Interconnectedness of Sovereign Yield curves

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Draft version

Abstract

Abstract: This paper is examining the linkages between the whole tenor structure of the yield curves of 12 sovereigns from all over the globe. The curves got decomposed to level, slope and curvature factors by the Nelson Siegel model. TBC...

1 Methodology

1.1 The Nelson and Siegel framework

Among the statistical models for interest rate, the influential model designed by Diebold-Li [Diebold & Li, 2006] is widely used in market applications. This model is a dynamic extension of the Nelson-Siegel model ([Nelson & Siegel, 1987]) for the cross-section fit for the yield curve. The Nelson-Siegel model corresponds to fitting the following equation for the yield curve observed in the market on a specific date:

$$y_{it}(m_{it}) = \beta_{1t} + \beta_{2t} \left(\frac{1 - e^{-\lambda \tau_t}}{\lambda \tau_t} \right) + \beta_{3t} \left(\frac{1 - e^{-\lambda \tau_t}}{\lambda \tau_t} - e^{-\lambda \tau_t} \right) + \epsilon_{it}$$
 (1)

where $y_{it}(m_{it})$ are the observed rates on a given date i and maturity t, and β_{1t} , β_{2t} , β_{3t} and τ_t are parameters. The Nelson-Siegel model is a parsimonious way of fitting the yield curve while managing to capture a part of the stylized facts in interest rate process, such as the exponential formats present in the yield curves. The parameters β_{it} have economic interpretations, where textbeta_{1t} presents a long-term level interpretation, β_{2t} short-term components, and β_{3t} medium-term components. It may also be interpreted as decompositions of Level, Slope and Curvature of the yield curve, according to the terminology developed by [Litterman & Scheinkman, 1991]. These components may be used directly in the immunization process of interest rate portfolios.

The purpose of these models is to allow fitting, and subsequent interpolations and extrapolations of the yield curve based on a parametric structure, which concurs with othernon-parametric fitting models such as smoothing-splines. Besides the parsimonious estimation,the [Nelson & Siegel, 1987] model has two additional advantages over non-parametric models. The first advantage is that the extrapolation of the curve has a better performance due to the exponential nature of this model. The second advantage is that the parameters β_{1t} , β_{2t} and β_{3t} have interpretation of level, slope and curvature compatible with the interpretation of three factors proposed by [Litterman & Scheinkman, 1991], a benchmark in literature. This makes theinterpretation and comparison of the results obtained in the curve fitting easier. The extension formulated by [Diebold & Li, 2006] renders the [Nelson & Siegel, 1987] model dynamic (adjusting the several days observed for the yield curve) by means of a procedure in 3 stages:

- The Nelson-Siegel model (with τ fixed, thus making the model linear in the parameters) is fitted by Ordinary Least Squares for each date, estimating the parameters β_{1t} , β_{2t} , β_{3t} .
- The dynamics of the system is modelled by a vector autoregressive (VAR) model for the parameters β_{1t} , β_{2t} and β_{3t} , estimated in the first stage.
- Forecasts for these parameters are made through the VAR model estimated for vectors β_{1t} , β_{2t} and β_{3t} . By substituting the forecasted parameters in Nelson-Siegel model given by equation '1' it is possible to forecast future interest rate curves.

According to [Diebold & Li, 2006], this dynamic formulation has the purpose of capturing the set of existing stylized facts in the term structure of interest rates, such as the fact that while the yield curve is crescent and concave, it may also assume inverted shapes like decreasing curves and slope changes. Other stylized facts captured by [Diebold & Li, 2006] models are the high persistence in the temporal dynamics (rates with same maturity are highly dependent on the past), and the fact that persistence in the long-term rates is higher than in the short-term rates.

