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theory Hnr_Base
  imports Base
begin

definition hnr where
  "hnr  $\Gamma$  c  $\Gamma'$  a = (
    case a of None  $\Rightarrow$  True |
    Some a  $\Rightarrow$   $\langle \Gamma \rangle$  c  $\langle \lambda r. \Gamma' a r \rangle_t$ 
  )"

lemma hnr_none [simp]: "hnr  $\Gamma$  c  $\Gamma'$  None"

named_theorems hnr_rule

lemma hnr_hoare: " $(\forall x. a = \text{Some } x \longrightarrow \langle \Gamma \rangle c \langle \lambda r. \Gamma' x r \rangle_t) \longleftrightarrow (\text{hnr } \Gamma c \Gamma' a)$ "

lemmas hnrI = hnr_hoare[THEN iffD1, rule_format]
lemmas hnrD = hnr_hoare[THEN iffD2, rule_format]

definition id_rel where "id_rel a c  $\equiv$  c = a"

abbreviation id_assn where "id_assn a c  $\equiv$   $\uparrow$ (id_rel a c)"

abbreviation array_assn where "array_assn xs xsi  $\equiv$  xsi  $\mapsto_a$  xs"

lemma hnr_post_cons:
  assumes
    "hnr  $\Gamma$  fi  $\Gamma'$  f"
    " $\bigwedge x \text{ xi}. \Gamma' x \text{ xi} \Rightarrow_A (\Gamma'' x \text{ xi})$ "
  shows
    "hnr  $\Gamma$  fi  $\Gamma''$  f"

lemma hnr_const: "hnr  $\Gamma$  (return x) ( $\lambda r \text{ ri}. \Gamma * \text{id\_assn } r \text{ ri}$ ) (Some x)"

lemma hnr_pass_general: "hnr ( $\Gamma x \text{ xi}$ ) (return xi)  $\Gamma$  (Some x)"

lemma hnr_pass: "hnr (id_assn x xi) (return xi) id_assn (Some x)"

end

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