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Linear Algebra Assignment

Q1. $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$ $\quad R_4 \rightarrow R_4 - 2R_3$

$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 0 & 4 & 5 & -1 \end{bmatrix}$ $\quad R_3 \rightarrow R_3 - 3R_1$, $R_2 \rightarrow R_2 - 2R_1$

$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 4 & 5 & -1 \end{bmatrix}$ $\quad R_4 \rightarrow R_4 + R_3$

$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$ $\quad R_2 \longleftrightarrow R_3$

$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & -3 & 2 \end{bmatrix}$ $\quad R_4 \rightarrow R_4 - R_3$

$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\therefore \boxed{\rho(A) = 3}$

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2. every matrix and vector space w is of the form: $\begin{bmatrix} a & b \\ b & c \end{bmatrix}$

linear transformation, $T : w \rightarrow P_2$ defined by

$$T\left(\begin{bmatrix} a & b \\ b & c \end{bmatrix}\right) = (a-b) + (b-c)x + (c-a)x$$

$$\therefore \text{Dimension} = 2+1 = 3$$

Standard basis for w is

$$E = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

(w is 2×2 matrix)

Also $T = \left(\begin{bmatrix} a & b \\ b & c \end{bmatrix}\right) = \begin{bmatrix} a-b \\ b-c \\ c-a \end{bmatrix}$

$$\therefore T\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$T\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$T\left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & +1 \\ 0 & 0 & 0 \end{bmatrix}$$

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Converting to row-echleon form

$$A = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_2 \to R_2 + R_1} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\rho(A) = 1$

$\rho(A) + N = D \Rightarrow 1 + N = 3 \Rightarrow \boxed{N = 2}$

Rank $\Rightarrow \rho(A) = 1$

Nullity $\Rightarrow N = 2$

3. $A - \lambda I = 0$

$$\begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix} = 0 \Rightarrow (2-\lambda)^2 - 1 = 0$$

$\Rightarrow 4 + \lambda^2 - 4\lambda - 1 = 0$

$\Rightarrow \lambda^2 + 4\lambda + 3 = 0$

$\Rightarrow \lambda^2 - 3\lambda - \lambda + 3 = 0$

$\Rightarrow \lambda(\lambda - 3) - 1(\lambda - 3) = 0$

$\boxed{\lambda = 1, 3} \rightarrow$ eigen values of A.

$\therefore$ for $A^{-1} = 1, 1/3$

Eigen space corresponding to $\lambda = 1$ & $\lambda = 3$

for $\lambda = 1$, $[A - \lambda I]x = [0] \Rightarrow [A - I]x = [0]$

$$\left(\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x \\ y \end{bmatrix} = [0]$$

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = [0]$$

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$x - y = 0 \Rightarrow x = y$
& $-x + y = 0$

If $x = k$, then $x = y = k$

$\therefore$ Eigen space $= \begin{bmatrix} k \\ k \end{bmatrix} = k \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Same for A^T

$\underline{\lambda = 3}$

$$[A - 3I][x] = [0]$$

$$\begin{bmatrix} 2-3 & -1 \\ -1 & 2-3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = [0]$$

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-x - y = 0 \Rightarrow -x = y$

if $x = k$, $y = -k$

$\therefore$ Eigen space $= \begin{bmatrix} k \\ -k \end{bmatrix} = k \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Eigen values for $A + kI$

$\lambda_1 + c, \lambda_2 + c$

$1 + 4, 3 + 4$

$5, 7$

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4. $3x - 0.1y - 0.2z = 7.85$
$0.1x + 7y - 0.3z = -19.3$
$0.3x - 0.2y + 10z = 71.4$

1^{st} iteration :- $y = 0, z = 0$

- $3x = 7.85 \Rightarrow \boxed{x = 2.61}$
- $0.1u - 0.3z + 19.3 = -7y$

$$y = \frac{0.1 \times 2.61 + 19.3}{-7}$$

$$\boxed{y = -2.79}$$

$$z = \frac{71.4 + 0.2y - 0.3x}{10}$$

$$\boxed{z = 7.00}$$

2^{nd} iteration

- $y = -2.79, \quad z = 7.00$
- $x = \frac{7.85 + 0.2z + 0.1y}{3} = \boxed{2.99 = x}$
- $y = \frac{0.1x - 0.3z + 19.3}{-7} \Rightarrow z = 7, x = 2.99$

$$\boxed{y = -2.49}$$

- $z = \frac{71.4 + 0.2y - 0.3x}{10}, \quad x = 2.99, y = -2.49$

$$\boxed{z = 7.0005 = 7}$$

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3^{rd} iteration

- $x = \frac{7.85 + 0.2z + 0.1y}{3}$, $y = -2.49$, $z = 7$

$\boxed{x = 3.00}$

- $y = \frac{0.1x - 0.3z + 19.3}{-7}$, $x = 3$, $z = 7$

$\boxed{y = -2.5}$

- $z = \frac{71.4 + 0.2y - 0.3x}{10}$ $\Rightarrow$ $y = -2.5$, $x = 3$

$\boxed{z = 7}$ //

5. $AX = B$

$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 2 & 17 & 4 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

Since $B = 0$ $\therefore$ Our system is Homogeneous.

