The smallest bimolecular mass-action CRNs admitting Andronov-Hopf bifurcation

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This Matematica Notebook contains the focal value computations in the paper which has the same title as this document.

Define the Modules that are used later on.

```
In[1]:= GetRHS[L_, n_, m_, j_] := Module[{f, indent, ntw, i, src, tgt, ksrc, ktgt},
    f = 0;
    If[j ≤ 9, indent = " ", indent = ""];
    ntw = StringJoin[indent, "(", ToString[j], ")    "];
    For[i = 1, i ≤ m, i++, {
      src = L[[2 n (i - 1) + 1 ;; 2 n (i - 1) + n]];
      tgt = L[[2 n (i - 1) + n + 1 ;; 2 n (i - 1) + 2 n]];
      f = f + κi x^src[[1]] y^src[[2]] z^src[[3]] (tgt - src); (* works only for n=3 *)
      ksrc = Position[cplxs, src][[1]][[1]];
      ktgt = Position[cplxs, tgt][[1]][[1]];
      ntw = StringJoin[ntw, srcstring[[ksrc]], " ⟶ ", tgtstring[[ktgt]], "    "];
     }];
    {f, ntw}
   ];

GetEquilibrium[f_] := Module[{},
    Normal[Simplify[Solve[f == 0 && κpositive && x > 0 && y > 0 && z > 0, {x, y, z}][[1]]]]
   ];

RescaleODE[f_, equilibrium_] :=
   Module[{X, Y, Z, fscaled, fκ1, fnew, αβγ, A, i, factor, κ2αβγ},
    X = x /. equilibrium;
    Y = y /. equilibrium;
    Z = z /. equilibrium;
    fscaled = Simplify[
```

```
      DiagonalMatrix[{1 / X, 1 / Y, 1 / Z}].(f /. {x → u X, y → v Y, z → w Z}), κpositive];
    fnew = fscaled /. {κ1 → 1, κ2 → 1, κ3 → 1, κ4 → 1};
    αβγ = Simplify[fscaled / fnew, κpositive];
    (* now we hide in α,β,γ all the common factors *)
    A = D[fnew, {{u, v, w}}] /. {u → 1, v → 1, w → 1};
    For[i = 1, i ≤ Length[A], i++, {
      factor = LCM[Denominator[A[[i]] / Max[Abs[A[[i]]]]] /. List → Sequence] / Max[Abs[A[[i]]]];
      fnew[[i]] = fnew[[i]] factor;
      αβγ[[i]] = αβγ[[i]] / factor;
     }];
    fnew = DiagonalMatrix[{α, β, γ}].fnew;
    κ2αβγ = {α → αβγ[[1]], β → αβγ[[2]], γ → αβγ[[3]]};
    {fnew, κ2αβγ}
   ];

Idx[set_, n_] := Module[{seq}, seq = (Table[Count[set, i], {i, n}] /. List → Sequence);
    seq];

GetDerivatives[f_] := Module[{equil, derivatives, order, deriv, i, j, k, A, B, CC, DD, EE},
    n = Length[f];
    equil = {u → 1, v → 1, w → 1};
    A = D[f, {{u, v, w}}] /. equil;
    derivatives = {};
    order = 2;
    For[i = 0, i ≤ order, i++, For[j = 0, j ≤ order - i, j++, For[k = 0, k ≤ order - i - j, k++,
       deriv = Simplify[D[f, {u, i}, {v, j}, {w, k}] /. equil];
       derivatives =
        Join[derivatives, Simplify[{F_{i,j,k} → deriv[[1]], G_{i,j,k} → deriv[[2]], H_{i,j,k} → deriv[[3]]}]];
      ]]];
    B[x_, y_] :=
     Sum[{F_{Idx[{k,l},n]}, G_{Idx[{k,l},n]}, H_{Idx[{k,l},n]}} x[[k]] × y[[l]] /. derivatives, {k, n}, {l, n}];
    (*CC[x_,y_,z_]:=Sum[{F_{Idx[{k,l,m},n]},G_{Idx[{k,l,m},n]},H_{Idx[{k,l,m},n]}}x[[k]]y[[l]]z[[m]]/.derivatives,
       {k,n},{l,n},{m,n}];*)
    (* because of the bimolecularity, all derivatives of order 3 and higher vanish *)
    CC[x_, y_, z_] := {0, 0, 0};
    DD[x_, y_, z_, s_] := {0, 0, 0};
    EE[x_, y_, z_, s_, t_] := {0, 0, 0};
    (* the reason for the double symbols CC,
    DD, EE is that C, D, E are protected in Mathematica *)
    {A, B, CC, DD, EE}
   ];
```

```
GetHopf[A_] := Module[{a, A2A1minusA0, RouthHurwitz, Hopf, HopfCondition},
   {a0, a1, a2, a3} =
    Simplify[CoefficientList[Collect[-CharacteristicPolynomial[A, λ], λ], λ]];
   A2A1minusA0 = Simplify[a1 a2 - a0];
   RouthHurwitz = Simplify[Solve[A2A1minusA0 == 0 && α > 0 && β > 0 && γ > 0][[1]]];
   Hopf = Normal[RouthHurwitz];
   HopfCondition = FullSimplify[RouthHurwitz[[1]][[2]][[2]]];
   varremain = Complement[{α, β, γ}, {Hopf[[1]][[1]]}];
   {Hopf, HopfCondition, varremain}
  ];

GetEigvectors[A_, om_] := Module[{n, mtx, pconj, q, qconj, normalize},
   n = Length[A];
   mtx = A - om ⅈ IdentityMatrix[n];
   q = NullSpace[mtx[[Range[1, n - 1]]]][[1]];
   mtx = A^T - om ⅈ IdentityMatrix[n];
   pconj = NullSpace[mtx[[Range[1, n - 1]]]][[1]];
   normalize = FullSimplify[pconj.q, αβγωpositive];
   pconj = pconj / normalize;
   qconj = FullSimplify[q*, αβγωpositive];
   {pconj, q, qconj}
  ];

GetL1[A_, B_, CC_] :=
  Module[{n, pconj, q, qconj, v1, v2, v3, c1, numer, denom, a, b, c, d, L1αβγω},
   n = Length[A];
   {pconj, q, qconj} = GetEigvectors[A, ω];
   v1 = CC[q, q, qconj];
   v2 = Simplify[B[q, Inverse[-A].B[q, qconj]]];
   v3 = Simplify[B[qconj, Inverse[2 I ω IdentityMatrix[n] - A].B[q, q]]];
   c1 = Simplify[pconj.(1/2 v1 + v2 + 1/2 v3)];
   (* we take the real part in a bit complicated way,
   it seems faster than the standard solution would be *)
   numer = Numerator[c1];
   denom = Denominator[c1];
   a = Simplify[ComplexExpand[Re[numer]]];
   b = Simplify[ComplexExpand[Im[numer]]];
   c = Simplify[ComplexExpand[Re[denom]]];
   d = Simplify[ComplexExpand[Im[denom]]];
   L1αβγω = Simplify[a c + b d];
   L1αβγω
  ];

AnalyseL1[A_, B_, CC_] :=
```

```
  Module[{Hopf, HopfCondition, varremain, L1αβγω, L1, L1neg, L1zer, L1pos, sign},
   {Hopf, HopfCondition, varremain} = GetHopf[A];
   L1αβγω = GetL1[A, B, CC];
   L1 = Simplify[L1αβγω /. {ω → Sqrt[Det[A]/Tr[A]]} /. Hopf];
   L1neg = Reduce[L1 < 0 && HopfCondition];
   L1zer = Reduce[L1 == 0 && HopfCondition];
   L1pos = Reduce[L1 > 0 && HopfCondition];
   sign = {Length[FindInstance[L1neg, varremain]],
     Length[FindInstance[L1zer, varremain]], Length[FindInstance[L1pos, varremain]]};
   {L1neg, L1zer, L1pos, Hopf, HopfCondition, varremain, sign, L1}
  ];

GetL2[A_, B_, CC_, DD_, EE_] :=
  Module[{n, Id, omega, invA, inv2, inv3, pconj, q, qconj, h, prec, c, invbig},
   n = Length[A];
   Id = IdentityMatrix[n];
   omega = Sqrt[Det[A]/Tr[A]];
   invA = Inverse[A];
   inv2 = Simplify[Inverse[2 omega I Id - A]];
   inv3 = Simplify[Inverse[3 omega I Id - A]];
   {pconj, q, qconj} = GetEigvectors[A, omega];
   q = q /. {ω → omega};
   pconj = pconj /. {ω → omega};
   h2,0 = Simplify[inv2.B[q, q]];
   h1,1 = Simplify[-invA.B[q, q*]];
   prec = Simplify[CC[q, q, q*] + 2 B[q, h1,1] + B[q*, h2,0]];
   c1 = Simplify[1 / 2 (pconj.prec)];
   invbig = Simplify[Inverse[Join[Join[omega I Id - A, {q}ᵀ, 2], {Join[pconj, {0}]}]]];
   h2,1 = Simplify[invbig.Join[Simplify[prec - 2 c1 q], {0}]][[1 ;; n]];
   h3,0 = Simplify[inv3.(CC[q, q, q] + 3 B[q, h2,0])];
   h3,1 = Simplify[inv2.(DD[q, q, q, q*] + 3 CC[q, q, h1,1] +
        3 CC[q, q*, h2,0] + 3 B[h2,0, h1,1] + B[q*, h3,0] + 3 B[q, h2,1] - 6 c1 h2,0)];
   h2,2 = Simplify[-invA.(DD[q, q, q*, q*] + 4 CC[q, q*, h1,1] + CC[q*, q*, h2,0] +
         CC[q, q, h2,0*] + 2 B[h1,1, h1,1] + 2 B[q, h2,1*] + 2 B[q*, h2,1] + B[h2,0*, h2,0])];
   c2 = Simplify[1 / 12 (pconj.(EE[q, q, q, q*, q*] + DD[q, q, q, h2,0*] + 3 DD[q, q*, q*, h2,0] +
          6 DD[q, q, q*, h1,1] + CC[q*, q*, h3,0] + 3 CC[q, q, h2,1*] + 6 CC[q, q*, h2,1] +
          3 CC[q, h2,0*, h2,0] + 6 CC[q, h1,1, h1,1] + 6 CC[q*, h2,0, h1,1] + 2 B[q*, h3,1] +
          3 B[q, h2,2] + B[h2,0*, h3,0] + 3 B[h2,1*, h2,0] + 6 B[h1,1, h2,1]))];
   ComplexExpand[Re[c2]]
  ];
```

```
PrintInfo[ntw_, fnew_, Hopf_, HopfCondition_, sign_] := Module[{text},
   Print["\n", Style[ntw, Blue]];
   Print[StringRepeat[" ", spaces], "ODE after rescaling and reparametrising: ",
    MatrixForm[{u̇, v̇, ẇ}], "=", MatrixForm[fnew]];
   Print[StringRepeat[" ", spaces], "pair of imaginary eigenvalues: ",
    Hopf[[1]][[1]], " = ", Hopf[[1]][[2]], " and ", HopfCondition];
   If[sign ≠ {1, 1, 1}, {
     text = Switch[sign, {1, 0, 0}, negstr, {0, 0, 1},
       posstr, {0, 1, 1}, Row[{posstr, " or ", zerstr}], {0, 1, 0}, zerstr];
     Print[StringRepeat[" ", spaces], "L₁ is ", text];
    }];
  ];

PrintSignL1[L1neg_, L1zer_, L1pos_] := Module[{},
   Print[StringRepeat[" ", spaces], "L₁ is ", negstr, ": "    , L1neg];
   Print[StringRepeat[" ", spaces], "L₁ is ", zerstr, "    : ", L1zer];
   Print[StringRepeat[" ", spaces], "L₁ is ", posstr, ": "    , L1pos];
  ];

ComputePrintL2[L1zer_, varremain_, Hopf_, fnew_, isnumerical_, κ2aβγ_] :=
  Module[{varremainsubst, varremainsubsts, αβγsubst, L1vanishαβγ, A, B, CC, DD, EE, L2},
   varremainsubsts = Solve[L1zer && varremain[[1]] == 1, varremain];
   For[i = 1, i ≤ Length[varremainsubsts], i++, {
     varremainsubst = varremainsubsts[[i]];
     αβγsubst = Join[varremainsubst, Hopf /. varremainsubst];
     If[isnumerical,
      {αβγsubst = N[αβγsubst, 20]},
      {L1vanishαβγ = Simplify[Reduce[L1zer && Hopf[[1]][[1]] == (Hopf[[1]][[2]] /.
              Normal[Solve[L1zer, varremain[[1]]][[1]]]) && αβγpositive], αβγpositive]}];
     {A, B, CC, DD, EE} = GetDerivatives[fnew /. αβγsubst];
     L2 = GetL2[A, B, CC, DD, EE];
     str = Switch[Sign[L2], -1, negstr, 1, posstr];
     If[isnumerical,
      Print[StringRepeat[" ", spaces],
       "L₁ vanishes for approximately ", N[αβγsubst, 5], ", and then L₂ is ", str],
      Print[StringRepeat[" ", spaces], "L₁ vanishes for ",
       L1vanishαβγ, " (or equivalently, ",
       Simplify[Reduce[(L1vanishαβγ /. κ2αβγ) && κpositive], κpositive],
       "), and then L₂ is ", str]
     ];
    }];
  ];

HopfTransversality[A_] := Module[{Hopf, HopfCondition,
    varremain, pconj, q, qconj, var, DA, pDAq, numer, denom, a, b, c, d, Dμ},
   {Hopf, HopfCondition, varremain} = GetHopf[A];
```

```
    var = Hopf[[1]][[1]];
    {pconj, q, qconj} = GetEigvectors[A, ω];
    DA = D[A, {var}];
    pDAq = Together[pconj.DA.q];
    numer = Numerator[pDAq];
    denom = Denominator[pDAq];
    a = Simplify[ComplexExpand[Re[numer]]];
    b = Simplify[ComplexExpand[Im[numer]]];
    c = Simplify[ComplexExpand[Re[denom]]];
    d = Simplify[ComplexExpand[Im[denom]]];
    Dμ = Simplify[a c + b d /. {ω → Sqrt[Det[A]/Tr[A]]} /. Hopf];
    If[Length[FindInstance[Dμ == 0 && HopfCondition, varremain]] == 0,
     Print[StringRepeat[" ", spaces], "transversality in ", var, ": ", Style[✓, 16]]];
   ];

cplxs =     {{0, 0, 0}, {1, 0, 0}, {0, 1, 0}, {0, 0, 1},
    {2, 0, 0}, {1, 1, 0}, {0, 2, 0}, {0, 1, 1}, {0, 0, 2}, {1, 0, 1}};
srcstring = {"  0", "  X", "  Y", "  Z", " 2X", "X+Y", " 2Y", "Y+Z", " 2Z", "X+Z"};
tgtstring = {"0  ", "X  ", "Y  ", "Z  ", "2X ", "X+Y", "2Y ", "Y+Z", "2Z ", "X+Z"};
κpositive = κ1 > 0 && κ2 > 0 && κ3 > 0 && κ4 > 0;
αβγωpositive = α > 0 && β > 0 && γ > 0 && ω > 0;
αβγpositive = α > 0 && β > 0 && γ > 0;
inputcsv = "C:/bboros/Dropbox/murad/s3r4/Hopf_86_plus_1.csv";
M = Import[inputcsv];
n = 3;
m = 4;
negstr = Style["negative", RGBColor[0, 0.5, 0]];
zerstr = Style["zero", RGBColor[0, 0, 0.8]];
posstr = Style["positive", RGBColor[1, 0, 0]];
spaces = 12;
```

Some checks

Here we check that whenever we have a pair of purely imaginary eigenvalues, the third eigenvalue is a negative real. This is done by verifying that the determinant of the Jacobian matrix is negative. Furthermore, we check the transversality of the Andronov-Hopf bifurcation (in one of α, β, γ, the new parameters).

