Interconnectedness of Sovereign Yield curves

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Draft version

Abstract

Abstract: This paper is examining the linkages between the whole tenor structure of the yield curves of 12 sovereigns from all over the globe. The curves got decomposed to level, slope and curvature factors by the Nelson Siegel model. TBC...

1 Methodology

1.1 The Nelson and Siegel framework

Among the statistical models for interest rate, the influential model designed by Diebold-Li [Diebold & Li, 2006] is widely used in market applications. This model is a dynamic extension of the Nelson-Siegel model ([Nelson & Siegel, 1987]) for the cross-section fit for the yield curve. The Nelson-Siegel model corresponds to fitting the following equation for the yield curve observed inthe market on a specific date:

$$y_{it}(m_{it}) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda\tau_t}}{\lambda\tau_t}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda\tau_t}}{\lambda\tau_t} - e^{-\lambda\tau_t}\right) + \epsilon_{it} \tag{1}$$

where $y_{it}(m_{it})$ are the observed rates on a given date i and maturity t, and β_{1t}, β_{2t}, β_{3t} and τ_t are parameters. The Nelson-Siegel model is a parsimonious way of fitting the yield curve while managing to capture a part of the stylized facts in interest rate process, such as the exponential formats present in the yield curves. The parameters β_{it} have economic interpretations, where textbeta$_{1t}$ presents a long-term level interpretation, β_{2t} short-term components, and β_{3t} medium-termcomponents. It may also be interpreted as decompositions of Level, Slope and Curvature of the yield curve, according to the terminology developed by [Litterman & Scheinkman, 1991]. These components may be used directly in the immunization process of interest rate portfolios.

The purpose of these models is to allow fitting, and subsequent interpolationsand extrapolations of the yield curve based on a parametric structure, which concurs with othernon-parametric fitting models such as smoothing-splines. Besides the parsimonious estimation,the [Nelson & Siegel, 1987] model has two additional advantages over non-parametric models. The first advantage is that the extrapolation of the curve has a better performance due to the exponential nature of this model. The second advantage is that the parameters β_{1t}, β_{2t} and β_{3t} have interpretation of level, slope and curvature compatible with the interpretation of three factors proposed by [Litterman & Scheinkman, 1991], a benchmark in literature. This makes theinterpretation and comparison of the results obtained in the curve fitting easier.The extension formulated by [Diebold & Li, 2006] renders the [Nelson & Siegel, 1987] model dynamic (adjusting the several days observed for the yield curve) by means of a procedure in 3 stages:

- The Nelson-Siegel model (with τfixed, thus making the model linear in the parameters) is fitted by Ordinary Least Squares for each date, estimating the parameters β_{1t}, β_{2t}, β_{3t}.
- The dynamics of the system is modelled by a vector autoregressive (VAR) model for the parameters β_{1t}, β_{2t} and β_{3t}, estimated in the first stage.
- Forecasts for these parameters are made through the VAR model estimated for vectors β_{1t}, β_{2t} and β_{3t}. By substituting the forecasted parameters in Nelson-Siegel model given by equation '1' it is possible to forecast future interest rate curves.

According to [Diebold & Li, 2006], this dynamic formulation has the purpose of capturing the set of existing stylized facts in the term structure of interest rates, such as the fact that while the yield curve is crescent and concave, it may also assume inverted shapes like decreasing curves and slope changes. Other stylized facts captured by [Diebold & Li, 2006] models are the high persistencein the temporal dynamics (rates with same maturity are highly dependent on the past), and the fact that persistence in the long-term rates is higher than in the short-term rates.

1.2 The Toda-Yamamoto model

The Toda–Yamamoto procedure begins from the following premise: The implementation of the classic Granger Causality test from a VAR (Vector AutoRegressive) model can lead to non-stationarity problems in the series, as it is necessary to confirm the type of existing cointegration. The authors of ... point out that the "conventional" Wald test produces integrated or cointegrated causal VAR models, which would inevitably lead to obtaining spurious Granger causality relationships. However, the Toda–Yamamoto procedure drastically avoids this handicap by developing a Modified Wald test (MWALD) for restrictions on the parameters of a VAR *(p)* model. This test is generated on a χ_p distribution, with $p = p + d_{max}$ (or number of time lags). In Wolfe-Rufael's words, the fundamental idea underlying this procedure is to "artificially augment the correct VAR order, p, by the maximal order of integration, say d_{max}. Once this is done, a $(p + d_{max})$-th order of VAR is estimated and the coefficients of the last lagged dmax vector are ignored". The resulting VAR $(p + d_{max})$ model is formulated in Equations (3) and (4):

