Quantum error correction I:

What are quantum errors?

Austin Fowler

Classical error correction fundamentals

Suppose you wish to store a bit, 0 or 1

Suppose your best bit has a probability p per unit time of flipping

How can you increase the chance of successful storage?

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How can you increase the chance of successful storage?

Answer: store multiple copies, periodically take majority vote (instant and perfect)

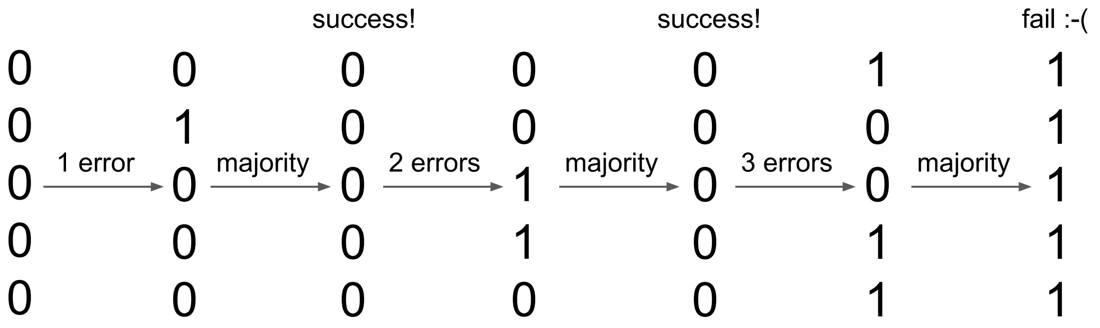
				success!				success!				fail :-(
0		0		0		0		0		1		1
0		1		0		0		0		0		1
0	$\xrightarrow{\text{1 error}}$	0	$\xrightarrow{\text{majority}}$	0	$\xrightarrow{\text{2 errors}}$	1	$\xrightarrow{\text{majority}}$	0	$\xrightarrow{\text{3 errors}}$	0	$\xrightarrow{\text{majority}}$	1
0		0		0		1		0		1		1
0		0		0		0		0		1		1

Classical error correction fundamentals

The state 00000 is called logical 0

The state 11111 is called logical 1

This is the classical repetition code, one of many possible error correction codes



Classical error correction fundamentals

The code distance is the number of bits that need to be flipped to convert logical 0 into logical 1

Code distance d = 5 in this example

A distance d code can only fail if at least (d+1)/2 errors occur, $p \rightarrow O(p^3)$

				success!				success!				fail :-(
0		0		0		0		0		1		1
0		1		0		0		0		0		1
0	$\xrightarrow{\text{1 error}}$	0	$\xrightarrow{\text{majority}}$	0	$\xrightarrow{\text{2 errors}}$	1	$\xrightarrow{\text{majority}}$	0	$\xrightarrow{\text{3 errors}}$	0	$\xrightarrow{\text{majority}}$	1
0		0		0		1		0		1		1
0		0		0		0		0		1		1