In[29]:=
```
t0 = SessionTime[];
For[j = 1, j ≤ Length[M], j++, {
   L = M[[j]][[1 ;; 2 n m]];

   {f, ntw} = GetRHS[L, n, m, j];
   equilibrium = GetEquilibrium[f];
   {fnew, ϰ2αβγ} = RescaleODE[f, equilibrium];
   {A, B, CC, DD, EE} = GetDerivatives[fnew];
   Print[Style[ntw, Blue]];
   Print[StringRepeat[" ", spaces], "det 𝐴 = ", Det[A]];
   HopfTransversality[A];
  }];
Print["\ntotal elapsed: ",
  Style[NumberForm[SessionTime[] - t0, {5, 2}], Magenta], " seconds"];
```

```
(1)     0 ⟶ X       X ⟶ Y     Y+Z ⟶ 2Z    X+Z ⟶ 0
          det 𝐴 =  -2 α β γ
          transversality in γ: ✓
(2)     0 ⟶ X       X ⟶ 2Y    Y+Z ⟶ 2Z    X+Z ⟶ 0
          det 𝐴 =  -3 α β γ
          transversality in γ: ✓
(3)     0 ⟶ X     X+Y ⟶ 2Y      Y ⟶ Z     X+Z ⟶ 0
          det 𝐴 =  -2 α β γ
          transversality in β: ✓
(4)     0 ⟶ X     X+Y ⟶ 2Y      Y ⟶ X+Z   X+Z ⟶ 0
          det 𝐴 =  -α β γ
          transversality in β: ✓
(5)     Z ⟶ X+Z   X+Y ⟶ 2Y    Y+Z ⟶ 0       0 ⟶ Z
          det 𝐴 =  -α β γ
          transversality in β: ✓
(6)     0 ⟶ X     X+Y ⟶ 2Y      Y ⟶ 2Z    X+Z ⟶ 0
          det 𝐴 =  -3 α β γ
          transversality in β: ✓
(7)     0 ⟶ X+Y   X+Z ⟶ Y+Z   Y+Z ⟶ 2Z      Z ⟶ 0
          det 𝐴 =  -2 α β γ
          transversality in γ: ✓
(8)     0 ⟶ X+Y   X+Z ⟶ Y     Y+Z ⟶ 2Z      Z ⟶ 0
          det 𝐴 =  -2 α β γ
          transversality in γ: ✓
```

(9) $0 \longrightarrow X+Y$ $X+Z \longrightarrow 2Y$ $Y+Z \longrightarrow 2Z$ $Z \longrightarrow 0$

det $A = -6\,\alpha\,\beta\,\gamma$

transversality in γ: ✓

(10) $0 \longrightarrow X$ $X+Z \longrightarrow Y+Z$ $Y+Z \longrightarrow 2Z$ $Z \longrightarrow 0$

det $A = -\alpha\,\beta\,\gamma$

transversality in γ: ✓

(11) $0 \longrightarrow X$ $X+Z \longrightarrow 2Y$ $Y+Z \longrightarrow 2Z$ $Z \longrightarrow 0$

det $A = -\alpha\,\beta\,\gamma$

transversality in γ: ✓

(12) $0 \longrightarrow X+Z$ $X+Y \longrightarrow 2Y$ $Y \longrightarrow Z$ $Y+Z \longrightarrow X$

det $A = -6\,\alpha\,\beta\,\gamma$

transversality in β: ✓

(13) $0 \longrightarrow X+Z$ $X+Y \longrightarrow 2Y$ $Y \longrightarrow 2Z$ $Y+Z \longrightarrow X$

det $A = -12\,\alpha\,\beta\,\gamma$

transversality in β: ✓

(14) $X \longrightarrow 2X$ $X+Z \longrightarrow Y+Z$ $Y \longrightarrow 2Z$ $2Z \longrightarrow 0$

det $A = -\alpha\,\beta\,\gamma$

transversality in α: ✓

(15) $X \longrightarrow 2X$ $X+Z \longrightarrow Y+Z$ $Y \longrightarrow Z$ $2Z \longrightarrow 0$

det $A = -\alpha\,\beta\,\gamma$

transversality in α: ✓

(16) $X \longrightarrow 2X$ $X+Z \longrightarrow Y+Z$ $Y \longrightarrow Z$ $2Z \longrightarrow Y$

det $A = -\alpha\,\beta\,\gamma$

transversality in α: ✓

(17) $X \longrightarrow 2X$ $X+Z \longrightarrow Y+Z$ $X+Y \longrightarrow Z$ $Z \longrightarrow 0$

det $A = -2\,\alpha\,\beta\,\gamma$

transversality in β: ✓

(18) $X \longrightarrow 2X$ $X+Z \longrightarrow Y+Z$ $X+Y \longrightarrow 2Z$ $Z \longrightarrow 0$

det $A = -2\,\alpha\,\beta\,\gamma$

transversality in β: ✓

(19) $X \longrightarrow 2X$ $X+Z \longrightarrow Y$ $X+Y \longrightarrow 2Z$ $Z \longrightarrow 0$

det $A = -2\,\alpha\,\beta\,\gamma$

transversality in β: ✓

(20) $X \longrightarrow 2X$ $X+Z \longrightarrow 2Y$ $X+Y \longrightarrow 2Z$ $Z \longrightarrow 0$

det $A = -9\,\alpha\,\beta\,\gamma$

transversality in β: ✓

(21) $X \longrightarrow 2X \quad X+Z \longrightarrow 2Y \quad X+Y \longrightarrow Z \quad Z \longrightarrow 0$

det $A = -3\,\alpha\,\beta\,\gamma$

transversality in β: ✓

(22) $X \longrightarrow 2X \quad X+Z \longrightarrow 2Y \quad Y \longrightarrow Z \quad 2Z \longrightarrow Y$

det $A = -3\,\alpha\,\beta\,\gamma$

transversality in α: ✓

(23) $X \longrightarrow 2X \quad X+Z \longrightarrow 2Y \quad Y \longrightarrow Z \quad 2Z \longrightarrow 0$

det $A = -\alpha\,\beta\,\gamma$

transversality in α: ✓

(24) $X \longrightarrow 2X \quad X+Z \longrightarrow 2Y \quad Y \longrightarrow 2Z \quad 2Z \longrightarrow 0$

det $A = -3\,\alpha\,\beta\,\gamma$

transversality in α: ✓

(25) $X \longrightarrow 2X \quad X+Z \longrightarrow Y \quad Y \longrightarrow 2Z \quad 2Z \longrightarrow 0$

det $A = -\alpha\,\beta\,\gamma$

transversality in α: ✓

(26) $X \longrightarrow 2X \quad 2X \longrightarrow 2Y \quad Y \longrightarrow 2Z \quad X+Z \longrightarrow 0$

det $A = -3\,\alpha\,\beta\,\gamma$

transversality in β: ✓

(27) $Z \longrightarrow 2X \quad X+Y \longrightarrow 2Y \quad Y \longrightarrow 0 \quad 2X \longrightarrow 2Z$

det $A = -\alpha\,\beta\,\gamma$

transversality in β: ✓

(28) $Y \longrightarrow 2X \quad X+Z \longrightarrow Y+Z \quad 2Y \longrightarrow Z \quad Z \longrightarrow 0$

det $A = -\alpha\,\beta\,\gamma$

transversality in α: ✓

(29) $Y \longrightarrow 2X \quad X+Z \longrightarrow Y+Z \quad 2Y \longrightarrow Z \quad Z \longrightarrow X$

det $A = -3\,\alpha\,\beta\,\gamma$

transversality in α: ✓

(30) $Y \longrightarrow 2X \quad X+Z \longrightarrow Y+Z \quad 2Y \longrightarrow Z \quad Z \longrightarrow Y$

det $A = -\alpha\,\beta\,\gamma$

transversality in α: ✓

(31) $Y \longrightarrow 2X \quad X+Z \longrightarrow Y+Z \quad 2Y \longrightarrow X+Z \quad Z \longrightarrow 0$

det $A = -3\,\alpha\,\beta\,\gamma$

transversality in α: ✓

(32) $Y \longrightarrow 2X \quad X+Z \longrightarrow Y+Z \quad 2Y \longrightarrow 2Z \quad Z \longrightarrow 0$

det $A = -\alpha\,\beta\,\gamma$

transversality in α: ✓

(33) Y ⟶ 2X   X+Z ⟶ 2Y   2Y ⟶ 2Z   Z ⟶ 0

det $A = -3\,\alpha\,\beta\,\gamma$

transversality in α: ✓

(34) Z ⟶ 2X   X+Y ⟶ 2Y   Y ⟶ Z   2Z ⟶ Y

det $A = -3\,\alpha\,\beta\,\gamma$

transversality in α: ✓

(35) 0 ⟶ X   2X ⟶ Y   Y+Z ⟶ 2Z   X+Z ⟶ 0

det $A = -6\,\alpha\,\beta\,\gamma$

transversality in γ: ✓

(36) 0 ⟶ X   2X ⟶ 2Y   Y+Z ⟶ 2Z   X+Z ⟶ 0

det $A = -4\,\alpha\,\beta\,\gamma$

transversality in γ: ✓

(37) 0 ⟶ X   X+Y ⟶ 2Y   Y+Z ⟶ 2Z   X+Z ⟶ X

det $A = -2\,\alpha\,\beta\,\gamma$

transversality in γ: ✓

(38) 0 ⟶ X+Y   X+Y ⟶ 2Y   Y+Z ⟶ 2Z   X+Z ⟶ X

det $A = -4\,\alpha\,\beta\,\gamma$

transversality in γ: ✓

(39) 0 ⟶ X+Z   X+Y ⟶ 2Y   Y+Z ⟶ Z   X+Z ⟶ X

det $A = -2\,\alpha\,\beta\,\gamma$

transversality in β: ✓

(40) X ⟶ 2X   X+Z ⟶ Y+Z   2Y ⟶ Z   2Z ⟶ 0

det $A = -\alpha\,\beta\,\gamma$

transversality in α: ✓

(41) X ⟶ 2X   X+Z ⟶ Y+Z   2Y ⟶ Z   2Z ⟶ Y

det $A = -3\,\alpha\,\beta\,\gamma$

transversality in α: ✓

(42) X ⟶ 2X   X+Z ⟶ Y+Z   2Y ⟶ Z   2Z ⟶ 2Y

det $A = -\alpha\,\beta\,\gamma$

transversality in α: ✓

(43) X ⟶ 2X   X+Z ⟶ Y+Z   2Y ⟶ Z   Y+Z ⟶ 0

det $A = -3\,\alpha\,\beta\,\gamma$

transversality in α: ✓

(44) $X \longrightarrow 2X \quad X+Z \longrightarrow Y+Z \quad 2Y \longrightarrow Z \quad Y+Z \longrightarrow Y$

det $A = -\alpha\,\beta\,\gamma$

transversality in α: ✓

(45) $X \longrightarrow 2X \quad X+Z \longrightarrow Y+Z \quad 2Y \longrightarrow Z \quad Y+Z \longrightarrow 2Y$

det $A = -\alpha\,\beta\,\gamma$

transversality in α: ✓

(46) $X \longrightarrow 2X \quad X+Z \longrightarrow Y+Z \quad 2Y \longrightarrow 2Z \quad 2Z \longrightarrow 0$

det $A = -\alpha\,\beta\,\gamma$

transversality in α: ✓

(47) $X \longrightarrow 2X \quad X+Z \longrightarrow Y+Z \quad 2Y \longrightarrow 2Z \quad 2Z \longrightarrow Y$

det $A = -\alpha\,\beta\,\gamma$

transversality in α: ✓

(48) $X \longrightarrow 2X \quad X+Z \longrightarrow Y+Z \quad 2Y \longrightarrow 2Z \quad Y+Z \longrightarrow 0$

det $A = -2\,\alpha\,\beta\,\gamma$

transversality in α: ✓

(49) $X \longrightarrow 2X \quad X+Z \longrightarrow Y+Z \quad 2Y \longrightarrow 2Z \quad Y+Z \longrightarrow Y$

det $A = -\alpha\,\beta\,\gamma$

transversality in α: ✓

(50) $X \longrightarrow 2X \quad X+Z \longrightarrow Y+Z \quad Y+Z \longrightarrow 2Z \quad 2Z \longrightarrow 0$

det $A = -\alpha\,\beta\,\gamma$

transversality in α: ✓

(51) $X \longrightarrow 2X \quad X+Z \longrightarrow Y+Z \quad Y+Z \longrightarrow 2Z \quad 2Z \longrightarrow Y$

det $A = -\alpha\,\beta\,\gamma$

transversality in α: ✓

(52) $X \longrightarrow 2X \quad 2X \longrightarrow Y \quad Y+Z \longrightarrow 2Z \quad X+Z \longrightarrow 0$

det $A = -3\,\alpha\,\beta\,\gamma$

transversality in γ: ✓

(53) $X \longrightarrow 2X \quad 2X \longrightarrow 2Y \quad Y+Z \longrightarrow 2Z \quad X+Z \longrightarrow 0$

det $A = -2\,\alpha\,\beta\,\gamma$

transversality in γ: ✓

(54) $X \longrightarrow 2X \quad X+Y \longrightarrow 2Y \quad 2Y \longrightarrow Z \quad X+Z \longrightarrow Y$

det $A = -2\,\alpha\,\beta\,\gamma$

transversality in α: ✓

(55) $X \longrightarrow 2X \quad X+Z \longrightarrow Y \quad X+Y \longrightarrow 2Z \quad 2Z \longrightarrow 0$

