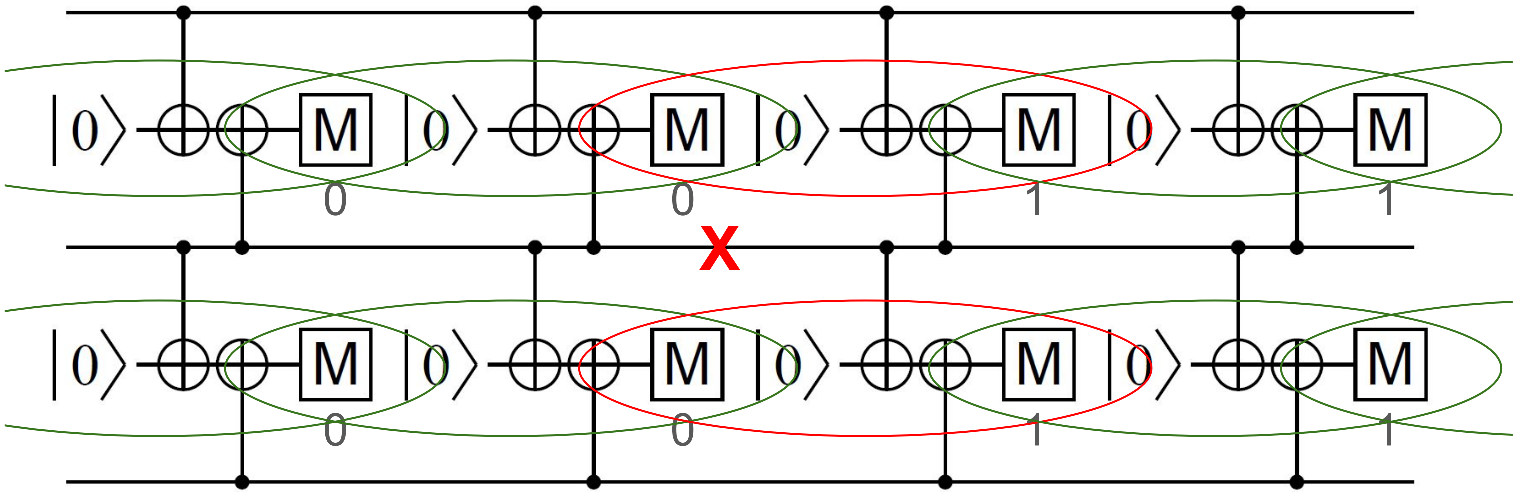

- Every sequential pair of measurements is a detector
- Note that it's not the specific value of the individual measurements that matters, just the parity of the set

