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E0 CSA 0610

Big omega Notation: prove that $g(n) = n^3+2n^2+4n$ is $\Omega(n^3)$

$g(n) \geq c.n^3$

$g(n) = n^3+2n^2+4n$

For finding constants c and n_0

$n^3+2n^2+4n \geq c.n^3$

Divide both sides with n^3

$1+\frac{2n^2}{n^3}+\frac{4n}{n^3} \geq c$

$1+ 2/n + 4/n^2 \geq c$

Here $2/n$ and $4/n^2$ approaches 0

$1+2/n+4/n^2 \approx 1$

Example $c = 1/2$ $(1 \geq 1/2, n \geq 1)$

$1+2/n+4/n^2 \geq 1/2$ $(n \geq 1, n_0 = 1)$

$1+2/n+\frac{4}{n^2} \geq 1$

$1+\frac{2}{n}+4/n^2 \geq 1/2$

Thus, $g(n) = n^3+2n^2+4n$ is indecded $\Omega(n^3)$

Big theta Notation: Determine whether $h(n) = 4n^2 + 3n$ is $\theta(n^2)$ or not

$$c_1 n^2 \le h(n) \le c_2 n^2$$

In upper bound $h(n)$ is $O(n^2)$

In Lower bound $h(n)$ is $\Omega(n^2)$

Upper Bound $(O(n^2))$:

$$h(n) = 4n^2 + 3n$$

$$h(n) \le c_2 n^2$$

$$4n^2 + 3n \le c_2 n^2 \Rightarrow 4n^2 + 3n \le 5n^2$$

let's $c_2 = 5$

Divide both sides by n^2

$$4 + 3/n \le 5$$

$h(n) = 4n^2 + 3n$ is $O(n^2)$ $(c_2 = 5, n_0 = 1)$

Lower bound:-

$$h(n) = 4n^2 + 3n$$

$$h(n) \ge c_1 n^2$$

$$4n^2 + 3n \ge c_1 n^2$$

let's both sides by n^2

$$4 + 3/n \ge 4$$

$h(n) = 4n^2+3n$ $(c_1=4, n_0=1)$

$h(n) = 4n^2+3n$ is $\theta(n^2)$

Let $f(n) = n^3-2n^2+n$ and $g(n) = n^2$ show wheter $f(n) = \Omega(g(n))$ is true or false and Justifly your answer

$f(n) \geq (g(n))$

substuting $f(n)$ and $g(n)$ into this inequality we get

$n^3-2n^2+n \geq c\cdot(-n^2)$

find c and n_0 holds $n \geq n_0$

$n^3-2n^2+n \geq -cn^2$

$n^3-2n^2+n+cn^2 \geq 0$

$n^3+(c-2)n^2+n \geq 0$

$n^3+(c-2)n^2+n \geq 0$

$n^3+(1-2)n^2+n = n^3-n^2+n \geq 0$

$f(n) = n^3-2n^2+n$ is $\Omega(g(n)) = \Omega(-n^2)$

Ther fore the statement $f(n) = \Omega(g(n))$ is True.

Determine whether $h(n) = n \log n + n$ is $\Theta(n \log n)$
Prove a rigoros proof for your conclusion

$$c_1 n \log n \leq h(n) \leq c_2 n \log n$$

Upper Bound:

$$h(n) \leq c_2 n \log n$$

$$h(n) = n \log n + n$$

$$n \log n + n \leq c_2 n \log n$$

Divide both sides by $n \log n$

$$1 + \frac{n}{n \log n} \leq 2$$

(Simplify

$$1 + \frac{1}{\log n} \leq c_2$$

$(c_2 = 2)$)

$$1 + \frac{1}{\log n} \leq 2$$

Then $h(n)$ is $O(n \log n)$ $(c_2 = 2, n_0 = 2)$

Lower Bound:

$$h(n) \geq c_1 n \log n$$

$$h(n) = n \log n + n$$

$$n \log n \geq c_1 n \log n$$

Divide both sides by $n \log n$

$$1+\frac{n}{n\log n} \geq c_1$$

$$1+\frac{1}{\log n} \geq c_1 \quad \text{(simpilly}$$

$$1+\frac{1}{\log n} \geq 1 \quad (c_1=1)$$

$$\frac{1}{\log n} \geq 0 \quad (c_1=1), n_0=1$$

$h(n)$ is $\Omega(n \log n)$

$h(n) = n \log n + n$ is $\theta(n \log n)$

Solve the following recurrence relations and find the order of growth of solutions

$T(n) = 4T(n/2) + n2, \; T(1)=1$

$T(n) = 4T(n/2) + n^2, \; T(1) = 1$

$T(n) = aT(n/b) + f(n)$

$a=4, b=2 \quad f(n)=n2$

Applying master theroem

$T(n) = aT(n/b) + f(n)$

$f(n) = O\left(n^{\log_b a - \epsilon}\right) \left[\begin{array}{l} E>0 \\ T(n) = \theta\left(n^{\log_b a}\right) \end{array}\right]$

$f(n) = \theta(n^{\log_b a}]$, then $T(n) = \theta(n^{\log_b a} \log n]$

$F(n) = \Omega(n^{\log_b a + \epsilon})$, then $T(n) = \Theta(n^{\log_b a} \log n)$
$F(n)$

calcuting $\log_b a$:

(Comparing $F(n)$ with $n^{\log_b a}$)

$\log_b a = \log_2 4 = 2$

(case 2)

$F(n) = n^2 = \Theta(n^2)$

$F(n) = \Theta(n^2) = \Theta(n^{\log_b a})$,

$T(n) = 4T(n/2) + n^2$

$T(n) = \Theta(n^{\log_b a} \log n) = \Theta(n^2 \log n)$

Order of growth

$T(n) = 4T(n/2) + n^2$ with $T(1) = 1$

is $\Theta(n^2 \log n)$