

# Week 8

AI Seminar Series

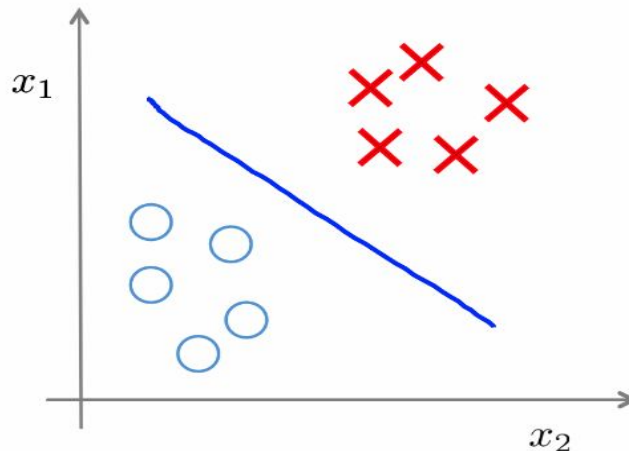


K-Means Clustering

Use-case jupyternotebooks

Credit: Slides from Andrew Ng's  
Coursera ML Course: lecture 13

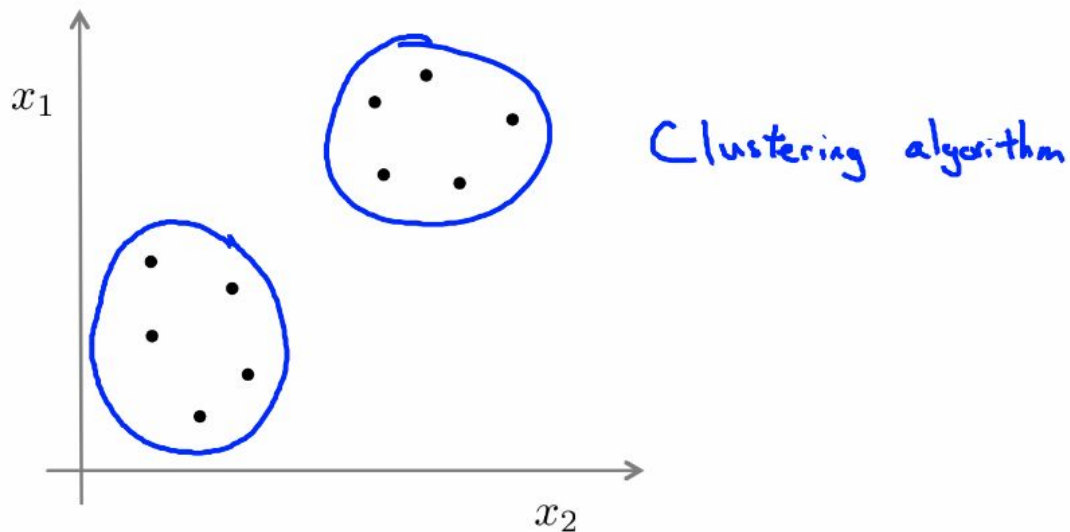
## Supervised learning



Training set:  $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), (x^{(3)}, y^{(3)}), \dots, (x^{(m)}, y^{(m)})\}$