# The quantum repetition code: one error



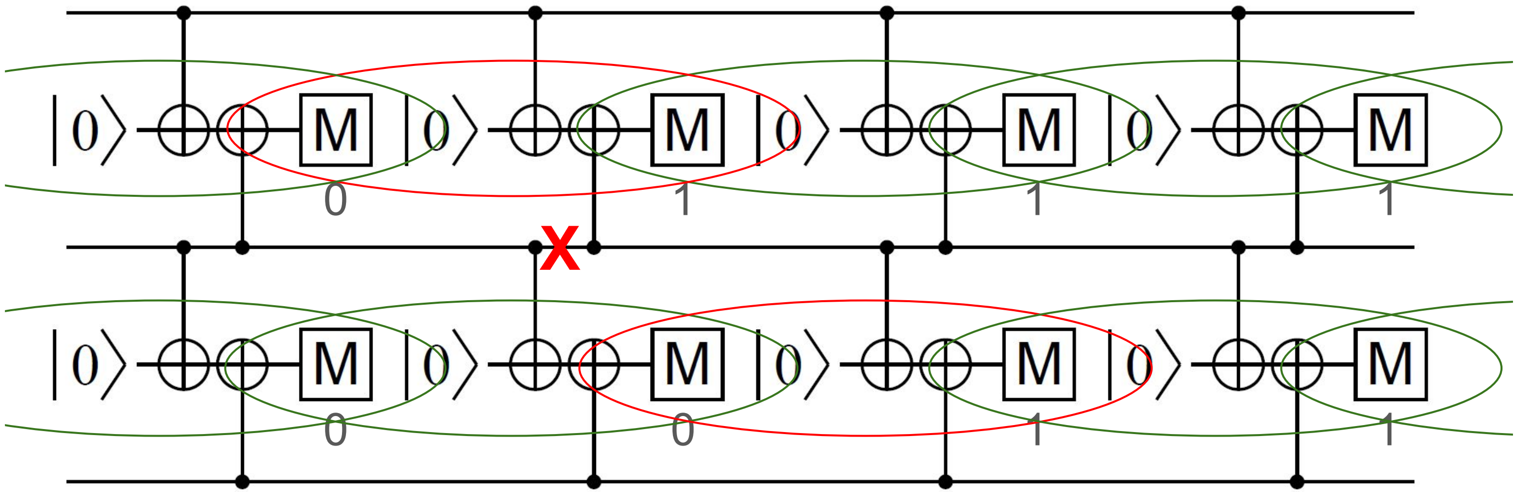

- **Detector:** a set of measurements with an expected parity
- **Detection event:** a detector with an unexpected parity
  - A single error can lead to **one** detection event

