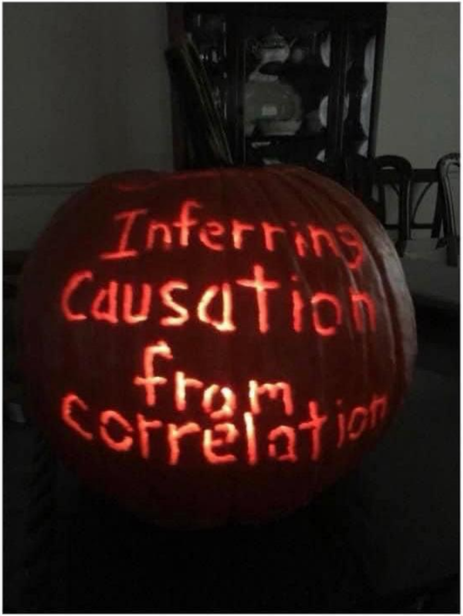
Nothing scares me more...



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Recitation 7: Logit and Probit

14.32 Fall 2023

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What are logit and probit?

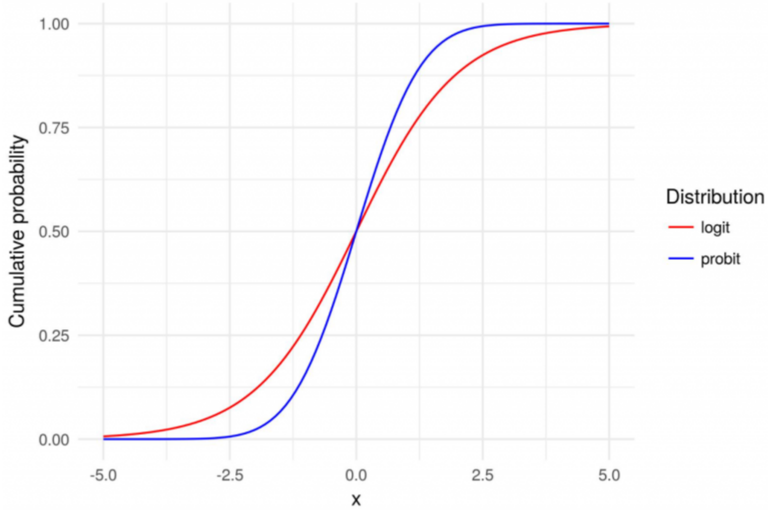
The model we consider is $P\left(Y=1 \mid X\right)=F\left(\alpha+\beta X\right)$, where F is called a *link function*.

A good link function takes us from the real numbers to the interval between 0 and 1: $\mathbb{R} \rightarrow [0,1]$.

We could in theory use any link function, but logit and probit are the most common:

- $F_{\text{logit}}\left(x\right)=\Lambda\left(x\right)=\frac{1}{1+e^{-x}}$
- $F_{\text{probit}}\left(x\right)=\Phi\left(x\right)=\int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}}dt$

Logit and probit functions



Picking between logit and probit

- In general there are no hard & fast rules, but rules of thumb
- Look at error terms!
- If many outliers, use logit as it is more robust
- Logit has a cleaner interpretation, as log odds
- When in doubt... common to default to logit
- Discussion a little more complex with dealing with multinomial logit/probit

Interpreting logit and probit models

In general we are interested in marginal effects

$$\frac{\partial P\left(Y=1 \mid X\right)}{\partial x_j} = \frac{\partial F\left(\beta_0 + \sum_{i=1}^{k} \beta_i x_i\right)}{\partial x_j} = f\left(\beta_0 + \sum_{i=1}^{k} \beta_i x_i\right) \beta_j$$

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Fundamental problem: effects we predict depend on the levels of our variables! We saw two approaches in class to deal with this.

Interpreting logit and probit models

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Fundamental problem: effects we predict depend on the levels of our variables! We saw two approaches in class to deal with this.

- Partial marginal effect at average

$$\frac{\partial P\left(Y=1 \mid X=\bar{x}\right)}{\partial x_j}=f\left(\beta_0+\sum_{i=1}^{k} \beta_i \bar{x}_i\right) \beta_j$$

Interpreting logit and probit models

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$$\frac{\partial P\left(Y=1 \mid X\right)}{\partial x_j} = \frac{\partial F\left(\beta_0 + \sum_{i=}^{k} \beta_i x_i\right)}{\partial x_j} = f\left(\beta_0 + \sum_{i=1}^{k} \beta_i x_i\right)\beta_j$$

Fundamental problem: effects we predict depend on the levels of our variables! We saw two approaches in class to deal with this.