1.2 The Toda-Yamamoto model

The Toda–Yamamoto procedure begins from the following premise: The implementation of the classic Granger Causality test from a VAR (Vector AutoRegressive) model can lead to non-stationarity problems in the series, as it is necessary to confirm the type of existing cointegration. The authors of ... point out that the "conventional" Wald test produces integrated or cointegrated causal VAR models, which would inevitably lead to obtaining spurious Granger causality relationships. However, the Toda–Yamamoto procedure drastically avoids this handicap by developing a Modified Wald test (MWALD) for restrictions on the parameters of a VAR (p) model. This test is generated on a χ_p distribution,

with $p = p + d_{max}$ (or number of time lags). In Wolfe-Rufael's words, the fundamental idea underlying this procedure is to "artificially augment the correct VAR order, p, by the maximal order of integration, say d_{max} . Once this is done, a $(p + d_{max})$ -th order of VAR is estimated and the coefficients of the last lagged dmax vector are ignored". The resulting VAR $(p + d_{max})$ model is formulated in Equations (3) and (4):

$$Y_{t} = \alpha_{0} + \sum_{i=1}^{k} \delta_{1i} Y_{t-i} + \sum_{j=k+1}^{d_{max}} \alpha_{1j} Y_{t-j} + \sum_{j=1}^{k} \theta_{1j} X_{t-j} + \sum_{j=k+1}^{d_{max}} \beta_{1j} X_{t-j} + \omega_{1t}$$

$$(2)$$

$$X_{t} = \alpha_{1} + \sum_{i=1}^{k} \delta_{2i} Y_{t-i} + \sum_{j=k+1}^{d_{max}} \alpha_{2j} Y_{t-j} + \sum_{j=1}^{k} \theta_{2j} X_{t-j} + \sum_{j=k+1}^{d_{max}} \beta_{2j} X_{t-j} + \omega_{2t}$$

$$(3)$$

where ω_{1t} and ω_{2t} are the VAR error terms and d_{max} is the maximum order of integration, according to the original specification of the Toda–Yamamoto procedure. Therefore, in Equation (3), causality in the sense of Granger between X and Y will be detected, provided that $\delta_{1i} \neq 0$ for every i, and, on an identical basis, Equation (4) will imply causality in the sense of Granger between X and Y, if $\delta_{2i} \neq 0$ for every i.

Once the VAR $(p + d_{max})$ model is obtained, the implementation of the Toda–Yamamoto procedure in practice requires the realization of three differentiated steps:

- Testing each time-series to conclude the maximum order of integration d_{max} of the variables by using, individually or jointly, the following tests: ADF (Augmented Dickey–Fuller), KPSS (Kwiatkowski–Phillips–Schmidt–Shin), and/or PPE (Phillips-Perron).
- Next, the optimal lag length (p) should be obtained based on the criteria: AIC (Akaike Information Criterion), FPE (Akaike's Final Prediction Error), SC (Schwartz), HQ (Hannan and Quinn), and LR (Likelihood-Ratio), seeking, as much as possible, an optimal length supported by the maximum degree of unanimity between criteria.
- Finally, the Granger causality test between the variables X and Y (in both directions) is properly performed by considering that the rejection of the null hypothesis implies the existence of causality in the sense of Granger according to the Toda–Yamamoto procedure and that a reciprocal rejection would indicate a bilateral causal relationship between the analyzed variables.

1.3 Multilayer causality based network

We denote the proposed multilayer causality based networks as $\overline{\Omega} = \{G^{[1]}, G^{[2]}, \dots G^{[L]}\}$ with L layers and N nodes, where $\{G^{[a]}\} = G(V, A^{[a]})$ is layer a of multilayer causality based networks, $V = \{1, 2, \dots, N\}$ is the set of nodes, and $A^{[a]}$ is the set of edges of layer a. On each layer, nodes represent a yield curve factor, and a directed edge indicates that there is a corresponding causality effect from the staring node to the terminal one. In our case L=3, and we assume that the first layer, the second layer and the third layer corresponds to level, slope and curvature layers, respectively. For any two factors $i,j \in V$, we draw a direct edge from i to j on the first (second, third) layer, if node i has a level (slope, curvature) causing effect on node j. $A^{[a]} = \{a_{ij}^{[a]}\}_{N \times N}$ is a directed binary connection matrix for all pairs of nodes i and j olayer a, where the element $a_{ij}^{[a]}$ in the matrix $A^{[a]}$ is defined as

$$\alpha_{ij} = \begin{cases} 1 & \text{if } i \neq j \text{ and } i \text{ has a corresponding causality effect on } j \text{ layer } \alpha \\ 0 & \text{otherwise} \end{cases}$$
 (4)

