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CSA0670 DAA

① Solve the following recurrence relation

a) $x(n) = x(n-1)+5$ for $n>1$ with $x(1)=0$

1) write down the first two terms of identify the pattern

$x(1) = 0$
$x(2) = x(1)+5 = 5$
$x(3) = x(2)+8 = 10$
$x(4) = x(3)+5 = 15$

2) identfify the pattern (or) the general term

→ The first term $x(1)=0$
The common difference $d=5$

The general formula for the nth tern of an AP is $x(n) = 0+(n-1)5 = 5(n-1)$

The solution is

$x(n) = 5(n-1)$

b) $x(n) = 3x(n-1)$ for $n>1$ with $x(1)=11$

1) write down the first two terns to identify

$x(1)=4$
$x(2) = 3x(1) = 3 \cdot 4 = 12$
$x(3) = 3x(2) = 36$
$x(4) = 3x(3) = 108$

2) Identify the general term

→ The first term $x(1)=4$
→ The common ration $=3$

The general formula for the n^{th} term of gp is

$x(n) = x(1) \cdot r^{n-1}$

substituting the given values

$$x(n) = 4 \cdot 3^{n-1}$$

The solution is

$$x(n) = 4 \cdot 3^{n-1}$$

c) $x(n) = x(n/2) + n$ for $n > 1$ with $x(1) = 1$ (solve for $n = 2k$)

for $n = 2^k$ we can write recurrence in term of k

1) substitute $n = 2^k$ in the recurrence

$$x(2^k) = x(2^{k-1}) + 2^k$$

2) write down the first few terms to identify the pattern

$$x(1) = 1$$
$$x(2) = x(2^1) = x(1) + 2 = 1 + 2 = 3$$
$$x(4) = x(2^2) = x(2) + 4 = 3 + 4 = 7$$
$$x(8) = x(2^3) = x(4) + 8 = 7 + 8 = 15$$

3) Identify the general term by finding the pattern

we observe that

$$x(2^k) = x(2^{k-1}) + 2^k$$

since $x(1) = 1$

$$x(2^k) = 2^k + 2^{k-1} + 2^{k-2}.$$

The geometric series with the term $a = 2$ and the last term 2^k Execept for the additional $+1$ term

The sum of a geometric series S with ratio $r = 2$ is give by

$$S = a\,\frac{r^n - 1}{r - 1}$$

where $a = 2$, $r = 2$ and $n = k$,

② Evalue the following recurences complexity

i) $T(n) = T(n/2) + 1$, where $n = 2k$ for all $k \geq 0$

The recurrence relation can be solved using algorithm method

1) Subsitute $n = 2k$

2) iterate the recurrence

for $k=0$: $T(2^0) = T(1) = T(1)$

$k=1$: $T(2^1) = T(1) + 1 =$

$k=2$: $T(2^2) = T(8) = T(n) + 1 = (T(1) + 2) + 1 = T(1) + 2$

$k=3$: $T(2^3) = T(8) = T(n) + 1 = (T(1) + 2) + 1 = T(1) + 3$

3) generalize the pattern

$T(2^k) = T(1) + 1 <$

since $n = 2^k$, $k = \log_2 n$

$T(n) = T(2^k) = T(1) + \log_2 n$

④ Assume $T(1)$ is constrat c

$T(n) = c + \log_2 n$ $T(n) = O(\log n)$

The solution

ii) $T(n) = T(n/3) + T(2n/3) + n$ where c is constant n

for divide and conque rearel

$T(n) = aT(n/2) + f(n)$

where $a = 2$, $b = 3$ and $f(n) = n$

Lets determine the value of $\log_b a$:

$\log_b a = \log_3 2$

Using the properties of logarithms

$$\log_3 2 = \frac{\log 2}{\log 3}$$

Now we compare $f(n) = cn$ with $n^{\log_3 2}$

$$f(n) = O(n)$$

$$n = n^1$$

since $\log_3 2$ we are in the third case of the master theorem $f(n) = O(n2)$ with $c > \log_b a$