# The quantum repetition code: one error



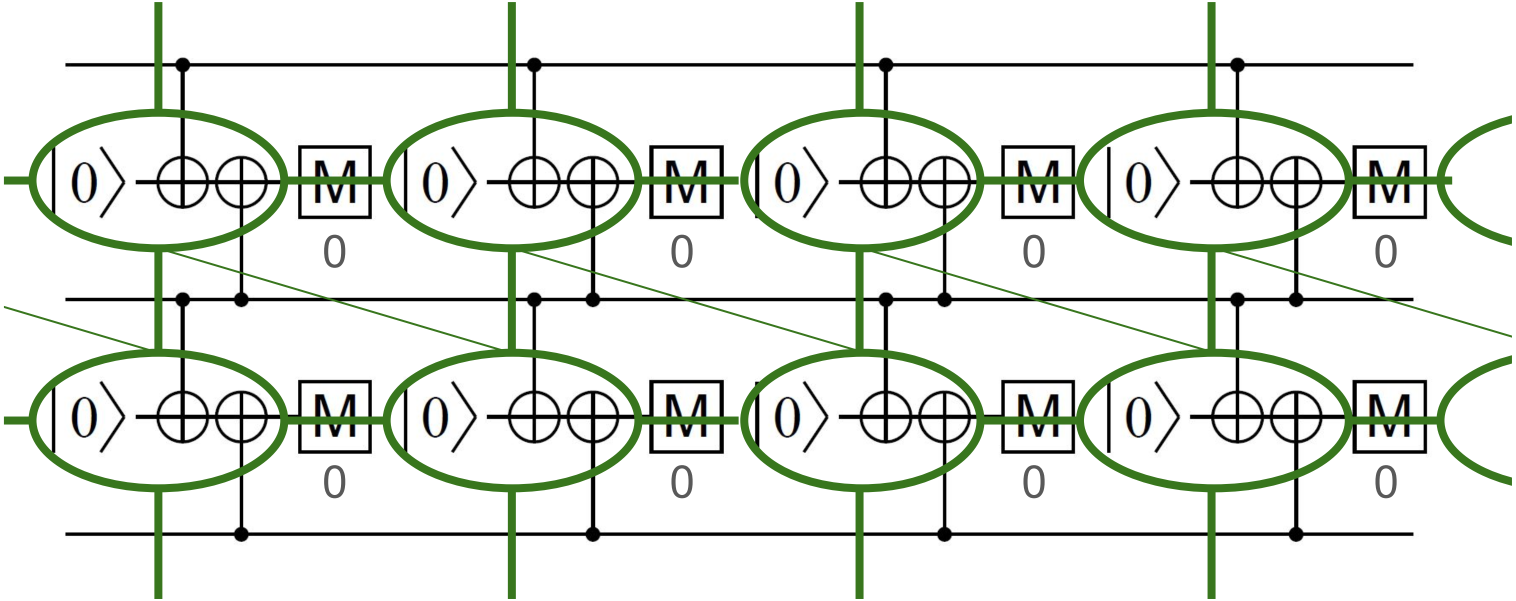

- **Detector:** a set of measurements with an expected parity
- **Detection event:** a detector with an unexpected parity
    - A single error can lead to **one** detection event
    - A single error can lead to **two** detection events