- Average partial effect

$$\frac{1}{n}\sum_{\ell=1}^{n} \frac{\partial P\left(Y=1 \mid X=x_\ell\right)}{\partial x_j} = \frac{1}{n}\sum_{\ell=1}^{n} f\left(\beta_0 + \sum_{i=1}^{k} \beta_i x_{i,\ell}\right)\beta_j$$

Example: predicting NCAA tournament qualification

Follow along with Stata code posted on Canvas if you wish!

Question: do expert rankings predict who can qualify for the NCAA final tournament?

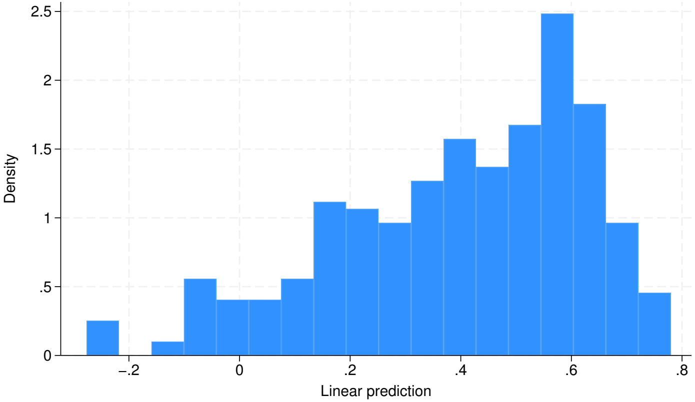
```
. * summary stats
. sum tourney prerpi postrpi_1 coachexper  power5
```

Variable	Obs	Mean	Std. dev.	Min	Max
tourney	336	.3988095	.4903837	0	1
prerpi	336	75.05655	64.12944	1	323
postrpi_1	336	86.59524	72.90892	1	316
coachexper	336	24.94048	8.843382	1	46
power5	336	.7619048	.4265529	0	1

Linear probability model

```
. * trying out a linear probability model
. reg tourney prerpi coachexper  power5, r

Linear regression                               Number of obs     =        336
                                                F(3, 332)         =      38.27
                                                Prob > F          =     0.0000
                                                R-squared         =     0.2066
                                                Root MSE          =     .43877

------------------------------------------------------------------------------
             |               Robust
     tourney | Coefficient  std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
      prerpi |  -.0033347   .0003496    -9.54   0.000    -.0040224   -.0026469
  coachexper |    .007508   .0028785     2.61   0.010     .0018456    .0131703
      power5 |  -.0562974   .0599212    -0.94   0.348    -.1741706    .0615757
       _cons |   .5047385   .0927727     5.44   0.000     .3222421    .6872349
------------------------------------------------------------------------------

. reg tourney prerpi postrpi_1 coachexper  power5, r

Linear regression                               Number of obs     =        336
                                                F(4, 331)         =      31.55
                                                Prob > F          =     0.0000
                                                R-squared         =     0.2181
                                                Root MSE          =     .43623

------------------------------------------------------------------------------
             |               Robust
     tourney | Coefficient  std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
      prerpi |   .0000206   .0012818     0.02   0.987    -.0025008    .0025421
   postrpi_1 |  -.0030031   .0011465    -2.62   0.009    -.0052584   -.0007478
  coachexper |   .0069718   .0028751     2.42   0.016     .0013161    .0126276
      power5 |  -.0276566   .0595865    -0.46   0.643    -.1448726    .0895595
       _cons |   .5045044   .0920316     5.48   0.000     .3234639    .6855449
------------------------------------------------------------------------------
```

Predicted values are negative!