det $A = -2\,\alpha\,\beta\,\gamma$

transversality in α: ✓

(56) X ⟶ 2X   X+Z ⟶ Y   X+Y ⟶ 2Z   Y+Z ⟶ Y

det $A = -2\,\alpha\,\beta\,\gamma$

transversality in α: ✓

(57) X ⟶ 2X   X+Z ⟶ 2Y   2Y ⟶ 2Z   2Z ⟶ 0

det $A = -2\,\alpha\,\beta\,\gamma$

transversality in α: ✓

(58) X ⟶ 2X   X+Z ⟶ 2Y   2Y ⟶ 2Z   2Z ⟶ Y

det $A = -3\,\alpha\,\beta\,\gamma$

transversality in α: ✓

(59) X ⟶ 2X   X+Z ⟶ 2Y   2Y ⟶ 2Z   Y+Z ⟶ 0

det $A = -6\,\alpha\,\beta\,\gamma$

transversality in α: ✓

(60) X ⟶ 2X   X+Z ⟶ 2Y   2Y ⟶ 2Z   Y+Z ⟶ Y

det $A = -\alpha\,\beta\,\gamma$

transversality in α: ✓

(61) X ⟶ 2X   X+Z ⟶ 2Y   X+Y ⟶ Z   2Z ⟶ 0

det $A = -3\,\alpha\,\beta\,\gamma$

transversality in α: ✓

(62) X ⟶ 2X   X+Z ⟶ 2Y   X+Y ⟶ Z   2Z ⟶ Y

det $A = -12\,\alpha\,\beta\,\gamma$

transversality in α: ✓

(63) X ⟶ 2X   X+Z ⟶ 2Y   X+Y ⟶ Z   Y+Z ⟶ Y

det $A = -3\,\alpha\,\beta\,\gamma$

transversality in α: ✓

(64) X ⟶ 2X   X+Z ⟶ 2Y   X+Y ⟶ 2Z   2Z ⟶ 0

det $A = -9\,\alpha\,\beta\,\gamma$

transversality in α: ✓

(65) X ⟶ 2X   X+Z ⟶ 2Y   X+Y ⟶ 2Z   Y+Z ⟶ Y

det $A = -9\,\alpha\,\beta\,\gamma$

transversality in α: ✓

(66) X ⟶ 2X   X+Z ⟶ 2Y   X+Y ⟶ X+Z   Y+Z ⟶ 0

det $A = -6\,\alpha\,\beta\,\gamma$

transversality in α: ✓

(67) X ⟶ 2X   X+Z ⟶ Y+Z   X+Y ⟶ Z   2Z ⟶ 0

det $A = -2\,\alpha\,\beta\,\gamma$

transversality in α: ✓

(68) X ⟶ 2X X+Z ⟶ Y+Z X+Y ⟶ Z 2Z ⟶ Y

det $A = -3\,\alpha\,\beta\,\gamma$

transversality in α: ✓

(69) X ⟶ 2X X+Z ⟶ Y+Z X+Y ⟶ Z Y+Z ⟶ Y

det $A = -2\,\alpha\,\beta\,\gamma$

transversality in α: ✓

(70) X ⟶ 2X X+Z ⟶ Y+Z X+Y ⟶ 2Z 2Z ⟶ 0

det $A = -2\,\alpha\,\beta\,\gamma$

transversality in α: ✓

(71) X ⟶ 2X X+Z ⟶ Y+Z X+Y ⟶ 2Z Y+Z ⟶ Y

det $A = -2\,\alpha\,\beta\,\gamma$

transversality in α: ✓

(72) X ⟶ 2X X+Z ⟶ 2Y 2Y ⟶ 0 X+Y ⟶ X+Z

det $A = -\alpha\,\beta\,\gamma$

transversality in α: ✓

(73) X ⟶ 2X X+Z ⟶ 2Y Y+Z ⟶ Z X+Y ⟶ X+Z

det $A = -\alpha\,\beta\,\gamma$

transversality in α: ✓

(74) X ⟶ 2X X+Z ⟶ 2Y Y+Z ⟶ Z 2Y ⟶ 2Z

det $A = -\alpha\,\beta\,\gamma$

transversality in α: ✓

(75) X ⟶ 2X X+Z ⟶ 2Y Y+Z ⟶ 2Z 2Z ⟶ 0

det $A = -\alpha\,\beta\,\gamma$

transversality in α: ✓

(76) X ⟶ 2X X+Z ⟶ 2Y Y+Z ⟶ 2Z 2Z ⟶ Y

det $A = -3\,\alpha\,\beta\,\gamma$

transversality in α: ✓

(77) X ⟶ 2X X+Z ⟶ Y+Z 2Y ⟶ X+Z Y+Z ⟶ 0

det $A = -6\,\alpha\,\beta\,\gamma$

transversality in γ: ✓

(78) X ⟶ 2X X+Y ⟶ 2Y 2Y ⟶ Z X+Z ⟶ 0

det $A = -3\,\alpha\,\beta\,\gamma$

transversality in α: ✓

(79) X ⟶ 2X   X+Y ⟶ 2Y   2Y ⟶ 2Z   X+Z ⟶ 0

det $A = -2\,\alpha\,\beta\,\gamma$

transversality in α: ✓

(80) Y ⟶ 2X   2X ⟶ 2Y   Y+Z ⟶ 2Z   X+Z ⟶ 0

det $A = -6\,\alpha\,\beta\,\gamma$

transversality in γ: ✓

(81) Y ⟶ 2X   X+Z ⟶ 2Y   Y+Z ⟶ 2Z   2Z ⟶ 0

det $A = -3\,\alpha\,\beta\,\gamma$

transversality in β: ✓

(82) Y ⟶ X+Y   2X ⟶ Y+Z   Y+Z ⟶ Z   X+Z ⟶ 0

det $A = -3\,\alpha\,\beta\,\gamma$

transversality in γ: ✓

(83) Y ⟶ X+Y   2X ⟶ Y+Z   Y+Z ⟶ 2Z   X+Z ⟶ 0

det $A = -4\,\alpha\,\beta\,\gamma$

transversality in γ: ✓

(84) Y ⟶ X+Y   2X ⟶ Y   Y+Z ⟶ 2Z   X+Z ⟶ 0

det $A = -3\,\alpha\,\beta\,\gamma$

transversality in γ: ✓

(85) Y ⟶ X+Y   2X ⟶ 2Y   Y+Z ⟶ 2Z   X+Z ⟶ 0

det $A = -2\,\alpha\,\beta\,\gamma$

transversality in γ: ✓

(86) Z ⟶ X+Z   2X ⟶ Y+Z   X+Y ⟶ 0   Y+Z ⟶ X+Y

det $A = -2\,\alpha\,\beta\,\gamma$

transversality in β: ✓

(87) 0 ⟶ X   X+Y ⟶ 2Y   Y+Z ⟶ 2Z   X+Z ⟶ 0

det $A = -4\,\alpha\,\beta\,\gamma$

transversality in β: ✓

total elapsed: 39.33 seconds

Calculate the focal values

In[32]:=
```
t0 = SessionTime[];
(*For[j=1,j≤Length[M],j++,{*)
For[j = 1, j ≤ 87, j++, {
   L = M[[j]][[1 ;; 2 n m]];

   {f, ntw} = GetRHS[L, n, m, j];
   equilibrium = GetEquilibrium[f];
   {fnew, κ2αβγ} = RescaleODE[f, equilibrium];
   {A, B, CC, DD, EE} = GetDerivatives[fnew];
   {L1neg, L1zer, L1pos, Hopf, HopfCondition, varremain, sign, L1} = AnalyseL1[A, B, CC];
   PrintInfo[ntw, fnew, Hopf, HopfCondition, sign];
   If[sign == {0, 1, 1}, {
     isnumerical = False;
     ComputePrintL2[L1zer, varremain, Hopf, fnew, isnumerical, κ2αβγ];
    }];
   If[sign == {1, 1, 1}, {
     PrintSignL1[L1neg, L1zer, L1pos];
     isnumerical = True;
     ComputePrintL2[L1zer, varremain, Hopf, fnew, isnumerical, κ2αβγ];
    }];
  }];
Print["\ntotal elapsed: ",
  Style[NumberForm[SessionTime[] - t0, {5, 2}], Magenta], " seconds"];
```

(1) $0 \longrightarrow X \quad X \longrightarrow Y \quad Y+Z \longrightarrow 2Z \quad X+Z \longrightarrow 0$

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} (2 - u - u\,w)\,\alpha \\ (u - v\,w)\,\beta \\ -((u - v)\,w\,\gamma) \end{pmatrix}$

pair of imaginary eigenvalues: $\gamma = \dfrac{2\,\alpha\,\beta\,(2\,\alpha + \beta)}{2\,\alpha^2 + \alpha\,\beta - \beta^2}$ and $0 < \beta < 2\,\alpha$

L_1 is negative

(2) $0 \longrightarrow X \quad X \longrightarrow 2Y \quad Y+Z \longrightarrow 2Z \quad X+Z \longrightarrow 0$

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} (3 - u - 2\,u\,w)\,\alpha \\ (u - v\,w)\,\beta \\ -((u - v)\,w\,\gamma) \end{pmatrix}$

pair of imaginary eigenvalues: $\gamma = \dfrac{3\,\alpha\,\beta\,(3\,\alpha + \beta)}{6\,\alpha^2 + 2\,\alpha\,\beta - \beta^2}$ and $0 < \beta < \alpha + \sqrt{7}\,\alpha$

L_1 is negative

(3) $0 \longrightarrow X \quad X+Y \longrightarrow 2Y \quad Y \longrightarrow Z \quad X+Z \longrightarrow 0$

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} (2 - u\,(v + w))\,\alpha \\ (-1 + u)\,v\,\beta \\ (v - u\,w)\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\beta = -\frac{\gamma\,(2\,\alpha + \gamma)}{2\,\alpha - \gamma}$ and $\alpha > 0\,\&\&\,\gamma > 2\,\alpha$

L_1 is negative

(4) 0 ⟶ X X+Y ⟶ 2Y Y ⟶ X+Z X+Z ⟶ 0

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} (1 - (-1 + u)\,v - u\,w)\,\alpha \\ (-1 + u)\,v\,\beta \\ (v - u\,w)\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\beta = 2\,\alpha + \gamma$ and $\alpha > 0\,\&\&\,\gamma > 0$

L_1 is negative

(5) Z ⟶ X+Z X+Y ⟶ 2Y Y+Z ⟶ 0 0 ⟶ Z

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} (-u\,v + w)\,\alpha \\ v\,(u - w)\,\beta \\ (1 - v\,w)\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\beta = -\frac{\alpha\,\gamma\,(\alpha + \gamma)}{\alpha^2 - \alpha\,\gamma - \gamma^2}$ and $\alpha > 0\,\&\&\,\sqrt{5}\,\alpha < \alpha + 2\,\gamma$

L_1 is negative

(6) 0 ⟶ X X+Y ⟶ 2Y Y ⟶ 2Z X+Z ⟶ 0

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} (3 - u\,(v + 2\,w))\,\alpha \\ (-1 + u)\,v\,\beta \\ (v - u\,w)\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\beta = -\frac{\gamma\,(3\,\alpha + \gamma)}{3\,\alpha - 2\,\gamma}$ and $\alpha > 0\,\&\&\,2\,\gamma > 3\,\alpha$

L_1 is negative: $\gamma > 0\,\&\&\,\mathrm{Root}\left[-\gamma^4 + 30\,\gamma^3\,\#1 + 45\,\gamma^2\,\#1^2 - 216\,\gamma\,\#1^3 + 162\,\#1^4\,\&,\ 2\right] < \alpha < \frac{2\,\gamma}{3}$

L_1 is zero : $\gamma > 0\,\&\&\,\alpha == \mathrm{Root}\left[-\gamma^4 + 30\,\gamma^3\,\#1 + 45\,\gamma^2\,\#1^2 - 216\,\gamma\,\#1^3 + 162\,\#1^4\,\&,\ 2\right]$

L_1 is positive: $\gamma > 0\,\&\&\,0 < \alpha < \mathrm{Root}\left[-\gamma^4 + 30\,\gamma^3\,\#1 + 45\,\gamma^2\,\#1^2 - 216\,\gamma\,\#1^3 + 162\,\#1^4\,\&,\ 2\right]$

L_1 vanishes for approximately
$\{\alpha \to 1.0000,\ \gamma \to 31.225,\ \beta \to 17.976\}$, and then L_2 is negative

(7) 0 ⟶ X+Y X+Z ⟶ Y+Z Y+Z ⟶ 2Z Z ⟶ 0

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} (1 - u\,w)\,\alpha \\ (1 + u\,w - 2\,v\,w)\,\beta \\ (-1 + v)\,w\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\gamma = \frac{2\,\alpha\,(\alpha + 2\,\beta)}{\alpha - 2\,\beta}$ and $\beta > 0\,\&\&\,\alpha > 2\,\beta$

L_1 is positive

(8) $\emptyset \longrightarrow X+Y \quad X+Z \longrightarrow Y \quad Y+Z \longrightarrow 2Z \quad Z \longrightarrow \emptyset$

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} (1 - u\,w)\,\alpha \\ (1 + u\,w - 2\,v\,w)\,\beta \\ -((1 + u - 2\,v)\,w\,\gamma) \end{pmatrix}$

pair of imaginary eigenvalues: $\gamma = \dfrac{2\,\alpha\,\beta\,(\alpha + 2\,\beta)}{\alpha^2 + 2\,\alpha\,\beta - 4\,\beta^2}$ and $\beta > 0\ \&\&\ \alpha + \sqrt{5}\,\alpha > 4\,\beta$

L_1 is positive or zero

L_1 vanishes for $4\,\beta == \gamma\ \&\&\ \alpha == 2\,\beta$

(or equivalently, $\kappa_2 == \kappa_3\ \&\&\ 2\,\kappa_1\,\kappa_3 == \kappa_4^2$), and then L_2 is negative

(9) $\emptyset \longrightarrow X+Y \quad X+Z \longrightarrow 2Y \quad Y+Z \longrightarrow 2Z \quad Z \longrightarrow \emptyset$

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} (1 - u\,w)\,\alpha \\ (1 + 2\,u\,w - 3\,v\,w)\,\beta \\ -((2 + u - 3\,v)\,w\,\gamma) \end{pmatrix}$

pair of imaginary eigenvalues: $\gamma = \dfrac{3\,\alpha\,\beta\,(\alpha + 3\,\beta)}{\alpha^2 + 6\,\alpha\,\beta - 9\,\beta^2}$ and $\beta > 0\ \&\&\ \alpha + \sqrt{2}\,\alpha > 3\,\beta$

L_1 is positive or zero

L_1 vanishes for $3\,\beta == \gamma\ \&\&\ \alpha == 3\,\beta$

(or equivalently, $\kappa_2 == \kappa_3\ \&\&\ 4\,\kappa_1\,\kappa_3 == \kappa_4^2$), and then L_2 is negative

(10) $\emptyset \longrightarrow X \quad X+Z \longrightarrow Y+Z \quad Y+Z \longrightarrow 2Z \quad Z \longrightarrow \emptyset$

