3 JA
4 Indigo
2 spicejet

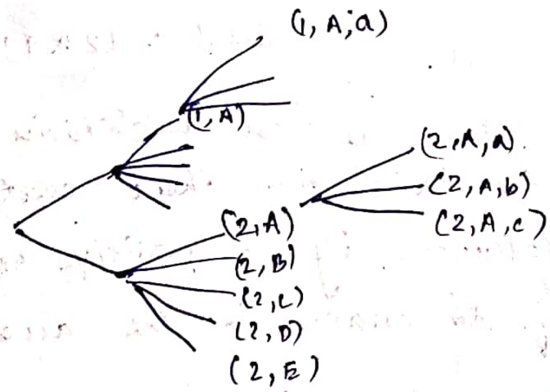
Total no. of Flights: 9 flights

$A \cap B = \phi$

$|A \cup B| = |A| + |B|$

2 cones
5 flavours
3 syrups

Total type of ~~fo~~ icecreams: $2 \times 5 \times 3$

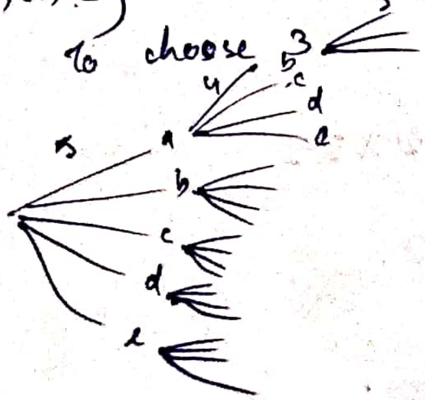


Product Rule:

Suppose that a procedure can be broken down into a sequence of two tasks. If there are N_1 ways to do first task & for each of these ways of doing the first task there are N_2 ways of doing second task. Then there are $N_1 \times N_2$ ways to do 2nd task.

$\{a, b, c, d, e\}$

To choose 3



$5 \times 4 \times 3 = 60 \quad ({}^5P_3)$

Q. A company has rented an office with 12 rooms and has only two employees (A & B). How many different ways to assign different rooms to 2 employees?

$^{12}P_2$

Sol^n For 1st person = 12 choices
2nd = 11

$\therefore$ Total = $12 \times 11 = 132$

Q. There is an auditorium. We have to assign the chairs such that seat begins with capital letter followed by a positive no. (1 to 100). What is the no. of chairs in the audi?

Total seat numbers =
$26 \times 100 = 2600$

$26^3 \times 10^3$

Q. m = 0;
for i → 1 to 100
  for j → 1 to 50
    for k → 1 to 25
      m++
print m

$100 \times 50 \times 25$

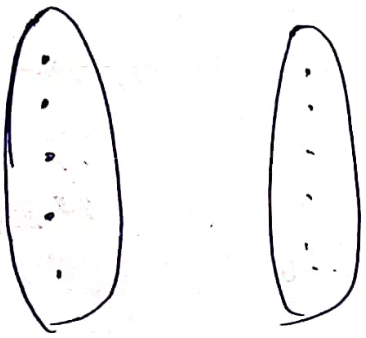
Password of 6 length, can be letter or digit.
36 36 3 . . .

Total alphanumeric: 36^6. ~~10×36^5~~

If only alphabets then 26^6

Total no. of password with atleast 1 no. $= 36^6 - 26^6$

A B



m element n elements,

$n-k\ 1$

Total no. of funct^ns =

$n \cdot n \cdot n \cdots = n^m$

If one to one funct^n

$\underset{1st}{n}\ \underset{2nd}{(n-1)}\ (n-2) \cdots \underset{m^{th}}{(n-m+1)}$

Sum Rule

A task can be done in either one of n_1 ways & one of n_2 ways where none of the set of n_1 ways is same as any of the set of n_2 ways. Then there are $(n_1 + n_2)$ ~~set of datas~~ ways to do the task. $A \cap B = \phi$

Resource Request Algorithm

① $Req^{est}(i) \leq Need(i)$

$\quad^{est}(i) \leq Avail(i)$

eg:

```
k=0
for i→1 to n
    k++
  for j→1 to m.
      k++
    for l→1 to p
        k++
  printf (k)
```

Total no. of points = n+m+p

eg:

180 students.
12 teachers

Total no. of persons = 192.

