1

1.1

$$\frac{x^{n+2}}{x^{n-2}} = \frac{x^n \cdot x^2}{\frac{x^n}{x^2}} = x^n \cdot x^2 \cdot \frac{x^2}{x^n} = x^2 \cdot x^2 = \boxed{x^4}$$

1.2

$$x^{-1} \cdot 8 = 2$$

$$\frac{1}{x} \cdot 8 = 2$$

$$\frac{8}{x} = 2$$

$$8 = 2x$$

$$\boxed{4 = x}$$

1.3.

$$a = 5$$

$$b = 10$$

$$(a^b)^0 = (5^{-10})^0 = \boxed{1}$$

1.4

$$\frac{\sqrt{4x}}{\sqrt{x}} = \frac{4x^{\frac{1}{2}}}{x^{\frac{1}{2}}} = \frac{4^{\frac{1}{2}} \cdot x^{\frac{1}{2}}}{x^{\frac{1}{2}}} = 4^{\frac{1}{2}} = \boxed{2}$$

1.5

$$x^2 + (x+1)^2 = (x+2)^2$$

$$x^2 + x^2 + 2x + 1 = x^2 + 4x + 4$$

$$2x^2 + 2x + 1 = x^2 + 4x + 4$$

$$x^2 + 2x + 1 = 4x + 4$$

$$x^2 = 2x + 3$$

$$0 = 2x + 3 - x^2$$

$$-x^2 + 2x + 3 = 0$$

$$\rightarrow \frac{-2 \pm \sqrt{(2)^2 - 4 \cdot (-1) \cdot 3}}{2 \cdot (-1)} =$$

$$= \frac{-2 \pm \sqrt{4+12}}{-2} = \frac{-2 \pm \sqrt{16}}{-2}$$

$$= \frac{-2 \pm 4}{-2} \quad \begin{matrix} x_1 = -1 \\ x_2 = 3 \end{matrix}$$

1.6

$2^x > 1024$

$2^x > 2^{10}$

$\boxed{x > 10}$

2.

2.1

$y = a + bx$

$f_{100} = 32 + y \cdot 100 = 212$

$y100 = 180$

$y = 1,8$

$\boxed{f_{100} = 32 + 1,8y}$

↓

$32 + 1,8C = F$

$1,8C = F - 32$

$C = \frac{F - 32}{1,8}$

$C = 32 + 1,8C$

$-0,8C = 32$

$\boxed{C = -40}$

$F = \frac{F - 32}{1,8}$

$1,8F = F - 32$

$0,8F = -32$

$\boxed{F = -40}$

2.2

$f(x) = 5x + 4$

$f(3) = y$

$f(3) = 15 + 4 \rightarrow \boxed{f(3) = 19}$

2.3

$$\frac{+4 \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1} = \frac{4 \pm \sqrt{16 - 12}}{2} = \frac{4 \pm 2}{2} \begin{matrix} \nearrow x_1 = 1 \\ \searrow x_2 = 3 \end{matrix}$$

2.4

$$10 \cdot (1.02)^{90} = \boxed{59.43 \text{ HUF}}$$

2.5

$$e^{\ln 5} = \boxed{5}$$

3.

3.1

$$\sum_{i=1}^{\infty} \frac{12}{6^i} = \frac{12 \cdot \left(\frac{1}{6}\right)^i}{1 - \frac{1}{6}} = \frac{\frac{12}{6}}{\frac{5}{6}} = \boxed{\frac{12}{5}}$$

3.2

$$\lim_{x \to 1} = \frac{6^{1-x}}{x} = \frac{6^{1-1}}{1} = \frac{6^0}{1} = \frac{1}{1} = \boxed{1}$$