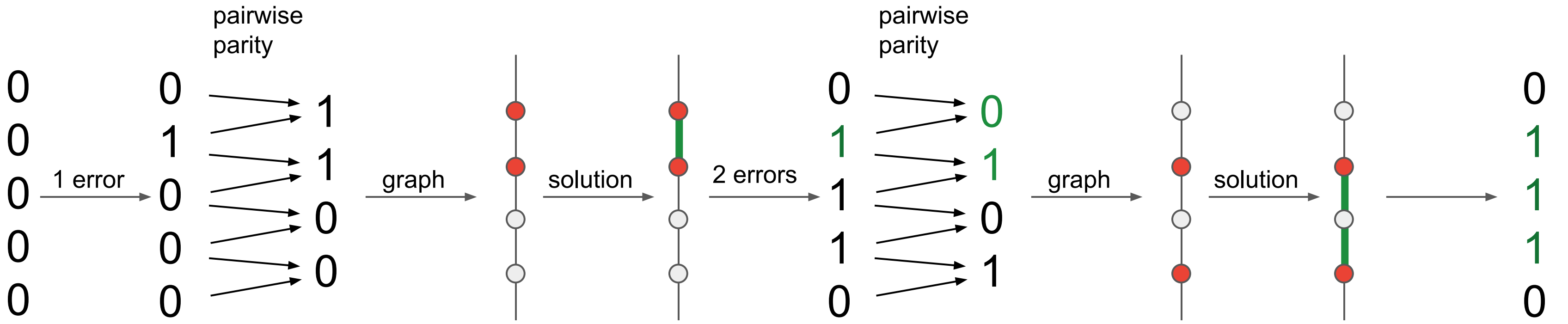
Classical error correction fundamentals

Don't actually need to measure bits directly

Pairwise parity measurements are sufficient

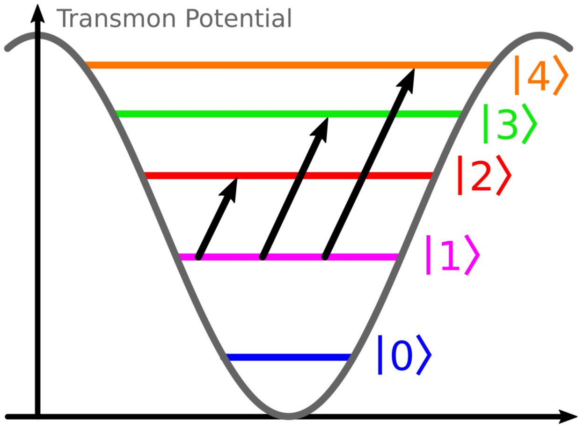
Use minimum weight perfect matching (MWPM) to decode instead of majority vote

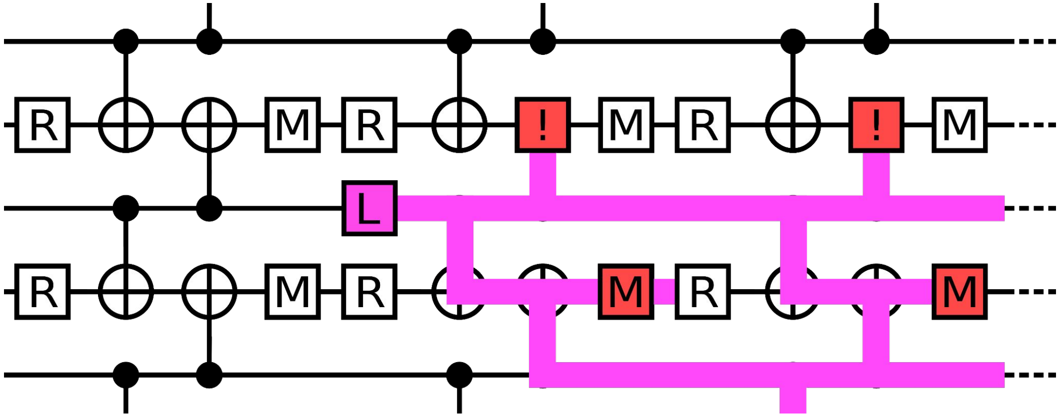
Don't need to apply corrections, track in software (green)



Quantum errors are more complex, eg: leakage

There is a lot more to deal with that just classical bit flips...