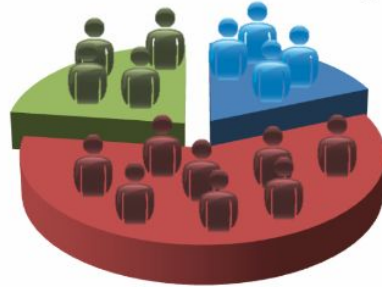
# Unsupervised learning



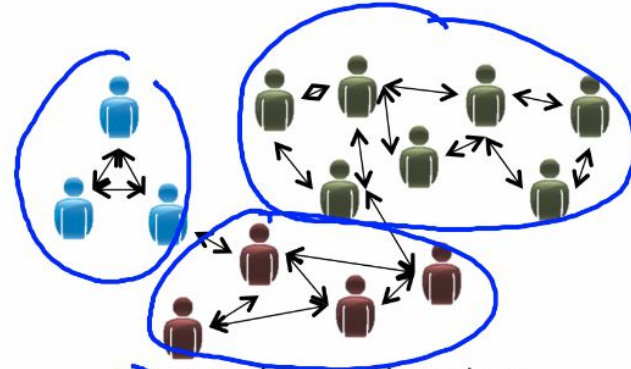
Training set:  $\{\underline{x^{(1)}}, \underline{x^{(2)}}, \underline{x^{(3)}}, \dots, \underline{x^{(m)}}\}$  ←