$$Y_t = \alpha_0 + \sum_{i=1}^{k} \delta_{1i} Y_{t-i} + \sum_{j=k+1}^{d_{max}} \alpha_{1j} Y_{t-j} + \sum_{j=1}^{k} \theta_{1j} X_{t-j} + \sum_{j=k+1}^{d_{max}} \beta_{1j} X_{t-j} + \omega_{1t} \tag{2}$$

$$X_t = \alpha_1 + \sum_{i=1}^{k} \delta_{2i} Y_{t-i} + \sum_{j=k+1}^{d_{max}} \alpha_{2j} Y_{t-j} + \sum_{j=1}^{k} \theta_{2j} X_{t-j} + \sum_{j=k+1}^{d_{max}} \beta_{2j} X_{t-j} + \omega_{2t} \tag{3}$$

where ω_{1t} and ω_{2t} are the VAR error terms and d_{max} is the maximum order of integration, according to the original specification of the Toda–Yamamoto procedure. Therefore, in Equation (3), causality in the sense of Granger between X and Y will be detected, provided that $\delta_{1i} \neq 0$ for every i, and, on an identical basis, Equation (4) will imply causality in the sense of Granger between X and Y , if $\delta_{2i} \neq 0$ for every i.

Once the VAR $(p + d_{max})$ model is obtained, the implementation of the Toda–Yamamoto procedure in practice requires the realization of three differentiated steps:

- Testing each time-series to conclude the maximum order of integration d_{max} of the variables by using, individually or jointly, the following tests: ADF (Augmented Dickey–Fuller), KPSS (Kwiatkowski–Phillips–Schmidt–Shin), and/or PPE (Phillips-Perron).
- Next, the optimal lag length (p) should be obtained based on the criteria: AIC (Akaike Information Criterion), FPE (Akaike's Final Prediction Error), SC (Schwartz), HQ (Hannan and Quinn), and LR (Likelihood-Ratio), seeking, as much as possible, an optimal length supported by the maximum degree of unanimity between criteria.
- Finally, the Granger causality test between the variables X and Y (in both directions) is properly performed by considering that the rejection of the null hypothesis implies the existence of causality in the sense of Granger according to the Toda–Yamamoto procedure and that a reciprocal rejection would indicate a bilateral causal relationship between the analyzed variables.

1.3 Multilayer causality based network

We denote the proposed mulitilayer causality based networks as $\overline{\Omega} = \{G^{[1]}, G^{[2]}, \ldots G^{[L]}\}$ with L layers and N nodes, where $\{G^{[a]}\} = G(V,\ A^{[a]})$ is layer a of multilayer causality based networks, $V = \{1, 2, \ldots, N\}$ is the set of nodes, and $A^{[a]}$ is the set of edges of layer a. On each layer, nodes represent a yield curve factor, and a directed edge indicates that there is a corresponding causality effect from the staring node to the terminal one. In our case L=3, and we assume that the first layer, the second layer and the third layer corresponds to level, slope and curvature layers, respectively. For any two factors $i,j \in V$, we draw a direct edge from i to j on the first (second, third) layer, if node i has a level (slope, curvature) causing effect on node j. $A^{[a]}$=$\{a_{ij}^{[a]}\}_{N \times N}$ is a directed binary connection matrix for all pairs of nodes i and j olayer a, where the element $a_{ij}^{[a]}$ in the matrix $A^{[a]}$ is defined as

$$\alpha_{ij} = \begin{cases} 1 & \text{if } i \neq j \text{ and } i \text{ has a correspopnding causality effect on } j \text{ layer } \alpha \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Thus, multilayer information spillover networks are simplified to a 3 dimensional $N \times N$ adjacency matrix by mathematical notation. Considering the unpredictability of the financial system and the dynamic changes of interconnectedness amon the yield curve nodes, we build time-varying multilayer causality based networks using rolling window analysis. TBC...

