

Transformations from Theoretical H-R Diagrams to C-M Diagrams: Effective Temperatures, Colors and Bolometric Corrections

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Summary. Complete $T_{\text{eff}}:B-V:BC$ scales are presented for supergiants and giants as well as for main sequence stars hotter than the sun. Revisions in the temperatures of hot stars and cool supergiants and recent improvements in the IR and UV flux measurements warrant a revision of the old scales. Separate $T_{\text{eff}}:B-V$ relations are found for each of the luminosity classes V, III and I. Bolometric corrections for supergiants are found to be significantly different at all temperatures from those for less luminous stars. The new compilation differs from Johnson's scales for all hot stars and for cool supergiants.

A detailed analysis is presented of the effects which the new scales have on the transformation of evolutionary tracks from theoretical H-R diagrams to C-M diagrams. Also presented is a comparison between the results obtained with Johnson's scales and those obtained with the new scales for these transformations.

It is found that during core helium-burning at high surface temperatures for relatively massive models a hook-like feature appears in the C-M diagrams. This blue-loop hook is especially of interest in explaining the distribution of hot stars in C-M diagrams of young clusters. Also very important for the interpretation of C-M diagrams is the result that the red giant phases of evolutionary tracks, transformed with the new scales, are about 0.12 mag bluer than if Johnson's scales are used.

Key words: effective temperatures — bolometric corrections — population I stars

Introduction

Customarily C-M diagrams of star clusters are compared to theoretical evolutionary tracks or isochrones by

means of a set of transformations involving relationships between effective temperature, color and bolometric correction. These allow a conversion from theoretical model luminosities to absolute visual magnitudes and from model effective temperatures to $B-V$ colors.

As everyone who has attempted such comparisons knows there is only one compilation of $T_{\text{eff}}:B-V:BC$ scales for supergiants, giants and main sequence stars, that published by Johnson (1966). Since then there have been a number of revisions to these scales, but mainly for hot main sequence stars and late-type giants and supergiants. At this time, due to recent observational advances, notably in the UV and IR, it is possible to construct complete scales based mainly on empirical measurements, rather than on theoretical model atmospheres, that represent substantial improvements over previous ones.

The improvements incorporated in this paper include the new temperatures, colors and bolometric corrections of Code, Davis, Bless and Brown (1974) for stars hotter than the sun, those of Lee (1970) and of Flower (1975, Paper I) for M-type supergiants and giants and the temperature-color relation of Paradijs (1973) for G and K supergiants.

$T_{\text{eff}}:B-V$ Scales

All attempts in the last decade to improve the hot end of the temperature-color scale have been limited to main sequence stars. The most widely used improvement to Johnson's scale, for comparisons between C-M diagrams and evolutionary tracks, is that by Morton and Adams (1968) for stars later than spectral type O and Morton (1969) for O stars. Code, Davis et al. (1974, hereafter called CDBB) have been able to obtain a new scale by relying mainly on new, essentially complete, empirical energy distributions with very little dependence on model atmospheres. They have determined effective temperatures, colors and bolometric corrections for 32 stars (including 9 supergiants), ranging in spectral type from O9.5 to F8, using the recent angular diameter measurements of Hanbury Brown, Davis

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and Allen (1974) in conjunction with empirical spectral energy distributions obtained from a combination of new OAO-2 UV measurements (Bless et al., 1976), new spectrophotometry (Davis and Webb, 1974) and broad-band infrared photometry.

The CDBB measurements of T_{eff} and $B-V$ for hot stars ($T_{\text{eff}} > 6000$ K) and Johnson's calibration for main sequence stars and giants are presented in Figure 1. Two important differences can be seen between the new measurements and Johnson's scales. First, a large disagreement exists between the hot supergiants and Johnson's supergiant scale. This, as will be shown later, is the result of the limited number of supergiants Johnson had available for his treatment. Secondly, the new data separates into three different temperature-color relations for the hottest stars, one for luminosity class V, one for luminosity classes IV, III and II and another for the Ia and Ib supergiants, as shown by the solid lines in Figure 1. There is a definite separation, $\Delta \log T_{\text{eff}} \approx 0.025$, between luminosity classes V and IV.

The curves through the new data shown in Figure 1 were derived from the following linear fits, shown in the inset to Figure 1, to the data for $T_{\text{eff}} > 11000$ K:

$$\begin{aligned} \text{V} \quad \log T_{\text{eff}} &= 3.832 - 2.204 (B-V)_0, \\ &-0.30 \leq B-V \leq 0.12 \end{aligned}$$

$$\begin{aligned} \text{IV, III, II} \quad \log T_{\text{eff}} &= 3.851 - 2.252 (B-V)_0, \\ &-0.29 \leq B-V \leq -0.11 \end{aligned} \quad (1)$$

$$\begin{aligned} \text{Ia, Ib} \quad \log T_{\text{eff}} &= 3.987 - 1.880 (B-V)_0, \\ &-0.25 \leq B-V \leq -0.05. \end{aligned}$$