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} (1 - u\,w)\,\alpha \\ (u - v)\,w\,\beta \\ (-1 + v)\,w\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\gamma = \alpha + \beta$ and $\alpha > 0\ \&\&\ \beta > 0$

L_1 is positive or zero

L_1 vanishes for $2\,\beta == \gamma\ \&\&\ \alpha == \beta$

(or equivalently, $\kappa_2 == \kappa_3\ \&\&\ 2\,\kappa_1\,\kappa_3 == \kappa_4^2$), and then L_2 is negative

(11) $\emptyset \longrightarrow X \quad X+Z \longrightarrow 2Y \quad Y+Z \longrightarrow 2Z \quad Z \longrightarrow \emptyset$

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} (1 - u\,w)\,\alpha \\ (u - v)\,w\,\beta \\ -((1 + u - 2\,v)\,w\,\gamma) \end{pmatrix}$

pair of imaginary eigenvalues: $\gamma = \dfrac{\beta\,(\alpha + \beta)}{\alpha + 2\,\beta}$ and $\alpha > 0\ \&\&\ \beta > 0$

L_1 is positive or zero

L_1 vanishes for $2\,\beta == 3\,\gamma\ \&\&\ \alpha == \beta$

(or equivalently, $\kappa_2 == \kappa_3\ \&\&\ 2\,\kappa_1\,\kappa_3 == 3\,\kappa_4^2$), and then L_2 is negative

(12) $0 \longrightarrow X+Z \quad X+Y \longrightarrow 2Y \quad Y \longrightarrow Z \quad Y+Z \longrightarrow X$

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} (1 - 3\,u\,v + 2\,v\,w)\,\alpha \\ v\,(-1 + 3\,u - 2\,w)\,\beta \\ (1 - v\,(-1 + 2\,w))\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\beta = -\dfrac{6\,\alpha\,\gamma\,(3\,\alpha + 2\,\gamma)}{9\,\alpha^2 - 6\,\alpha\,\gamma - 4\,\gamma^2}$ and $\alpha > 0\;\&\&\;3 \times \left(-1 + \sqrt{5}\right)\alpha < 4\,\gamma$

L_1 is positive or zero

L_1 vanishes for $\beta == 4\,\gamma\;\&\&\;3\,\alpha == 2\,\gamma$
(or equivalently, $\kappa_2 == \kappa_4\;\&\&\;\kappa_3^2 == 2\,\kappa_1\,\kappa_4$), and then L_2 is negative

(13) $0 \longrightarrow X+Z \quad X+Y \longrightarrow 2Y \quad Y \longrightarrow 2Z \quad Y+Z \longrightarrow X$

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} (1 - 4\,u\,v + 3\,v\,w)\,\alpha \\ v\,(-1 + 4\,u - 3\,w)\,\beta \\ (1 - v\,(-2 + 3\,w))\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\beta = -\dfrac{12\,\alpha\,\gamma\,(4\,\alpha + 3\,\gamma)}{16\,\alpha^2 - 12\,\alpha\,\gamma - 9\,\gamma^2}$ and $\alpha > 0\;\&\&\;2 \times \left(-1 + \sqrt{5}\right)\alpha < 3\,\gamma$

L_1 is positive or zero

L_1 vanishes for $\beta == 6\,\gamma\;\&\&\;4\,\alpha == 3\,\gamma$
(or equivalently, $\kappa_2 == \kappa_4\;\&\&\;\kappa_3^2 == 2\,\kappa_1\,\kappa_4$), and then L_2 is negative

(14) $X \longrightarrow 2X \quad X+Z \longrightarrow Y+Z \quad Y \longrightarrow 2Z \quad 2Z \longrightarrow 0$

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-1 + w)\,\alpha \\ (-v + u\,w)\,\beta \\ \left(v - w^2\right)\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = \beta + 2\,\gamma$ and $\beta > 0\;\&\&\;\gamma > 0$

L_1 is negative

(15) $X \longrightarrow 2X \quad X+Z \longrightarrow Y+Z \quad Y \longrightarrow Z \quad 2Z \longrightarrow 0$

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-1 + w)\,\alpha \\ (-v + u\,w)\,\beta \\ \left(v - w^2\right)\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = \beta + 2\,\gamma$ and $\beta > 0\;\&\&\;\gamma > 0$

L_1 is negative

(16) $X \longrightarrow 2X \quad X+Z \longrightarrow Y+Z \quad Y \longrightarrow Z \quad 2Z \longrightarrow Y$

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-1 + w)\,\alpha \\ (-2\,v + w\,(u + w))\,\beta \\ \left(v - w^2\right)\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = 2\,(\beta + \gamma)$ and $\beta > 0\;\&\&\;\gamma > 0$

L_1 is negative

(17) $X \longrightarrow 2X \quad X+Z \longrightarrow Y+Z \quad X+Y \longrightarrow Z \quad Z \longrightarrow 0$

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-2+v+w)\,\alpha \\ -u\,(v-w)\,\beta \\ (u\,v-w)\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\beta = \gamma$ and $\alpha > 0\ \&\&\ \gamma > 0$

L_1 is negative

(18) $X \longrightarrow 2X \quad X+Z \longrightarrow Y+Z \quad X+Y \longrightarrow 2Z \quad Z \longrightarrow 0$

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-2+v+w)\,\alpha \\ -u\,(v-w)\,\beta \\ (u\,v-w)\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\beta = \gamma$ and $\alpha > 0\ \&\&\ \gamma > 0$

L_1 is negative

(19) $X \longrightarrow 2X \quad X+Z \longrightarrow Y \quad X+Y \longrightarrow 2Z \quad Z \longrightarrow 0$

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-2+v+w)\,\alpha \\ -u\,(v-w)\,\beta \\ (2\,u\,v-w-u\,w)\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\beta = 2\,\gamma$ and $\alpha > 0\ \&\&\ \gamma > 0$

L_1 is negative

(20) $X \longrightarrow 2X \quad X+Z \longrightarrow 2Y \quad X+Y \longrightarrow 2Z \quad Z \longrightarrow 0$

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-3+2\,v+w)\,\alpha \\ -u\,(v-w)\,\beta \\ (4\,u\,v-3\,w-u\,w)\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\beta = 2\,\gamma$ and $\alpha > 0\ \&\&\ \gamma > 0$

L_1 is negative

(21) $X \longrightarrow 2X \quad X+Z \longrightarrow 2Y \quad X+Y \longrightarrow Z \quad Z \longrightarrow 0$

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-3+2\,v+w)\,\alpha \\ -u\,(v-w)\,\beta \\ (2\,u\,v-w-u\,w)\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\beta = \gamma$ and $\alpha > 0\ \&\&\ \gamma > 0$

L_1 is negative

(22) $X \longrightarrow 2X \quad X+Z \longrightarrow 2Y \quad Y \longrightarrow Z \quad 2Z \longrightarrow Y$

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-1+w)\,\alpha \\ (-3\,v+w\,(2\,u+w))\,\beta \\ (3\,v-w\,(u+2\,w))\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = \frac{3\beta(3\beta+5\gamma)}{6\beta+5\gamma}$ and $\beta > 0 \,\&\&\, \gamma > 0$

L_1 is negative:

$\gamma > 0 \,\&\&\, \beta > \text{Root}\left[-2500\gamma^5 - 5400\gamma^4 \#1 - 1710\gamma^3 \#1^2 + 5751\gamma^2 \#1^3 + 5670\gamma \#1^4 + 1701 \#1^5 \&, 1\right]$

L_1 is zero :

$\gamma > 0 \,\&\&\, \beta == \text{Root}\left[-2500\gamma^5 - 5400\gamma^4 \#1 - 1710\gamma^3 \#1^2 + 5751\gamma^2 \#1^3 + 5670\gamma \#1^4 + 1701 \#1^5 \&, 1\right]$

L_1 is positive:

$\gamma > 0 \,\&\&\, 0 < \beta < \text{Root}\left[-2500\gamma^5 - 5400\gamma^4 \#1 - 1710\gamma^3 \#1^2 + 5751\gamma^2 \#1^3 + 5670\gamma \#1^4 + 1701 \#1^5 \&, 1\right]$

L_1 vanishes for approximately

$\{\beta \to 1.0000, \gamma \to 1.1189, \alpha \to 2.2238\}$, and then L_2 is positive

(23) X ⟶ 2X X+Z ⟶ 2Y Y ⟶ Z 2Z ⟶ 0

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u(-1+w)\alpha \\ (-v+u w)\beta \\ (2v - w(u+w))\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = \frac{\beta(\beta+3\gamma)}{2\beta+3\gamma}$ and $\beta > 0 \,\&\&\, \gamma > 0$

L_1 is negative:

$\gamma > 0 \,\&\&\, \beta > \text{Root}\left[-162\gamma^5 - 180\gamma^4 \#1 - 78\gamma^3 \#1^2 + 25\gamma^2 \#1^3 + 28\gamma \#1^4 + 7 \#1^5 \&, 1\right]$

L_1 is zero :

$\gamma > 0 \,\&\&\, \beta == \text{Root}\left[-162\gamma^5 - 180\gamma^4 \#1 - 78\gamma^3 \#1^2 + 25\gamma^2 \#1^3 + 28\gamma \#1^4 + 7 \#1^5 \&, 1\right]$

L_1 is positive:

$\gamma > 0 \,\&\&\, 0 < \beta < \text{Root}\left[-162\gamma^5 - 180\gamma^4 \#1 - 78\gamma^3 \#1^2 + 25\gamma^2 \#1^3 + 28\gamma \#1^4 + 7 \#1^5 \&, 1\right]$

L_1 vanishes for approximately

$\{\beta \to 1.0000, \gamma \to 0.50788, \alpha \to 0.71620\}$, and then L_2 is negative

(24) X ⟶ 2X X+Z ⟶ 2Y Y ⟶ 2Z 2Z ⟶ 0

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u(-1+w)\alpha \\ (-v+u w)\beta \\ (4v - w(u+3w))\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = \frac{3\beta(\beta+7\gamma)}{4\beta+7\gamma}$ and $\beta > 0 \,\&\&\, \gamma > 0$

L_1 is negative:

$\gamma > 0 \,\&\&\, \beta > \text{Root}\left[-4802\gamma^5 - 2156\gamma^4 \#1 + 84\gamma^3 \#1^2 + 421\gamma^2 \#1^3 + 118\gamma \#1^4 + 15 \#1^5 \&, 1\right]$

L_1 is zero :

$\gamma > 0 \,\&\&\, \beta == \text{Root}\left[-4802\gamma^5 - 2156\gamma^4 \#1 + 84\gamma^3 \#1^2 + 421\gamma^2 \#1^3 + 118\gamma \#1^4 + 15 \#1^5 \&, 1\right]$

L_1 is positive:

$\gamma > 0 \,\&\&\, 0 < \beta < \text{Root}\left[-4802\gamma^5 - 2156\gamma^4 \#1 + 84\gamma^3 \#1^2 + 421\gamma^2 \#1^3 + 118\gamma \#1^4 + 15 \#1^5 \&, 1\right]$

L_1 vanishes for approximately

$\{\beta \to 1.0000, \gamma \to 0.43588, \alpha \to 1.7236\}$, and then L_2 is negative

(25) X ⟶ 2X X+Z ⟶ Y Y ⟶ 2Z 2Z ⟶ 0

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-1+w)\,\alpha \\ (-v+u\,w)\,\beta \\ (2\,v-w\,(u+w))\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = \dfrac{\beta\,(\beta+3\,\gamma)}{2\,\beta+3\,\gamma}$ and $\beta>0\,\&\&\,\gamma>0$

L_1 is negative:
$\gamma>0\,\&\&\,\beta>\mathrm{Root}\left[-162\,\gamma^5-180\,\gamma^4\,\#1-78\,\gamma^3\,\#1^2+25\,\gamma^2\,\#1^3+28\,\gamma\,\#1^4+7\,\#1^5\,\&,\,1\right]$

L_1 is zero :
$\gamma>0\,\&\&\,\beta==\mathrm{Root}\left[-162\,\gamma^5-180\,\gamma^4\,\#1-78\,\gamma^3\,\#1^2+25\,\gamma^2\,\#1^3+28\,\gamma\,\#1^4+7\,\#1^5\,\&,\,1\right]$

L_1 is positive:
$\gamma>0\,\&\&\,0<\beta<\mathrm{Root}\left[-162\,\gamma^5-180\,\gamma^4\,\#1-78\,\gamma^3\,\#1^2+25\,\gamma^2\,\#1^3+28\,\gamma\,\#1^4+7\,\#1^5\,\&,\,1\right]$

L_1 vanishes for approximately
$\{\beta\to1.0000,\ \gamma\to0.50788,\ \alpha\to0.71620\}$, and then L_2 is negative

(26) X ⟶ 2X 2X ⟶ 2Y Y ⟶ 2Z X+Z ⟶ 0

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-3+u+2\,w)\,\alpha \\ (u^2-v)\,\beta \\ (v-u\,w)\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\beta = -\dfrac{\alpha^2-2\,\alpha\,\gamma+\gamma^2-\sqrt{\alpha^4+14\,\alpha^2\,\gamma^2+\gamma^4}}{2\,(\alpha+\gamma)}$ and $\alpha>0\,\&\&\,\gamma>0$

L_1 is negative: $\gamma>0\,\&\&$
$\mathrm{Root}\left[4\,\gamma^8-12\,\gamma^7\,\#1+81\,\gamma^6\,\#1^2-204\,\gamma^5\,\#1^3+226\,\gamma^4\,\#1^4-204\,\gamma^3\,\#1^5+81\,\gamma^2\,\#1^6-12\,\gamma\,\#1^7+4\,\#1^8\,\&,\,1\right]<\alpha<$
$\mathrm{Root}\left[4\,\gamma^8-12\,\gamma^7\,\#1+81\,\gamma^6\,\#1^2-204\,\gamma^5\,\#1^3+226\,\gamma^4\,\#1^4-204\,\gamma^3\,\#1^5+81\,\gamma^2\,\#1^6-12\,\gamma\,\#1^7+4\,\#1^8\,\&,\,2\right]$

L_1 is zero : $\gamma>0\,\&\&\,\big(\alpha==$
$\mathrm{Root}\left[4\,\gamma^8-12\,\gamma^7\,\#1+81\,\gamma^6\,\#1^2-204\,\gamma^5\,\#1^3+226\,\gamma^4\,\#1^4-204\,\gamma^3\,\#1^5+81\,\gamma^2\,\#1^6-12\,\gamma\,\#1^7+4\,\#1^8\,\&,\,1\right]\;||$
$\alpha==\mathrm{Root}\left[4\,\gamma^8-12\,\gamma^7\,\#1+81\,\gamma^6\,\#1^2-204\,\gamma^5\,\#1^3+226\,\gamma^4\,\#1^4-204\,\gamma^3\,\#1^5+81\,\gamma^2\,\#1^6-12\,\gamma\,\#1^7+4\,\#1^8\,\&,\,2\right]\big)$