# The quantum repetition code: one error



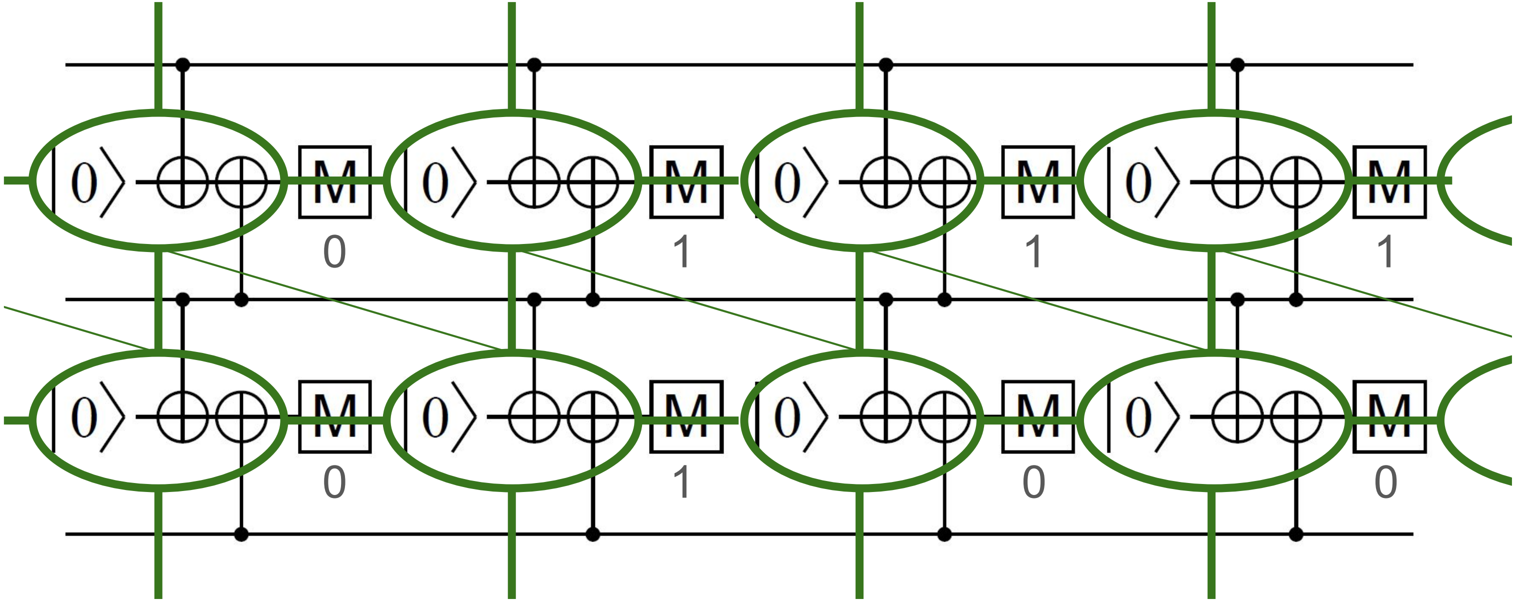

- **Detector:** a set of measurements with an expected parity
- **Detection event:** a detector with an unexpected parity
  - A single error can lead to **two** detection events in two **different** detection rounds
  - This is rare as there is very small region in circuit where this can happen