Thus, multilayer information spillover networks are simplified to a 3 dimensional $N \times N$ adjacency matrix by mathematical notation. Considering the unpredictability of the financial system and the dynamic changes of interconnectedness amon the yield curve nodes, we build time-varying multilayer causality based networks using rolling window analysis. TBC...

2 Data

We obtain daily sovereign yield data from Bloomberg for twelve advanced countries. Our scope is covering multiple reginos of the globe, therefore we picked countries from differenct continents. Eventually four regions were identified with three sovereign each: Pacific (Australia, China, Japan); Americas (Canada, Mexico, United States of America), Eurozone (France, Germany, Italy) and the Non-Eurozone (Great Britain, Norway, Switzerland). We are denoting them with the three letter country codes obtained from The World Bank. Therse are AUS, CHN, JPN, CAD, MEX, USA, FRA, DEU, ITA, GBR, NOR, CHE respectively. The empirical analysis focuses on whole tenor structure of

government yields with fifteen maturities: 3, 6, 12, 24, 36, 48, 60, 72, 84, 96, 120, 180, 240 and 360 months. Below is the table consists of the corresponding Bloomberg iddentification codes for the sovereigns.

Table 1: This is the eaxemple table

	-
Series ID	Description
AUS	Australia Sovereign (IYC 1)
$_{\rm CHN}$	China (IYC 299) Zero Coupon Yields
$_{ m JPN}$	Japan Šovereign (IYC 18) Zero Coupon Yields
CAN	Canada Sovereign (IYC 7) Zero Coupon Yields
MEX	Mexico (IYC 251) Zero Coupon Yields
USA	Treasury Actives (IYC 25) Zero Coupon Yields
FRA	France Sovereign (IYC 14) Zero Coupon Yields
GER	German Sovereign (IYC 16) Zero Coupon Yields
ITA	Italy Sovereign (IYC 40) Zero Coupon Yields
GBR	United Kingdom (IYC 22) Zero Coupon Yields
NOR	Norway (IYC 78) Zero Coupon Yields
CHE	Switzerland (IYĆ 82) Zero Coupon Yields

The data spawns from July 1 2004 to December 31 2019 which means 4045 data points alltogether. This unusual choose of starting date is the reason of this being the first observation date for the Chinese yield curve. Additionally we were not keen on examining the recent consequences of the Covid 19 outbreak thus we ended our analysis period at the end of 2019.

The sovereign bond yields of the mentioned sovereigns are denominated in local currency terms. Local currency debt indicates divergent interest rate cycles of the economy. The local currency bonds reflect the domestic monetary and economic policy stance. Thus the co-movement of local currency bond yields reveals the convergence of monetary policy and business cycles. Moreover the local currency denominated debt possesses greater liquidity and better credit quality compared to the USD dominated debt. The input data used in this study reflects extracted latent factors of each country using their zero coupon yield rates that mirror their domestic term structure. Table 1 presents the descriptive statistics of bond yields at representative maturities for countries considered in the study. The yield curves are upward sloping for all sample countries (CHECK CRYSIS).

The Dynamic Nelson Siegel model was used to extract the latent factors, Level, Slope and Curvature, for each country separately, following Diebold et al. (2006) and Diebold et al. (2008). The level representing long-term interest rates indicates the expected inflation in the long run; the slope representing short-term interest rates reflects the reaction of monetary policy to the cyclical state of the economy; the curvature implies medium-term interest rates (Aguiar-Conraria et al., 2012).