Alpha numerical

Total no. of passwords of max 2 length.

~~36+36²+36³+36⁴+36⁵~~

$\text{Total 1st char} + \text{Total 2char} = 36 + 36^2$

Only alpha

$\underline{26}$

$\underline{26}.\underline{36} = 26 \times 36$

$\text{Total} = 26 + 26.36$

Passwords of 6,7,8 length

$P_6 = 36^6 \quad P_7 = 36^7 \quad P_8 = 36^8$

$\text{Total} = 36^6 + 36^7 + 36^8$.

If ~~only~~ ~~alphabets~~ numbers present

$P_6 = 36^6 - 26^6$

$P_7 = 36^7 - 26^7$

$P_8 = 36^8 - 26^8$

Total: Sum of P_6, P_7, P_8

	0	1	2	3	4 . . . 8	16	24	31
Class A	0	NID				Hid		
B	1	0	Nid				Hid	
C	1	1	0	Nid				hid
X: D	1	1	1	0				
X: E	1	1	1	1	0			

No. of public IP addresses

~~$2^{24}-2$~~

~~$2^{16}-2$~~

~~$2^{8}-2$~~

Restrictions:-

⇒ All 0 not possible

⇒ Net id of A can't be all 1's.

⇒ All 0's & all 1's not possible in host ID's

Net ID

$2^7 - 1$

$2^{24} - 2$

$3 + 9 - 6$

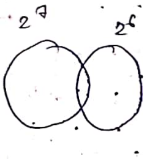
$x_A = (2^7 - 1) \times (2^{24} - 2)$

$x_B = (2^{14}) \times (2^{16} - 2)$

$x_C = (2^{21}) \times (2^8 - 2$

$\text{Total public IP} = x_A + x_B + x_C$

Q. How many bit string of length 8 are there that starts with 1 or ends with 2 zero.



$$\frac{1}{1}\ \frac{2}{0/1}\ \frac{2}{0/1}\ \frac{2}{0/1}\ \frac{2}{0/1}\ \frac{2}{0/1}\ \frac{1}{0}\ \frac{1}{0}$$

$A = \underline{1}\,\_\ \_\ \_\ \_\ \_\ \_\ \_ = 2^7$

$B = \_\ \_\ \_\ \_\ \_\ \_\ \underline{0}\ \underline{0} = 2^6$

$\therefore$ Tot,

$A \cap B = \underline{1}\,\_\ \_\ \_\ \_\ \_\ \underline{0}\ \underline{0} = 2^5$

$|A \cup B| = |A| + |B| - |A \cap B|$

$= 2^7 + 2^6 - 2^5$

350 → Total no. of applicant
220 → CSE students
147 → Maths students
51 → Both CSE Maths

No. of applicants who are not eligible (neither maths nor CSE)

$\overline{A \cup B} = 350 - (220 + 147 - 51)$

$= 350 - (367 - 51) = 350 - 316$

$= 34$

Permutation

Act of arranging members of a set into a sequence or order.

<Template 1> n objects, can't repeat

No. of r-permutation ?

1st 2nd 3rd 4th . . . kth . . . rth

$\underline{n}\ \underline{n-1}\ \underline{n-2}\ \_\ \_\ \_\ \underline{n-k+1}\ \dots\ \underline{n-r+1}$

$\underbrace{\qquad\qquad\qquad\qquad}_{r\text{ - positions}}$

$$= \frac{[n \times (n-1) \times (n-2) \times \dots \times (n-r+1)] \times [(n-r) \times (n-r-1) \times (n-r-2) \times \dots 1]}{(n-r) \times (n-r-1) \times (n-r-2) \times \dots \times 1}$$

$$= \frac{\lfloor n}{\lfloor n-r}$$

$$= {}^nP_r$$

Q. $n = 15$ total
$r = 11$ to be chosen

${}^nP_r = {}^{15}P_{11}$

Q. 5 letters, 1st vowel.

$\underline{5}\ \overbrace{\_\ \_\ \_\ \_}^{{}^{25}P_4} = 5 \times {}^{25}P_4$

< Template 2> n objects, can repeat

No. of r-perm ?