# The quantum repetition code: no errors



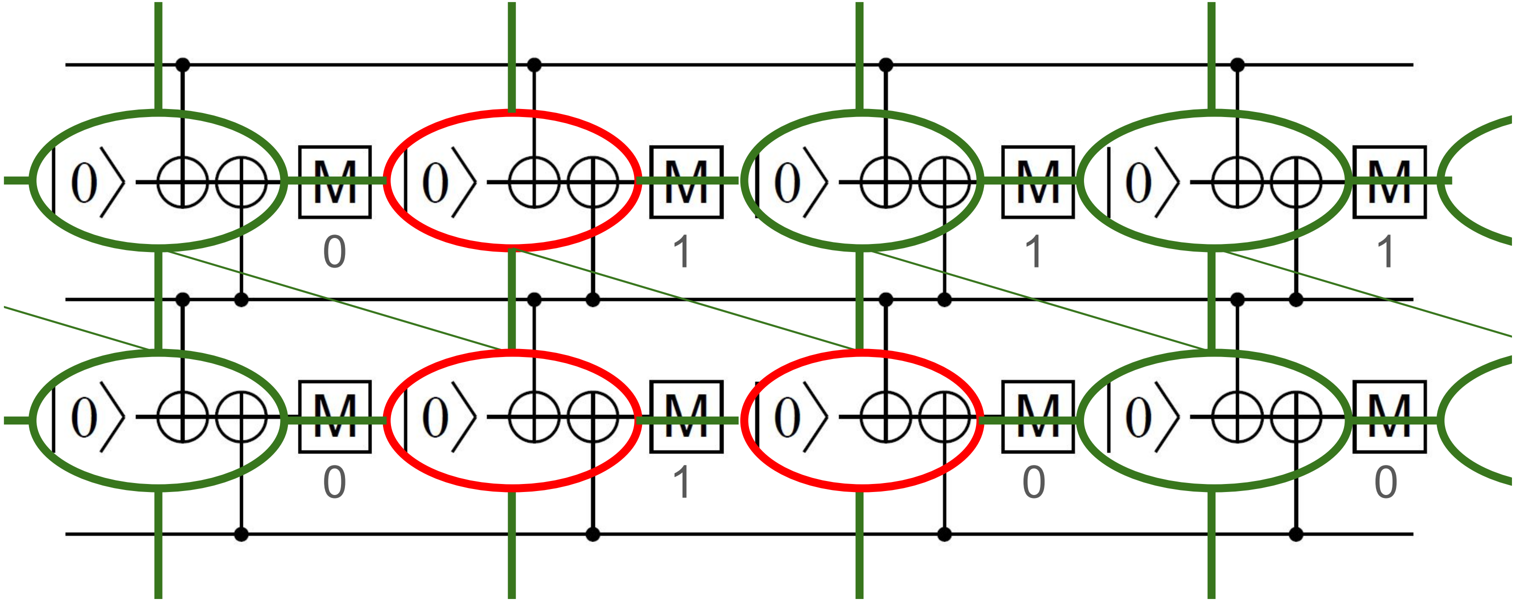

- Let's shrink the detector bubbles so they only touch their measurements
  - Can represent each possible set of detection events with an edge
  - Build a graph, thin edges means few errors lead to that set of detection events