Due to the uncertainty in the temperature of ζ Pup (Conti 1973, Castor 1974, Mihalas and Hummer 1974), it was not included in the linear fits. Although the temperatures derived from the linear fits are tabulated for very high temperatures, it is probably best to use Conti's (1973) O-star temperatures for $T_{\text{eff}} > 30000$ K. Since the data do not show any separation for the stars cooler than 10000 K, a temperature-color relation was derived from a least squares fit (6th order) to luminosity class V, IV and III stars, including the sun ($T_{\text{eff}} = 5770$ K, $B-V = 0.63$, $BC = -0.07$), measured by CDBB. The derived scales for the hot stars are given in Tables 1 and 2 along with a temperature calibration for the spectral type assuming the color-spectral type relation of FitzGerald (1970) for stars later than B4 and the spectral type temperature relations of Conti (1973) and Panagia (1973) to be correct. The spectral types have only been roughly matched with $B-V$ and serve only as a guide to the spectral type. The separation of the CDBB data is supported by the results obtained from the grid of model atmospheres by Kurucz et al. (1972) who predict exactly such a division from their models with $\log g = 4, 3$ and 2.

The main guide for the scales of effective temperature for cool giants and supergiants has been Johnson's compilation which is based on extensive IR observations. Lee (1970) and Paper I give improvements for

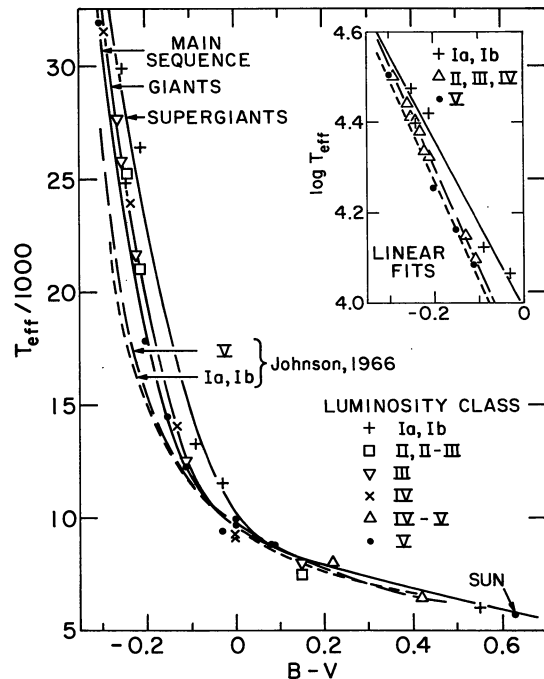


Fig. 1. A comparison is shown between the new effective temperature determinations for 30 stars from Code et al. (1974), the scales derived from this data (solid lines for each of the luminosity classes V, III and I) and the temperature scales of Johnson (1966). The insert shows the linear fits [Eq. (1)] to the new data of Code et al. (1974)

M-type giants and supergiants. Böhm-Vitense (1972) has constructed a theoretical scale for supergiants and Paradijs (1973) has derived a scale from an analysis of line spectra of seven supergiants; these provide important comparisons with Johnson's scales.

The scales for cool supergiants are plotted in Figure 2 (scale a). A rather large discrepancy, ~ 500 K, is shown between the empirical scale of Johnson and the theoretical one of Böhm-Vitense. This temperature difference is related to that found earlier for the hot supergiants; it is partly due to a strong selection effect in luminosity class present in Johnson's sample of stars with known angular diameters, and partly due to a revision of the effective temperatures of the supergiants used by Johnson.

The difficulty encountered by Johnson was a lack of stars with measured angular diameters, only 14. He therefore used a color index to interpolate between his data to derive effective temperatures for other stars. There was, unfortunately, a strong bias in his sample; all stars hotter than 9000 K were main sequence stars while all cooler than 6000 K (no stars with T_{eff} between 9000 and 6000 K were available) were giants except for the two supergiants α Ori and α Sco. Perhaps it doesn't seem surprising that the temperatures of the hot end of his supergiant scale are almost identical to those of his main sequence scale (see Fig. 1).

A recent redetermination of the temperatures of α Ori and α Sco by Dyck et al. (1974) tends to reduce the discrepancy between Johnson and Böhm-Vitense; the new temperatures are 540 and 270 K cooler re-