1st 2nd 3rd 4th . . k^{th} . r^{th}

$\underline{n}$ $\underline{n}$ $\underline{n}$ ___ ___ ___ ___ ___ $\underline{n}$

$= n^r$

Q. No. of 5 letter English words ?

$\underline{26}$ $\underline{26}$ _ _ . $\underline{26}$

9X

$\frac{\lfloor n}{\lfloor n-r \lfloor r}$

$= 26^5$

Q. No. of 5 digit

$\underline{9}$ $\underline{10}$ $\underline{10}$ $\underline{10}$ $\underline{10}$

$= 9 \times 10^4$

< Template 3> n objects, can't repeat

No. of r comb ?

^rC_n

$= \frac{\lfloor n}{r}$

1st 2nd 3rd . . k^{th} r^{th}

___ ___ ___ ___ ___ ___ ___ ___ . $= \frac{^nP_r}{r!} = \frac{n!}{r!\, n-r!}$

$= {}^nC_r$

Q. 10 ~~committee~~ members

5 member committee to be chose

Raju must be chosen

$\underline{\text{Raju}}$ ___ ___ ___ ___

$^9C_4$

Q. 5 member commitee of 3 teachers & 2 student.

Teachers = 10

Students = 15

If Raju present, Teacher

Student ⟶ Rani won't be present.

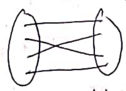
No raju, no rani

$^{10}C_3 \times {}^{15}C_2$

___ ___ ___ ___ ___ $\underline{\text{Raju}}$ ___ ___ ___ ___ ___ ___ ___ ___ $\underline{\text{Rani}}$

$({}^9C_3 \times {}^{14}C_2) + ({}^9C_2 \times {}^{14}C_2) + ({}^9C_3 \times {}^{14}C_1)$

One to one correspondance



$|A| = n$ $|B| = m$

$n > m$?

$n < m$?

$n = m$? ⟵ Bijective.

2^9

Natural no. set $N \{0, 1, 2 \ldots\}$

Even $E \{0, 2, 4 \ldots\}$

natural $f(x) = 2x$ even

(1, 2, 3, ...) (2, 4, 6, ...)

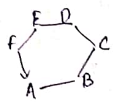
one to one ✓

onto ✓

∴ none

	a_1	a_2	......	a_n
ϕ	0	0	...	0
$\{a_n\}$	0	0	...	1
$\{a_1\}$	1	0	.....	0
⋮				
$\{a_1, a_2, a_3, a_n\}$	1	1	.....	1

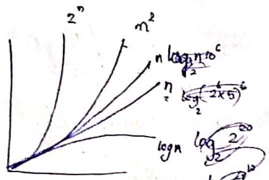
2^{20}



A person to visit all cities in how many way?

6 !

	1m	1b
$\log n$	20	30
n	10^6	10^9
n^2	10^{12}	10^{18}
2^n	2^{1m}	2^{1b}



$\log_2 10^6$

$\log_2 (2 \times 5)^6$

$\log_2 2^{20}$

$= \log_2 2^{10}$

$= \log_2 2^{20}$ $\quad 2^{20} = 4^{10}$

$4^{10} = 8^5$

$8^5 = 16^{2.5}$

$= \log_2 4^{10}$

$= \log$

<T3> n obj, no repeatation

2^n $\quad \{a,b,c\} = 2^3 = 8 = \{\phi, a, b, c, \{a,b\}, \{b,c\}, \{a,c\}, \{a,b,c\}\}$

<T4> n different object, repeatation allowed

r combination?

$x_1 + x_2 + x_3 = 2$

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

space as 1

$\{a,b,c\}$

	x_1	x_2	x_3		Binary sequence
$\{a,a\}$	2	0	0	[00] [ ] [ ]	0011
$\{a,b\}$	1	1	0	[0] [0] [ ]	0101
$\{b,c\}$	0	1	1	[ ] [0] [0]	1010
$\{b,b\}$	0	2	0	[ ] [00] [ ]	1001
⋮					
$\{c,c\}$	0	0	2	[ ] [ ] [00]	1100

Ways to arrange 1 in

_ _ _ _

$= {}^4C_2$ $\quad$ 4 = places, 2 1's

Ways for 0

$= {}^2C_2$ $\quad$ 2 places left, 2 zero's

$1_a \_ 1_b \_$
$1_b \_ 1_a \_$ } doesn't matter so combination

Total no. of ways $= {}^4C_2 \times {}^2C_2$

$= 6$ ways.

