Introduction to Crumble

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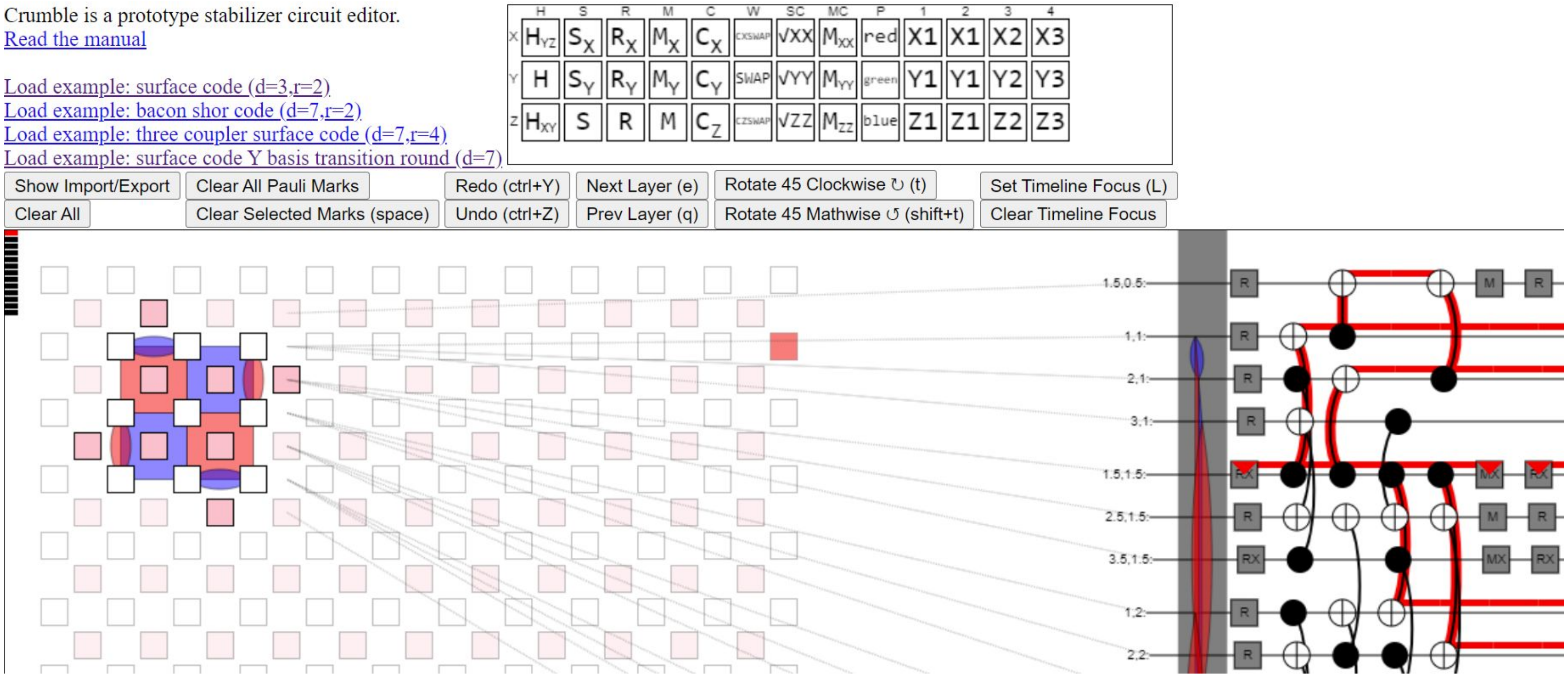
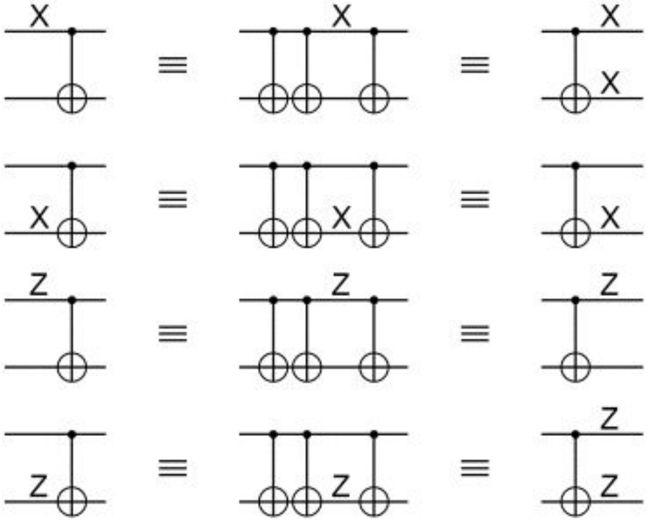
What is Crumble?