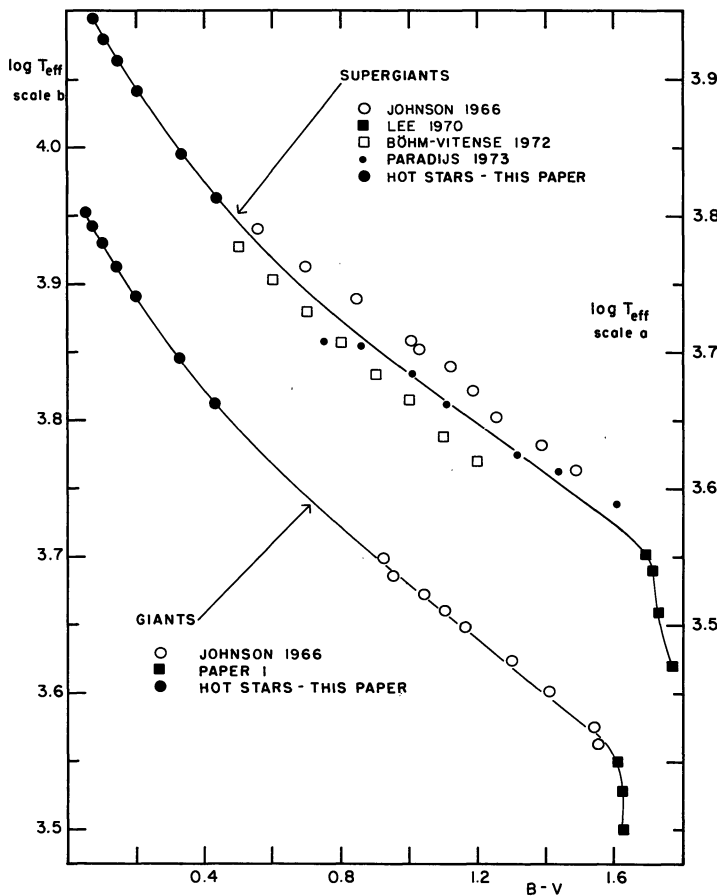


Fig. 2. Scale a shows the curve (solid line) defining the new $T_{\text{eff}}:B-V$ scale for cool supergiants. This curve is from a linear fit for $B-V < 1.14$ to the temperatures of Paradijs (1973). Scale b gives the smooth curve representing the $T_{\text{eff}}:B-V$ scale for giants

spectively than those used by Johnson. These temperatures are in good agreement with the lower temperatures, compared to Johnson's, derived by Paradijs (1973) from an analysis of the line spectra of seven supergiants. The adopted $T_{\text{eff}}:B-V$ relation for supergiants, shown in Figure 2 and given in Table 1, was determined from a linear fit in $\log T_{\text{eff}}$ and $B-V$ to Paradijs's data for $B-V \leq 1.40$, joined to the relation for hot stars and to Lee's scale for cool supergiants. The linear fits to all the scales are listed below along with Kraft's (1961):

$$\begin{aligned}
 \log T_{\text{eff}} &= 3.886 - 0.175 (B-V)_0 & 0.30 \leq B-V \leq 1.10 & \text{Kraft} \\
 \log T_{\text{eff}} &= 3.898 - 0.191 (B-V)_0 & 0.55 \leq B-V \leq 1.50 & \text{Johnson} \\
 \log T_{\text{eff}} &= 3.859 - 0.175 (B-V)_0 & 0.80 \leq B-V \leq 1.40 & \text{Paradijs} \\
 \log T_{\text{eff}} &= 3.889 - 0.226 (B-V)_0 & 0.50 \leq B-V \leq 1.20 & \text{Böhm-Vitense.}
 \end{aligned}
 \tag{2}$$

Note that Kraft's scale, which has been widely used to obtain temperatures for Cepheids, is almost identical to Johnson's. It seems certain that Johnson's scale for supergiants is at least a few hundred degrees too hot. It should also be noted that the relation given by Bell and Parsons (1974) for supergiants with solar

abundances is not as steep as the scale we've adopted. Their scale in comparison is cooler at the blue end and hotter at the red end.

For giants cooler than the sun we have available Johnson's scale for G to M stars and the scale in Paper I for M0 through M9.8 giants. Johnson's calibration is based on temperatures of a half-dozen giants with measured angular diameters. Recent angular diameter determinations for these giants (Gezari et al., 1972) confirm those he used. The scale in Paper I is based on the IR observations of Dyck et al. (1974) using Lee's (1970) colors.

Figure 2 (scale b) shows the $T_{\text{eff}}:B-V$ relation of Paper I, Johnson's scale for giants and our scale for hot stars. The adopted relation was found by connecting, in $\log T_{\text{eff}}$ and $B-V$, Johnson's giant scale, $\log T_{\text{eff}} = 3.883 - 0.201 (B-V)$ for $0.90 \leq B-V \leq 1.55$, to that for the hot giants and to the scale presented in Paper I. It is also tabulated in Table 1.

BC: $B-V$ Scales

Recently there have been three determinations of bolometric corrections for hot main sequence stars: Davis and Webb (1970) for late B to F stars; Beeckmans et al. (1974) for B stars using UV measurements from the European TD-1 satellite; CDBB for O to F stars using UV data from OAO-2. CDBB have also derived bolometric corrections for giants and supergiants. All