n - boxes

r - balls

Starting with filling 1's

$(n-1)$ 1's

r 0's

$n + r - 1$

$\Rightarrow {}^{n+r-1}C_{n-1} \times {}^rC_r$

Starting with filling 0's $\Rightarrow {}^{n+r-1}C_r \times {}^{n-1}C_{n-1}$

Both are same $= \dfrac{(n+r-1)!}{(n-1)!\, r!}$

${}^{n+r-1}C_r \times {}^{n-1}C_r$

$\langle T5 \rangle$ $n, \langle a_1, t_1 \rangle, \langle a_2, t_2 \rangle, \langle a_3, t_3 \rangle, \ldots \langle a_k, t_k \rangle$

a _ a _ _ _ a _ a a

10 spaces

5 a's doesn't matter if all different a's in same places so combination.

$a_1 \; a_2 \; a_3 \; a_4 \; a_5$

$a_5 \; a_1 \; a_2 \; a_3 \; a_4$

a	b	c
5	3	2
T_a	T_b	T_c

$T_a \times T_b \times T_c$

$^{10}C_5 \times {}^5C_3 \times {}^2C_2$

$\frac{10!}{5!\,3!\,2!}$

$\frac{10!}{3!\,7!} = \frac{10\times9\times8}{3\times2}$

$^nC_{t_1} \times {}^{n-t_1}C_{t_2} \times {}^{n-t_1-t_2}C_{t_3} \times \ldots {}^{n-t_1-t_2\cdots t_{k-1}}C_{t_k}$

$= \frac{n!}{t_1!\,\cancel{(n-t_1)!}} \times \frac{\cancel{(n-t_1)!}}{t_2!\,\cancel{(n-t_1-t_2)!}} \times \frac{\cancel{(n-t_1-t_2)!}}{t_3!\,\cancel{(n-t_1-t_2-t_3)!}} \times \ldots \times \frac{\cancel{(n-t_1-t_2\cdots t_{k-1})!}}{t_k!\,(n-t_1-t_2\cdots t_{k-1}-t_k)!}$

$= \frac{n!}{t_1!\,t_2!\,t_3!\ldots t_k!\,(n-t_1-t_2\cdots t_k)!}$

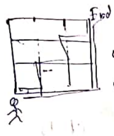
$= \frac{n!}{t_1!\,t_2!\,t_3!\cdots t_k!\,(n-(t_1+t_2\cdots t_k))!}$ $\because n = t_1+t_2\cdots t_k$

$= \frac{n!}{t_1!\,t_2!\ldots t_k!\,0!} = \frac{n!}{t_1!\,t_2!\,t_3!\ldots t_k!}$

Q. How many 10 places Binary sequence can be filled with 7 1's & 3 zero's?

_ _ _ _ _ _ _ _ _ _

$^{10}C_7 \times {}^3C_3$ or $\frac{10!}{7!\,3!}$



up = 3

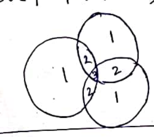
right = 3

$\frac{6!}{3!\,3!}$

$\frac{10}{3!\,7!}$

Principle of Exclusion or Inclusion :

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |C \cap A| + |A \cap B \cap C|$



Closure

A set 'A' with respect to op '×' if $\forall a,b \in A$

$a * b \in A$

Associative

A set 'A' w.r.t. op '*'

if $\forall a,b,c \in A$

$(a * b) * c = a * (b * c)$

Identity A set 'A' w.r.t to op '*' is said to satisfy Identity property if $\forall a \in A$

$\exists e \in A$

$a * e = e * a = a$

$a + \boxed{0} = \boxed{0} + a = a$

$a \times \boxed{1} = \boxed{1} \times a = a$

Inverse A set 'A' w.r.t op '*'

if $\forall a \in A$

$\exists a^{-1} \in A$

st $a * a^{-1} = a^{-1} * a = e$

$a + \boxed{-a} = \boxed{-a} + a = 0$

$a \times \boxed{1/a} = \boxed{1/a} \times a = 1$

G

$(N,+)$

$(N,-)$

$(N,\times)$

$(N,\div)$

$(Z,+)$

$Z,-$

$Z,\times$

	G	AG
(N,+)	✗	✗
(N,−)	✗	✗
(N,×)	✗	✗
(N,÷)	✗	✗
(Z,+)	✓	✓
(Z,−)	✗	✗
(Z,×)	✗	✗
(Z,÷)	✗	✗

	G	AG
(R,+)	✓	✓
(R,−)	✗	✗
(R,×)	✗	✗
(R,÷)	✗	✗
(e,+)	✓	✓
(e,×)	✗	✗
(0,+)	✗	✗
(0,×)	✗	✗

	G	AG
(M,+)	✓	✓
(M,×)	✗	✗

Commutative property –

A set "A" wrt '$*$' is said to satisfy commutative property if

$\forall a, b \in A$

$a * b = b * a$

If group satisfies then called Abelian group.