Table 1: This is the eaxemple table

Maturity	Mean	Std. Dev	Minimum	Maximum	ρ(1)	ρ(10)
Germany						
1 year	0.0256	1.6275	-0.9690	4.6900	1.0000	0.9960
5 years	0.0257	1.6324	-0.9410	4.7670	0.9990	0.9930
10 years	0.0243	1.5451	-0.7220	4.6860	0.9930	0.9640
30 years	0.0228	1.4482	-0.2440	5.1950	0.9990	0.9880
Italy						
1 year	0.0241	1.5327	-0.4840	8.3940	0.9980	0.9860
5 years	0.0239	1.5199	0.2370	7.8950	0.9980	0.9850
10 years	0.0218	1.3876	0.8750	7.4920	0.9980	0.9860
30 years	0.0188	1.1971	2.0430	7.5840	0.9980	0.9850
France						
1 year	0.0250	1.5891	-0.8010	4.6570	1.0000	0.9960
5 years	0.0247	1.5730	-0.7730	4.9100	0.9990	0.9930
10 years	0.0231	1.4670	-0.4150	4.8510	0.9990	0.9910
30 years	0.0194	1.2351	0.4190	5.1160	0.9990	0.9860
USA						
1 year	0.0254	1.6128	0.0540	5.3230	1.0000	0.9980
5 years	0.0194	1.2315	0.5590	5.3010	0.9990	0.9890
10 years	0.0163	1.0394	1.3890	5.3880	0.9980	0.9830
30 years	0.0151	0.9618	1.9920	5.8390	0.9970	0.9760
Canada						
1 year	0.0193	1.2301	0.3000	4.8090	1.0000	0.9950
5 years	0.0180	1.1463	0.4840	4.8010	0.9990	0.9890
10 years	0.0173	1.0976	0.9830	5.0760	0.9990	0.9870
30 years	0.0159	1.0118	1.3060	5.6120	0.9980	0.9850
Mexico						
1 year	0.0318	2.0250	1.5120	10.5700	0.9790	0.9880
5 years	0.0238	1.5108	3.7860	10.8970	0.9860	0.9810
10 years	0.0213	1.3524	4.6190	12.4130	0.9920	0.9700
30 years	0.0195	1.2384	5.8730	12.7260	0.9930	0.9470
Japan						
1 year	0.0043	0.2715	-0.3710	0.8500	0.9990	0.9920
5 years	0.0076	0.4852	-0.3960	1.6310	0.9990	0.9900
10 years	0.0103	0.6563	-0.2850	2.0500	0.9990	0.9900
30 years	0.0120	0.7606	0.0530	3.2950	0.9980	0.9850
China						
1 year	0.0115	0.7296	0.9570	4.3820	0.9920	0.9660
5 years	0.0093	0.5939	1.7820	4.8740	0.9970	0.9730
10 years	0.0090	0.5702	2.4810	5.5030	0.9930	0.9640
30 years	0.0097	0.6149	2.4700	6.0090		
Australia						
1 year	0.0279	1.7749	0.6750	7.3760	0.9990	0.9920
5 years	0.0264	1.6769	0.6390	6.9600	0.9990	0.9900
10 years	0.0235	1.4971	0.8850	6.8730	0.9990	0.9880
30 years	0.0190	1.2069	1.5580	6.8880	0.9980	0.9830
Norway						
1 year	0.0222	1.4100	0.1990	6.2430	0.9990	0.9940
5 years	0.0200	1.2734	0.5450	5.3350	0.9990	0.9910
10 years	0.0188	1.1935	0.8880	5.2760	0.9990	0.9890
30 years	0.0175	1.1127	0.8820	5.2730	0.9990	0.9870
United Kingdom						
1 year	0.0308	1.9558	0.0240	5.8830	0.9990	0.9950
5 years	0.0259	1.6444	0.1610	5.8210	0.9990	0.9910
10 years	0.0223	1.4181	0.4000	5.5430	0.9990	0.9890
30 years	0.0175	1.1099	0.9390	5.0700	0.9990	0.9870
Switzerland						
1 year	0.0195	1.2405	-1.1650	3.3750	1.0000	0.9960
5 years	0.0187	1.1921	-1.1960	3.2000	0.9990	0.9930
10 years	0.0188	1.1976	-1.1380	3.4550	0.9990	0.9910
30 years	0.0173	1.1006	-0.6440	3.7330	0.9980	0.9860