# Applications

## Applications of clustering



→ Market segmentation



→ Social network analysis

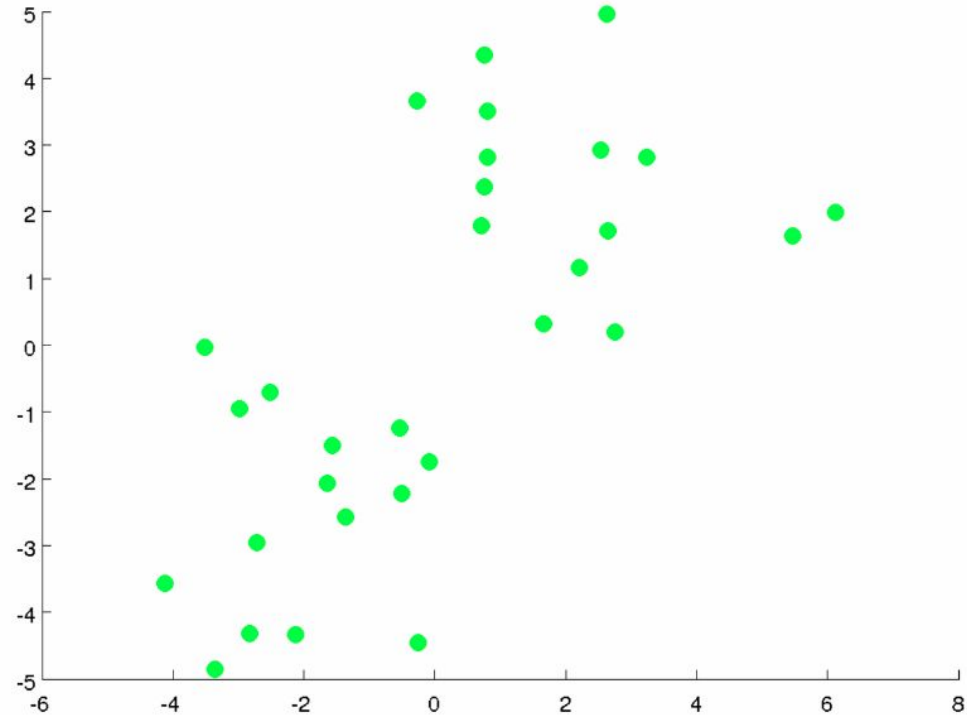


→ Organize computing clusters



→ Astronomical data analysis

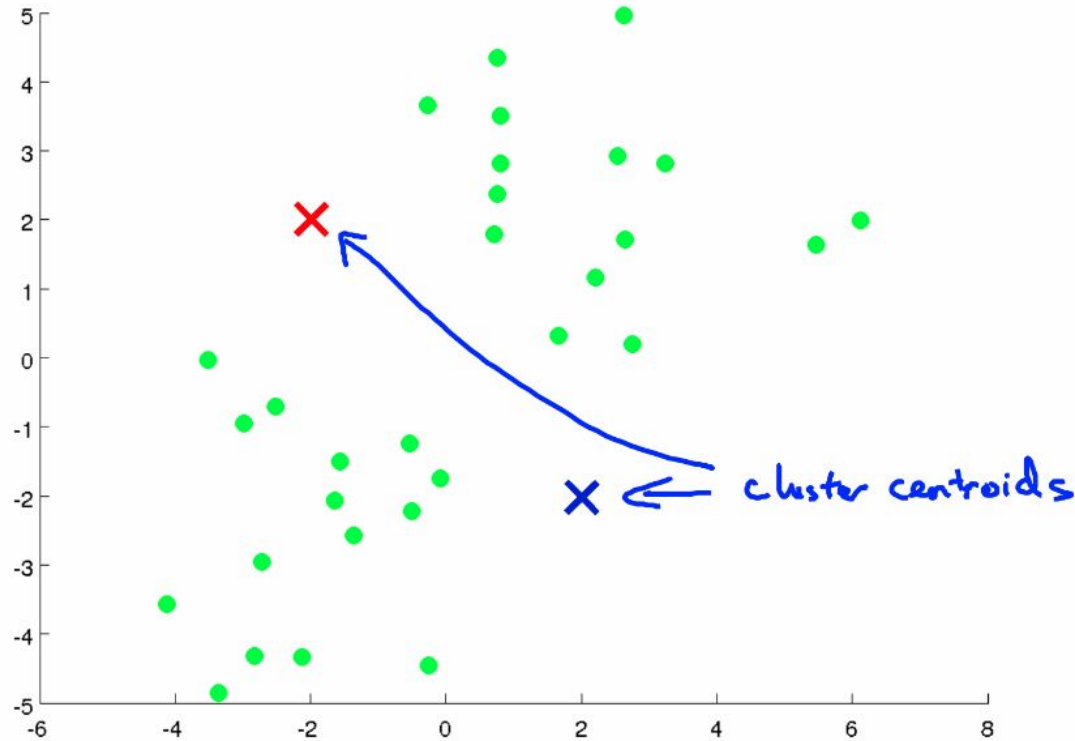
# K-Means Clustering



Credit: Slides from Andrew Ng's Coursera ML Course: lecture 13

<https://github.com/vkosuri/CourseraMachineLearning/blob/master/home/week-8/lectures/pdf/Lecture13.pdf>

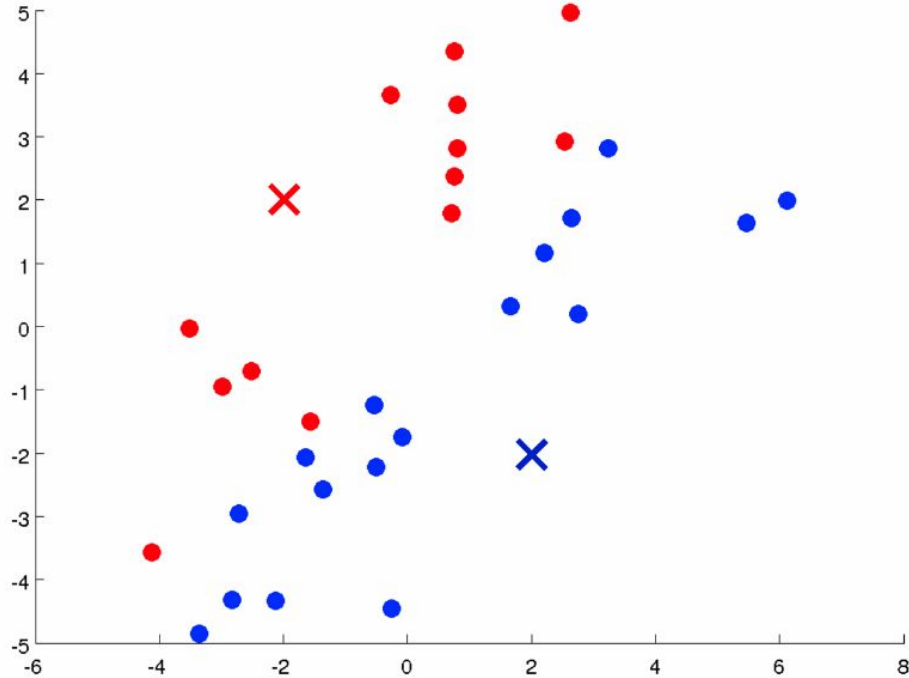
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# K-means Clustering