L_1 is positive: $\gamma>0\,\&\&\,\big(0<\alpha<$
$\mathrm{Root}\left[4\,\gamma^8-12\,\gamma^7\,\#1+81\,\gamma^6\,\#1^2-204\,\gamma^5\,\#1^3+226\,\gamma^4\,\#1^4-204\,\gamma^3\,\#1^5+81\,\gamma^2\,\#1^6-12\,\gamma\,\#1^7+4\,\#1^8\,\&,\,1\right]\;||$
$\alpha>\mathrm{Root}\left[4\,\gamma^8-12\,\gamma^7\,\#1+81\,\gamma^6\,\#1^2-204\,\gamma^5\,\#1^3+226\,\gamma^4\,\#1^4-204\,\gamma^3\,\#1^5+81\,\gamma^2\,\#1^6-12\,\gamma\,\#1^7+4\,\#1^8\,\&,\,2\right]\big)$

L_1 vanishes for approximately
$\{\alpha\to1.0000,\ \gamma\to0.56578,\ \beta\to0.69438\}$, and then L_2 is negative

L_1 vanishes for approximately
$\{\alpha\to1.0000,\ \gamma\to1.7675,\ \beta\to1.2273\}$, and then L_2 is negative

(27) Z ⟶ 2X X+Y ⟶ 2Y Y ⟶ 0 2X ⟶ 2Z

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} (-u^2-u\,v+2\,w)\,\alpha \\ (-1+u)\,v\,\beta \\ (u^2-w)\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\beta = \gamma+\dfrac{\gamma^2}{3\,\alpha}$ and $\alpha>0\,\&\&\,\gamma>0$

L_1 is negative:
$\gamma>0\,\&\&\,\alpha>\mathrm{Root}\left[-\gamma^5+6\,\gamma^4\,\#1+141\,\gamma^3\,\#1^2+360\,\gamma^2\,\#1^3+252\,\gamma\,\#1^4+162\,\#1^5\,\&,\,3\right]$

L_1 is zero :

$\gamma > 0$ && $\alpha == \text{Root}\left[-\gamma^5 + 6\gamma^4 \#1 + 141\gamma^3 \#1^2 + 360\gamma^2 \#1^3 + 252\gamma \#1^4 + 162 \#1^5 \&, 3\right]$

L_1 is positive:

$\gamma > 0$ && $0 < \alpha < \text{Root}\left[-\gamma^5 + 6\gamma^4 \#1 + 141\gamma^3 \#1^2 + 360\gamma^2 \#1^3 + 252\gamma \#1^4 + 162 \#1^5 \&, 3\right]$

L_1 vanishes for approximately

$\{\alpha \to 1.0000, \gamma \to 16.163, \beta \to 103.25\}$, and then L_2 is negative

(28) Y ⟶ 2X X+Z ⟶ Y+Z 2Y ⟶ Z Z ⟶ 0

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} (v - u\,w)\,\alpha \\ \left(-v - v^2 + 2\,u\,w\right)\beta \\ \left(v^2 - w\right)\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha =$

$-\dfrac{3\beta^2 + 2\beta\gamma + \gamma^2 - \sqrt{9\beta^4 + 24\beta^3\gamma + 26\beta^2\gamma^2 + 8\beta\gamma^3 + \gamma^4}}{2(\beta + \gamma)}$ and $\beta > 0$ && $\gamma > 0$

L_1 is positive

(29) Y ⟶ 2X X+Z ⟶ Y+Z 2Y ⟶ Z Z ⟶ X

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} (2\,v + w - 3\,u\,w)\,\alpha \\ \left(-v - 2\,v^2 + 3\,u\,w\right)\beta \\ \left(v^2 - w\right)\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha =$

$-\dfrac{15\beta^2 + 6\beta\gamma + \gamma^2 - \sqrt{225\beta^4 + 240\beta^3\gamma + 98\beta^2\gamma^2 + 16\beta\gamma^3 + \gamma^4}}{6 \times (3\beta + \gamma)}$ and $\beta > 0$ && $\gamma > 0$

L_1 is positive

(30) Y ⟶ 2X X+Z ⟶ Y+Z 2Y ⟶ Z Z ⟶ Y

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} (v - u\,w)\,\alpha \\ \left(-v - 2\,v^2 + (1 + 2\,u)\,w\right)\beta \\ \left(v^2 - w\right)\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha =$

$-\dfrac{15\beta^2 + 6\beta\gamma + \gamma^2 - \sqrt{225\beta^4 + 240\beta^3\gamma + 98\beta^2\gamma^2 + 16\beta\gamma^3 + \gamma^4}}{6\beta + 2\gamma}$ and $\beta > 0$ && $\gamma > 0$

L_1 is positive

(31) Y ⟶ 2X X+Z ⟶ Y+Z 2Y ⟶ X+Z Z ⟶ 0

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} \left(2\,v + v^2 - 3\,u\,w\right)\alpha \\ \left(-v - 2\,v^2 + 3\,u\,w\right)\beta \\ \left(v^2 - w\right)\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha =$

$-\dfrac{5\beta^2 + 4\beta\gamma + \gamma^2 - \sqrt{25\beta^4 + 60\beta^3\gamma + 50\beta^2\gamma^2 + 12\beta\gamma^3 + \gamma^4}}{6(\beta + \gamma)}$ and $\beta > 0$ && $\gamma > 0$

L_1 is positive

(32) $Y \longrightarrow 2X \quad X+Z \longrightarrow Y+Z \quad 2Y \longrightarrow 2Z \quad Z \longrightarrow 0$

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} (v - u\,w)\,\alpha \\ \left(-v - v^2 + 2\,u\,w\right)\beta \\ \left(v^2 - w\right)\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = -\dfrac{3\,\beta^2 + 2\,\beta\,\gamma + \gamma^2 - \sqrt{9\,\beta^4 + 24\,\beta^3\,\gamma + 26\,\beta^2\,\gamma^2 + 8\,\beta\,\gamma^3 + \gamma^4}}{2\,(\beta + \gamma)}$ and $\beta > 0\ \&\&\ \gamma > 0$

L_1 is positive

(33) $Y \longrightarrow 2X \quad X+Z \longrightarrow 2Y \quad 2Y \longrightarrow 2Z \quad Z \longrightarrow 0$

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} (v - u\,w)\,\alpha \\ \left(-v - 3\,v^2 + 4\,u\,w\right)\beta \\ \left(3\,v^2 - (1 + 2\,u)\,w\right)\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = \dfrac{-21\,\beta^2 - 10\,\beta\,\gamma - 3\,\gamma^2 + \sqrt{441\,\beta^4 + 672\,\beta^3\,\gamma + 418\,\beta^2\,\gamma^2 + 96\,\beta\,\gamma^3 + 9\,\gamma^4}}{2 \times (3\,\beta + \gamma)}$ and $\beta > 0\ \&\&\ \gamma > 0$

L_1 is positive

(34) $Z \longrightarrow 2X \quad X+Y \longrightarrow 2Y \quad Y \longrightarrow Z \quad 2Z \longrightarrow Y$

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} (-u\,v + w)\,\alpha \\ \left((-3 + 2\,u)\,v + w^2\right)\beta \\ (3\,v - w\,(1 + 2\,w))\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = \dfrac{-3\,\beta^2 - 16\,\beta\,\gamma - 25\,\gamma^2 + \sqrt{9\,\beta^4 + 108\,\beta^3\,\gamma + 486\,\beta^2\,\gamma^2 + 900\,\beta\,\gamma^3 + 625\,\gamma^4}}{6\,\beta + 10\,\gamma}$ and $\beta > 0\ \&\&\ \gamma > 0$

L_1 is positive

(35) $0 \longrightarrow X \quad 2X \longrightarrow Y \quad Y+Z \longrightarrow 2Z \quad X+Z \longrightarrow 0$

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} \left(3 - 2\,u^2 - u\,w\right)\alpha \\ \left(u^2 - v\,w\right)\beta \\ -((u - v)\,w\,\gamma) \end{pmatrix}$

pair of imaginary eigenvalues: $\gamma = \dfrac{5\,\alpha\,\beta\,(5\,\alpha + \beta)}{5\,\alpha^2 + 2\,\alpha\,\beta - \beta^2}$ and $0 < \beta < \alpha + \sqrt{6}\,\alpha$

L_1 is negative

(36) $0 \longrightarrow X \quad 2X \longrightarrow 2Y \quad Y+Z \longrightarrow 2Z \quad X+Z \longrightarrow 0$

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} \left(2 - u^2 - u\,w\right)\alpha \\ \left(u^2 - v\,w\right)\beta \\ -((u - v)\,w\,\gamma) \end{pmatrix}$

pair of imaginary eigenvalues: $\gamma = \frac{3\alpha\beta(3\alpha+\beta)}{3\alpha^2+2\alpha\beta-\beta^2}$ and $0<\beta<3\alpha$

L_1 is negative

(37) $0 \longrightarrow X \quad X+Y \longrightarrow 2Y \quad Y+Z \longrightarrow 2Z \quad X+Z \longrightarrow X$

ODE after rescaling and reparametrising: $\begin{pmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\end{pmatrix} = \begin{pmatrix}(1-uv)\,\alpha\\ v\,(u-w)\,\beta\\ (-u+v)\,w\,\gamma\end{pmatrix}$

pair of imaginary eigenvalues: $\gamma = \alpha$ and $\alpha>0 \,\&\&\, \beta>0$

L_1 is negative

(38) $0 \longrightarrow X+Y \quad X+Y \longrightarrow 2Y \quad Y+Z \longrightarrow 2Z \quad X+Z \longrightarrow X$

ODE after rescaling and reparametrising: $\begin{pmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\end{pmatrix} = \begin{pmatrix}(1-uv)\,\alpha\\ (1+uv-2vw)\,\beta\\ (-u+v)\,w\,\gamma\end{pmatrix}$

pair of imaginary eigenvalues: $\gamma = \frac{\alpha(\alpha+\beta)}{\alpha-\beta}$ and $0<\beta<\alpha$

L_1 is negative

(39) $0 \longrightarrow X+Z \quad X+Y \longrightarrow 2Y \quad Y+Z \longrightarrow Z \quad X+Z \longrightarrow X$

ODE after rescaling and reparametrising: $\begin{pmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\end{pmatrix} = \begin{pmatrix}(1-uv)\,\alpha\\ v\,(u-w)\,\beta\\ (1-uw)\,\gamma\end{pmatrix}$

pair of imaginary eigenvalues: $\beta = -\frac{\gamma(\alpha+\gamma)}{\alpha-\gamma}$ and $\alpha>0 \,\&\&\, \gamma>\alpha$

L_1 is negative

(40) $X \longrightarrow 2X \quad X+Z \longrightarrow Y+Z \quad 2Y \longrightarrow Z \quad 2Z \longrightarrow 0$

ODE after rescaling and reparametrising: $\begin{pmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\end{pmatrix} = \begin{pmatrix}-u\,(-1+w)\,\alpha\\ \left(-v^2+uw\right)\beta\\ \frac{1}{2}\left(v^2-w^2\right)\gamma\end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = 2\beta+\gamma$ and $\beta>0 \,\&\&\, \gamma>0$

L_1 is negative

(41) $X \longrightarrow 2X \quad X+Z \longrightarrow Y+Z \quad 2Y \longrightarrow Z \quad 2Z \longrightarrow Y$

ODE after rescaling and reparametrising: $\begin{pmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\end{pmatrix} = \begin{pmatrix}-u\,(-1+w)\,\alpha\\ \left(-4v^2+w\,(3u+w)\right)\beta\\ \frac{1}{2}\left(v^2-w^2\right)\gamma\end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = 8\beta+\gamma$ and $\beta>0 \,\&\&\, \gamma>0$

L_1 is negative

(42) $X \longrightarrow 2X \quad X+Z \longrightarrow Y+Z \quad 2Y \longrightarrow Z \quad 2Z \longrightarrow 2Y$

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-1+w)\,\alpha \\ \left(-2\,v^2 + w\,(u+w)\right)\beta \\ \frac{1}{2}\left(v^2 - w^2\right)\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = 4\,\beta + \gamma$ and $\beta > 0\,\&\&\,\gamma > 0$

L_1 is negative

(43) $X \longrightarrow 2X \quad X+Z \longrightarrow Y+Z \quad 2Y \longrightarrow Z \quad Y+Z \longrightarrow 0$

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-1+w)\,\alpha \\ \left(-2\,v^2 + 3\,u\,w - v\,w\right)\beta \\ v\,(v-w)\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = 5\,\beta + \gamma$ and $\beta > 0\,\&\&\,\gamma > 0$

L_1 is negative

(44) $X \longrightarrow 2X \quad X+Z \longrightarrow Y+Z \quad 2Y \longrightarrow Z \quad Y+Z \longrightarrow Y$

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-1+w)\,\alpha \\ \left(-v^2 + u\,w\right)\beta \\ v\,(v-w)\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = 2\,\beta + \gamma$ and $\beta > 0\,\&\&\,\gamma > 0$

L_1 is negative

(45) $X \longrightarrow 2X \quad X+Z \longrightarrow Y+Z \quad 2Y \longrightarrow Z \quad Y+Z \longrightarrow 2Y$

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-1+w)\,\alpha \\ \left(-2\,v^2 + u\,w + v\,w\right)\beta \\ v\,(v-w)\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = 3\,\beta + \gamma$ and $\beta > 0\,\&\&\,\gamma > 0$

L_1 is negative

(46) $X \longrightarrow 2X \quad X+Z \longrightarrow Y+Z \quad 2Y \longrightarrow 2Z \quad 2Z \longrightarrow 0$

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-1+w)\,\alpha \\ \left(-v^2 + u\,w\right)\beta \\ \frac{1}{2}\left(v^2 - w^2\right)\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = 2\,\beta + \gamma$ and $\beta > 0\,\&\&\,\gamma > 0$

L_1 is negative

(47) $X \longrightarrow 2X \quad X+Z \longrightarrow Y+Z \quad 2Y \longrightarrow 2Z \quad 2Z \longrightarrow Y$

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-1+w)\,\alpha \\ \left(-2\,v^2 + w\,(u+w)\right)\beta \\ \frac{1}{2}\left(v^2 - w^2\right)\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = 4\beta + \gamma$ and $\beta > 0$ && $\gamma > 0$

L_1 is negative

(48) X ⟶ 2X X+Z ⟶ Y+Z 2Y ⟶ 2Z Y+Z ⟶ 0

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-1+w)\,\alpha \\ \left(-v^2 + 2\,u\,w - v\,w\right)\beta \\ v\,(v-w)\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = 3\beta + \gamma$ and $\beta > 0$ && $\gamma > 0$

L_1 is negative

(49) X ⟶ 2X X+Z ⟶ Y+Z 2Y ⟶ 2Z Y+Z ⟶ Y

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-1+w)\,\alpha \\ \left(-v^2 + u\,w\right)\beta \\ v\,(v-w)\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = 2\beta + \gamma$ and $\beta > 0$ && $\gamma > 0$