Table 2: This is the eaxemple table

Factor	Mean	Std. Dev	Minimum	Maximum	Jarque-Bera t-statistics	P values
Germany						
Level	2.915	1.535	-0.343	5.413	347.041	0
Slope	-1.856	1.062	-4.544	0.140	210.098	0
Curvature	-3.723	1.720	-7.147	0.732	234.599	0
Italy						
Level	4.784	1.292	1.980	7.998	106.101	0
Slope	-3.427	1.570	-7.009	-0.435	183.385	0
Curvature	-4.251	2.236	-8.604	4.752	194.464	0
France						
Level	3.391	1.367	0.263	5.484	417.881	0
Slope	-2.237	1.191	-4.731	0.016	162.987	0
Curvature	-4.294	1.956	-7.820	1.073	210.076	0
USA						
Level	3.957	0.988	1.881	5.867	323.945	0
Slope	-2.408	1.550	-5.519	0.710	139.147	0
Curvature	-3.633	2.498	-9.577	0.723	228.960	0
Canada						
Level	3.431	1.102	1.232	5.897	274.286	0
Slope	-1.726	1.228	-4.839	0.583	251.083	0
Curvature	-2.493	1.627	-6.257	1.307	206.333	0
Mexico						
Level	8.579	1.282	5.550	13.413	834.740	0
Slope	-2.356	1.825	-6.135	0.674	340.695	0
Curvature	-4.157	2.852	-14.835	0.489	321.622	0
Japan						
Level	1.703	0.828	-0.018	3.256	406.313	0
Slope	-1.280	0.625	-2.827	-0.019	155.924	0
Curvature	-3.694	1.283	-6.033	-0.874	278.292	0
China						
Level	4.024	0.616	2.704	6.523	2047.454	0
Slope	-1.531	0.809	-3.869	1.648	275.315	0
Curvature	-1.243	0.924	-5.198	1.259	1128.360	0
Australia						
Level	4.629	1.230	1.398	6.773	262.680	0
Slope	-0.898	0.980	-3.868	1.003	105.808	0
Curvature	-2.081	1.839	-6.590	2.247	296.577	0
Norway						
Level	3.215	1.124	1.024	5.228	333.917	0
Slope	-1.223	1.072	-4.041	2.262	167.102	0
Curvature	-1.586	1.223	-4.681	1.726	337.556	0
United Kingdom						
Level	3.663	1.232	0.867	5.798	329.213	0
Slope	-1.759	1.746	-5.418	1.346	211.589	0
Curvature	-3.329	3.021	-8.767	3.647	159.960	0
Switzerland						
Level	1.773	1.176	-0.756	3.856	289.824	0
Slope	-1.181	0.699	-3.323	0.909	219.716	0
Curvature	-2.940	1.211	-7.766	0.616	543.782	0

Table 3: ADF

Country	GER		ITA		FRA		US		CAN		MXN	
	value	P	value	P	value	P	value	P	value	P	value	P
Level	-2.3040	0.4496	-1.4617	0.8063	-2.1607	0.5103	-3.6572	0.0271	-3.0599	0.1295	-3.9732	0.0103
Slope	-2.0878	0.5412	-2.2616	0.4676	-1.6409	0.7304	-1.5000	0.7900	-1.5719	0.7596	-1.8809	0.6288
Curvature	-2.2176	0.4862	-3.5924	0.0334	-2.3566	0.4273	-1.6211	0.7388	-2.3508	0.4298	-2.7054	0.2796

Country	JPY		CHN		AUS		NEK		UK		SWI	
	value	P	value	P	value	P	value	P	value	P	value	P
Level	-2.9719	0.1668	-3.6926	0.0243	-2.9007	0.1969	-2.6295	0.3118	-2.1593	0.5109	-2.4985	0.3672
Slope	-4.8506	0.0100	-3.6893	0.0245	-2.3030	0.4500	-2.7244	0.2716	-0.9488	0.9470	-3.1354	0.0990
Curvature	-2.0320	0.5648	-5.5214	0.0100	-2.6118	0.3193	-3.6953	0.0242	-1.6153	0.7412	-3.1377	0.0986