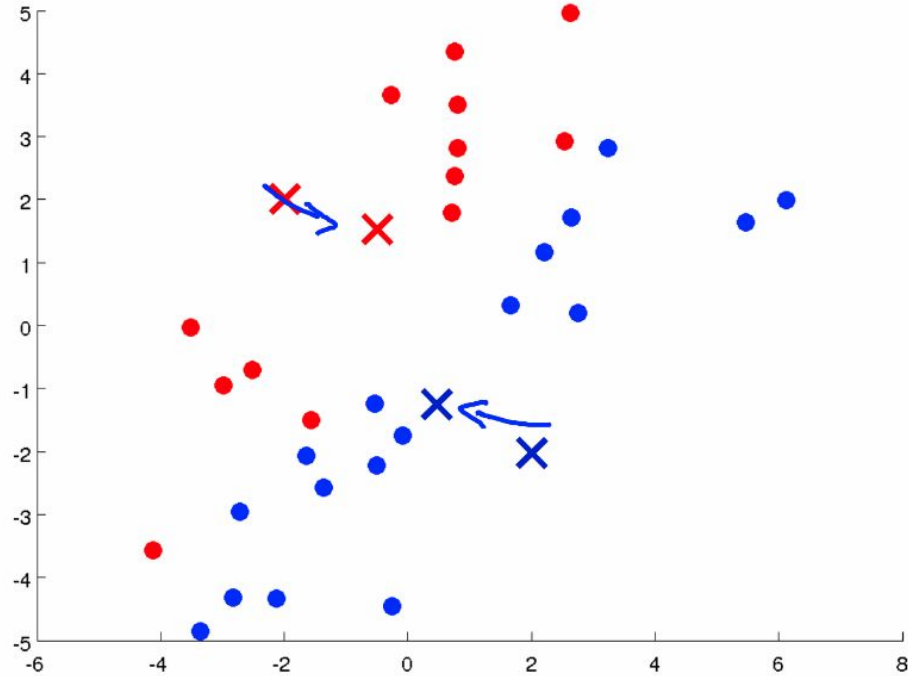


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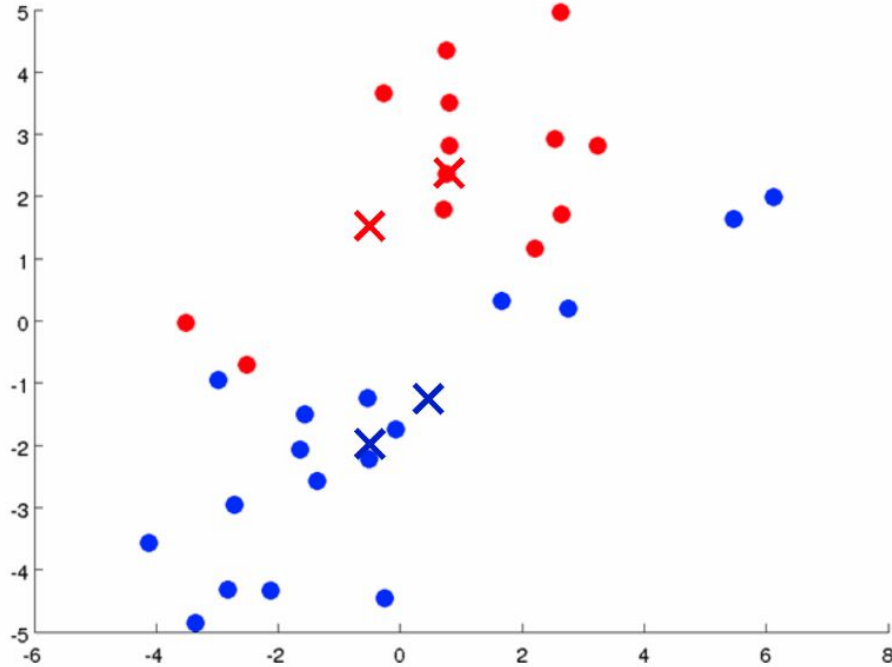
# K-means Clustering



