

**lemma** hnr\_case\_tuple [hnr\_rule]:

**assumes**

" $\bigwedge a\ ai\ b\ bi.$

hnr

( $\Gamma * \text{id\_assn } x\ xi * \text{id\_assn } a\ ai * \text{id\_assn } b\ bi$ )

( $ci\ ai\ bi$ )

( $\Gamma_a\ a\ ai\ b\ bi$ )

( $c\ a\ b$ )"

" $\bigwedge a\ ai\ b\ bi\ ri\ r.$  Keep\_Drop ( $\Gamma_a\ a\ ai\ b\ bi\ r\ ri$ ) ( $\Gamma_a'\ r\ ri$ ) ( $\Gamma\text{Drop } a\ ai\ b\ bi\ r\ ri$ )"

" $\bigwedge r\ ri.$  Norm ( $\Gamma_a'\ r\ ri$ ) ( $\Gamma_a''\ r\ ri$ )"

**shows**

"hnr ( $\Gamma * \text{id\_assn } x\ xi$ ) ( $\text{case } xi\ \text{of } (ai, bi) \Rightarrow ci\ ai\ bi$ )  $\Gamma_a''$  ( $\text{case } x\ \text{of } (a, b) \Rightarrow c$ )"

**lemma** hnr\_case\_sum [hnr\_rule]:

**assumes**

" $\bigwedge s'\ si'. \text{ hnr } (\Gamma * \text{id\_assn } s\ si * \text{id\_assn } s'\ si') (cli\ si') (\Gamma_a\ s'\ si') (cl\ s')$ "

" $\bigwedge l'\ li'\ ri\ r.$  Keep\_Drop ( $\Gamma_a\ l'\ li'\ r\ ri$ ) ( $\Gamma_a'\ r\ ri$ ) ( $\text{Drop}_a\ l'\ li'\ r\ ri$ )"

" $\bigwedge r\ ri.$  Norm ( $\Gamma_a'\ r\ ri$ ) ( $\Gamma_a''\ r\ ri$ )"

" $\bigwedge s'\ si'. \text{ hnr } (\Gamma * \text{id\_assn } s\ si * \text{id\_assn } s'\ si') (cri\ si') (\Gamma_b\ s'\ si') (cr\ s')$ "

" $\bigwedge r'\ ri'\ ri\ r.$  Keep\_Drop ( $\Gamma_b\ r'\ ri'\ r\ ri$ ) ( $\Gamma_b'\ r\ ri$ ) ( $\text{Drop}_b\ r'\ ri'\ r\ ri$ )"

" $\bigwedge r\ ri.$  Norm ( $\Gamma_b'\ r\ ri$ ) ( $\Gamma_b''\ r\ ri$ )"

" $\bigwedge r\ ri.$  Merge ( $\Gamma_a''\ r\ ri$ ) ( $\Gamma_b''\ r\ ri$ ) ( $\Gamma_c\ r\ ri$ )"

**shows**

"hnr

( $\Gamma * \text{id\_assn } s\ si$ )

( $\text{case } si\ \text{of } \text{Inl } l \Rightarrow cli\ l \mid \text{Inr } r \Rightarrow cri\ r$ )

$\Gamma_c$

( $\text{case } s\ \text{of } \text{Inl } l \Rightarrow cl\ l \mid \text{Inr } r \Rightarrow cr\ r$ )"

**lemma** hnr\_case\_nat[hnr\_rule]:

**assumes**

"hnr ( $\Gamma * \text{id\_assn } n\ ni$ )  $ci0\ \Gamma_a\ c0$ "

" $\bigwedge n'\ ni'. \text{ hnr } (\Gamma * \text{id\_assn } n\ ni * \text{id\_assn } n'\ ni') (ci\ ni') (\Gamma_b\ n'\ ni') (c\ n')$ "

" $\bigwedge n\ ni\ ri\ r.$  Keep\_Drop ( $\Gamma_b\ n\ ni\ r\ ri$ ) ( $\Gamma_b'\ r\ ri$ ) ( $\text{Drop } n\ ni\ r\ ri$ )"

" $\bigwedge r\ ri.$  Norm ( $\Gamma_b'\ r\ ri$ ) ( $\Gamma_b''\ r\ ri$ )"

" $\bigwedge r\ ri.$  Merge ( $\Gamma_a\ r\ ri$ ) ( $\Gamma_b''\ r\ ri$ ) ( $\Gamma_c\ r\ ri$ )"

**shows**

"hnr

( $\Gamma * \text{id\_assn } n\ ni$ )

( $\text{case } ni\ \text{of } 0 \Rightarrow ci0 \mid \text{Suc } n' \Rightarrow ci\ n'$ )

$\Gamma_c$

( $\text{case } n\ \text{of } 0 \Rightarrow c0 \mid \text{Suc } n' \Rightarrow c\ n'$ )"

**lemma** hnr\_case\_list [hnr\_rule]:

**assumes**

"hnr ( $\Gamma * \text{id\_assn } xs\ xsi$ )  $ci0\ \Gamma_a\ c0$ "

" $\bigwedge x'\ xi'\ xs'\ xsi'.$

hnr

( $\Gamma * \text{id\_assn } xs\ xsi * \text{id\_assn } x'\ xi' * \text{id\_assn } xs'\ xsi'$ )

( $ci\ xi'\ xsi'$ )

( $\Gamma_b\ x'\ xi'\ xs'\ xsi'$ )

( $c\ x'\ xs'$ )"

" $\bigwedge x\ xi\ xs\ xsi\ ri\ r.$  Keep\_Drop ( $\Gamma_b\ x\ xi\ xs\ xsi\ r\ ri$ ) ( $\Gamma_b'\ r\ ri$ ) ( $\text{Drop } x\ xi\ xs\ xsi\ r\ ri$ )"

```
"^r ri. Norm ( $\Gamma_b'$  r ri) ( $\Gamma_b''$  r ri)"
```

```
"^r ri. Merge ( $\Gamma_a$  r ri) ( $\Gamma_b'$  r ri) ( $\Gamma_c$  r ri)"
```

shows

```
"hnr
```

```
( $\Gamma$  * id_assn xs xsi)
```

```
(case xs of []  $\Rightarrow$  ci0 | x#xs  $\Rightarrow$  ci x xs)
```

```
 $\Gamma_c$ 
```

```
(case xs of []  $\Rightarrow$  c0 | x#xs  $\Rightarrow$  c x xs)"
```