At first check for group, if not group then not Abelian too.

Cayley Table :-

×	1	−1	i	−i
1	$\boxed{1}$	−1	i	−i
−1	−1	$\boxed{1}$	−i	i
i	i	−i	−1	$\boxed{1}$
−i	−i	i	$\boxed{1}$	−1

$$\begin{bmatrix} 2 & 3 & 5 \\ 1 & 5 & 6 \\ 7 & 2 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 & 5 \\ 1 & 5 & 6 \\ 7 & 2 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 3/2 & 5/2 \\ 1 & & \\ & & \end{bmatrix} = \begin{bmatrix} 1/2 & 0 & 0 \\ & & \\ & & \end{bmatrix} A$$

$(a * x) = (a * y)$.

$\Rightarrow a^{-1} * (a * x) = a^{-1} * (a * y)$

$\Rightarrow (a^{-1} * a) * x = (a^{-1} * a) * y$

$\Rightarrow e * x = e * y$

$\Rightarrow x = y$

*	x	y
a	z	z

$x * a = y * a$

$\Rightarrow (x * a) * a^{-1} = (y * a) * a^{-1}$

$x = y$

*	a
x	z
y	z

Order of the group :

No. of elements in the set.

for this ex → $|G| = 1$

*	e
e	e

*	e	a
e	$\boxed{e}$	a
a	a	$\boxed{e}$

Has to be here ∵ each element must be in each row & each column

Also symmetric

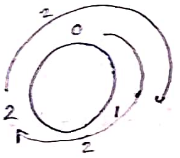
It is a group Abelian group

*	e	a	b
e	e	a	b
a	a	b	e
b	b	e	a

Isomorphic same just elements different

modulo 3

+	0	1	2
0	0	1	2
1	1	2%3=2	0
2	2	0	4%3=1



$0 + 0 = 0 + 0$

$1 + 2 = 2 + 1$

$2 + 1 = 1 + 2$

Sub-group

A sub-group of group 'G' wrt operatⁿ * is a subset of G which is also a group.

$G = (R, +)$

$H = (Z, +)$

$H \le G$ (sub group ≤ group)

1) G

2) {e} [Trivial sub-group]

$Z = \{ \ldots -3, -2, -1, 0, 1, 2, 3 \ldots \}$

$Za = \{ \ldots -3a, -2a, -a, 0, a, 2a, 3a \ldots \}$

$Z3 = \{ \ldots -9, -6, -3, 0, 3, 6, 9 \ldots \}$

$H = (Za, +)$

[✓] clos

[–] Asso

[✓] e

[–] inv

Lagrange's Theorem

$H \le G \longrightarrow |H|$ divides $|G|$

$|G| = 323 = 19 \times 17$

order of $H \rightarrow 1, 17, 19, 323$

|H|

1. {e}

2. G

$G = (A, *)$

$H = (B, *)$

$B \subseteq A$

$H \le G$

Additive Notation : +

" Identity : 0

Multiplicative Notation : X

" Identity : 1

Q. Let G be a group with operation × multiplicat°

Pick $x \in G$

What is the smallest group that contains x?

Sol^n: $\langle x \rangle = \{ \dots x^{-3}, x^{-2}, x^{-1}, 1, x^1, x^2, x^3 \dots \}$

means this group is generated by x

closed ✓
associative ✓
Identity ✓
Inverse ✓

complex variable

$\langle i \rangle = \{ -\frac{1}{i}, \frac{1}{i}, 1, i, -1, -i \}$ $\qquad -i^2 = +1$

Let G be a group with op^n +

Pick $x \in G$

What is smallest group that contains x?