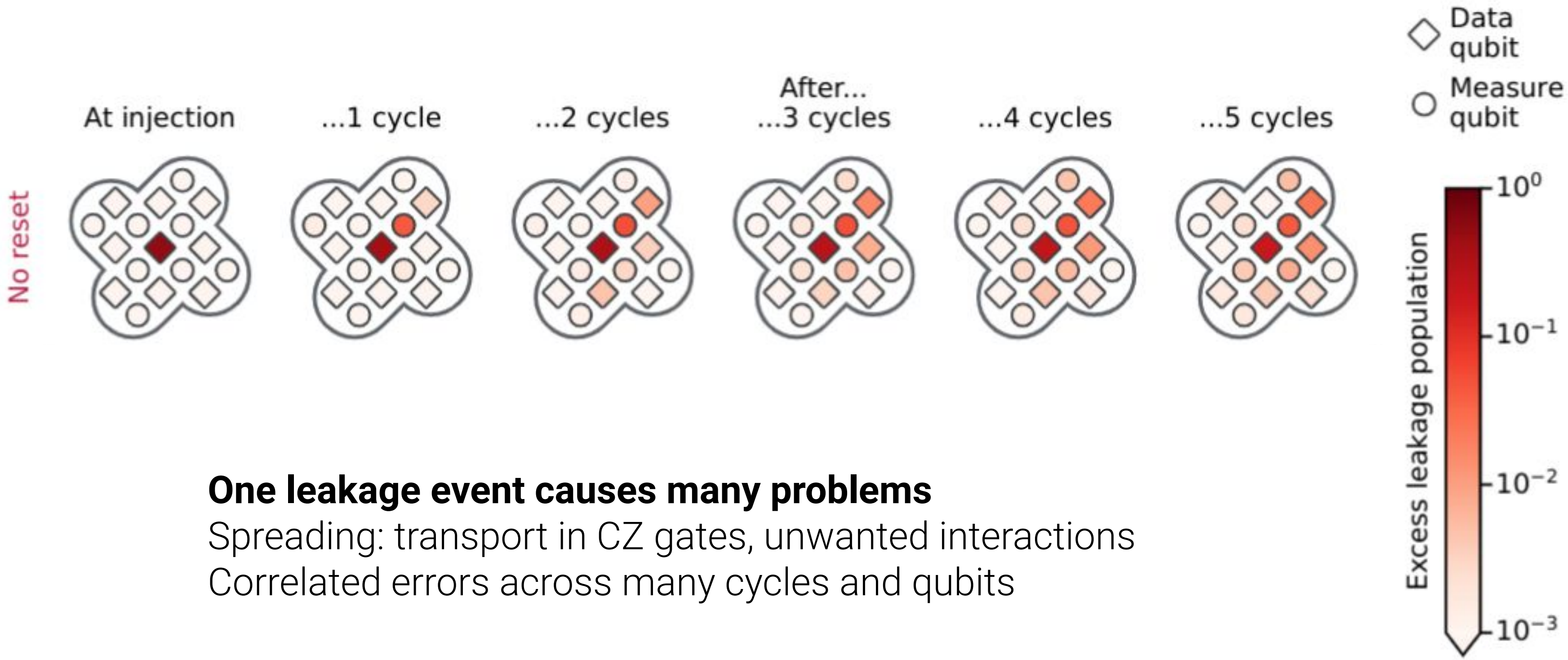




[arXiv:2211.04728](https://arxiv.org/abs/2211.04728)