Table 1. Temperature Calibration for Giants and Supergiants

Supergiants					Giants				
$B-V$	MK	T_{eff}	$\log T_{\text{eff}}$	BC	$B-V$	MK	T_{eff}	$\log T_{\text{eff}}$	BC
-0.33	O 5.5	40495	4.607	-3.73	-0.33	O 6	39280	4.594	-3.50
-0.31		37135	4.570	-3.46	-0.31	O 7.5	35410	4.549	-3.25
-0.29	O 7.5	34060	4.532	-3.20	-0.30	O 8.5	33620	4.527	-3.12
-0.28	O 9	32615	4.513	-3.06	-0.29	O 9.5	31920	4.504	-2.99
-0.26		29910	4.476	-2.79	-0.28		30310	4.482	-2.87
-0.25	B 0	28640	4.457	-2.68	-0.26		27320	4.437	-2.60
-0.23		26265	4.419	-2.40	-0.25		25940	4.414	-2.48
-0.20		23065	4.363	-2.02	-0.23		23385	4.369	-2.22
-0.18		21155	4.325	-1.76	-0.21	B 1	21080	4.324	-1.97
-0.17	B 1	20260	4.307	-1.67	-0.20		20015	4.301	-1.84
-0.16		19400	4.288	-1.56	-0.18		18045	4.256	-1.58
-0.12	B 3	16315	4.213	-1.16	-0.17	B 3	17135	4.234	-1.46
-0.11	B 4	15625	4.194	-1.05	-0.16		16265	4.211	-1.34
-0.08	B 5	13720	4.137	-0.82	-0.12	B 7	13220	4.121	-1.07
-0.05	B 7	12050	4.081	-0.64	-0.11	B 8	12550	4.099	-0.72
-0.025	B 8	10915	4.038	-0.51	-0.08	B 9	11400	4.057	-0.46
0.00	B 9	10255	4.011	-0.38	-0.05	B 9.5	10545	4.023	-0.32
0.05	A 2	9120	3.960	-0.17	-0.025	A 0	10000	4.000	-0.24
0.07	A 4	8790	3.944	-0.10	0.00	A 1	9570	3.981	-0.19
0.10	A 5	8510	3.930	0.00	0.05	A 2	8995	3.954	-0.09
0.14	A 8	8205	3.914	0.09	0.07		8790	3.944	-0.06
0.20	F 0	7800	3.892	0.14	0.10	A 3	8510	3.930	-0.03
0.33	F 5	7000	3.845	0.13	0.14	A 5	8205	3.914	-0.02
0.50	F 8	6210	3.793	0.08	0.20		7800	3.892	0.00
0.60		5835	3.766	0.04	0.33	F 1	7000	3.845	0.01
0.70		5525	3.743	-0.01	0.43	F 5	6500	3.813	-0.01
0.88	G 2	5070	3.705	-0.12	0.50		6210	3.793	-0.03
0.92	G 3	4990	3.698	-0.14	0.60		5875	3.769	-0.06
1.04	G 6	4755	3.677	-0.22	0.70	G 1	5585	3.747	-0.10
1.16	G 9	4530	3.656	-0.35	0.80		5300	3.724	-0.16
1.30	K 2	4280	3.632	-0.46	0.88	G 5	5085	3.706	-0.22
1.45	K 3	4030	3.605	-0.67	0.92	G 6	4990	3.698	-0.26
1.54	K 4	3875	3.588	-0.84	1.04		4720	3.674	-0.37
1.61	K 7	3750	3.574	-1.00	1.16	K 2	4465	3.650	-0.49
1.70		3500	3.544	-1.43	1.30		4185	3.622	-0.66
1.72		3250	3.512	-1.72	1.45	K 4	3905	3.592	-0.92
1.76	M 4	3000	3.477	-2.50	1.54	K 8	3750	3.574	-1.19
1.80		2800	3.447	-3.36	1.61		3500	3.544	-1.66

of these measurements are shown in Figure 3 with smooth curves through the main sequence stars and giants and through the supergiants. The smoothed relations are given in Tables 1 and 2 along with the temperature-color relations.

The supergiants follow a relation differing from that for the less luminous stars (no separation between luminosity classes, V, IV, and III). Although for stars with temperatures greater than 20000 K it is difficult to discern the separation, the cool supergiants clearly show a significant difference. It is in the right direction for the F and G stars. These supergiants may be expected to have less negative bolometric corrections because the lower atmospheric pressures in supergiants reduce the absorption due to the negative hydrogen ion, decreasing the opacity in the visual relative to the

UV and IR, and thus allowing more flux through in the visual.

The six hottest supergiants, on the other hand, show more negative corrections; they are redder for a given BC than the less luminous stars. Since all six are redder and since no significant reddening has been detected for the two most conspicuous supergiants, α Ori (B 8 Ia, $B-V = -0.03$, $T_{\text{eff}} = 11500$ K) and η CMa (B 5 Ia, $B-V = -0.09$, $T_{\text{eff}} = 13310$ K), reddening is probably not the cause of the differences between the scales.

Two explanations for the differences shown in Figure 3 seem reasonable. The first is simply a result of the change in the energy distribution of a star caused by a shift in the dominant opacity source along the luminosity sequence from main sequence stars to supergiants, that is, as the surface gravity decreases.

Table 2. Temperature Calibration for Main Sequence Stars

Main sequence				
$B-V$	MK	T_{eff}	$\log T_{\text{eff}}$	BC
-0.33	O8	36250	4.559	-3.50
-0.31	O9.5	32750	4.515	-3.25
-0.30		31130	4.493	-3.12
-0.29	B0	29590	4.471	-2.99
-0.28		28127	4.449	-2.87
-0.26	B0.5	25412	4.405	-2.60
-0.25		24155	4.383	-2.48
-0.23	B1.5	21825	4.339	-2.22
-0.21		19715	4.295	-1.97
-0.20	B3	18740	4.273	-1.84
-0.18	B4	16930	4.229	-1.58
-0.17		16095	4.207	-1.46
-0.16	B5	15300	4.185	-1.34
-0.12		12490	4.096	-1.07
-0.11	B8	12200	4.086	-0.72
-0.08	B9	11200	4.049	-0.46
-0.05	B9.5	10545	4.023	-0.32
-0.025		10000	4.000	-0.24
0.00	A0	9570	3.981	-0.19
0.05	A2	8995	3.954	-0.09
0.07	A3	8790	3.944	-0.06
0.14	A5	8200	3.914	-0.02
0.20	A7	7800	3.892	0.00
0.33	F1	7000	3.845	0.01
0.43	F4	6500	3.813	-0.01

Changing the surface gravity in this manner, from high to low, results in a flattening in the opacity distribution and thus the energy distribution as electron scattering becomes the dominant opacity source in hot supergiants. The models of Kurucz et al. (1972) show just such a change. Their models, for instance, with $T_{\text{eff}}=16000$ K and $\log g$ between 4.5 and 2.0 give a difference in $B-V$ of 0.06 mag, about the same as between the two scales shown in Figure 3.