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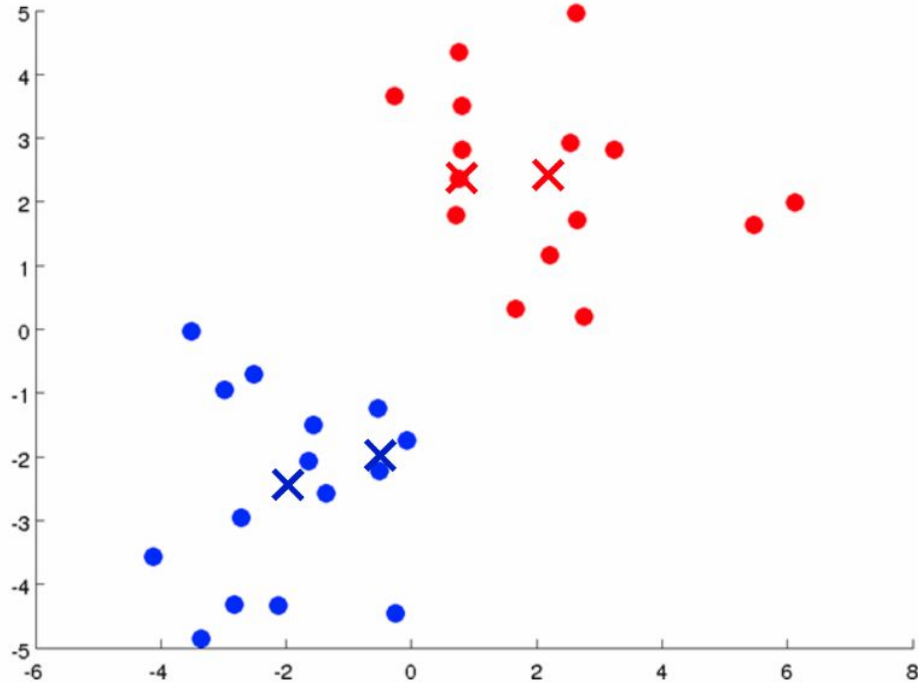
# K-means Clustering



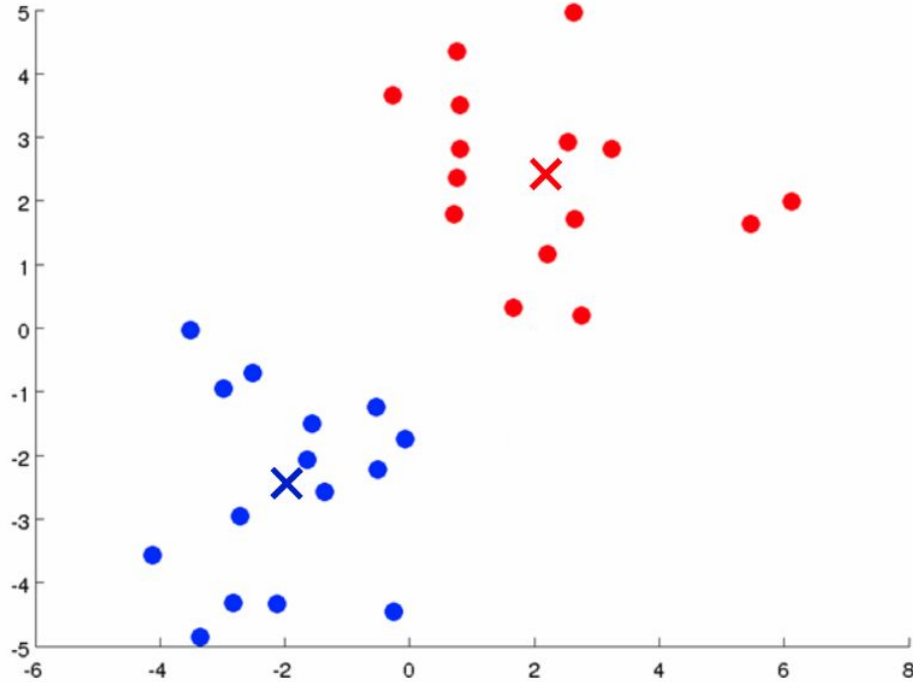
Credit: Slides from Andrew Ng's Coursera ML Course: lecture 13