Table 2: This is the eaxemple table

3.5			2.51		(4)	(4.0)
Maturity	Mean	Std. Dev	Minimum	Maximum	$\rho(1)$	ρ(10)
Germany						
1 year	0.0256	1.6275	-0.9690	4.6900	1.0000	0.9960
5 years	0.0257	1.6324	-0.9410	4.7670	0.9990	0.9930
10 years	0.0243	1.5451	-0.7220	4.6860	0.9930	0.9640
30 years	0.0228	1.4482	-0.2440	5.1950	0.9990	0.9880
Italy						
1 year	0.0241	1.5327	-0.4840	8.3940	0.9980	0.9860
5 years	0.0239	1.5199	0.2370	7.8950	0.9980	0.9850
10 years	0.0218	1.3876	0.8750	7.4920	0.9980	0.9860
30 years	0.0188	1.1971	2.0430	7.5840	0.9980	0.9850
France						
1 year	0.0250	1.5891	-0.8010	4.6570	1.0000	0.9960
5 years	0.0247	1.5730	-0.7730	4.9100	0.9990	0.9930
10 years	0.0231	1.4670	-0.4150	4.8510	0.9990	0.9910
30 years	0.0194	1.2351	0.4190	5.1160	0.9990	0.9860
•	0.0101	1.2001	0.1100	3.1130	0.0000	0.0000
USA	0.0054	1 (100	0.0540	r 2020	1 0000	0.0000
1 year	0.0254	1.6128	0.0540	5.3230	1.0000	0.9980
5 years	0.0194	1.2315	0.5590	5.3010	0.9990	0.9890
10 years	0.0163	1.0394	1.3890	5.3880	0.9980	0.9830
30 years $Canada$	0.0151	0.9618	1.9920	5.8390	0.9970	0.9760
1 year	0.0193	1.2301	0.3000	4.8090	1.0000	0.9950
5 years	0.0133 0.0180	1.1463	0.4840	4.8010	0.9990	0.9890
10 years	0.0130 0.0173	1.1405 1.0976	0.9830	5.0760	0.9990	0.9870
30 years	0.0179	1.0118	1.3060	5.6120	0.9980	0.9850
·	0.0100	1.0110	1.0000	0.0120	0.0000	0.0000
Mexico	0.0010	2.0250	1 5100	10 5500	0.0500	0.0000
1 year	0.0318	2.0250	1.5120	10.5700	0.9790	0.9880
5 years	0.0238	1.5108	3.7860	10.8970	0.9860	0.9810
10 years	0.0213	1.3524	4.6190	12.4130	0.9920	0.9700
30 years	0.0195	1.2384	5.8730	12.7260	0.9930	0.9470
Japan 1 year	0.0043	0.2715	-0.3710	0.8500	0.9990	0.9920
5 years	0.0045 0.0076	0.2713 0.4852	-0.3960	1.6310	0.9990	0.9920
10 years	0.0103	0.4562 0.6563	-0.2850	2.0500	0.9990	0.9900
30 years	0.0100	0.7606	0.0530	3.2950	0.9980	0.9850
*	0.0120	0.1000	0.0000	0.2000	0.0000	0.0000
China	0 0445	0 = 200		4 0000	0.0000	0.0000
1 year	0.0115	0.7296	0.9570	4.3820	0.9920	0.9660
5 years	0.0093	0.5939	1.7820	4.8740	0.9970	0.9730
10 years	0.0090	0.5702	2.4810	5.5030	0.9930	0.9640
30 years	0.0097	0.6149	2.4700	6.0090		
Australia						
1 year	0.0279	1.7749	0.6750	7.3760	0.9990	0.9920
5 years	0.0264	1.6769	0.6390	6.9600	0.9990	0.9900
10 years	0.0235	1.4971	0.8850	6.8730	0.9990	0.9880
30 years	0.0190	1.2069	1.5580	6.8880	0.9980	0.9830
Norway						
1 year	0.0222	1.4100	0.1990	6.2430	0.9990	0.9940
5 years	0.0200	1.2734	0.5450	5.3350	0.9990	0.9910
10 years	0.0188	1.1935	0.8880	5.2760	0.9990	0.9890
30 years	0.0175	1.1127	0.8820	5.2730	0.9990	0.9870
United Kingdom	0.0308	1.9558	0.0240	5.8830	0.9990	0.9950
1 year 5 years	0.0308 0.0259	1.9558 1.6444	$0.0240 \\ 0.1610$	5.8210	0.9990	0.9930 0.9910
10 years	0.0239 0.0223	1.4444 1.4181	0.1010 0.4000	5.5430	0.9990	0.9810 0.9890
30 years	0.0225 0.0175	1.1099	0.9390	5.0700	0.9990	0.9870
	0.0210	000	2.0000	2.0.00	0.0000	2.0010
Switzerland	0.0105	1 0 405	1 1050	9.9550	1 0000	0.0000
1 year	0.0195	1.2405	-1.1650	$\frac{3.3750}{2.2000}$	1.0000	0.9960
5 years	0.0187	1.1921	-1.1960	$\frac{3.2000}{2.4550}$	0.9990	0.9930
10 years	0.0188	1.1976	-1.1380 0.6440	$\frac{3.4550}{3.7330}$	0.9990	0.9910
30 years	0.0173	1.1006	-0.6440	3.7330	0.9980	0.9860