Secondly, it is possible that the positions in Figure 3 of supergiants relative to the less luminous stars are a result of extended atmospheres. It is well known that supergiants, both early- and late-type, exhibit properties indicative of extended atmospheres (Wright, 1974). A qualitative explanation can be found in two recent attempts at constructing extended, spherical atmospheres: Cassinelli (1971) for central stars of planetary nebulae and Mihalas and Hummer (1974) for O stars. Cassinelli found that the energy distribution of spherical atmospheres are flat compared to plane-parallel atmospheres. Mihalas and Hummer found the same effect; the UV and IR fluxes increase relative to the visual flux as their models became more extended. This would result in more negative bolometric corrections. They also found that when they compared $b-y$ colors of extended atmospheres to those of plane-parallel atmospheres, the spherical models were redder, an intrinsic reddening.

A recent analysis of bolometric corrections for late-type stars, M0 through M9.8, was presented in Paper I. Corrections were derived as a function of spectral type from the IR observations of Dyck et al. (1974). This scale is shown in Figure 4 for M0 through M6 giants using Lee's (1970) colors. It is easily seen that $B-V$ is not a good indicator of BC for these late-type giants. Bolometric corrections for evolutionary models at the corresponding low temperatures of these stars are, however, easily found using the T_{eff} :BC relation given in Paper I.

The BC: $B-V$ relation for giants is found by drawing a smooth curve through the M giant data, the hot giant data and Johnson's data, as shown in Figure 4. The entire scale is given in Table 1. We believe that this scale is quite reliable because Johnson's corrections are based on extensive IR observations of G and K giants where there are virtually no uncorrectable effects on the observed energy distributions caused by either the earth's atmosphere or by molecular absorption in the stellar atmosphere.

The scale for supergiants was derived in the same manner as was done for the giants; i.e., by joining Johnson's supergiant data to the hot end shown in Figure 3 and to the cool end represented by Lee's scale. The discordant points in Johnson's data at $B-V=0.6$ are probably caused by the older UV data available in 1965 which were generally lower than more recent

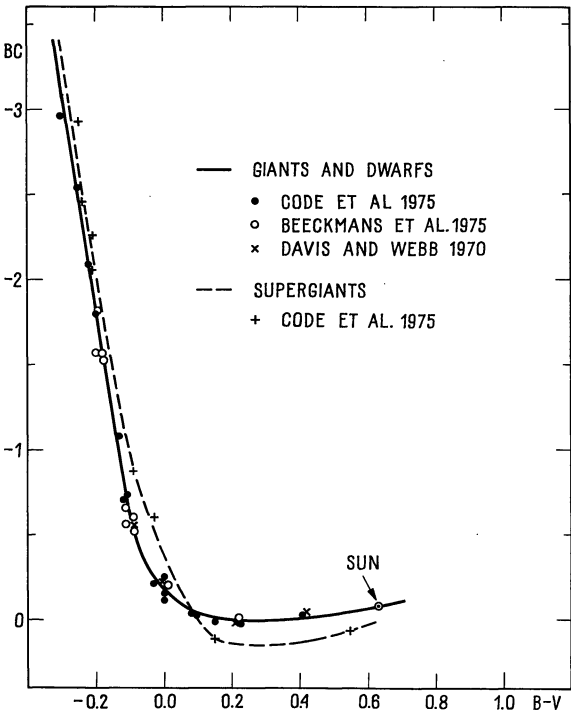


Fig. 3. The new BC: $B-V$ scales for hot stars given in Tables 1 and 2 were derived from these curves through the new satellite data. Separate scales are indicated for supergiants and for less luminous stars

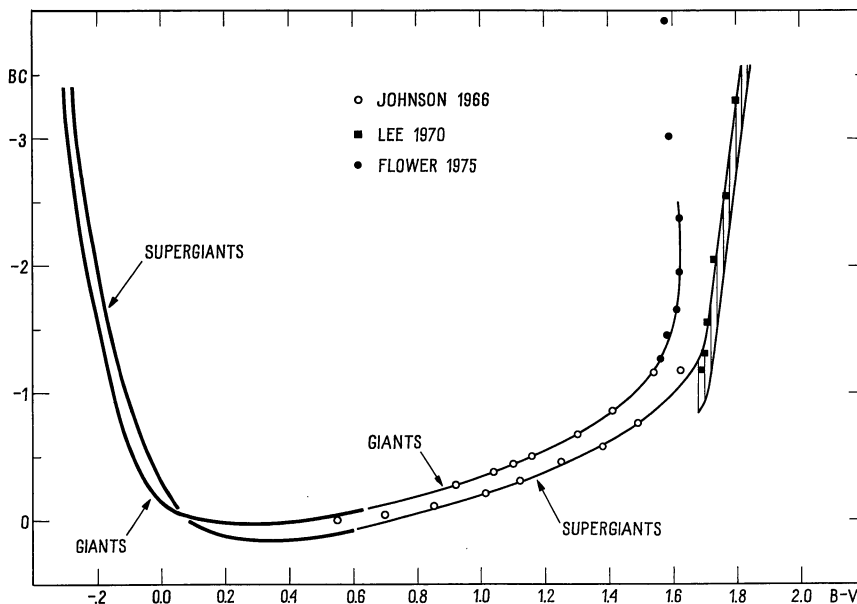


Fig. 4. The complete $BC:B-V$ scales are shown for both giants and supergiants. The heavy lines are the curves from Figure 3 representing the hot stars. The hatched area depicts the 0.5 mag reduction (in absolute value) of BC for supergiants found in Paper I