# The quantum repetition code: unknown errors



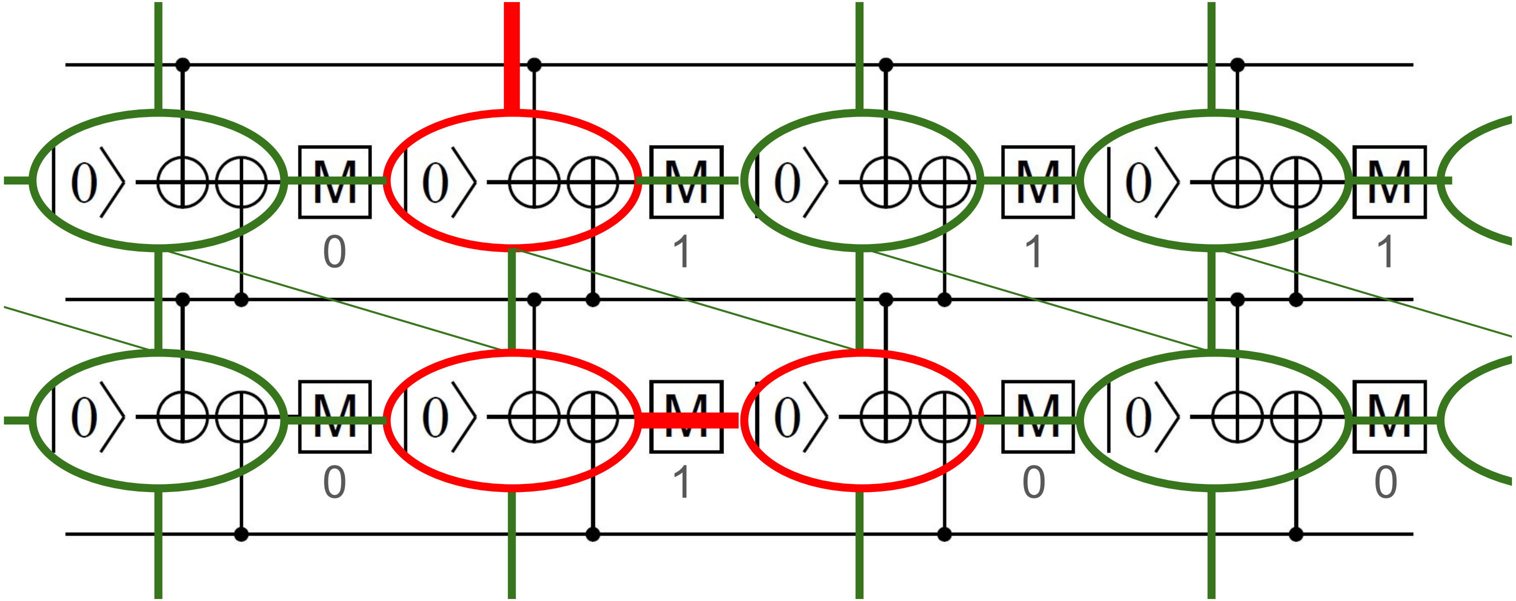

- Suppose we run an experiment and observed this pattern of measurements

# The quantum repetition code: unknown errors



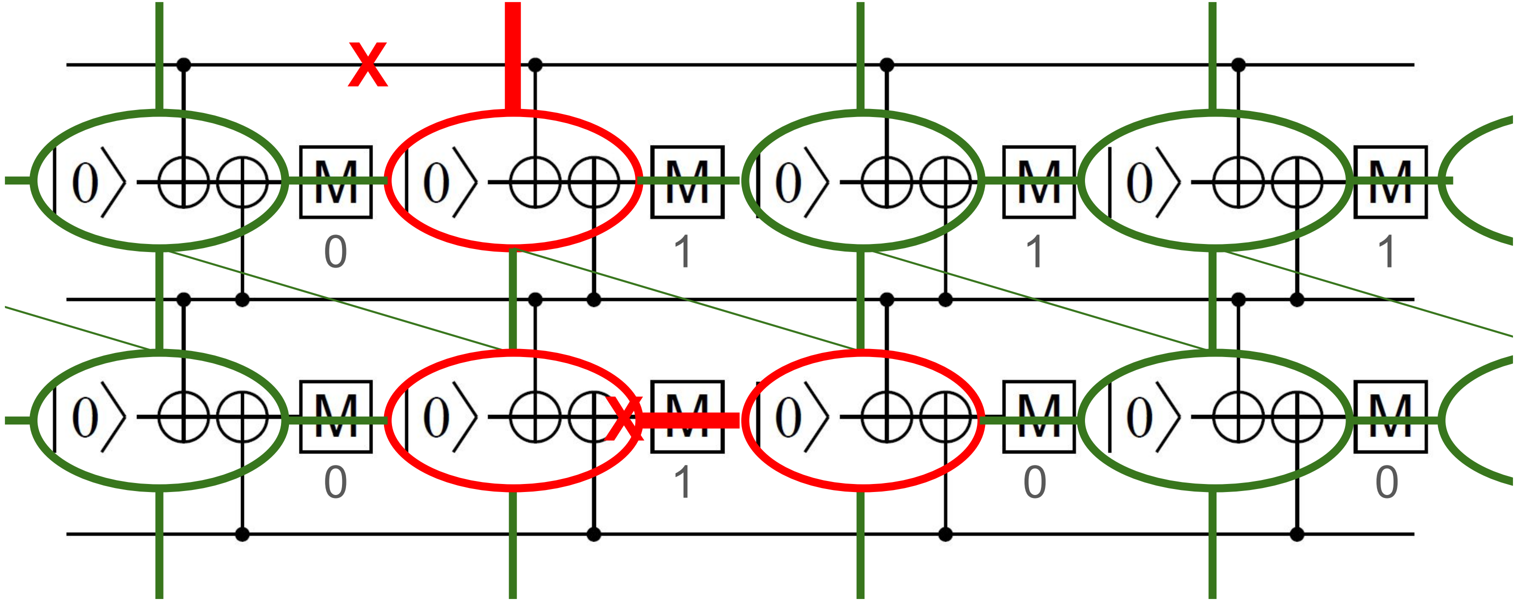

- Suppose we run an experiment and observed this pattern of measurements
- Highlight the detectors that are detection events