Like Stim, a software tool written by Craig Gidney: https://algassert.com/crumble

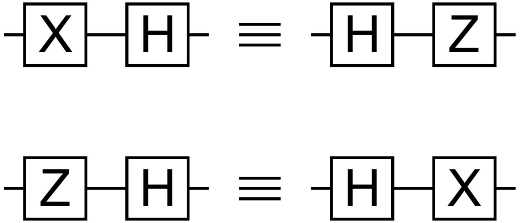


What is Crumble used for?

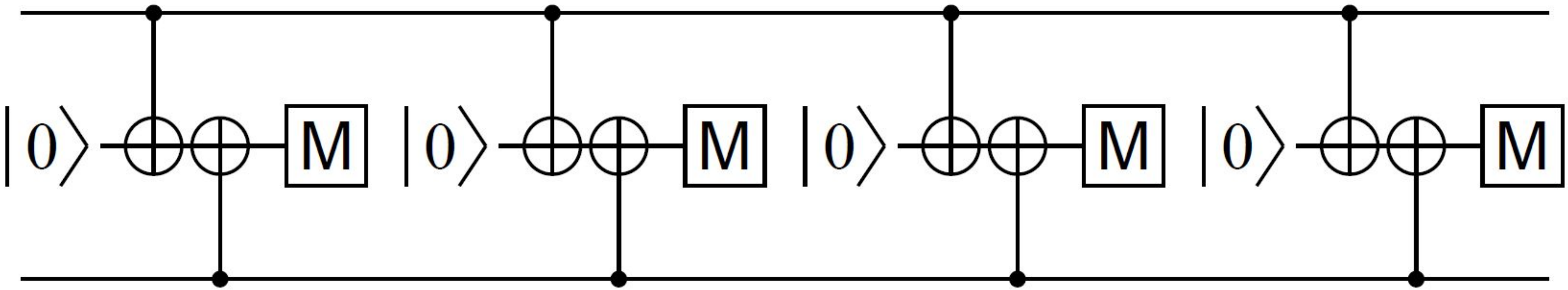


CNOT gates copy X errors on control, and Z errors on target.

Crumble automates the propagation of Pauli operators through circuits.