Logit

```
. logit tourney prerpi coachexper  power5, r

Iteration 0:  Log pseudolikelihood = -225.96874
Iteration 1:  Log pseudolikelihood = -184.44301
Iteration 2:  Log pseudolikelihood = -182.21799
Iteration 3:  Log pseudolikelihood = -182.19819
Iteration 4:  Log pseudolikelihood = -182.19819

Logistic regression                                     Number of obs =    336
                                                        Wald chi2(3)  =  42.88
                                                        Prob > chi2   = 0.0000
Log pseudolikelihood = -182.19819                       Pseudo R2     = 0.1937

------------------------------------------------------------------------------
             |               Robust
     tourney | Coefficient  std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
      prerpi |  -.0224412   .0039588    -5.67   0.000    -.0302003   -.0146821
  coachexper |   .0313247   .0155884     2.01   0.044     .0007721    .0618774
      power5 |  -.1632856    .337554    -0.48   0.629    -.8248793     .498308
       _cons |   .3455622   .5024758     0.69   0.492    -.6392723    1.330397
------------------------------------------------------------------------------
```

```
. logit tourney prerpi postrpi_1 coachexper  power5, r

Iteration 0:  Log pseudolikelihood = -225.96874
Iteration 1:  Log pseudolikelihood = -183.76753
Iteration 2:  Log pseudolikelihood = -181.59773
Iteration 3:  Log pseudolikelihood = -181.56745
Iteration 4:  Log pseudolikelihood = -181.56745

Logistic regression                                     Number of obs =    336
                                                        Wald chi2(4)  =  46.05
                                                        Prob > chi2   = 0.0000
Log pseudolikelihood = -181.56745                       Pseudo R2     = 0.1965

------------------------------------------------------------------------------
             |               Robust
     tourney | Coefficient  std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
      prerpi |  -.0103837   .0101899    -1.02   0.308    -.0303555    .0095882
   postrpi_1 |  -.0099673   .0078573    -1.27   0.205    -.0253674    .0054328
  coachexper |   .0308211   .0156325     1.97   0.049     .0001819    .0614603
      power5 |  -.0838392   .3295318    -0.25   0.799    -.7297097    .5620312
       _cons |   .2729752   .4997466     0.55   0.585    -.7065101    1.252461
------------------------------------------------------------------------------
```

What is the predicted probability for LSU?

$$P(\text{tourney} = 1 \mid X) = \Lambda(0.2730 - 0.0104\text{prerpi} - 0.0100\text{postrpi\_1} \\ + 0.0308\text{coachexper} - 0.0838\text{power5})$$

LSU in 2015-16 had a prerpi of 45, postrpi_1 of 65, coachexper of 31 and is in the power5. What is the predicted probability of them reaching the playoffs?

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LSU in 2015-16 had a prerpi of 45, postrpi_1 of 65, coachexper of 31 and is in the power5. What is the predicted probability of them reaching the playoffs?

$$P_{LSU}\,(\text{tourney} = 1) = \Lambda\,(0.2730 - 0.0104 \times 45 - 0.0100 \times 65 \\ +0.0308 \times 31 - 0.0838 \times 1) \\ = \Lambda\,(0.026) \\ = 0.5065$$

How would a more experienced coach affect LSU?

$$P_{LSU}\left(\text{tourney} = 1\right) = \Lambda\left(0.026\right)$$

LSU in 2015-16 had a prerpi of 45, postrpi_1 of 65, coachexper of 31 and is in the power5. What is their marginal effect of a better coach?

How would a more experienced coach affect LSU?

$$P_{LSU}\,(\text{tourney} = 1) = \Lambda\,(0.026)$$

LSU in 2015-16 had a prerpi of 45, postrpi_1 of 65, coachexper of 31 and is in the power5. What is their marginal effect of a better coach?

$$\frac{\partial P\,(Y = 1 \mid X = \text{LSU})}{\partial \text{coachexper}} = \lambda\,(0.026) \times \beta_{\text{coachexper}}$$

$$= 0.25 \times 0.0308 = 0.77\%$$

What is the partial marginal effect of better coaching at the average?

What is the partial marginal effect of better coaching at the average?

$$
\begin{aligned}
\frac{\partial P\left(Y=1 \mid X=\bar{x}\right)}{\partial x_{\text{coachexper}}} =& \lambda\left(\beta_0+\sum_{i=1}^{k} \beta_i \bar{x}_i\right) \beta_{\text{coachexper}} \\
=& \lambda\Big(0.2730-0.0104 \overline{\text{prerpi}}-0.0100 \overline{\text{postrpi\_1}} \\
& +0.0308 \overline{\text{coachexper}}-0.0838 \overline{\text{power5}}\Big) \times 0.0308 \\
=& \lambda\left(0.2730-0.0104 \times 75.06-0.0100 \times 86.60\right. \\
& \left.+0.0308 \times 24.94-0.0838 \times 0.76\right) \times 0.0308 \\
=& \lambda\left(-0.6692\right) \times 0.0308 \\
=& 0.224 \times 0.0308 \\
=& 0.69\%
\end{aligned}
$$

What is the partial marginal effect of better coaching at the average?

```
. margins, dydx(coachexper) atmeans

Conditional marginal effects                          Number of obs = 336
Model VCE: Robust

Expression: Pr(tourney), predict()
dy/dx wrt:  coachexper
At: prerpi     = 75.05655 (mean)
    postrpi_1  = 86.59524 (mean)
    coachexper = 24.94048 (mean)
    power5     = .7619048 (mean)

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
  coachexper |   .0069132   .0035621     1.94   0.052    -.0000684    .0138947
------------------------------------------------------------------------------
```

Addendum: logit and probit are ultimately quite similar