14.32: Econometrics

Problem Set 7

No due date

1. True, False, Uncertain with Explanation:

   (a) The first difference of the logarithm of $Y_t$ approximately equals the growth rate of $Y$ when the growth rate is small.

   (b) An autoregression is a regression of a dependent variable on lags of regressors.

   (c) To construct a good forecast one should use only exogenous variables as regressors.

   (d) One should not use OLS to estimate dynamic effect if data is exogeneous, but not strictly exogeneous.

2. Consider the standard AR(1) process $Y_t = \beta_0 + \beta_1 Y_{t-1} + u_t$, where the usual assumptions hold. Assume that $y_t$ is $Y_t$ with the mean removed, that is, $y_t = Y_t - EY_t$. Assume that $Y_t$ is stationary.

   (a) Do mean and variance of $Y_t$ depend on $t$? Why?

   (b) Find the mean and variance of $Y_t$.

   (c) Show that $y_t = \beta_1 y_{t-1} + u_t$.

   (d) Show that the $r$-period ahead forecast $y_{T+r|T} = \beta_1^r y_T$. If $0 < \beta_1 < 1$, how does the $r$-period ahead forecast behave as $r$ becomes large? What is the forecast of $Y_{T+r|T}$ for large $r$?

   (e) The half-life is the number of periods it takes a time series with zero mean to halve its current value (in expectation), i.e., the solution $r$ to $y_{T+r|T} = 0.5 y_T$. Find the formula for the half-life in the present case. Could you explain why sometimes $\beta_1$ is called "persistence".

3. It sure would be great if you could use time series econometrics to forecast stock prices. If so, you could buy stocks when you forecast their prices will rise (that is, when you forecast positive returns) and sell stocks (go short) when you forecast their prices will drop (forecast negative returns). You won't be right all the time, but if your forecasts were right on average then you would make

money on average, at least if you ignore the transaction costs involved with buying and selling stocks.

In this problem set, you will use monthly data on the excess return of stock prices over the period 1931 - 2002 to see if you can forecast stock returns based on their past behavior. The stock prices are measured as a broadly - based index of stock prices constructed by the Center for Research in Security Prices. The monthly excess return is $R1_t = 100 \times (\ln(\frac{P_t + Div_t}{P_{t-1}}) - ln(TBill_t))$, where $Div_t$ are the dividends paid on the stocks in the index, and $TBill_t$ is the gross return on a 30-day Treasury Bill.

You will use autoregressive methods to forecast both one-month returns and multi-month returns. In addition you will examine "momentum" forecasts, in which recent strong returns are taken as a signal that future returns will be strong (so the forecaster "rides the wave" or "follows the momentum" of the market).

(a) *Autoregressive forecasts.* Estimate AR(1), AR(2), and AR(4) models of one-month returns. Construct a table which reports the coefficients, the F-statistics and p-values testing the hypothesis that the lag(s) of the one-month return do not help to predict the one-month return, along with the adjusted $R^2$'s of the regressions. What are your conclusions on whether autoregressive models are helpful in predicting stock returns?

(b) *6-month return autoregressions.* Now suppose you want to forecast stock returns over the next six months, that is, six months into the future. For this you define a return over the past six months: $R6_t = \frac{1}{6}(R1_t + R1_{t-1} + ... + R1_{t-5})$. Consider the regression,

$$R6_t = \beta_0 + \beta_6 R6_{t-6} + \beta_7 R6_{t-7} + \beta_8 R6_{t-8} + u_t. \tag{1}$$

The most recent regressor in the above regression is dated $t-6$. Why wouldn't you use a more recent 6-month return, for example, why wouldn't you include $R6_{t-1}$ and $R6_{t-2}$ as regressors?

(c) Under the null hypothesis that lagged returns cannot be used to forecast future returns, are the regressors in Equation (1) plausibly exogenous?

(d) Under the null hypothesis that lagged returns cannot be used to forecast future returns, are the regressors in Equation (1) plausibly strictly exogenous?

(e) Is autoregression (1) helpful in predicting 6-month returns?

(f) *Momentum forecasts.* Generate the following variables:

- $M6_t = 1$ if $R1_t > R6_{t-1}$, and $= 0$ otherwise. The variable indicates that the stock return exceeds the average over the previous 6 months.
- $Mpos_t = 1$ if $R1_t > 0$, and $= 0$ otherwise. The variable indicates a positive return over the last month.
- $Mpos2_t = 1$ if $Mpos_1 = 1$ and $Mpos_{t-1} = 1$, and $= 0$ otherwise. The variable indicates positive returns over both previous two months.

Regress $R1_t$ on each of variables $M6_{t-1}$, $Mpos_{t-1}$ and $Mpos2_{t-1}$ separately and on all of them together. Report the F-statistics and p-values testing the hypothesis that all coefficients (except the intercept) are zeros, along with the adjusted $R^2$s of the regressions. So, can you forecast the market?