Leakage in the surface code

Start the center qubit in |2⟩ and observe



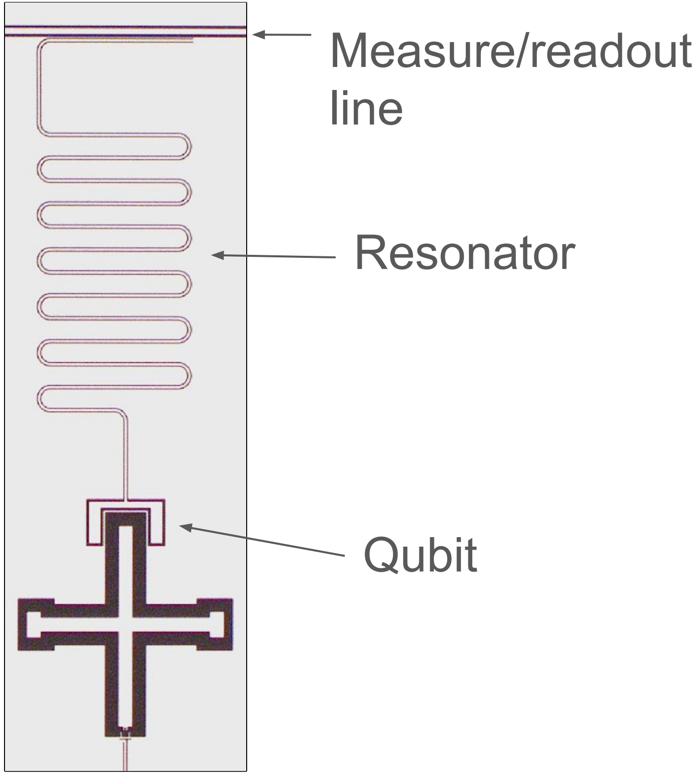
One leakage event causes many problems

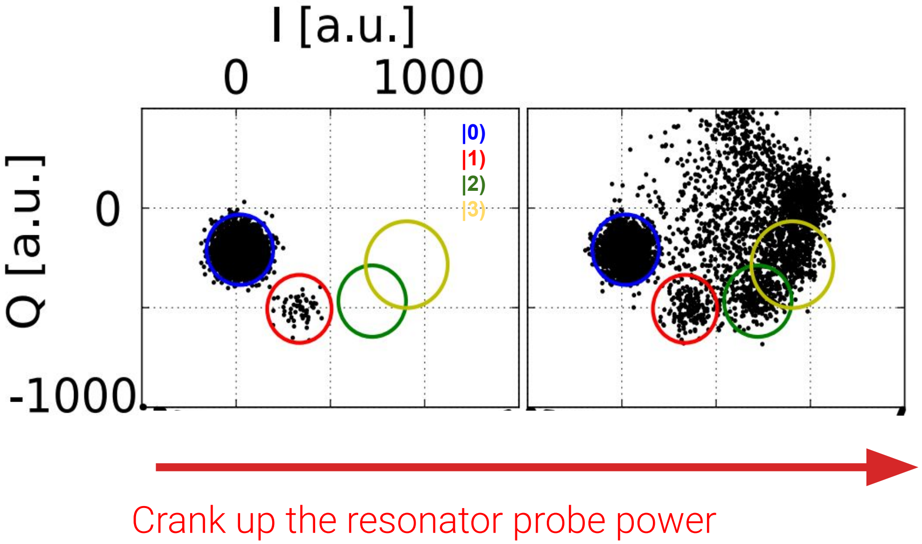
Spreading: transport in CZ gates, unwanted interactions