Table 4: KPSS

Country	GER		ITA		FRA		US		CAN		MXN	
	value	P	value	P	value	P	value	P	value	P	value	P
Level	32.897	0.010	14.577	0.010	29.872	0.010	26.849	0.010	32.182	0.010	15.235	0.010
Slope	4.255	0.010	7.608	0.010	4.501	0.010	5.301	0.010	5.403	0.010	4.780	0.010
Curvature	8.958	0.010	10.208	0.010	15.213	0.010	7.339	0.010	4.590	0.010	4.926	0.010

Country	JPY		CHN		AUS		NEK		UK		SWI	
	value	P	value	P	value	P	value	P	value	P	value	P
Level	33.938	0.010	3.546	0.010	29.172	0.010	30.064	0.010	28.697	0.010	31.964	0.010
Slope	33.208	0.010	14.992	0.010	7.317	0.010	1.251	0.010	7.531	0.010	5.485	0.010
Curvature	15.946	0.010	2.912	0.010	24.672	0.010	11.552	0.010	13.325	0.010	5.669	0.010

Table 5: ADF(1)

Country	GER		ITA		FRA		US		CAN		MXN	
	value	P	value	P	value	P	value	P	value	P	value	P
Level	-17.051	0.010	-16.015	0.010	-15.986	0.010	-15.788	0.010	-16.353	0.010	-16.557	0.010
Slope	-15.899	0.010	-14.628	0.010	-14.593	0.010	-16.547	0.010	-16.323	0.010	-15.194	0.010
Curvature	-18.514	0.010	-18.671	0.010	-17.646	0.010	-17.454	0.010	-16.016	0.010	-18.880	0.010

Country	JPY		CHN		AUS		NEK		UK		SWI	
	value	P	value	P	value	P	value	P	value	P	value	P
Level	-16.555	0.010	-16.990	0.010	-15.609	0.010	-16.432	0.010	-16.484	0.010	-15.573	0.010
Slope			-18.902	0.010	-17.872	0.010	-17.605	0.010	-16.116	0.010	-14.598	0.010
Curvature	-17.512	0.010			-18.061	0.010	-17.041	0.010	-16.451	0.010	-14.266	0.010

Table 6: KPSS(1)

Country	GER		ITA		FRA		US		CAN		MXN	
	value	P	value	P	value	P	value	P	value	P	value	P
Level	0.039	0.100	0.127	0.100	0.058	0.100	0.026	0.100	0.060	0.100	0.083	0.100
Slope	0.083	0.100	0.073	0.100	0.118	0.100	0.158	0.100	0.171	0.100	0.091	0.100
Curvature	0.050	0.100	0.015	0.100	0.048	0.100	0.084	0.100	0.076	0.100	0.011	0.100

Country	JPY		CHN		AUS		NEK		UK		SWI	
	value	P	value	P	value	P	value	P	value	P	value	P
Level	0.026	0.100	0.156	0.100	0.049	0.100	0.036	0.100	0.073	0.100	0.042	0.100
Slope	0.027	0.100	0.029	0.100	0.042	0.100	0.084	0.100	0.242	0.100	0.137	0.100
Curvature	0.059	0.100	0.045	0.100	0.037	0.100	0.030	0.100	0.160	0.100	0.045	0.100

Table 7: Engle-Granger test

	Level	Slope	Curvature
Level	75.694%	44.444%	64.583%
Slope	29.167%	74.306%	71.528%
Curvature	29.167%	78.472%	75.000%

Table 8: Edge counts

	Level	Slope	Curvature	All
Level	29	38	52	119
Slope	30	41	46	117
Curvature	25	34	23	82
All	84	113	121	318

Table 9: Edge ratio

	Level	Slope	Curvature	All
Level	21.970%	26.389%	36.111%	84.470%
Slope	20.833%	31.061%	31.944%	83.838%
Curvature	17.361%	23.611%	17.424%	58.396%
All	60.164%	81.061%	85.480%	25.238%

Table 10: top nodes

Top 5 all				Top 5 in		Top 5 out		Top 5 net	
Node	All	In	Out	Node	In	Node	Out	Node	Net
USA_L	30	7	23	FRA_C	19	USA_L	23	USA_L	16
AUS_S	28	11	17	NOR_S	16	USA_S	18	USA_S	12
FRA_S	28	14	14	NOR_C	16	USA_C	17	USA_C	10
NOR_S	27	16	11	MEX_L	14	AUS_S	17	DEU_L	9
FRA_C	27	19	8	FRA_S	14	DEU_L	16	CAN_L	7