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# K-means Clustering



# K-means Clustering





Credit: Slides from Andrew Ng's Coursera ML Course: lecture 13

<https://github.com/vkosuri/CourseraMachineLearning/blob/master/home/week-8/lectures/pdf/Lecture13.pdf>

## K-means algorithm

Input:

- $K$  (number of clusters) 
- Training set  $\{x^{(1)}, x^{(2)}, \dots, x^{(m)}\}$  

$x^{(i)} \in \mathbb{R}^n$  (drop  $x_0 = 1$  convention)

## K-means algorithm

$$\mu_1 \quad \mu_2$$

Randomly initialize  $K$  cluster centroids  $\underline{\mu}_1, \underline{\mu}_2, \dots, \underline{\mu}_K \in \mathbb{R}^n$

Repeat {

Cluster  
assignment  
step

for  $i = 1$  to  $m$

$\underline{c}^{(i)} :=$  index (from 1 to  $K$ ) of cluster centroid  
closest to  $x^{(i)}$

$$\min_k \|x^{(i)} - \mu_k\|^2$$

for  $k = 1$  to  $K$

$\rightarrow \mu_k :=$  average (mean) of points assigned to cluster  $k$

$$x^{(1)}, x^{(5)}, x^{(6)}, x^{(10)}$$

$$\rightarrow c^{(1)}=2, c^{(5)}=2, c^{(6)}=2, c^{(10)}=2$$

$$\mu_2 = \frac{1}{4} [x^{(1)} + x^{(5)} + x^{(6)} + x^{(10)}] \in \mathbb{R}^n$$

# K-means Clustering

## K-means optimization objective

→  $c^{(i)}$  = index of cluster  $(1, 2, \dots, K)$  to which example  $x^{(i)}$  is currently assigned

→  $\mu_k$  = cluster centroid  $\underline{k}$  ( $\mu_k \in \mathbb{R}^n$ )

$\mu_{c^{(i)}}$  = cluster centroid of cluster to which example  $x^{(i)}$  has been assigned

$K$   $k \in \{1, 2, \dots, K\}$   
 $x^{(i)} \rightarrow \underline{5}$   $c^{(i)} = \underline{5}$   $\mu_{c^{(i)}} = \mu_5$

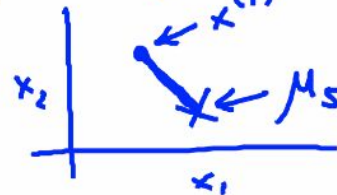
Optimization objective:

→ 
$$J(c^{(1)}, \dots, c^{(m)}, \mu_1, \dots, \mu_K) = \frac{1}{m} \sum_{i=1}^m \boxed{\|x^{(i)} - \mu_{c^{(i)}}\|^2} \leftarrow$$