Table 2 presents the descriptive statistics of the estimated latent level, slope and curvature factors. The average level factor was positive for all of the countries and was highest for Mexico and lowest for Japan. The average slope showed the typical pattern of ascending yield curves (negative values) (Aguiar-Conraria et al., 2012). The slope is interpreted as the difference between short-term interest rates (3 months), and the long-term interest rates (120 months 360?). Slope was negative for all of the countries, indicating that long-term rates were higher than the short-term rates. The average slope was highest (in absoluter terms) in the US (-3.427) and lowest (in absoluter terms) in Australia (-0.898). The positive values for the slope indicate brief episodes associated with restrictive monetary policies. The curvature also takes negative values for all counties. It was highest in France and lowest in China.

The raw series of level, slope and curvature were tested for normality using Jarque-Bera test. All 36 values failed to meet the criterion, thus we can state that neither time series follow normal distribution. Additionally the factor values were tested fot unit roots using the Augmented Dickey-Fuller (ADF) and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests. China curvature and Japan slope found to be stationary (according to ADF) on standard 95% confidence level. Differentiating the remaining time series became stationary according to both unit root tests.

Undiferentiated time series were tested for cointegration with pairwise Engle Granger test. Table 4 consists of the ratio of the cointegrated and non-cointegrated time series grouuped by the factors. Taken with self results are more that 70%, additionally Curvature - Slope and Slope-Curvature pairs exceeds this level too. These results indicate the need of the Toda-Yamamoto approach.