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 2 & 17 & 4 \end{bmatrix} \xrightarrow[\substack{R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1}]{R_4 \to R_4 - R_1} \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & -2 \\ 0 & 14 & 2 \end{bmatrix}$$

$R_4 \to R_4 + R_3$

6.

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$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & -2 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_3 \to R_3 - 2R_2} \begin{bmatrix} 2 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 2 < n = 3$

$\therefore$ infinite sol^n

$1x + 3y + 2z = 0$

$-7y - z = 0$

$\boxed{z = 7y}$

$1x + 3y + 14y = 0$

$x + 17y = 0$

$\boxed{x = -17y}$,,

if $y = k$, $x = -17k$, $z = 7k$

$$\therefore X = k \begin{bmatrix} -17 \\ 1 \\ 7 \end{bmatrix}$$

6. $T(a + bx + cx^2) = (a+1) + (b+1)x + (c+1)x^2$

① $T(P_1) + T(P_2) = T(P_1 + P_2)$

$T(a_1 + b_1 x + c_1 x^2) + T(a_2 + b_2 x + c_2 x^2)$

$= (a_1 + 1) + (b_1 + 1)x + (c_1 + 1)x^2 + (a_2 + 1) + (b_2 + 1)x + (c_2 + 1)x^2$

$= (a_1 + a_2 + 2) + (b_1 + b_2 + 2)x + (c_1 + c_2 + 2)x^2$

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$$T(P_1+P_2) = T\left((a_1+a_2)+(b_1+b_2)x+(c_1+c_2)x^2\right)$$
$$= (a_1+a_2+1)+(b_1+b_2+1)x+(c_1+c_2+1)$$

$$T(P_1)+T(P_2) \neq T(P_1+P_2)$$

$\therefore$ T is not linear transformation.

8.

7\. Form a matrix with vector S as its column & row reduce to check for linear independence.

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{bmatrix} \xrightarrow[R_2 \to R_2 - 2R_1]{R_3 \to R_3 - 3R_1} \begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & -9 & 9 \end{bmatrix}$$

$$\xrightarrow{R_3 \to R_3 - 9/5 R_2} \begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\rho(A) = 2 < n = 3$$

infinite, linearly dependent, don't span $V_3(R)$ and not basis.

let's Determine dimension & basis of subspace spanned by S.

Dimension $\to$ no. of linearly independent vectors in S.

1^{st} & 2^{nd} row or L.I but 3^{rd} not.

Dimension = 2 and basis for the subspace spanned by S is
$\therefore \{(1,2,3), (3,1,0)\}$

8. Initial = $x^{(0)} = 1$, $y^{(0)} = 1$, $z^{(0)} = 1$

$$x^{k} = \frac{1}{a_{11}}\left(b_1 + a_{12}\, y^{(k-1)} + a_{13}\, z^{(k-1)}\right)$$

Similarly y^k & z^k

1st iteration

$$x^{(1)} = \frac{1}{3}(23 + 6.1 - 2.1) = \boxed{9}$$

$$y^{(1)} = \frac{1}{(-1)}(-15 + 1 + 1) = \boxed{-10}$$

$$z^{(1)} = \frac{1}{7}(16 + 1 - 3 + 1) = \boxed{2}$$

2nd iteration

$$x^{(2)} = \frac{1}{3}(23 + 6.10 - 2.2) = \boxed{-36}$$

$$y^{(2)} = \frac{1}{1}(-15 + 4(9) + 2) = \boxed{26}$$

$$z^{(2)} = \frac{1}{7}(16 - (-36) + 3(-10)) = \boxed{8}$$

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3rd iteration

$$x^{(3)} = \frac{1}{3}\left(23 + 6(26) - 2(8)\right) = \boxed{47}$$

$$y^{(3)} = \frac{1}{1}\left(-15 + 4(-36) + 8\right) = \boxed{-168}$$

$$z^{(3)} = \frac{1}{7}\left(16 - 47 + 3(26)\right) = \boxed{11}$$

9. Matrix operations are extensively used in image processing. Like transpose of matrix is used to rotate the image in various directions and the blur matrix is used to to blur certain area of image. Apart from this, images are made up of matrix itself. Images are made up of pixels which are arranged in grid to produce image.

~~10.~~

10. linear transformation plays very important role in computer vision. In linear transformation is extremely used in manipulating image for various purpose.

One example is rotating image with angle about x - axis.
for this purpose, we use famous rotation matrix in 2D to do this task.
Here, $T : V \to W$,
where $T(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

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If we have to rotate (x, y) about θ, then new x' and y' are -

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

In this way we perform this basic operation for each pixel of the image and find the rotated image.

This transformation is also used in image registration, object detection and image alignment.