Correlated errors across many cycles and qubits

arXiv:2211.04728

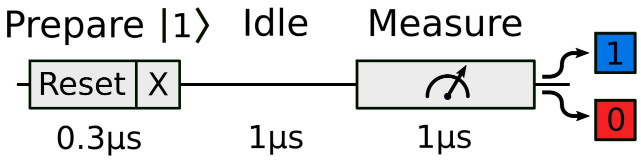
Measurement-induced state transitions

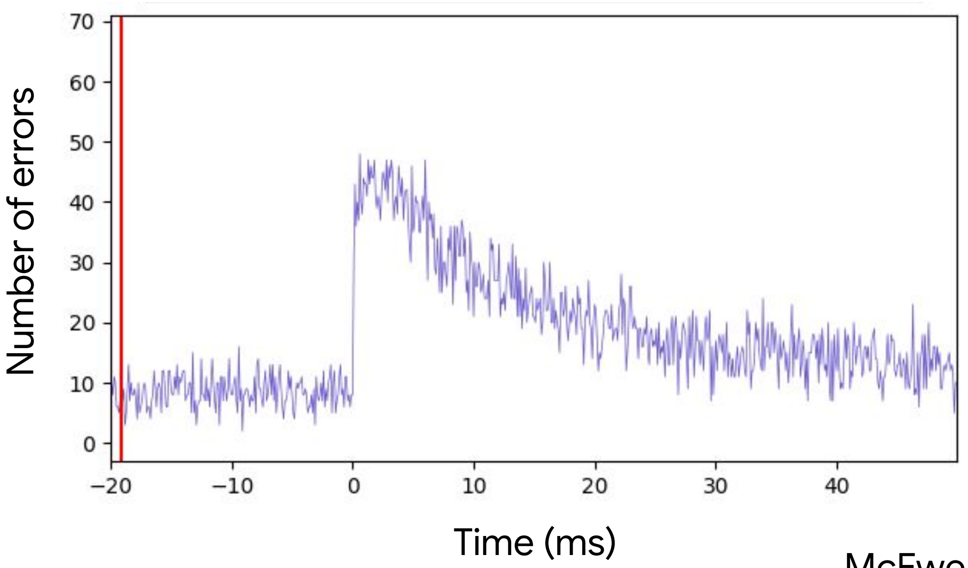


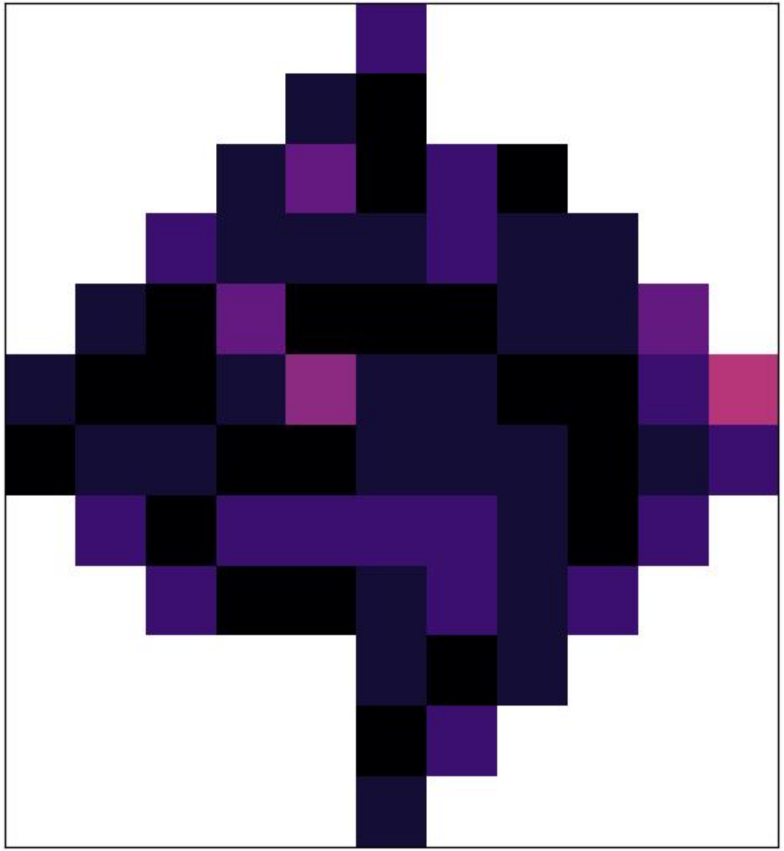


High-energy impacts

Repetitive correlated sampling



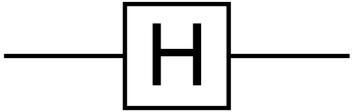




McEwen et al., *Nat. Phys.* **18** 107-111 (2022), arXiv:2104.05219

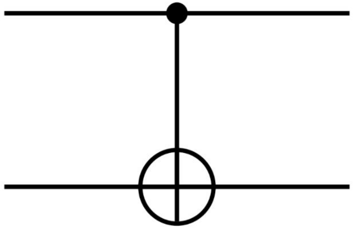
Real quantum computers are not ideal!

Hadamard



$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = |+\rangle$$
$$H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = |-\rangle$$

Controlled-X



$$CX(1,0)|00\rangle = |00\rangle$$
$$CX(1,0)|01\rangle = |01\rangle$$
$$CX(1,0)|10\rangle = |11\rangle$$
$$CX(1,0)|11\rangle = |10\rangle$$

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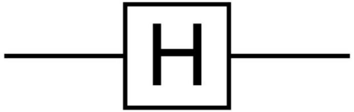
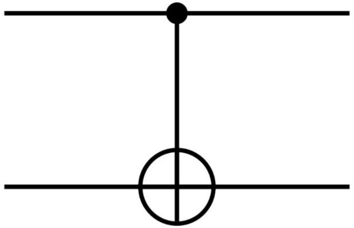


$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Controlled-X



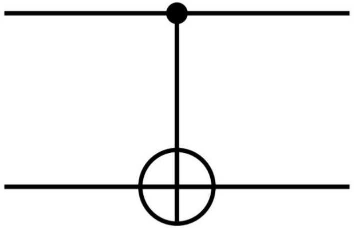
$$|00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \ |01\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \ etc$$

Real quantum computers are not ideal!