Table 3: This is the eaxemple table

Factor	Mean	Std. Dev	Minimum	Maximum	Jarque-Bera t-statistics	P values
Germany Level Slope Curvature	2.915 -1.856 -3.723	1.535 1.062 1.720	-0.343 -4.544 -7.147	5.413 0.140 0.732	347.041 210.098 234.599	0 0 0
Italy Level Slope Curvature	4.784 -3.427 -4.251	1.292 1.570 2.236	1.980 -7.009 -8.604	7.998 -0.435 4.752	106.101 183.385 194.464	0 0 0
France Level Slope Curvature	3.391 -2.237 -4.294	1.367 1.191 1.956	0.263 -4.731 -7.820	5.484 0.016 1.073	417.881 162.987 210.076	0 0 0
USA Level Slope Curvature	3.957 -2.408 -3.633	0.988 1.550 2.498	1.881 -5.519 -9.577	5.867 0.710 0.723	323.945 139.147 228.960	0 0 0
Canada Level Slope Curvature	3.431 -1.726 -2.493	1.102 1.228 1.627	1.232 -4.839 -6.257	5.897 0.583 1.307	274.286 251.083 206.333	0 0 0
Mexico Level Slope Curvature	8.579 -2.356 -4.157	1.282 1.825 2.852	5.550 -6.135 -14.835	13.413 0.674 0.489	834.740 340.695 321.622	0 0 0
Japan Level Slope Curvature	1.703 -1.280 -3.694	0.828 0.625 1.283	-0.018 -2.827 -6.033	3.256 -0.019 -0.874	$406.313 \\ 155.924 \\ 278.292$	0 0 0
China Level Slope Curvature	4.024 -1.531 -1.243	0.616 0.809 0.924	2.704 -3.869 -5.198	6.523 1.648 1.259	2047.454 275.315 1128.360	0 0 0
Australia Level Slope Curvature	4.629 -0.898 -2.081	1.230 0.980 1.839	1.398 -3.868 -6.590	6.773 1.003 2.247	262.680 105.808 296.577	0 0 0
Norway Level Slope Curvature	3.215 -1.223 -1.586	1.124 1.072 1.223	1.024 -4.041 -4.681	5.228 2.262 1.726	333.917 167.102 337.556	0 0 0
United Kingdom Level Slope Curvature	3.663 -1.759 -3.329	1.232 1.746 3.021	0.867 -5.418 -8.767	5.798 1.346 3.647	$\begin{array}{c} 329.213 \\ 211.589 \\ 159.960 \end{array}$	0 0 0
Switzerland Level Slope Curvature	1.773 -1.181 -2.940	1.176 0.699 1.211	-0.756 -3.323 -7.766	3.856 0.909 0.616	289.824 219.716 543.782	0 0 0

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Country	GE	ER	IT	Ά	FR	άA	U	S	CA	N	MX	N
	value	P	value	Ρ	value	Ρ	value	Р	value	Ρ	value	P
Level Slope Curvature	-2.304 -2.088 -2.218	$0.450 \\ 0.541 \\ 0.486$	-1.461 -2.262 -3.592	$0.806 \\ 0.468 \\ 0.033$	-2.161 -1.641 -2.357	$0.510 \\ 0.730 \\ 0.427$	-3.657 -1.500 -1.621	0.027 0.790 0.739	-3.060 -1.572 -2.351	$0.129 \\ 0.760 \\ 0.430$	-3.9732 -1.881 -2.705	0.010 0.629 0.279
Country	JP	Ϋ́	CH	IN	AU	JS	NE	EK	U.	K	SW	
	value	Р	value	P								
Level Slope Curvature	-2.972 -4.851 -2.032	$0.167 \\ 0.010 \\ 0.565$	-3.693 -3.689 -5.521	$0.024 \\ 0.025 \\ 0.010$	-2.901 -2.303 -2.612	0.197 0.450 0.319	-2.630 -2.724 -3.695	$0.312 \\ 0.272 \\ 0.024$	-2.159 -0.949 -1.615	0.511 0.947 0.741	-2.499 -3.135 -3.138	0.367 0.099 0.099

Table 5: KPSS

Country	GE	ER	IT	A	FR	A	U	S	CA	N	MX	IN
	value	P	value	Ρ	value	Ρ	value	P	value	P	value	P
Level Slope Curvature	32.897 4.255 8.958	$0.010 \\ 0.010 \\ 0.010$	14.577 7.608 10.208	$0.010 \\ 0.010 \\ 0.010$	29.872 4.501 15.213	$0.010 \\ 0.010 \\ 0.010$	26.849 5.301 7.339	$0.010 \\ 0.010 \\ 0.010$	32.182 5.403 4.590	$0.010 \\ 0.010 \\ 0.010$	15.235 4.780 4.926	0.010 0.010 0.010
Country	JP value	Y P	CH value	IN P	AU value	JS P	NE value	EK P	Ul value	K P	SV value	VI P
Level Slope Curvature	33.938 33.208 15.946	$0.010 \\ 0.010 \\ 0.010$	3.546 14.992 2.912	$0.010 \\ 0.010 \\ 0.010$	29.172 7.317 24.672	$0.010 \\ 0.010 \\ 0.010$	30.064 1.251 11.552	$0.010 \\ 0.010 \\ 0.010$	28.697 7.531 13.325	$0.010 \\ 0.010 \\ 0.010$	31.964 5.485 5.669	0.010 0.010 0.010