L_1 is negative

(50) X ⟶ 2X X+Z ⟶ Y+Z Y+Z ⟶ 2Z 2Z ⟶ 0

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-1+w)\,\alpha \\ (u-v)\,w\,\beta \\ (v-w)\,w\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = \beta + \gamma$ and $\beta > 0$ && $\gamma > 0$

L_1 is negative

(51) X ⟶ 2X X+Z ⟶ Y+Z Y+Z ⟶ 2Z 2Z ⟶ Y

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-1+w)\,\alpha \\ w\,(u-2\,v+w)\,\beta \\ (v-w)\,w\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = 2\beta + \gamma$ and $\beta > 0$ && $\gamma > 0$

L_1 is negative

(52) X ⟶ 2X 2X ⟶ Y Y+Z ⟶ 2Z X+Z ⟶ 0

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-3+2\,u+w)\,\alpha \\ \left(u^2 - v\,w\right)\beta \\ -((u-v)\,w\,\gamma) \end{pmatrix}$

pair of imaginary eigenvalues: $\gamma = \dfrac{2\,\alpha\,\beta\,(2\,\alpha+\beta)}{2\,\alpha^2 + 2\,\alpha\,\beta - \beta^2}$ and $0 < \beta < \alpha + \sqrt{3}\,\alpha$

L_1 is negative

(53) X ⟶ 2X 2X ⟶ 2Y Y+Z ⟶ 2Z X+Z ⟶ 0

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-2+u+w)\,\alpha \\ \left(u^2 - v\,w\right)\beta \\ -((u-v)\,w\,\gamma) \end{pmatrix}$

pair of imaginary eigenvalues: $\gamma = \frac{\alpha\,\beta\,(\alpha+\beta)}{\alpha^2 + 2\,\alpha\,\beta - \beta^2}$ and $0 < \beta < \alpha + \sqrt{2}\,\alpha$

L_1 is negative

(54) X ⟶ 2X   X+Y ⟶ 2Y   2Y ⟶ Z   X+Z ⟶ Y

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-2+v+w)\,\alpha \\ \left(-2\,v^2 + u\,(v+w)\right)\beta \\ \left(v^2 - u\,w\right)\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = -\frac{\beta\,\gamma\,(3\,\beta+\gamma)}{6\,\beta^2 - 3\,\beta\,\gamma - \gamma^2}$ and $\beta > 0\ \&\&\ \left(-3 + \sqrt{33}\right)\beta < 2\,\gamma$

L_1 is negative: $\gamma > 0\ \&\&$
$\text{Root}\left[\gamma^8 + 15\,\gamma^7\,\#1 + 73\,\gamma^6\,\#1^2 + 119\,\gamma^5\,\#1^3 - 123\,\gamma^4\,\#1^4 - 751\,\gamma^3\,\#1^5 - 453\,\gamma^2\,\#1^6 + 1017\,\gamma\,\#1^7 + 486\,\#1^8\,\&,\ 5\right] <$
$\beta < \frac{2\,\gamma}{-3 + \sqrt{33}}$

L_1 is zero : $\gamma > 0\ \&\&$
$\beta == \text{Root}\left[\gamma^8 + 15\,\gamma^7\,\#1 + 73\,\gamma^6\,\#1^2 + 119\,\gamma^5\,\#1^3 - 123\,\gamma^4\,\#1^4 - 751\,\gamma^3\,\#1^5 - 453\,\gamma^2\,\#1^6 + 1017\,\gamma\,\#1^7 + 486\,\#1^8\,\&,\ 5\right]$

L_1 is positive: $\gamma > 0\ \&\&\ 0 < \beta <$
$\text{Root}\left[\gamma^8 + 15\,\gamma^7\,\#1 + 73\,\gamma^6\,\#1^2 + 119\,\gamma^5\,\#1^3 - 123\,\gamma^4\,\#1^4 - 751\,\gamma^3\,\#1^5 - 453\,\gamma^2\,\#1^6 + 1017\,\gamma\,\#1^7 + 486\,\#1^8\,\&,\ 5\right]$

L_1 vanishes for approximately
$\{\beta \to 1.0000,\ \gamma \to 1.6046,\ \alpha \to 5.3215\}$, and then L_2 is negative

(55) X ⟶ 2X   X+Z ⟶ Y   X+Y ⟶ 2Z   2Z ⟶ 0

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-2+v+w)\,\alpha \\ -u\,(v-w)\,\beta \\ \left(2\,u\,v - u\,w - w^2\right)\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = \frac{\beta\,(\beta + 3\,\gamma)}{\beta - 3\,\gamma}$ and $\gamma > 0\ \&\&\ \beta > 3\,\gamma$

L_1 is negative:
$\gamma > 0\ \&\&\ 3\,\gamma < \beta < \text{Root}\left[972\,\gamma^7 + 1800\,\gamma^6\,\#1 + 549\,\gamma^5\,\#1^2 - 411\,\gamma^4\,\#1^3 - 170\,\gamma^3\,\#1^4 + 2\,\gamma^2\,\#1^5 + 9\,\gamma\,\#1^6 + \#1^7\,\&,\ 5\right]$

L_1 is zero :
$\gamma > 0\ \&\&\ \beta == \text{Root}\left[972\,\gamma^7 + 1800\,\gamma^6\,\#1 + 549\,\gamma^5\,\#1^2 - 411\,\gamma^4\,\#1^3 - 170\,\gamma^3\,\#1^4 + 2\,\gamma^2\,\#1^5 + 9\,\gamma\,\#1^6 + \#1^7\,\&,\ 5\right]$

L_1 is positive:
$\gamma > 0\ \&\&\ \beta > \text{Root}\left[972\,\gamma^7 + 1800\,\gamma^6\,\#1 + 549\,\gamma^5\,\#1^2 - 411\,\gamma^4\,\#1^3 - 170\,\gamma^3\,\#1^4 + 2\,\gamma^2\,\#1^5 + 9\,\gamma\,\#1^6 + \#1^7\,\&,\ 5\right]$

L_1 vanishes for approximately
$\{\beta \to 1.0000,\ \gamma \to 0.25592,\ \alpha \to 7.6121\}$, and then L_2 is negative

(56) X ⟶ 2X   X+Z ⟶ Y   X+Y ⟶ 2Z   Y+Z ⟶ Y

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-2+v+w)\,\alpha \\ -u\,(v-w)\,\beta \\ (2\,u\,v-u\,w-v\,w)\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = \dfrac{\beta\,(\beta+2\,\gamma)}{\beta-2\,\gamma}$ and $\gamma>0$ && $\beta>2\,\gamma$

L_1 is negative:
$\gamma>0$ && $2\,\gamma<\beta<$ Root$[64\,\gamma^7+192\,\gamma^6\,\#1+64\,\gamma^5\,\#1^2-184\,\gamma^4\,\#1^3-64\,\gamma^3\,\#1^4+22\,\gamma^2\,\#1^5+11\,\gamma\,\#1^6+\#1^7\,\&,\,3]$

L_1 is zero :
$\gamma>0$ && $\beta==$ Root$[64\,\gamma^7+192\,\gamma^6\,\#1+64\,\gamma^5\,\#1^2-184\,\gamma^4\,\#1^3-64\,\gamma^3\,\#1^4+22\,\gamma^2\,\#1^5+11\,\gamma\,\#1^6+\#1^7\,\&,\,3]$

L_1 is positive:
$\gamma>0$ && $\beta>$ Root$[64\,\gamma^7+192\,\gamma^6\,\#1+64\,\gamma^5\,\#1^2-184\,\gamma^4\,\#1^3-64\,\gamma^3\,\#1^4+22\,\gamma^2\,\#1^5+11\,\gamma\,\#1^6+\#1^7\,\&,\,3]$

L_1 vanishes for approximately
$\{\beta\to 1.0000,\ \gamma\to 0.45114,\ \alpha\to 19.466\}$, and then L_2 is negative

(57) X ⟶ 2X X+Z ⟶ 2Y 2Y ⟶ 2Z 2Z ⟶ 0

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-1+w)\,\alpha \\ \left(-v^2+u\,w\right)\beta \\ \left(2\,v^2-w\,(u+w)\right)\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = \dfrac{2\,\beta\,(2\,\beta+3\,\gamma)}{4\,\beta+3\,\gamma}$ and $\beta>0$ && $\gamma>0$

L_1 is negative:
$\gamma>0$ && $\beta>$ Root$[-1053\,\gamma^6-1071\,\gamma^5\,\#1+1248\,\gamma^4\,\#1^2+1996\,\gamma^3\,\#1^3+1376\,\gamma^2\,\#1^4+672\,\gamma\,\#1^5+128\,\#1^6\,\&,\,4]$

L_1 is zero :
$\gamma>0$ && $\beta==$ Root$[-1053\,\gamma^6-1071\,\gamma^5\,\#1+1248\,\gamma^4\,\#1^2+1996\,\gamma^3\,\#1^3+1376\,\gamma^2\,\#1^4+672\,\gamma\,\#1^5+128\,\#1^6\,\&,\,4]$

L_1 is positive:
$\gamma>0$ && $0<\beta<$ Root$[-1053\,\gamma^6-1071\,\gamma^5\,\#1+1248\,\gamma^4\,\#1^2+1996\,\gamma^3\,\#1^3+1376\,\gamma^2\,\#1^4+672\,\gamma\,\#1^5+128\,\#1^6\,\&,\,4]$

L_1 vanishes for approximately
$\{\beta\to 1.0000,\ \gamma\to 1.4137,\ \alpha\to 1.5146\}$, and then L_2 is negative

(58) X ⟶ 2X X+Z ⟶ 2Y 2Y ⟶ 2Z 2Z ⟶ Y

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-1+w)\,\alpha \\ \frac{1}{2}\times\left(-3\,v^2+w\,(2\,u+w)\right)\beta \\ \left(3\,v^2-w\,(u+2\,w)\right)\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = \dfrac{3\,\beta\,(3\,\beta+5\,\gamma)}{6\,\beta+5\,\gamma}$ and $\beta>0$ && $\gamma>0$

L_1 is negative: $\gamma>0$ &&
$\beta>$ Root$[-26\,875\,\gamma^6-37\,175\,\gamma^5\,\#1+31\,605\,\gamma^4\,\#1^2+85\,257\,\gamma^3\,\#1^3+64\,746\,\gamma^2\,\#1^4+22\,518\,\gamma\,\#1^5+2916\,\#1^6\,\&,\,4]$

L_1 is zero : $\gamma>0$ &&
$\beta==$ Root$[-26\,875\,\gamma^6-37\,175\,\gamma^5\,\#1+31\,605\,\gamma^4\,\#1^2+85\,257\,\gamma^3\,\#1^3+64\,746\,\gamma^2\,\#1^4+22\,518\,\gamma\,\#1^5+2916\,\#1^6\,\&,\,4]$

L_1 is positive: $\gamma>0$ && $0<\beta<$
Root$[-26\,875\,\gamma^6-37\,175\,\gamma^5\,\#1+31\,605\,\gamma^4\,\#1^2+85\,257\,\gamma^3\,\#1^3+64\,746\,\gamma^2\,\#1^4+22\,518\,\gamma\,\#1^5+2916\,\#1^6\,\&,\,4]$

L_1 vanishes for approximately
$\{\beta \to 1.0000,\ \gamma \to 1.5398,\ \alpha \to 2.3430\}$, and then L_2 is positive

(59) $X \longrightarrow 2X \quad X+Z \longrightarrow 2Y \quad 2Y \longrightarrow 2Z \quad Y+Z \longrightarrow 0$

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-1+w)\,\alpha \\ \left(-3\,v^2+4\,u\,w-v\,w\right)\beta \\ \left(3\,v^2-2\,u\,w-v\,w\right)\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = \dfrac{3\,\beta\,(7\,\beta+3\,\gamma)}{10\,\beta+3\,\gamma}$ and $\beta > 0\ \&\&\ \gamma > 0$

L_1 is negative:
$\gamma > 0\ \&\&\ \beta > \text{Root}\left[-405\,\gamma^5 - 6210\,\gamma^4\,\#1 - 27\,954\,\gamma^3\,\#1^2 - 50\,034\,\gamma^2\,\#1^3 - 26\,517\,\gamma\,\#1^4 + 5320\,\#1^5\,\&,\ 3\right]$

L_1 is zero :
$\gamma > 0\ \&\&\ \beta == \text{Root}\left[-405\,\gamma^5 - 6210\,\gamma^4\,\#1 - 27\,954\,\gamma^3\,\#1^2 - 50\,034\,\gamma^2\,\#1^3 - 26\,517\,\gamma\,\#1^4 + 5320\,\#1^5\,\&,\ 3\right]$

L_1 is positive:
$\gamma > 0\ \&\&\ 0 < \beta < \text{Root}\left[-405\,\gamma^5 - 6210\,\gamma^4\,\#1 - 27\,954\,\gamma^3\,\#1^2 - 50\,034\,\gamma^2\,\#1^3 - 26\,517\,\gamma\,\#1^4 + 5320\,\#1^5\,\&,\ 3\right]$

L_1 vanishes for approximately
$\{\beta \to 1.0000,\ \gamma \to 0.15273,\ \alpha \to 2.1394\}$, and then L_2 is negative

(60) $X \longrightarrow 2X \quad X+Z \longrightarrow 2Y \quad 2Y \longrightarrow 2Z \quad Y+Z \longrightarrow Y$

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-1+w)\,\alpha \\ \left(-v^2+u\,w\right)\beta \\ \left(2\,v^2-u\,w-v\,w\right)\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = \dfrac{2\,\beta\,(\beta+\gamma)}{3\,\beta+2\,\gamma}$ and $\beta > 0\ \&\&\ \gamma > 0$

L_1 is negative: $\gamma > 0\ \&\&\ \beta > \text{Root}\left[-13\,\gamma^4 - 33\,\gamma^3\,\#1 - 24\,\gamma^2\,\#1^2 + 3\,\#1^4\,\&,\ 2\right]$

L_1 is zero : $\gamma > 0\ \&\&\ \beta == \text{Root}\left[-13\,\gamma^4 - 33\,\gamma^3\,\#1 - 24\,\gamma^2\,\#1^2 + 3\,\#1^4\,\&,\ 2\right]$

L_1 is positive: $\gamma > 0\ \&\&\ 0 < \beta < \text{Root}\left[-13\,\gamma^4 - 33\,\gamma^3\,\#1 - 24\,\gamma^2\,\#1^2 + 3\,\#1^4\,\&,\ 2\right]$

L_1 vanishes for approximately
$\{\beta \to 1.0000,\ \gamma \to 0.29357,\ \alpha \to 0.72123\}$, and then L_2 is negative

(61) $X \longrightarrow 2X \quad X+Z \longrightarrow 2Y \quad X+Y \longrightarrow Z \quad 2Z \longrightarrow 0$