The solution is:

~~$f(n) = O(n2)$~~ with $T(n) = O(f(n) = O((n) = O(n)$

(8) ~~consider~~ ~~the~~ ~~following~~ ~~recurrence~~ ~~algorithm~~ consider the following recurrence algorithm:

d) $x(n) = x(n/3) + 1$ min for $n > 1$ with $x(1) = 1$ (save for $n = 3^k$)

for $n = 3^k$

1) substitute $n = 3^k$ in the recursion

$$x(3^k) = x(3^{k-1}) + 1$$

2) write down the first few terms

$$x(1) = 1$$

$$x(3) = x(3^1) = x(1) + 1 = 1 + 1 = 2$$

$$x(9) = x(3^2) = x(3) + 1 = 2 + 1 = 3$$

$$x(27) = x(3^3) = x(9) + 1 = 3 + 1 = 4$$

3) identify the general term

we observe that

$$x(3^k) = x(3^{k-1}) + 1$$

Summing up the series

$$x(3^k) = 1 + 1 + \cdots + 1$$

$$x(3^k) = k + 1$$

The solution is

$$x(3^k) = k + 1$$

3) consider the following recurrence algorithm

min [A [0 ... n-2])

if n=1 return A[0]

Else temp = min [|A 10 .. n-2]

if temp<= A[n-1) return temp

Else

Return A[n-1)

a) what does this algorithm compute?

The given algorithm min [A(0 ... n-1)] computes the minimum value in the array "A" from index "0" for "n-1" if does this by recurresively finding the minimum value in the sub array A[0 .. n-2] and then comparing it overall maximum value

b) set up recurrence relation for the algorithm basic operation count and solve it

The solution is

$T(n) = n$

this means the algorithm performs n basic parameters for an input array of size "n"

4) Analysie the order of ~~growth~~ growth

i) $f(n) = 2n^2+5$ and $g(n) = 7n$ use the $\Omega(g(n))$ notation

To analyze the order of growth and use the Ω notation, we need to compare the given function $f(n)$ and $g(n)$

given functions:

$F(n) = 2n^2+5$

$g(n) = 7n$

Order of growth using $\Omega(g(n))$ Notation

The notation $\Omega(g(n))$ describes a lower bound on the growth rate that for sufficiently large n, $f(n)$, grows at least as fas as $g(n)$

$$F(n) = c \cdot g(n)$$

Less analysie $F(n) = 2n^2+5$ with respect to $g(n) = 7n$

1) identifly Dominant terms

→ The dominant terms in $F(n)$ is $2n^2$ since it grows faster then the constant terms as n increase

→ The dominant term in $g(n)$ is $7n$

2) establish the inequality

→ we want to find constant c and n_0 such that $2n^2+5 \geq c \cdot 7n$ for all $n \geq n_0$

3) simplify the inequality

⇒ ignore the lower order return 5 for larger

$2n^2 \geq 7cm$

⇒ Divide both sides by n

$2n \geq 7c$

⇒ solve for n:

$n \geq 7c/2$

4) chosse constants

Let $c = 1$

$n \geq \frac{7.1}{2} = 3.5$

∴ for $n \geq n$ the inequality hodes:

$2n^2+5 \geq 7n$ for all $n \geq n$

we have shown that there Exist constants $c = 1$ and $n_0 = n$ such that for all $n \geq n_0$

$2n^2+5 \geq 7n$

Thus, we can concwsude that:-

$f(n) = 2n^2 5 = \Omega(7m)$

in Ω notation the dominat term $2n^2$ in $f(n)$ clearly grows faster than th Hence

$f(n) = \Omega(n^2)$

However for the specific comparison asked

$f(n) = \Omega(7A)$ is also correct-

showing that $f(n)$ grows at least as fast as $7n$.