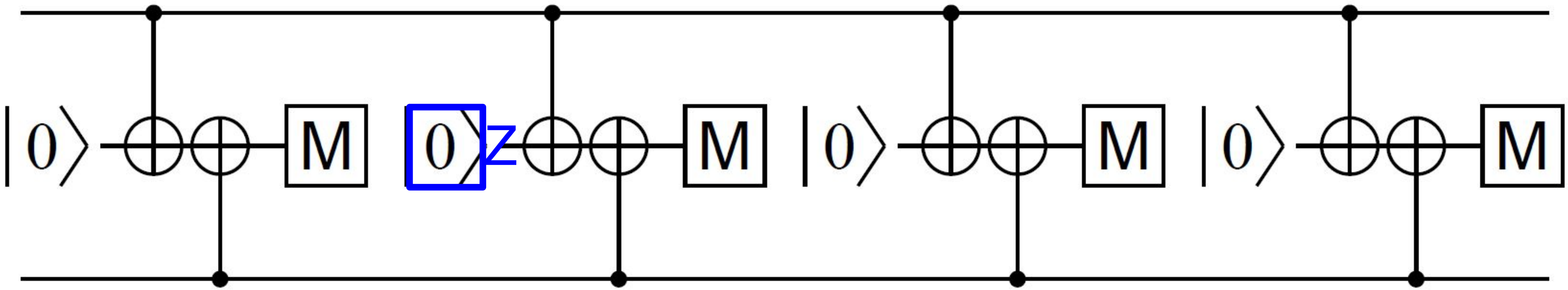


Hand propagation



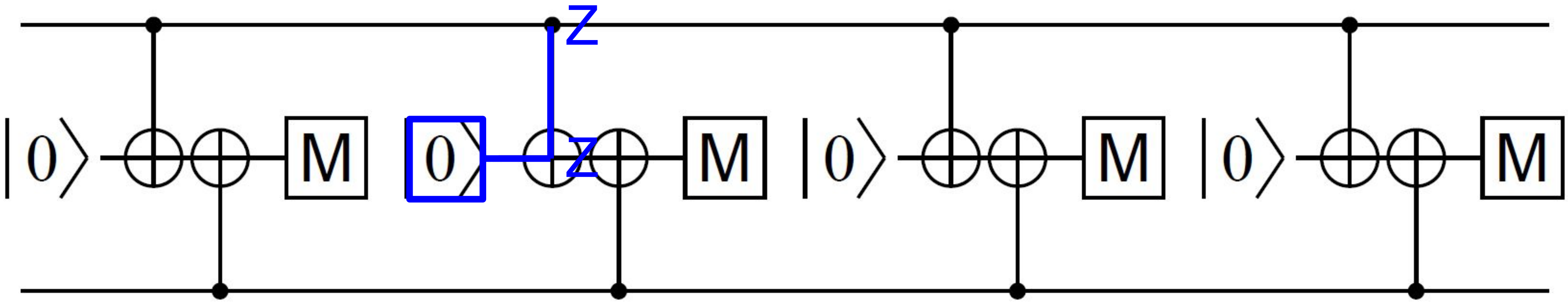
Let's trace some Pauli operators through this circuit, making no assumptions about the initial states of the data qubits.

Hand propagation



After reset, this qubit is in the +1 eigenstate of Z.
The blue Z represents this known eigenstate.

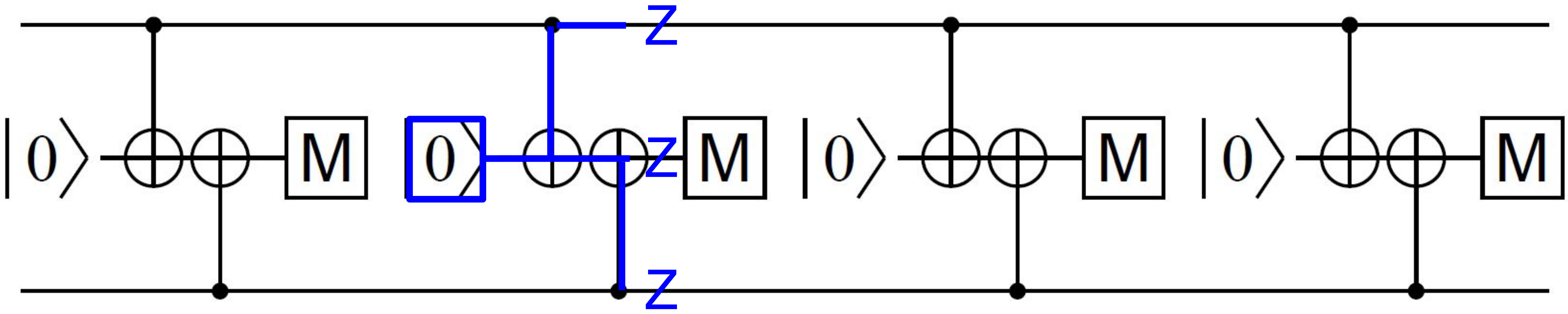
Hand propagation



This means that after the CNOT gate, the indicated qubits will be in the +1 eigenstate of ZZ