Table 6: ADF(1)

Country	GE	R	IT	A	FR	A	US	3	CA	N	MX	.N
	value	P	value	P	value	P	value	P	value	P	value	Р
Level	-17.051	0.010	-16.015	0.010	-15.986	0.010	-15.788	0.010	-16.353	0.010	-16.557	0.010
Slope	-15.899	0.010	-14.628	0.010	-14.593	0.010	-16.547	0.010	-16.323	0.010	-15.194	0.010
Curvature	-18.514	0.010	-18.671	0.010	-17.646	0.010	-17.454	0.010	-16.016	0.010	-18.880	0.010
Country	JP	Y	СН	N	AU	S	NE	<u></u> К	UF	ζ	SW	
Country	JP value	Y P	CH value	N P	AU value	S P	NE value	K P	UI value	γ P	SW	T P
Country Level Slope	_		_				_		_			_

Table 7: KPSS(1)

Country	GI	ΞR	ΙΊ	ΓA	FF	RA	U	TS.	CA	AN	M	XN
	value	Р	value	Р	value	Ρ	value	Р	value	Ρ	value	P
Level Slope Curvature	$0.039 \\ 0.083 \\ 0.050$	$0.100 \\ 0.100 \\ 0.100$	$0.127 \\ 0.073 \\ 0.015$	$0.100 \\ 0.100 \\ 0.100$	$0.058 \\ 0.118 \\ 0.048$	$0.100 \\ 0.100 \\ 0.100$	$0.026 \\ 0.158 \\ 0.084$	$0.100 \\ 0.100 \\ 0.100$	$0.060 \\ 0.171 \\ 0.076$	$0.100 \\ 0.100 \\ 0.100$	$0.083 \\ 0.091 \\ 0.011$	$0.100 \\ 0.100 \\ 0.100$
Country	JF value	PY P	CI value	HN P	Al value	US P	NI value	EK P	U value	K P	SV value	VI P
Level Slope Curvature	$0.026 \\ 0.027 \\ 0.059$	$0.100 \\ 0.100 \\ 0.100$	$0.156 \\ 0.029 \\ 0.045$	$0.100 \\ 0.100 \\ 0.100$	0.049 0.042 0.037	$0.100 \\ 0.100 \\ 0.100$	$0.036 \\ 0.084 \\ 0.030$	$0.100 \\ 0.100 \\ 0.100$	0.073 0.242 0.160	0.100 0.100 0.100	$0.042 \\ 0.137 \\ 0.045$	0.100 0.100 0.100

Table 8: Engle-Granger test

	Level	Slope	Curvature
Level	75.694%	44.444%	64.583%
Slope	29.167%	74.306%	71.528%
Curvature	29.167%	78.472%	75.000%

Table 9: Edge counts

	Level	Slope	Curvature	All
Level	29	38	52	119
Slope	30	41	46	117
Curvature	25	34	23	82
All	84	113	121	318

Table 10: Edge ratio

Level Slope Curvature	Level 21.970% 20.833% 17.361%	Slope 26.389% 31.061% 23.611%	Curvature 36.111% 31.944% 17.424%	All 84.470% 83.838% 58.396%
All	60.164%	81.061%	85.480%	25.238%

Table 11: top nodes

	Top 5	all		Top 5 i	in	Top 5	out	Top 5	net
Node	All	In	Out	Node	In	Node	Out	Node	Net
USA_L AUS_S FRA_S NOR_S	30 28 28 27	7 11 14 16	23 17 14 11	FRA_C NOR_S NOR_C MEX_L	19 16 16 14	USA_L USA_S USA_C AUS_S	23 18 17 17	USA_L USA_S USA_C DEU_L	16 12 10
FRA_C	$\frac{27}{27}$	19	8	FRA_S	14	DEU_L	16	CAN_L	7