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-3+2\,v+w)\,\alpha \\ -u\,(v-w)\,\beta \\ \left(2\,u\,v-u\,w-w^2\right)\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = \dfrac{\beta\,(\beta+3\,\gamma)}{2\,\beta-3\,\gamma}$ and $0 < \gamma < \dfrac{2\,\beta}{3}$

L_1 is negative:
$\gamma > 0\ \&\&\ \dfrac{3\,\gamma}{2} < \beta < \text{Root}\left[486\,\gamma^7 + 702\,\gamma^6\,\#1 - 288\,\gamma^5\,\#1^2 - 579\,\gamma^4\,\#1^3 - 57\,\gamma^3\,\#1^4 + 71\,\gamma^2\,\#1^5 + 23\,\gamma\,\#1^6 + 2\,\#1^7\,\&,\ 5\right]$

L_1 is zero :
$\gamma > 0\ \&\&\ \beta == \text{Root}\left[486\,\gamma^7 + 702\,\gamma^6\,\#1 - 288\,\gamma^5\,\#1^2 - 579\,\gamma^4\,\#1^3 - 57\,\gamma^3\,\#1^4 + 71\,\gamma^2\,\#1^5 + 23\,\gamma\,\#1^6 + 2\,\#1^7\,\&,\ 5\right]$

L_1 is positive:
$\gamma > 0\ \&\&\ \beta > \text{Root}[486\gamma^7 + 702\gamma^6 \#1 - 288\gamma^5 \#1^2 - 579\gamma^4 \#1^3 - 57\gamma^3 \#1^4 + 71\gamma^2 \#1^5 + 23\gamma \#1^6 + 2\#1^7 \&, 5]$

L_1 vanishes for approximately
$\{\beta \to 1.0000, \gamma \to 0.45108, \alpha \to 3.6386\}$, and then L_2 is negative

(62) X ⟶ 2X X+Z ⟶ 2Y X+Y ⟶ Z 2Z ⟶ Y

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u(-4 + 3v + w)\alpha \\ (-3uv + 2uw + w^2)\beta \\ (3uv - uw - 2w^2)\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = \dfrac{3\beta\gamma(3\beta + 5\gamma)}{9\beta^2 + 21\beta\gamma - 10\gamma^2}$ and $\gamma > 0\ \&\&\ 3 \times \left(7 + \sqrt{89}\right)\beta > 20\gamma$

L_1 is negative:
$\gamma > 0\ \&\&\ \dfrac{20\gamma}{21 + 3\sqrt{89}} < \beta < \text{Root}[6250\gamma^8 + 775\gamma^7 \#1 - 47625\gamma^6 \#1^2 - 38313\gamma^5 \#1^3 + 68607\gamma^4 \#1^4 + 121581\gamma^3 \#1^5 + 75573\gamma^2 \#1^6 + 21141\gamma \#1^7 + 2187\#1^8 \&, 6]$

L_1 is zero :
$\gamma > 0\ \&\&\ \beta == \text{Root}[6250\gamma^8 + 775\gamma^7 \#1 - 47625\gamma^6 \#1^2 - 38313\gamma^5 \#1^3 + 68607\gamma^4 \#1^4 + 121581\gamma^3 \#1^5 + 75573\gamma^2 \#1^6 + 21141\gamma \#1^7 + 2187\#1^8 \&, 6]$

L_1 is positive:
$\gamma > 0\ \&\&\ \beta > \text{Root}[6250\gamma^8 + 775\gamma^7 \#1 - 47625\gamma^6 \#1^2 - 38313\gamma^5 \#1^3 + 68607\gamma^4 \#1^4 + 121581\gamma^3 \#1^5 + 75573\gamma^2 \#1^6 + 21141\gamma \#1^7 + 2187\#1^8 \&, 6]$

L_1 vanishes for approximately
$\{\beta \to 1.0000, \gamma \to 1.8885, \alpha \to 5.4248\}$, and then L_2 is negative

(63) X ⟶ 2X X+Z ⟶ 2Y X+Y ⟶ Z Y+Z ⟶ Y

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u(-3 + 2v + w)\alpha \\ -u(v - w)\beta \\ (2uv - uw - vw)\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = \dfrac{\beta(\beta + 2\gamma)}{2(\beta - \gamma)}$ and $0 < \gamma < \beta$

L_1 is negative:
$\gamma > 0\ \&\&\ \gamma < \beta < \text{Root}[32\gamma^7 + 80\gamma^6 \#1 - 60\gamma^5 \#1^2 - 208\gamma^4 \#1^3 - \gamma^3 \#1^4 + 69\gamma^2 \#1^5 + 23\gamma \#1^6 + 2\#1^7 \&, 3]$

L_1 is zero :
$\gamma > 0\ \&\&\ \beta == \text{Root}[32\gamma^7 + 80\gamma^6 \#1 - 60\gamma^5 \#1^2 - 208\gamma^4 \#1^3 - \gamma^3 \#1^4 + 69\gamma^2 \#1^5 + 23\gamma \#1^6 + 2\#1^7 \&, 3]$

L_1 is positive:
$\gamma > 0\ \&\&\ \beta > \text{Root}[32\gamma^7 + 80\gamma^6 \#1 - 60\gamma^5 \#1^2 - 208\gamma^4 \#1^3 - \gamma^3 \#1^4 + 69\gamma^2 \#1^5 + 23\gamma \#1^6 + 2\#1^7 \&, 3]$

L_1 vanishes for approximately
$\{\beta \to 1.0000, \gamma \to 0.72543, \alpha \to 4.4630\}$, and then L_2 is negative

(64) X ⟶ 2X X+Z ⟶ 2Y X+Y ⟶ 2Z 2Z ⟶ 0

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-3+2\,v+w)\,\alpha \\ -u\,(v-w)\,\beta \\ (4\,u\,v-u\,w-3\,w^2)\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = \dfrac{\beta\,(\beta+7\,\gamma)}{2\,\beta-7\,\gamma}$ and $0<\gamma<\dfrac{2\,\beta}{7}$

L_1 is negative: $\gamma>0\,\&\&\,\dfrac{7\,\gamma}{2}<\beta<$
$\text{Root}\left[134\,456\,\gamma^7+99\,176\,\gamma^6\,\#1-20\,685\,\gamma^5\,\#1^2-17\,204\,\gamma^4\,\#1^3-344\,\gamma^3\,\#1^4+402\,\gamma^2\,\#1^5+53\,\gamma\,\#1^6+2\,\#1^7\,\&,\,5\right]$

L_1 is zero : $\gamma>0\,\&\&\,\beta==$
$\text{Root}\left[134\,456\,\gamma^7+99\,176\,\gamma^6\,\#1-20\,685\,\gamma^5\,\#1^2-17\,204\,\gamma^4\,\#1^3-344\,\gamma^3\,\#1^4+402\,\gamma^2\,\#1^5+53\,\gamma\,\#1^6+2\,\#1^7\,\&,\,5\right]$

L_1 is positive: $\gamma>0\,\&\&\,\beta>$
$\text{Root}\left[134\,456\,\gamma^7+99\,176\,\gamma^6\,\#1-20\,685\,\gamma^5\,\#1^2-17\,204\,\gamma^4\,\#1^3-344\,\gamma^3\,\#1^4+402\,\gamma^2\,\#1^5+53\,\gamma\,\#1^6+2\,\#1^7\,\&,\,5\right]$

L_1 vanishes for approximately
$\{\beta\to 1.0000,\ \gamma\to 0.19949,\ \alpha\to 3.9703\}$, and then L_2 is negative

(65) X ⟶ 2X X+Z ⟶ 2Y X+Y ⟶ 2Z Y+Z ⟶ Y

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-3+2\,v+w)\,\alpha \\ -u\,(v-w)\,\beta \\ (4\,u\,v-u\,w-3\,v\,w)\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = \dfrac{\beta\,(\beta+4\,\gamma)}{2\,(\beta-2\,\gamma)}$ and $\gamma>0\,\&\&\,\beta>2\,\gamma$

L_1 is negative: $\gamma>0\,\&\&$
$2\,\gamma<\beta<\text{Root}\left[2048\,\gamma^7+4352\,\gamma^6\,\#1-1056\,\gamma^5\,\#1^2-5432\,\gamma^4\,\#1^3-74\,\gamma^3\,\#1^4+348\,\gamma^2\,\#1^5+53\,\gamma\,\#1^6+2\,\#1^7\,\&,\,3\right]$

L_1 is zero :
$\gamma>0\,\&\&\,\beta==\text{Root}\left[2048\,\gamma^7+4352\,\gamma^6\,\#1-1056\,\gamma^5\,\#1^2-5432\,\gamma^4\,\#1^3-74\,\gamma^3\,\#1^4+348\,\gamma^2\,\#1^5+53\,\gamma\,\#1^6+2\,\#1^7\,\&,\,3\right]$

L_1 is positive:
$\gamma>0\,\&\&\,\beta>\text{Root}\left[2048\,\gamma^7+4352\,\gamma^6\,\#1-1056\,\gamma^5\,\#1^2-5432\,\gamma^4\,\#1^3-74\,\gamma^3\,\#1^4+348\,\gamma^2\,\#1^5+53\,\gamma\,\#1^6+2\,\#1^7\,\&,\,3\right]$

L_1 vanishes for approximately
$\{\beta\to 1.0000,\ \gamma\to 0.31247,\ \alpha\to 2.9993\}$, and then L_2 is negative

(66) X ⟶ 2X X+Z ⟶ 2Y X+Y ⟶ X+Z Y+Z ⟶ 0

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-1+w)\,\alpha \\ (-3\,u\,v+4\,u\,w-v\,w)\,\beta \\ (3\,u\,v-2\,u\,w-v\,w)\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = \dfrac{6\,\beta\,(4\,\beta+3\,\gamma)}{2\,\beta-3\,\gamma}$ and $0<\gamma<\dfrac{2\,\beta}{3}$

L_1 is negative:
$\gamma>0\,\&\&\,\beta>\text{Root}\left[243\,\gamma^6+1944\,\gamma^5\,\#1+4185\,\gamma^4\,\#1^2+234\,\gamma^3\,\#1^3-3504\,\gamma^2\,\#1^4-288\,\gamma\,\#1^5+256\,\#1^6\,\&,\,4\right]$

L_1 is zero :
$\gamma>0\,\&\&\,\beta==\text{Root}\left[243\,\gamma^6+1944\,\gamma^5\,\#1+4185\,\gamma^4\,\#1^2+234\,\gamma^3\,\#1^3-3504\,\gamma^2\,\#1^4-288\,\gamma\,\#1^5+256\,\#1^6\,\&,\,4\right]$

L_1 is positive:

$\gamma > 0 \;\&\&\; \frac{3\gamma}{2} < \beta < \text{Root}[243\gamma^6 + 1944\gamma^5 \#1 + 4185\gamma^4 \#1^2 + 234\gamma^3 \#1^3 - 3504\gamma^2 \#1^4 - 288\gamma \#1^5 + 256 \#1^6 \&, 4]$

L_1 vanishes for approximately

$\{\beta \to 1.0000, \gamma \to 0.24226, \alpha \to 22.275\}$, and then L_2 is negative

(67) X ⟶ 2X X+Z ⟶ Y+Z X+Y ⟶ Z 2Z ⟶ 0

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-2+v+w)\,\alpha \\ -u\,(v-w)\,\beta \\ (u\,v - w^2)\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = \frac{\beta\,(\beta + 2\gamma)}{\beta - 2\gamma}$ and $\gamma > 0 \;\&\&\; \beta > 2\gamma$

L_1 is negative:

$\gamma > 0 \;\&\&\; 2\gamma < \beta < \text{Root}[32\gamma^7 + 112\gamma^6 \#1 + 32\gamma^5 \#1^2 - 96\gamma^4 \#1^3 - 34\gamma^3 \#1^4 + 5\gamma^2 \#1^5 + 6\gamma \#1^6 + \#1^7 \&, 5]$

L_1 is zero :

$\gamma > 0 \;\&\&\; \beta == \text{Root}[32\gamma^7 + 112\gamma^6 \#1 + 32\gamma^5 \#1^2 - 96\gamma^4 \#1^3 - 34\gamma^3 \#1^4 + 5\gamma^2 \#1^5 + 6\gamma \#1^6 + \#1^7 \&, 5]$

L_1 is positive:

$\gamma > 0 \;\&\&\; \beta > \text{Root}[32\gamma^7 + 112\gamma^6 \#1 + 32\gamma^5 \#1^2 - 96\gamma^4 \#1^3 - 34\gamma^3 \#1^4 + 5\gamma^2 \#1^5 + 6\gamma \#1^6 + \#1^7 \&, 5]$

L_1 vanishes for approximately

$\{\beta \to 1.0000, \gamma \to 0.42012, \alpha \to 11.519\}$, and then L_2 is negative

(68) X ⟶ 2X X+Z ⟶ Y+Z X+Y ⟶ Z 2Z ⟶ Y

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-3+2\,v+w)\,\alpha \\ (-2\,u\,v + u\,w + w^2)\,\beta \\ (u\,v - w^2)\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = \frac{2\beta\gamma\,(\beta+\gamma)}{4\beta^2 + 5\beta\gamma - 2\gamma^2}$ and $\gamma > 0 \;\&\&\; (5 + \sqrt{57})\,\beta > 4\gamma$

L_1 is negative: $\gamma > 0 \;\&\&\; \frac{4\gamma}{5+\sqrt{57}} < \beta <$

$\text{Root}[4\gamma^8 + 10\gamma^7 \#1 - 62\gamma^6 \#1^2 - 151\gamma^5 \#1^3 + 89\gamma^4 \#1^4 + 563\gamma^3 \#1^5 + 688\gamma^2 \#1^6 + 352\gamma \#1^7 + 64 \#1^8 \&, 6]$

L_1 is zero : $\gamma > 0 \;\&\&$

$\beta == \text{Root}[4\gamma^8 + 10\gamma^7 \#1 - 62\gamma^6 \#1^2 - 151\gamma^5 \#1^3 + 89\gamma^4 \#1^4 + 563\gamma^3 \#1^5 + 688\gamma^2 \#1^6 + 352\gamma \#1^7 + 64 \#1^8 \&, 6]$

L_1 is positive: $\gamma > 0 \;\&\&$

$\beta > \text{Root}[4\gamma^8 + 10\gamma^7 \#1 - 62\gamma^6 \#1^2 - 151\gamma^5 \#1^3 + 89\gamma^4 \#1^4 + 563\gamma^3 \#1^5 + 688\gamma^2 \#1^6 + 352\gamma \#1^7 + 64 \#1^8 \&, 6]$

L_1 vanishes for approximately

$\{\beta \to 1.0000, \gamma \to 2.4828, \alpha \to 4.2334\}$, and then L_2 is negative