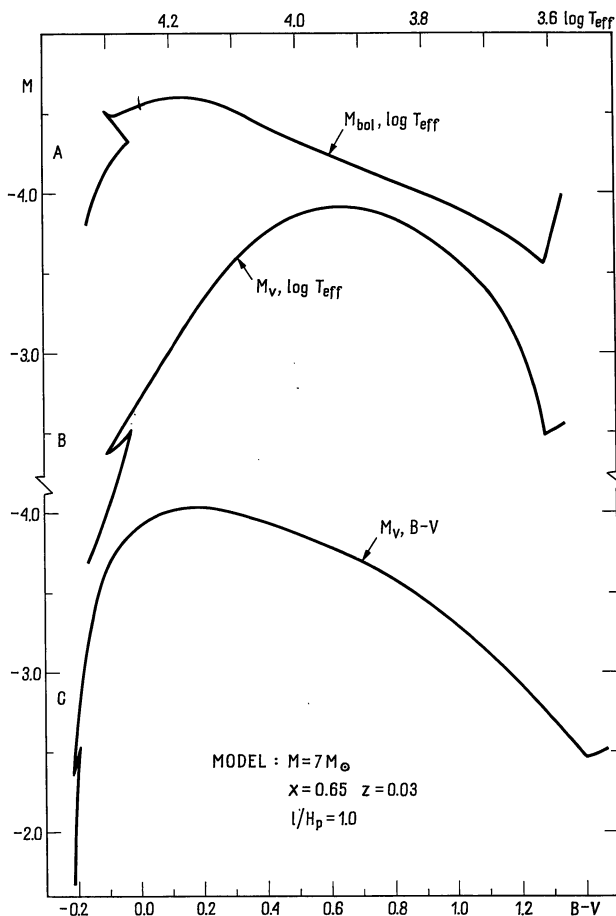


Fig. 5. Shown here is the two-step transformation on a $7M_{\odot}$ evolutionary track: step 1 is M_{bol} to M_v ; step 2 is $\log T_{\text{eff}}$ to $B-V$. The upper abscissa is scaled in $\log T_{\text{eff}}$ while the lower one is in $B-V$

data, resulting in more negative values of BC . This relation is also given in Table 1.

The hatched area in Figure 4 represents the 0.50 mag difference found in Paper I between the bolometric corrections of giants and supergiants (Lee's corrections for supergiants are practically the same as for giants in Paper I, the only difference being their colors). The smaller absolute value of BC for supergiants was shown to be consistent with current atmosphere calculations for very late-type stars.

We must also be aware that this scale, as well as the $T_{\text{eff}}:B-V$ scale for supergiants, does not reach as red as some supergiants in the Galaxy (η and χ Persei) or in the Large Magellanic Cloud (NGC 2100) where values as great as $B-V=2.0$ are found. It seems at this point that the intrinsic colors of late-type supergiants are poorly known. Several factors, from inaccurate spectral types to circumstellar effects, are probably responsible for the uncertainty.

It appears, however, that attempts in the last ten years to determine bolometric corrections for Population I stars have been successful. The $BC:B-V$ scale for giants is very well determined and little error in the interpretation of $C-M$ diagrams based on comparisons with evolutionary calculations should result from using it.

Uncertainty still remains for the supergiant scale. Improvements at the hot end of the supergiant temperature calibration will most likely result from improved model atmospheres rather than from more refined observations. CDBB believe that modifications of the present absolute UV calibrations will be minor. On the other hand, more information on bolometric

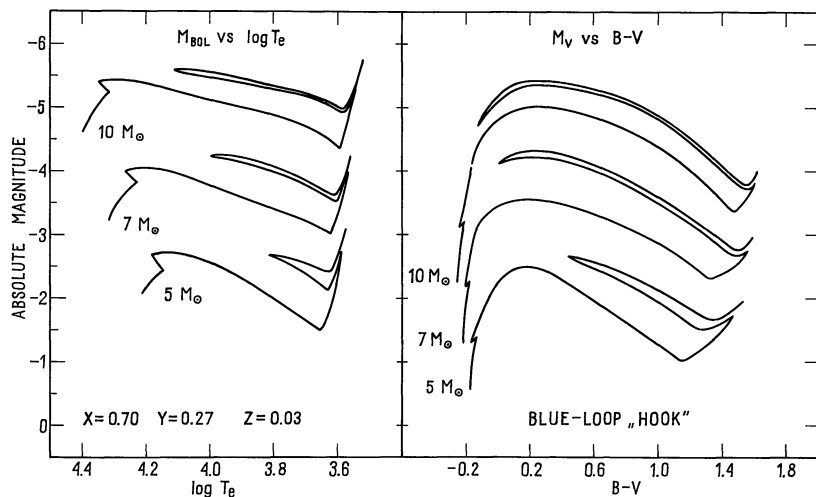


Fig. 6. In this transformation of a set of evolutionary tracks we see in the M_b vs. $B-V$ diagram a steep post-main sequence evolution, a stubby Hayashi line and the appearance of a hook-like feature for the massive models during their blue, core helium burning phase

corrections of supergiants most certainly can be obtained from IR observations of Magellanic Cloud supergiants. We still have, however, the difficulty with the $B-V$ colors of supergiants; we cannot compare the very reddest observed supergiants in clusters with theoretical models of very cool, luminous stars.