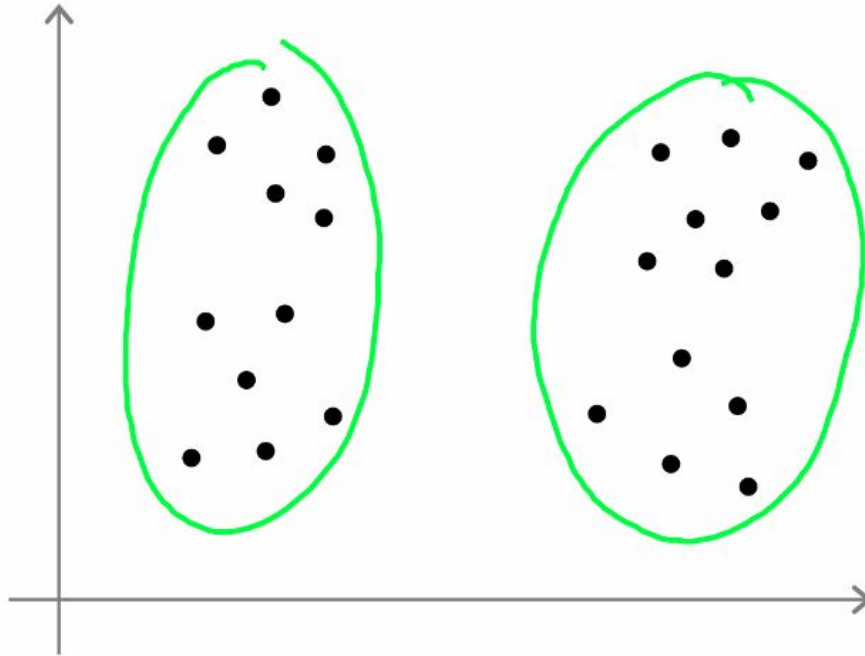
→ 
$$\min_{c^{(1)}, \dots, c^{(m)}, \mu_1, \dots, \mu_K} J(c^{(1)}, \dots, c^{(m)}, \mu_1, \dots, \mu_K)$$

→  $\mu_1, \dots, \mu_K$

Distortion



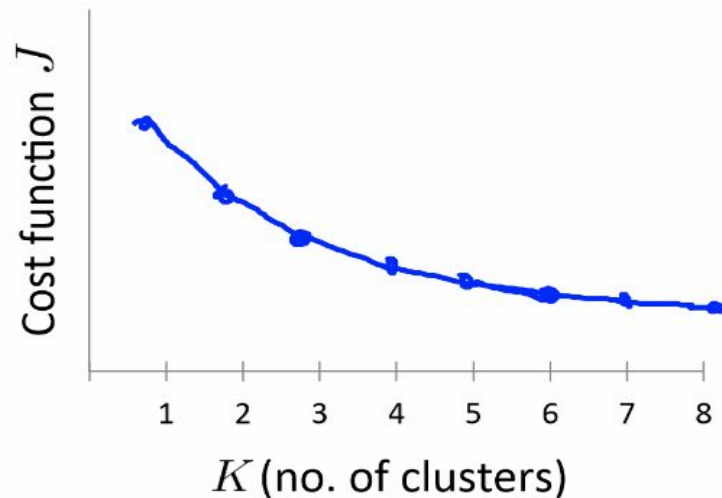
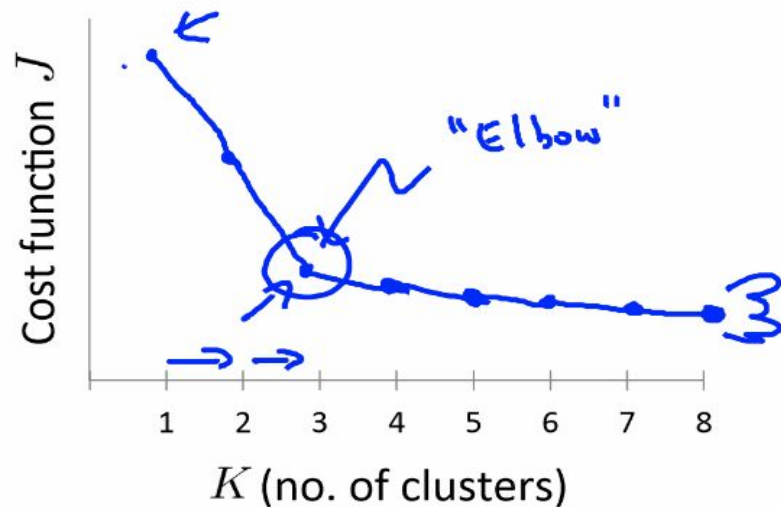
**What is the right value of K?**