(69) X ⟶ 2X X+Z ⟶ Y+Z X+Y ⟶ Z Y+Z ⟶ Y

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u\,(-2+v+w)\,\alpha \\ -u\,(v-w)\,\beta \\ v\,(u-w)\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = \frac{\beta(\beta+\gamma)}{\beta-\gamma}$ and $0 < \gamma < \beta$

L_1 is negative:
$\gamma > 0 \,\&\&\, \gamma < \beta < \text{Root}[\gamma^6 + 2\gamma^5 \#1 - 31\gamma^4 \#1^2 - 10\gamma^3 \#1^3 + 13\gamma^2 \#1^4 + 8\gamma \#1^5 + \#1^6 \&, 4]$

L_1 is zero :
$\gamma > 0 \,\&\&\, \beta == \text{Root}[\gamma^6 + 2\gamma^5 \#1 - 31\gamma^4 \#1^2 - 10\gamma^3 \#1^3 + 13\gamma^2 \#1^4 + 8\gamma \#1^5 + \#1^6 \&, 4]$

L_1 is positive:
$\gamma > 0 \,\&\&\, \beta > \text{Root}[\gamma^6 + 2\gamma^5 \#1 - 31\gamma^4 \#1^2 - 10\gamma^3 \#1^3 + 13\gamma^2 \#1^4 + 8\gamma \#1^5 + \#1^6 \&, 4]$

L_1 vanishes for approximately
$\{\beta \to 1.0000, \gamma \to 0.77372, \alpha \to 7.8385\}$, and then L_2 is negative

(70) X ⟶ 2X X+Z ⟶ Y+Z X+Y ⟶ 2Z 2Z ⟶ 0

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u(-2+v+w)\alpha \\ -u(v-w)\beta \\ (u v - w^2)\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = \frac{\beta(\beta+2\gamma)}{\beta-2\gamma}$ and $\gamma > 0 \,\&\&\, \beta > 2\gamma$

L_1 is negative:
$\gamma > 0 \,\&\&\, 2\gamma < \beta < \text{Root}[32\gamma^7 + 112\gamma^6 \#1 + 32\gamma^5 \#1^2 - 96\gamma^4 \#1^3 - 34\gamma^3 \#1^4 + 5\gamma^2 \#1^5 + 6\gamma \#1^6 + \#1^7 \&, 5]$

L_1 is zero :
$\gamma > 0 \,\&\&\, \beta == \text{Root}[32\gamma^7 + 112\gamma^6 \#1 + 32\gamma^5 \#1^2 - 96\gamma^4 \#1^3 - 34\gamma^3 \#1^4 + 5\gamma^2 \#1^5 + 6\gamma \#1^6 + \#1^7 \&, 5]$

L_1 is positive:
$\gamma > 0 \,\&\&\, \beta > \text{Root}[32\gamma^7 + 112\gamma^6 \#1 + 32\gamma^5 \#1^2 - 96\gamma^4 \#1^3 - 34\gamma^3 \#1^4 + 5\gamma^2 \#1^5 + 6\gamma \#1^6 + \#1^7 \&, 5]$

L_1 vanishes for approximately
$\{\beta \to 1.0000, \gamma \to 0.42012, \alpha \to 11.519\}$, and then L_2 is negative

(71) X ⟶ 2X X+Z ⟶ Y+Z X+Y ⟶ 2Z Y+Z ⟶ Y

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -u(-2+v+w)\alpha \\ -u(v-w)\beta \\ v(u-w)\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = \frac{\beta(\beta+\gamma)}{\beta-\gamma}$ and $0 < \gamma < \beta$

L_1 is negative:
$\gamma > 0 \,\&\&\, \gamma < \beta < \text{Root}[\gamma^6 + 2\gamma^5 \#1 - 31\gamma^4 \#1^2 - 10\gamma^3 \#1^3 + 13\gamma^2 \#1^4 + 8\gamma \#1^5 + \#1^6 \&, 4]$

L_1 is zero :
$\gamma > 0 \,\&\&\, \beta == \text{Root}[\gamma^6 + 2\gamma^5 \#1 - 31\gamma^4 \#1^2 - 10\gamma^3 \#1^3 + 13\gamma^2 \#1^4 + 8\gamma \#1^5 + \#1^6 \&, 4]$

L_1 is positive:
$\gamma > 0 \,\&\&\, \beta > \text{Root}[\gamma^6 + 2\gamma^5 \#1 - 31\gamma^4 \#1^2 - 10\gamma^3 \#1^3 + 13\gamma^2 \#1^4 + 8\gamma \#1^5 + \#1^6 \&, 4]$

L_1 vanishes for approximately
$\{\beta \to 1.0000, \gamma \to 0.77372, \alpha \to 7.8385\}$, and then L_2 is negative

(72) X ⟶ 2X X+Z ⟶ 2Y 2Y ⟶ 0 X+Y ⟶ X+Z

ODE after rescaling and reparametrising: $\begin{pmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\end{pmatrix} = \begin{pmatrix} -u\,(-1+w)\,\alpha \\ \left(-v^2 - u\,(v-2\,w)\right)\beta \\ u\,(v-w)\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = 3\,\beta + \gamma$ and $\beta > 0\ \&\&\ \gamma > 0$

L_1 is positive

(73) X ⟶ 2X X+Z ⟶ 2Y Y+Z ⟶ Z X+Y ⟶ X+Z

ODE after rescaling and reparametrising: $\begin{pmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\end{pmatrix} = \begin{pmatrix} -u\,(-1+w)\,\alpha \\ (-u\,(v-2\,w) - v\,w)\,\beta \\ u\,(v-w)\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = 2\,\beta + \gamma$ and $\beta > 0\ \&\&\ \gamma > 0$

L_1 is positive

(74) X ⟶ 2X X+Z ⟶ 2Y Y+Z ⟶ Z 2Y ⟶ 2Z

ODE after rescaling and reparametrising: $\begin{pmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\end{pmatrix} = \begin{pmatrix} -u\,(-1+w)\,\alpha \\ \left(-v^2 + 2\,u\,w - v\,w\right)\beta \\ \left(v^2 - u\,w\right)\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = \dfrac{\beta\,(3\,\beta + \gamma)}{4\,\beta + \gamma}$ and $\beta > 0\ \&\&\ \gamma > 0$

L_1 is positive

(75) X ⟶ 2X X+Z ⟶ 2Y Y+Z ⟶ 2Z 2Z ⟶ 0

ODE after rescaling and reparametrising: $\begin{pmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\end{pmatrix} = \begin{pmatrix} -u\,(-1+w)\,\alpha \\ (u-v)\,w\,\beta \\ -w\,(u - 2\,v + w)\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = \dfrac{\beta\,(\beta + \gamma)}{2\,\beta + \gamma}$ and $\beta > 0\ \&\&\ \gamma > 0$

L_1 is positive

(76) X ⟶ 2X X+Z ⟶ 2Y Y+Z ⟶ 2Z 2Z ⟶ Y

ODE after rescaling and reparametrising: $\begin{pmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\end{pmatrix} = \begin{pmatrix} -u\,(-1+w)\,\alpha \\ w\,(2\,u - 3\,v + w)\,\beta \\ -w\,(u - 3\,v + 2\,w)\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = \dfrac{3\,\beta\,(3\,\beta + 2\,\gamma)}{2\times(3\,\beta + \gamma)}$ and $\beta > 0\ \&\&\ \gamma > 0$

L_1 is positive

(77) X ⟶ 2X X+Z ⟶ Y+Z 2Y ⟶ X+Z Y+Z ⟶ 0

ODE after rescaling and reparametrising: $\begin{pmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\end{pmatrix} = \begin{pmatrix} \left(v^2 + u\,(2 - 3\,w)\right)\alpha \\ \left(-2\,v^2 + 3\,u\,w - v\,w\right)\beta \\ v\,(v-w)\,\gamma \end{pmatrix}$

pair of imaginary eigenvalues: γ =

$$-\frac{\alpha^2+\alpha\beta+15\beta^2-\sqrt{\alpha^4+6\alpha^3\beta+63\alpha^2\beta^2+90\alpha\beta^3+225\beta^4}}{2\alpha+6\beta} \text{ and } \alpha>0\,\&\&\,\beta>0$$

L_1 is positive

(78) X ⟶ 2X   X+Y ⟶ 2Y   2Y ⟶ Z   X+Z ⟶ 0

ODE after rescaling and reparametrising: $\begin{pmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\end{pmatrix}=\begin{pmatrix}-u(-3+2v+w)\alpha\\ (u-v)v\beta\\ (v^2-uw)\gamma\end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = -\frac{\beta\gamma(\beta+\gamma)}{2\beta^2-2\beta\gamma-\gamma^2}$ and $\beta>0\,\&\&\,\beta+\gamma>\sqrt{3}\,\beta$

L_1 is positive

(79) X ⟶ 2X   X+Y ⟶ 2Y   2Y ⟶ 2Z   X+Z ⟶ 0

ODE after rescaling and reparametrising: $\begin{pmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\end{pmatrix}=\begin{pmatrix}-u(-2+v+w)\alpha\\ (u-v)v\beta\\ (v^2-uw)\gamma\end{pmatrix}$

pair of imaginary eigenvalues: $\alpha = -\frac{\beta\gamma(\beta+\gamma)}{\beta^2-2\beta\gamma-\gamma^2}$ and $\beta>0\,\&\&\,\beta+\gamma>\sqrt{2}\,\beta$

L_1 is positive

(80) Y ⟶ 2X   2X ⟶ 2Y   Y+Z ⟶ 2Z   X+Z ⟶ 0

ODE after rescaling and reparametrising: $\begin{pmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\end{pmatrix}=\begin{pmatrix}(-3u^2+4v-uw)\alpha\\ (3u^2-v(2+w))\beta\\ -((u-v)w\gamma)\end{pmatrix}$

pair of imaginary eigenvalues: $\gamma = -\frac{3\alpha\beta(7\alpha+3\beta)}{7\alpha^2+2\alpha\beta-3\beta^2}$ and $\alpha>0\,\&\&\,\alpha+\sqrt{22}\,\alpha<3\beta$

L_1 is positive

(81) Y ⟶ 2X   X+Z ⟶ 2Y   Y+Z ⟶ 2Z   2Z ⟶ 0

ODE after rescaling and reparametrising: $\begin{pmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\end{pmatrix}=\begin{pmatrix}(v-uw)\alpha\\ (-v+4uw-3vw)\beta\\ -w(2u-3v+w)\gamma\end{pmatrix}$

pair of imaginary eigenvalues: $\beta = \frac{1}{8}\times\left(6\alpha-\gamma+\sqrt{52\alpha^2+4\alpha\gamma+\gamma^2}\right)$ and $\alpha>0\,\&\&\,\gamma>0$

L_1 is positive

(82) Y ⟶ X+Y   2X ⟶ Y+Z   Y+Z ⟶ Z   X+Z ⟶ 0

ODE after rescaling and reparametrising: $\begin{pmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\end{pmatrix}=\begin{pmatrix}(-2u^2+3v-uw)\alpha\\ (u^2-vw)\beta\\ u(u-w)\gamma\end{pmatrix}$

pair of imaginary eigenvalues: $\gamma = -\frac{30\alpha^2 + 7\alpha\beta + \beta^2 - \sqrt{900\alpha^4 + 540\alpha^3\beta + 153\alpha^2\beta^2 + 18\alpha\beta^3 + \beta^4}}{2\times(6\alpha + \beta)}$ and $\alpha > 0$ && $\beta > 0$

L_1 is positive

(83) Y ⟶ X+Y 2X ⟶ Y+Z Y+Z ⟶ 2Z X+Z ⟶ 0

ODE after rescaling and reparametrising: $\begin{pmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\end{pmatrix} = \begin{pmatrix}(-u^2 + 2v - uw)\,\alpha\\ (u^2 - vw)\,\beta\\ (u^2 - 2uw + vw)\,\gamma\end{pmatrix}$

pair of imaginary eigenvalues: $\gamma = \frac{-9\alpha^2 - 4\alpha\beta - 2\beta^2 + \sqrt{81\alpha^4 + 108\alpha^3\beta + 88\alpha^2\beta^2 + 24\alpha\beta^3 + 4\beta^4}}{6\alpha + 4\beta}$ and $\alpha > 0$ && $\beta > 0$

L_1 is positive

(84) Y ⟶ X+Y 2X ⟶ Y Y+Z ⟶ 2Z X+Z ⟶ 0

ODE after rescaling and reparametrising: $\begin{pmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\end{pmatrix} = \begin{pmatrix}(-2u^2 + 3v - uw)\,\alpha\\ (u^2 - vw)\,\beta\\ -((u - v)\,w\,\gamma)\end{pmatrix}$

pair of imaginary eigenvalues: $\gamma = \frac{\alpha\beta\,(5\alpha + \beta)}{-5\alpha^2 + \alpha\beta + \beta^2}$ and $\alpha > 0$ && $\sqrt{21}\,\alpha < \alpha + 2\beta$

L_1 is positive

(85) Y ⟶ X+Y 2X ⟶ 2Y Y+Z ⟶ 2Z X+Z ⟶ 0

ODE after rescaling and reparametrising: $\begin{pmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\end{pmatrix} = \begin{pmatrix}(-u^2 + 2v - uw)\,\alpha\\ (u^2 - vw)\,\beta\\ -((u - v)\,w\,\gamma)\end{pmatrix}$

pair of imaginary eigenvalues: $\gamma = -\frac{\alpha\beta\,(3\alpha + \beta)}{3\alpha^2 - \beta^2}$ and $\alpha > 0$ && $\beta > \sqrt{3}\,\alpha$

L_1 is positive

(86) Z ⟶ X+Z 2X ⟶ Y+Z X+Y ⟶ 0 Y+Z ⟶ X+Y

ODE after rescaling and reparametrising: $\begin{pmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\end{pmatrix} = \begin{pmatrix}(-2u^2 - uv + (2 + v)\,w)\,\alpha\\ u\,(u - v)\,\beta\\ (u^2 - vw)\,\gamma\end{pmatrix}$

pair of imaginary eigenvalues: $\beta = -\frac{25\alpha^2 + 7\alpha\gamma + \gamma^2 - \sqrt{625\alpha^4 + 450\alpha^3\gamma + 139\alpha^2\gamma^2 + 18\alpha\gamma^3 + \gamma^4}}{2\times(5\alpha + \gamma)}$ and $\alpha > 0$ && $\gamma > 0$

L_1 is positive

(87) 0 ⟶ X X+Y ⟶ 2Y Y+Z ⟶ 2Z X+Z ⟶ 0

ODE after rescaling and reparametrising: $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} (2 - u (v + w)) \alpha \\ v (u - w) \beta \\ (-u + v) w \gamma \end{pmatrix}$

pair of imaginary eigenvalues: $\beta = \frac{\alpha \gamma}{\alpha - \gamma}$ and $0 < \gamma < \alpha$

L_1 is zero

total elapsed: 205.59 seconds