These new scales will now be used to study their effect on the transformation of theoretical evolutionary tracks to C-M diagrams.

The Transformations

Since the major motivation in constructing the $T_{\text{eff}}:B-V:BC$ scales was to use them for comparing theoretical evolutionary tracks or isochrones with observed C-M diagrams, a rather detailed discussion of the transformations will now be presented.

The effects of the transformation scales on an evolutionary track for pre-core-helium burning phases of a $7M_{\odot}$ model (Flower et al., 1976) is illustrated in Figure 5. Track A is the evolutionary path on the $M_{\text{bol}}:\log T_{\text{eff}}$ -plane. Track B shows the results of converting bolometric magnitudes of the models to M_v without also converting $\log T_{\text{eff}}$ to $B-V$. It is seen that the post-main sequence gap, a manifestation of hydrogen exhaustion in the core, prominent on track A virtually disappears on track B. This is caused by the very steep nature of the $T_{\text{eff}}:BC$ relation at high temperatures; BC becomes more negative, as temperatures increase, faster than the models become more luminous.

This effect is not as severe for less massive models because, as Figure 4 shows, BC varies much slower with temperature at cooler temperatures.

More obvious is the shift to cooler temperatures of the maximum absolute magnitude attained during hydrogen shell burning. This maximum no longer corresponds to the luminosity maximum occurring during thick hydrogen shell burning as depicted in track A. This shift is due to the bolometric corrections

reaching a minimum at about $\log T_{\text{eff}} = 3.9$ or $B-V = 0.20$ mag. At cooler temperatures model luminosities continue to decrease causing M_v to do likewise.

Track C shows the results of the complete transformation of the theoretical evolutionary track to observable parameters, M_v and $B-V$. Immediately noticeable is the shift, now to the left, of the maximum absolute magnitude reached by the models and the steepness of the track during core hydrogen burning. These result from the steep $\log T_{\text{eff}}:B-V$ relation at high temperatures; a large change in the model's effective temperature corresponds to a small change in $B-V$. Of course, the opposite is true for temperatures less than about 10000 K which is reflected in the gentler slope found during thin shell burning phases.

Three evolutionary tracks computed by Paczyński (1970) are presented in Figure 6. It shows that in a C-M diagram the tracks appear stretched in magnitude at the hot end and squashed at the cool end compared to their appearance in a theoretical H-R diagram. In a color-magnitude diagram a model would appear to evolve at constant $B-V$ during core hydrogen-burning as its luminosity increases. During its climb up the Hayashi track, however, $B-V$ increases significantly. The squashed Hayashi track arises because as a model becomes redder (cooler) the bolometric corrections are increasing, thus reducing the magnitude difference between the base and the tip of the giant branch.

One other result of the nature of the $T_{\text{eff}}:B-V:BC$ scales is the appearance of a hook-like feature during core helium-burning phases. This blue-loop "hook" is very prominent in the $10M_{\odot}$ track. During core helium burning, models evolve toward higher temperatures at nearly constant luminosity, but for temperatures greater than 8000 K the bolometric corrections begin to rise very steeply (see Fig. 3), accounting for the sudden drop in M_v .

The blue-loop "hook" is even more dramatically displayed in Figure 7 which presents the results of