Hadamard

$$H|\Psi\rangle = H(\alpha|0\rangle + \beta|1\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1 \\ 1 & -1\end{pmatrix}\begin{pmatrix}\alpha \\ \beta\end{pmatrix}$$

Controlled-X

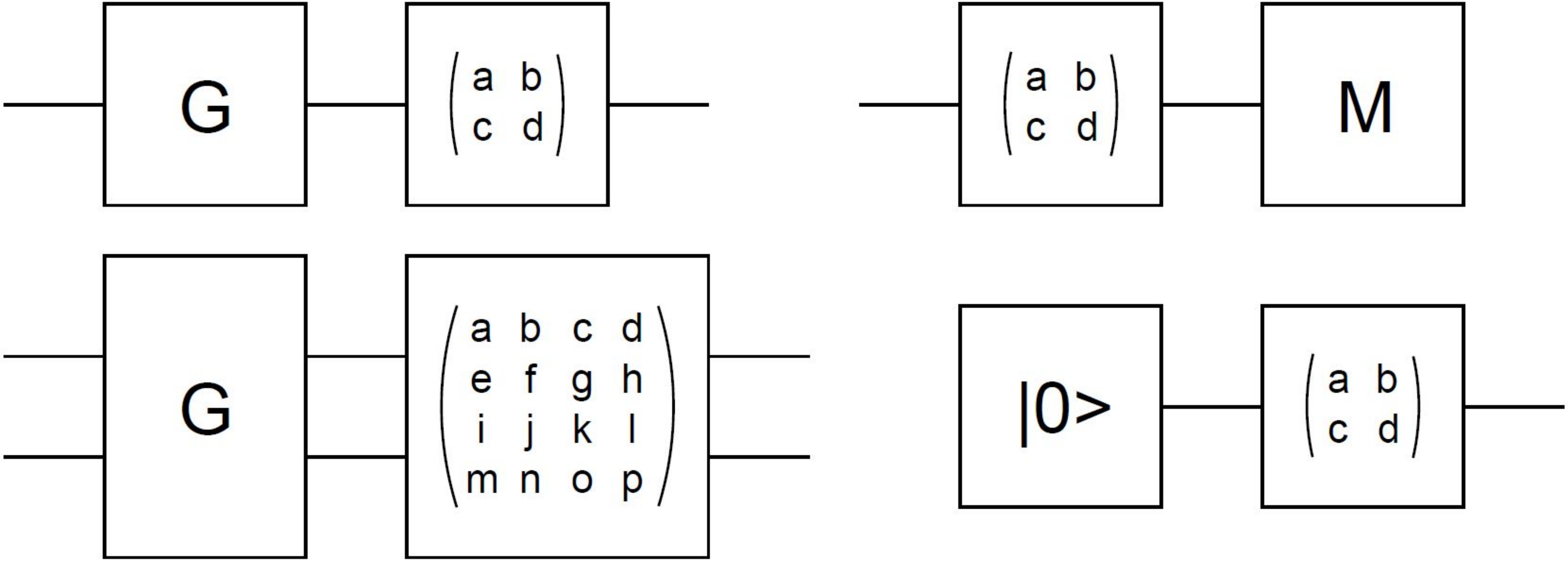


$$CX(1,0)|\Psi\rangle$$

$$= CX(1,0)(\alpha_0|00\rangle + \alpha_1|01\rangle + \alpha_2|10\rangle + \alpha_3|11\rangle) = \begin{pmatrix}1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0\end{pmatrix}\begin{pmatrix}\alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3\end{pmatrix}$$

Real quantum computers are not ideal!

This is the error model we actually want (and all QEC can handle):



Lots of physics and engineering still required to build a device with this error model.

Decomposing errors

- It would be nice if we only need to worry about these errors:

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

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- Can write arbitrary errors as a linear combinations of X and Z errors:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \frac{a+d}{2} I + \frac{b+c}{2} X + \frac{a-d}{2} Z + \frac{b-c}{2} ZX$$

Decomposing errors

Can do something similar with two-qubit noise:

$$\begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{pmatrix} = a' I \otimes I + b' I \otimes X + c' I \otimes Z + d' I \otimes XZ + e' X \otimes I + \ldots + p' XZ \otimes XZ$$

$$I \otimes XZ = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

If we can build a quantum device with only 1- and 2-qubit computational basis errors not handled at the hardware level, then detecting bit- and phase-flips and tracking them in software is enough.

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