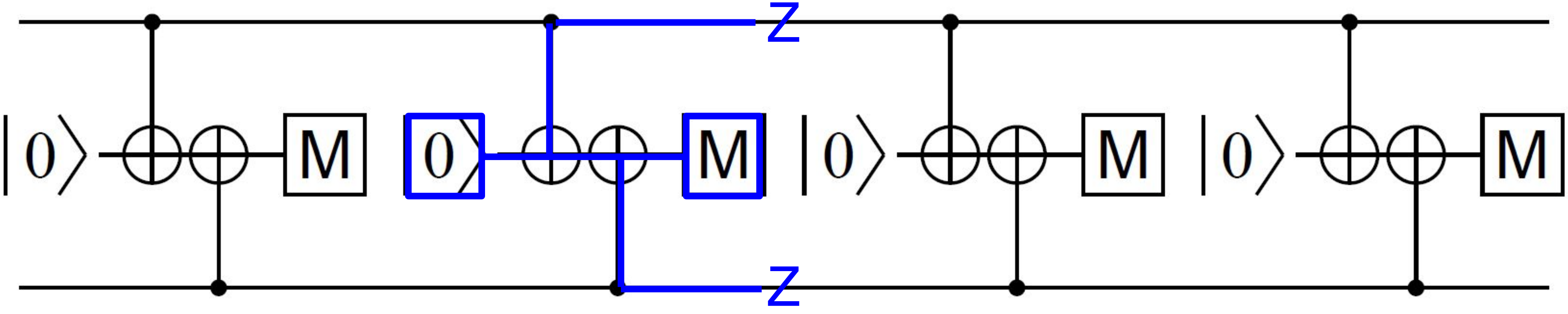
Exercise: prove this is true

Hand propagation



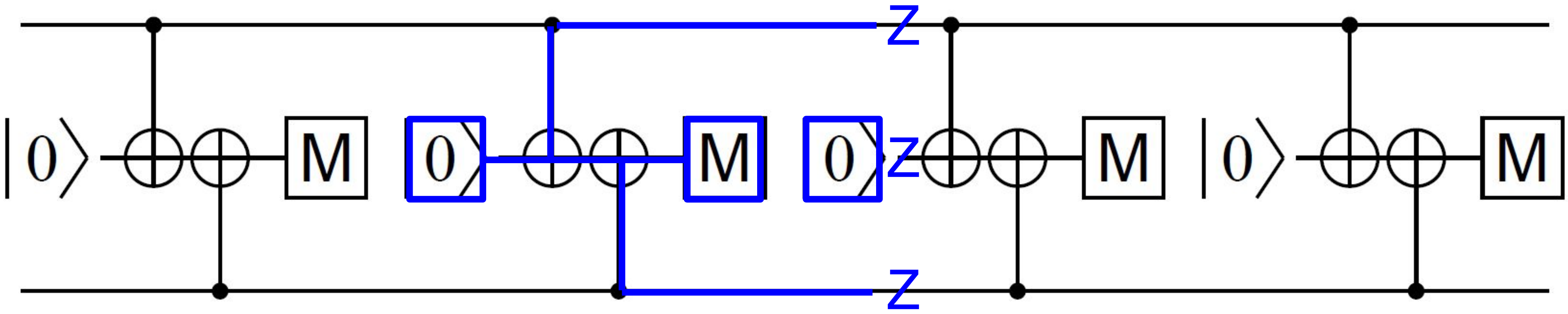
Continuing the logic, at this point in time the circuit has prepared the +1 eigenstate of ZZZ.