$\langle x \rangle = \{ \dots -3x, -2x, -x, 0, x, 2x, 3x \dots \}$

$\langle 1 \rangle = \{ \dots -5, -4, -3, -2, -1, 0, 1, 2, 3, \dots \}$

$= \mathbb{Z}$

$G = \mathbb{Z}/n\mathbb{Z}$ = Integers mod n under +

mod 12

$\langle 1 \rangle = \{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 \}$

$1 + \underline{11} = 0$ ← Inverse ← Identity

$2 + \underline{10} = 0$

$6 + \underline{6} = 0$

mod n

$\langle 1 \rangle = \{ 0, 1, 2, \dots, n-2, n-1 \}$

$\langle 0 \rangle = \{0\}$, op^n + , cannot be '×' since 1 is not there

$\langle 1 \rangle = \{1\}$ op^n × cannot be '+' since 0 is not there

$\langle e \rangle = \{ e \}$ trivial one

Every infinite cyclic group is isomorphic to $\mathbb{Z}$

$\{\dots -3, -2, -1, 0, 1 \dots\}$

Every finite " " " " to $\{0, 1, \dots n-1\}$

$G = (\mathbb{R}, +)$

$H = (\mathbb{Z}, +)$

$\mathbb{Z}/n\mathbb{Z} \subseteq \mathbb{Z}$ $\qquad \mathbb{Z} \subseteq \mathbb{R}$

$H \subseteq G$ $\qquad H \leq G$

Symmetric Groups:-

$\langle 1 \rangle = \mathbb{Z}$ → Isomorphic to all cyclic infinite groups

$\{1, 2, 3\}$ Permutation = 6

$S_3 = \overset{a}{(123)}\ (213)\ \overset{f}{(312)}$

$(132)\ {}^{b}(231)\ {}^{g}(321)$

$b(1) = 2$
$b(2) = 3$
$b(3) = 1$

$f(1) = 3$	$g(1) = 3$
$f(2) = 1$	$g(2) = 2$
$f(3) = 2$	$g(3) = 1$

[✓] Closure
[ ] Associative

$(f \circ g)(x) = f(g(x))$

$(f \circ g)(1) = 2$	$(a \circ b)(1) = 2$
$(f \circ g)(2) = 1$	$(a \circ b)(2) = 3$
$(f \circ g)(3) = 3$	$(a \circ b)(3) = 1$

2(a)

$((f \circ g) \circ a)(x) = (f \circ (g \circ a))(x)$

$\Rightarrow (f \circ g)\, a(x) \quad = f(g \circ a(x))$

$\Rightarrow f(g(a(x))) \quad = f(g(a(x)))$

Associative

$(f \circ (g \circ h))(x) = f \circ g(h(x))$

$= f(g(h(x)))$

$((f \circ g) \circ h)(x) = f(g(\,)) \circ h(x)$

$= f(g(h(x)))$

closer

$(f \circ b)(1) = f(b(1)) = f(2) = 1$

$(f \circ b)(2) = f(b(2)) = f(3) = 2$

$(f \circ b)(3) = f(b(3)) = f(1) = 3$

$g(3,2,1)$

$a \circ (f \circ b) =$

$(f \circ b) \circ a =$

$g \circ (f \circ b) = 3 \quad 2 \quad 1$

$(f \circ b) \circ g = 3 \quad 2 \quad 1$

$1 \circ g = g(1)$

$2 \circ g = g(2)$

$3 \circ g = g(3)$

✓ [Identity]

$g \circ (f \circ b) = (f \circ b) \circ g = g$

$c \circ (f \circ g) =$

$a * e = e * a = a$

$(f \circ g)(x) = f(g(x))$

$(f \circ b)(1) = 1$

$(f \circ b)(2) = 2$

$(f \circ b)(3) = 3$ satisfies Identity Property

$(c \circ c)(x) = c(c(x)) =$

$c(c(1)) = 1$

$c(c(2)) = 2$

$c(c(3)) = 3$

[✓] Inverse

[✗] Commutative

$(f \circ g)(1) = 2$	$(g \circ f)(1) = 1$
$(f \circ g)(2) = 1$	$(g \circ f)(2) = 3$
$(f \circ g)(3) = 3$	$(g \circ f)(3) \quad 2$

not commutative

This is a group but not abelian group.

If 2 elements $S = \{1, 2\}$

then abelian group.

If $n > 2$ then not abelian.