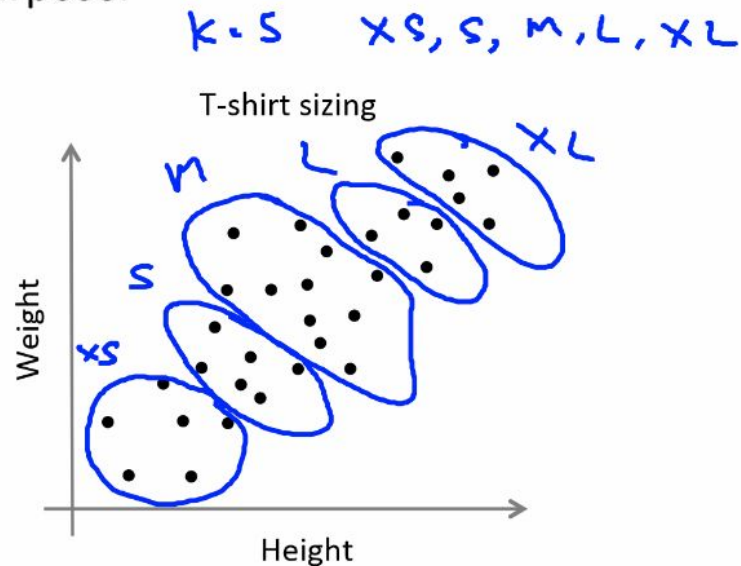
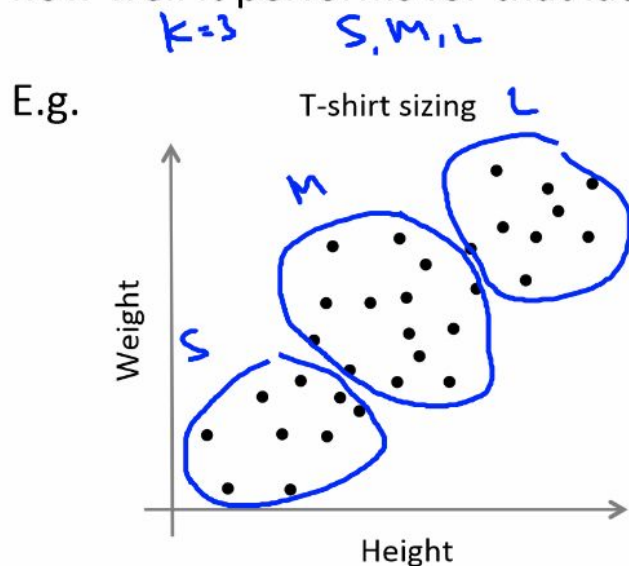
## Choosing the value of $K$

Elbow method:



## Choosing the value of K

Sometimes, you're running K-means to get clusters to use for some later/downstream purpose. Evaluate K-means based on a metric for how well it performs for that later purpose.



## `sklearn.metrics.silhouette_score`

```
sklearn.metrics.silhouette_score(X, labels, *, metric='euclidean', sample_size=None, random_state=None, **kws) \[source\]
```

Compute the mean Silhouette Coefficient of all samples.

The Silhouette Coefficient is calculated using the mean intra-cluster distance ( $a$ ) and the mean nearest-cluster distance ( $b$ ) for each sample. The Silhouette Coefficient for a sample is  $(b - a) / \max(a, b)$ . To clarify,  $b$  is the distance between a sample and the nearest cluster that the sample is not a part of. Note that Silhouette Coefficient is only defined if number of labels is  $2 \leq n\_labels \leq n\_samples - 1$ .

This function returns the mean Silhouette Coefficient over all samples. To obtain the values for each sample, use `silhouette_samples`.

The best value is 1 and the worst value is -1. Values near 0 indicate overlapping clusters. Negative values generally indicate that a sample has been assigned to the wrong cluster, as a different cluster is more similar.

# Use-cases

# Inputs

- > What topics would you like to see more working examples?
- > Ideas/presentation on projects?
- > What topics could be covered in coming class?
- > Any ML work you are currently leveraging in your project?