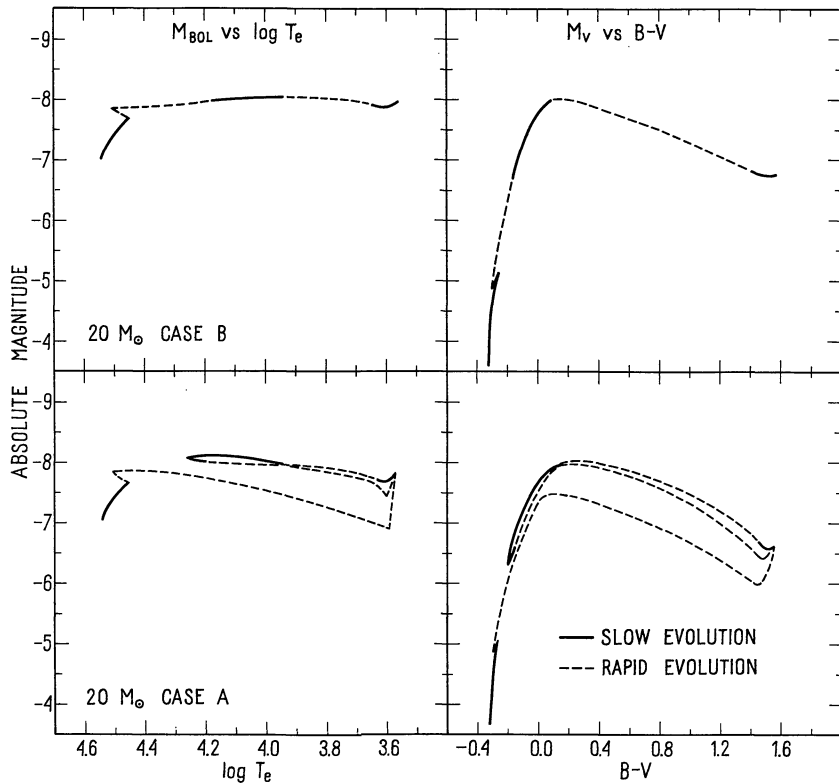


Fig. 7. Two tracks for a $20M_{\odot}$ model by Chiosi and Summa (1970), differing only in the relation for convective instability criteria, are shown transformed to a C-M diagram using the scale for supergiants from Table 1. For case B the model burns helium in its core only as a blue supergiant, for case A the model burns helium in its core as both a red supergiant and a blue supergiant

applying the transformations to tracks of two $20M_{\odot}$ models given by Chiosi and Summa (1970). Case A refers to the inclusion of the chemical composition gradient while in case B it is neglected in the treatment of convective instability. The thick, solid lines represent the slow (nuclear time scale) evolutionary phases. This is where the models spend most of their time and represents the most likely distribution of stars in a C-M diagram of a cluster whose members are the same mass as these models.

The basic behavioral difference between these two tracks in the theoretical H-R diagram is that evolution in case A burns helium in its core at higher temperatures, hotter by ~ 0.1 in $\log T_{\text{eff}}$, than in case B. However, after transforming these tracks to a C-M diagram, the difference is reduced to ~ 0.05 mag in $B-V$, a difference that would be difficult to detect observationally (consider, for instance, the C-M diagram of h and χ Persei). In both cases the evolutionary path occupies essentially the same position in the C-M diagram; both would give the same distribution of stars.

The blue-loop “hook” feature shown in Figure 7 can be used to understand some features found in the C-M diagrams of young clusters. Robertson (1973) has summarized some of their major observational properties. He found, however, some difficulty in explaining the existence of a group of stars about a magnitude above the main sequence. From the discussion of Figure 7 the suggestion that this group represents core helium

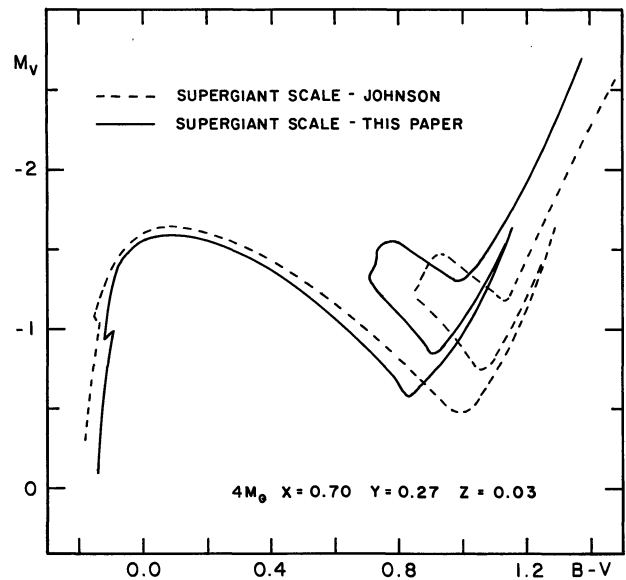


Fig. 8. The solid line is an evolutionary track of a $4M_{\odot}$ model transformed to a C-M diagram using the supergiant scale in Table 1; the dashed line is the same track transformed using Johnson's supergiant scale. The difference in $B-V$ between the two tracks is 0.12 mag

burning stars is consistent with C-M diagrams of young clusters like NGC 330 and NGC 2100.

Finally, a comparison will now be made between the scales listed in Table 1 and Johnson's scales.

Using Johnson's main sequence scale to transform the core hydrogen burning phases will result in evolutionary paths that are about 0.05 mag bluer than if the main sequence scale in Table 2 is used. Figure 1 shows this clearly. On the other hand, since the scale for giants in Table 1 depends heavily on Johnson's scale, no differences are expected. There will be some difference, again about 0.05 mag, when using Johnson's giant scale for cool models since the temperatures and bolometric corrections have been revised in Paper I. However, when applying the two supergiant scales to theoretical evolutionary tracks a very significant difference results. In Figure 8 an evolutionary track for a $4M_{\odot}$ model (Flower et al., 1976) is transformed using both Johnson's scale and the scale in Table 1 for supergiants. The older $T_{\text{eff}}:B-V:BC$ transformation results in a red giant branch about 0.12 mag redder than the transformation using the new scale. The reason for this is clearly shown in Figure 2; the adopted scale in Table 1 is shifted by about 0.12 mag to the blue of Johnson's scale.

One last comment should be made concerning the use of these scales. They have been derived from observations of Pop I stars. Consequently, they are totally inadequate for use with Pop II objects like globular clusters. For Pop II transformations, theoretical $T_{\text{eff}}:B-V:BC$ scales based on model atmospheres must be used since there are no convenient metal poor objects available for direct empirical measurements of effective temperatures and bolometric corrections.

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