Hand propagation



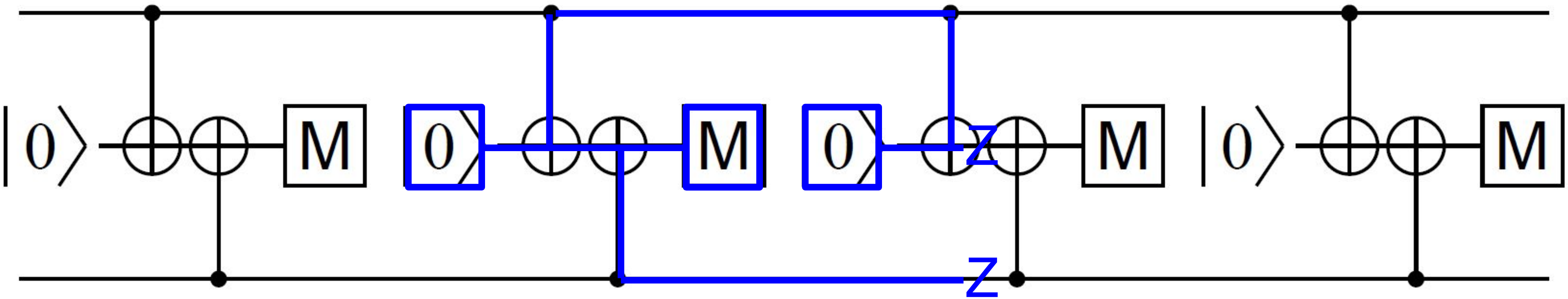
- Previously, all three qubits known to be in +1 eigenstate of ZZZ
- Measurement tells us either +1 or -1 eigenstate of Z for middle qubit.
- Therefore, after measurement, other two qubits known to be in eigenstate of ZZ with the **same sign as the measured qubit**.

Hand propagation



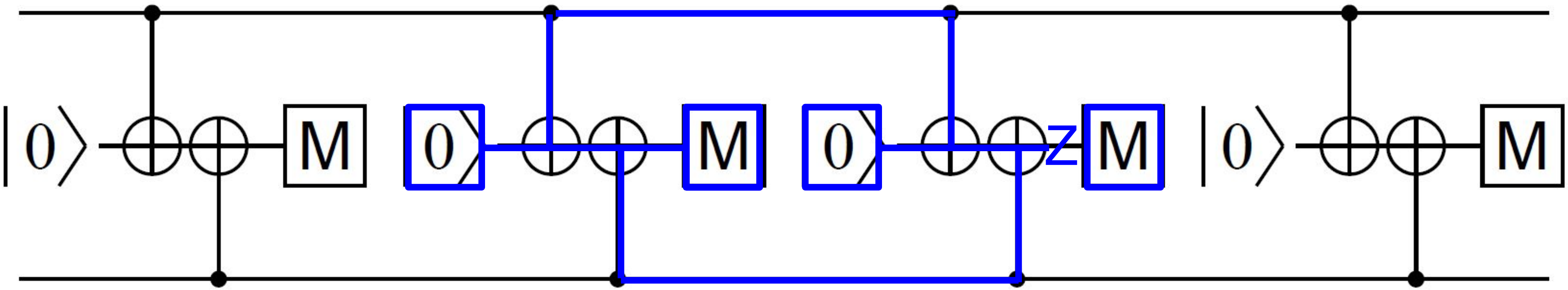
The reset prepares another +1 eigenstate of Z, so the eigenstate of ZZZ will be the same +/- reported by the blue highlighted measurement.