# The quantum repetition code: unknown errors



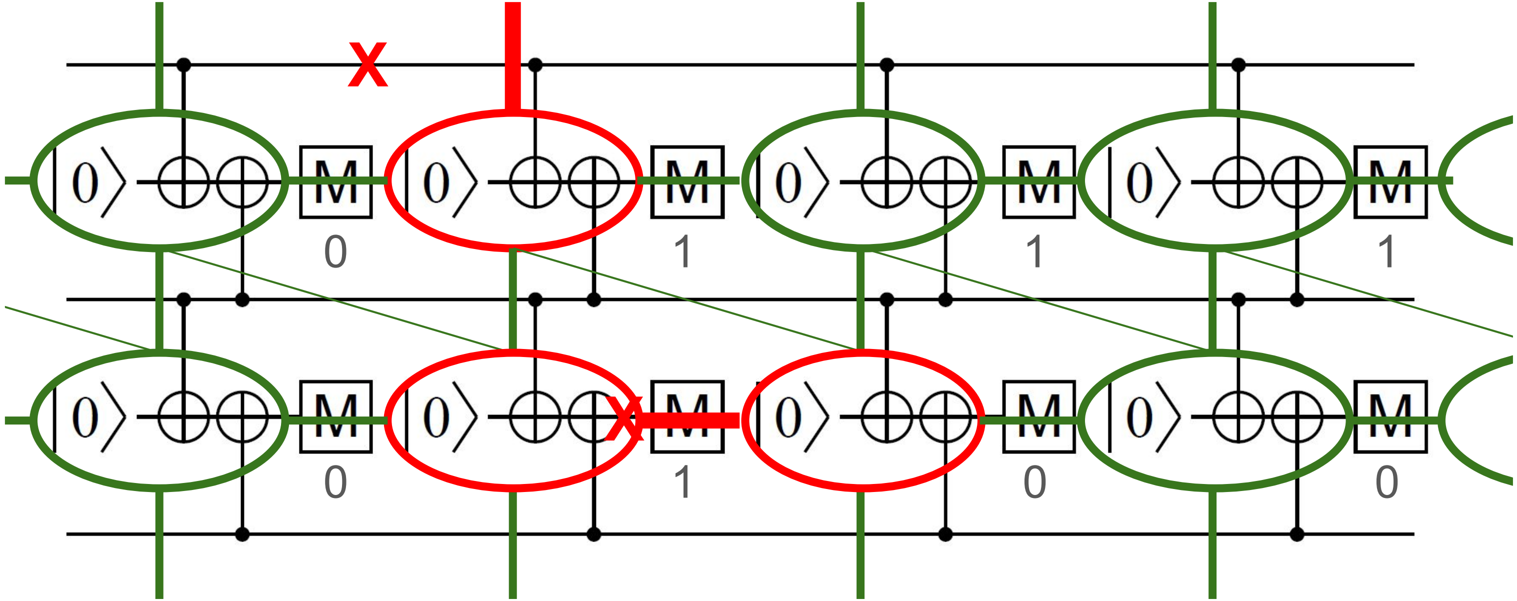

- Suppose we run an experiment and observed this pattern of measurements
- Highlight the detectors that are detection events
- Use minimum weight perfect matching (maximum probability, wt = -ln $p_{edge}$)
  - Won't chose the low probability diagonal edge in this case

# The quantum repetition code: inferred errors



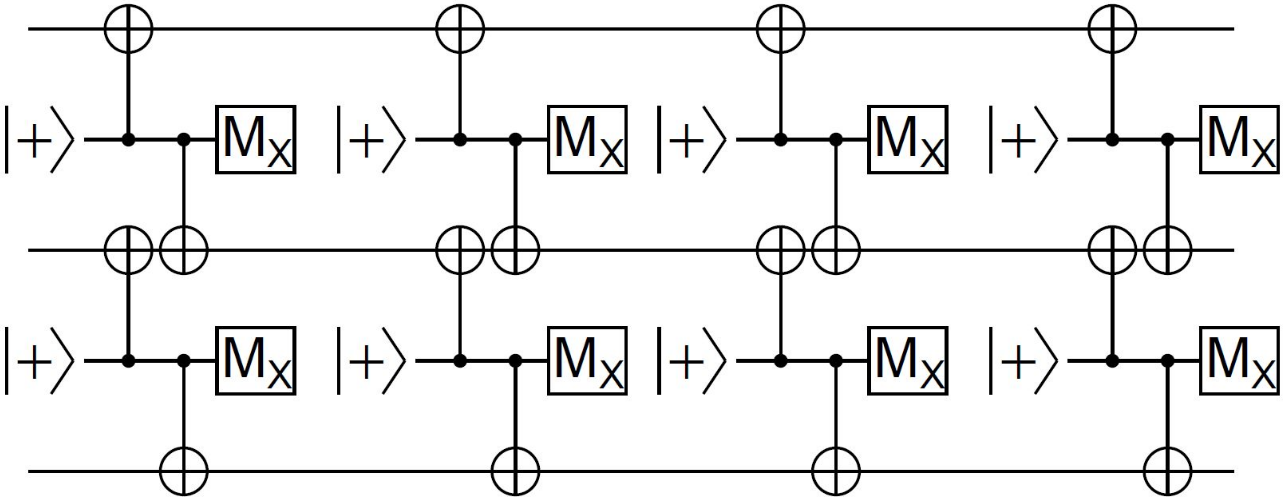

- Suppose we run an experiment and observed this pattern of measurements
- Highlight the detectors that are detection events
- Use minimum weight perfect matching (maximum probability, wt = -ln $p_{edge}$)
- Infer the presence of errors

# The quantum repetition code: performance




- Provided errors are rare, random, and independent, d data qubits enables the detection and correction (in software) of up to (d-1)/2 X errors
- Logical errors exponentially suppressed with code distance due to low probability of many errors on a single graph path from top to bottom

# The quantum repetition code: no errors




- Can use the Z detection circuit to protect states $\alpha|+++\rangle + \beta|---\rangle$
- Next time, the math we will need to study more complex codes, namely stabilizers