3.3

$$f(x) = x^5 - 8$$

$$x = -3$$

$$f'(x) = 5 \cdot x^4$$

$$f'(-3) = 5 \cdot (-3)^4$$

$$f'(-3) = 5 \cdot 81$$

$$f'(-3) = 405$$

3.4

$$\frac{x^3+2x-1}{x-2} = \frac{3x^2+2\cdot(x-2)-x^3+2x-1\cdot(1)}{(x-2)^2} =$$

$$= \frac{3x^3-6x^2+2x-4-x^3+2x+1}{x^2-4x+4} =$$

$$= \frac{3x^3-6x^2+2x-4-x^3-2x-1}{x^2-4x-4}$$

$$= \frac{2x^3-6x^2-3}{x^2-4x-4}$$

3.5

$f(x) = 4x^4 + 4x^2$

$f'(x) = 16x^3 + 8x$

$f''(x) = 48x^2 + 8$

3.6

$$f(x) = \frac{\ln x}{e^x}$$

$$f'(x) = \frac{\frac{1}{x}\cdot e^x - \ln x \cdot e^x}{(e^x)^2} =$$

$$= \frac{e^x\cdot(\frac{1}{x}-\ln x)}{(e^x)^2} = \frac{\frac{\ln x}{x}}{e^x}$$

3.7

$f(x) = 3x^2 - 5x + 2 \longrightarrow f(x) = 0$

$f'(x) = 6x - 5 = 0$

$3x^2 - 5x + 2 = 0$

$6x = 5$

$x = \frac{5}{6}$

$$\frac{5 \pm \sqrt{(-5)^2 - 4\cdot 3\cdot 2}}{6} =$$

$$= \frac{5 \pm \sqrt{25-24}}{6} = \frac{5 \pm 1}{6} \rightarrow x_1 = 1$$

$$x_2 = \frac{4}{6}$$

$f''(x) = 6x - 5$

$f''(x) = 6 > 0$ convex $\Rightarrow$ local minimum

3.8

$f(x,y) = x^2 + y^3 \qquad f(2,3)$

$f(x,y) = 2^2 + 3^3$

$f(x,y) = 4 + 9 = 13$

$x = 4$

$y = 9$

3.9

$f(x,y) = \ln(x - y)$

$x - y > 0$

$x > y$

$x \in \mathbb{R}^+$

3.10

$f(x) = x^5 + x \cdot y^3$

$f'(x) = 5x^4 + y^3$

3.11

$f(x,y) = x^2 \cdot y^2 + 10$

$f'_x(x,y) = 2xy^2$

$f''_x(x,y) = 2y^2 = 0 \rightarrow y = 0$

$f'_y(x,y) = 2yx^2$

$f''_y(x,y) = 2x^2 = 0 \rightarrow x = 0$

3.12 $x^2 y^2$ max/min.

$x+y=10 \rightarrow x+y-10=0$

Лагранжиан:

$$L = f(x,y) - \lambda \cdot g(x,y)$$

$$L = x^2 \cdot y^2 - \lambda \cdot (x+y-10)$$

① $\frac{\partial L}{\partial x} = y^2 \cdot 2x - \lambda = 0$

② $\frac{\partial L}{\partial y} = 2y \cdot x^2 - \lambda = 0$

$y^2 \cdot 2x = x^2 \cdot 2y$

$\frac{y^2 \cdot 2x}{2y} = x^2$

$\frac{y^2}{2y} = \frac{x^2}{2x}$

$\frac{y}{2} = \frac{x}{2}$

$y = x$

③ $\frac{\partial L}{\partial \lambda} = x+y-10=0$

$x + x - 10 = 0$

$2x = 10$

$x = 5$

$y = 5$

4.

4.1

$$\begin{array}{c|ccc} & 1 & 1 & 7 \\ & 2 & 8 & 2 \\ \hline 2\ 6 & 54 & 50 & 26 \\ 5\ 1 & 7 & 13 & 37 \\ 1\ 9 & 19 & 73 & 25 \end{array} = A \cdot B$$

4.2

$$\begin{array}{c|cc} & 2 & 2 \\ & 4 & 6 \\ & 1 & 3 \\ \hline 1\ 9\ 1 & 39 & 59 \\ 2\ 1\ 2 & 10 & 16 \end{array} = B \cdot A$$

4.3

$$A^T = \begin{bmatrix} 7.1 & 2 & 4 \\ 9.1 & 7.8 & 4.44 \\ 4.7 & 1.1 & 0 \end{bmatrix}$$

4.4

det. A = a · e − b · c

dt A = 1.8 − 2.9 = 8 − 18 = **−10**

5.

5.1

dice 2x

( ) ( )

$\Omega = \{(1,1), (1,2), (1,3) \dots (6,6)\}$

\# = **36**

5.2

Don't use

5.2

$P[A] = 0{,}01$

$P[-; \bar{A}] = 0{,}995$

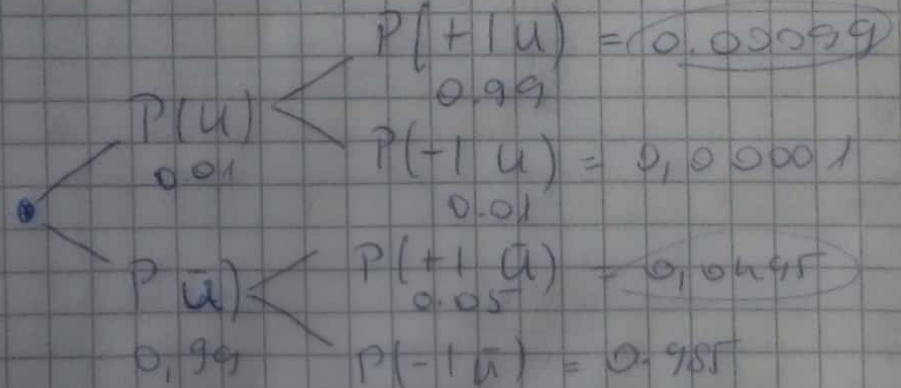
$P[+; \bar{A}] = 1 - P[-; \bar{A}] = 0{,}005$

$P[+|A] = 0.99$

$$P[A|+] = \frac{P[+|A] \cdot P[A]}{P[+|A] \cdot P[A] + P[+|\bar{A}] \cdot P[\bar{A}]} =$$

$$= \frac{0.99 \cdot 0.01}{(0.99 \cdot 0.01) + (0.05 \cdot 0.995)} = 0.167 \approx \boxed{16.7\%}$$

5.3



$$P = \frac{0{,}01 \cdot 0{,}99}{0{,}00099 + 0.04995} = \boxed{0{,}196 \approx 19{,}6\%}$$