Hand propagation



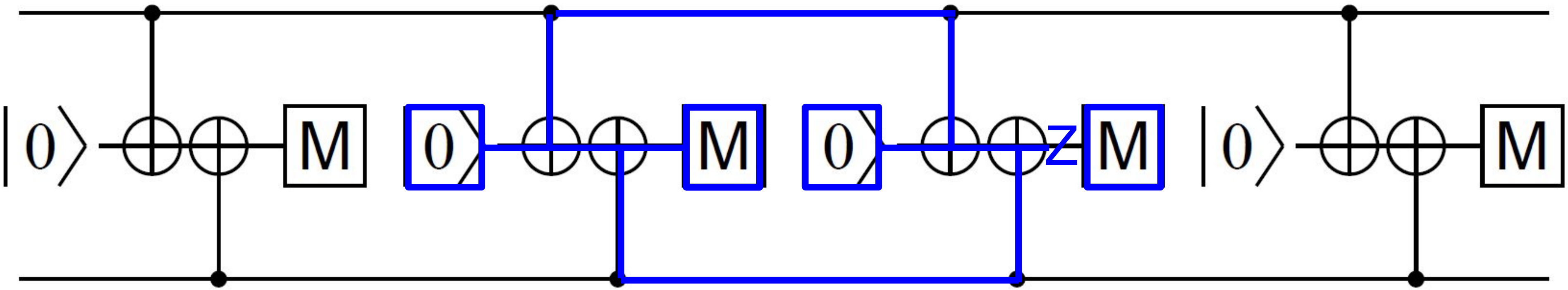
The CNOT cancels one of the Zs.

Hand propagation



Another CNOT cancels another Z. We can now see that the eigenstate of Z at this point in the circuit should match the left highlighted measurement. The right highlighted measurement should therefore give the same result as the first.

Detectors and detection regions



Since the two measurements should be the same, they form a detector. The blue region is called the detection region. An X error anywhere in the detection region will be detected by the detector.

Let's use Crumble to do the same thing

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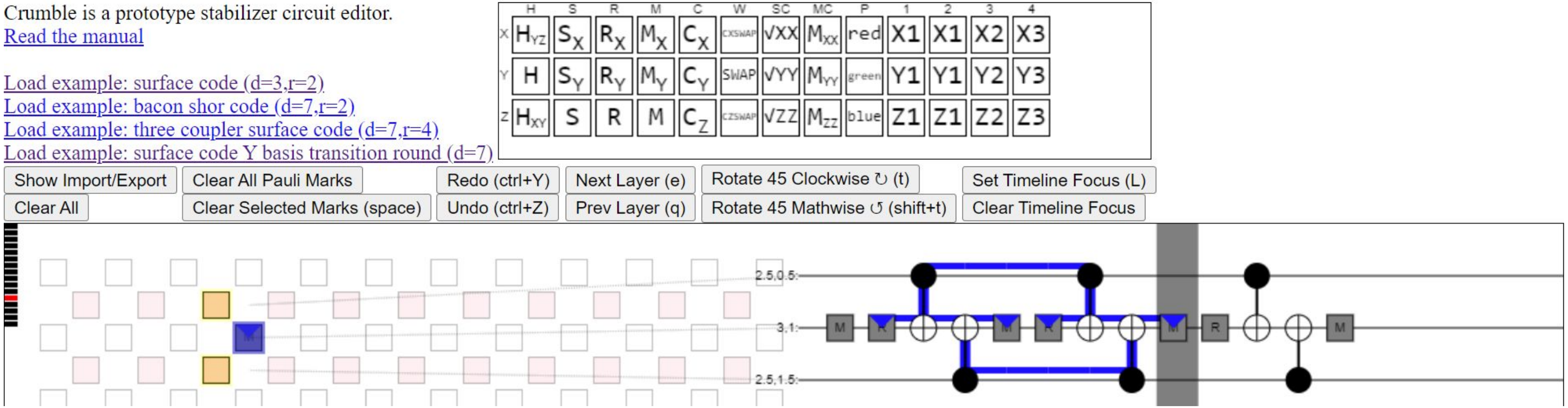

And to round out this course we'll also look at the surface code and part of the circuit required to do [Y-basis logical measurement](#).