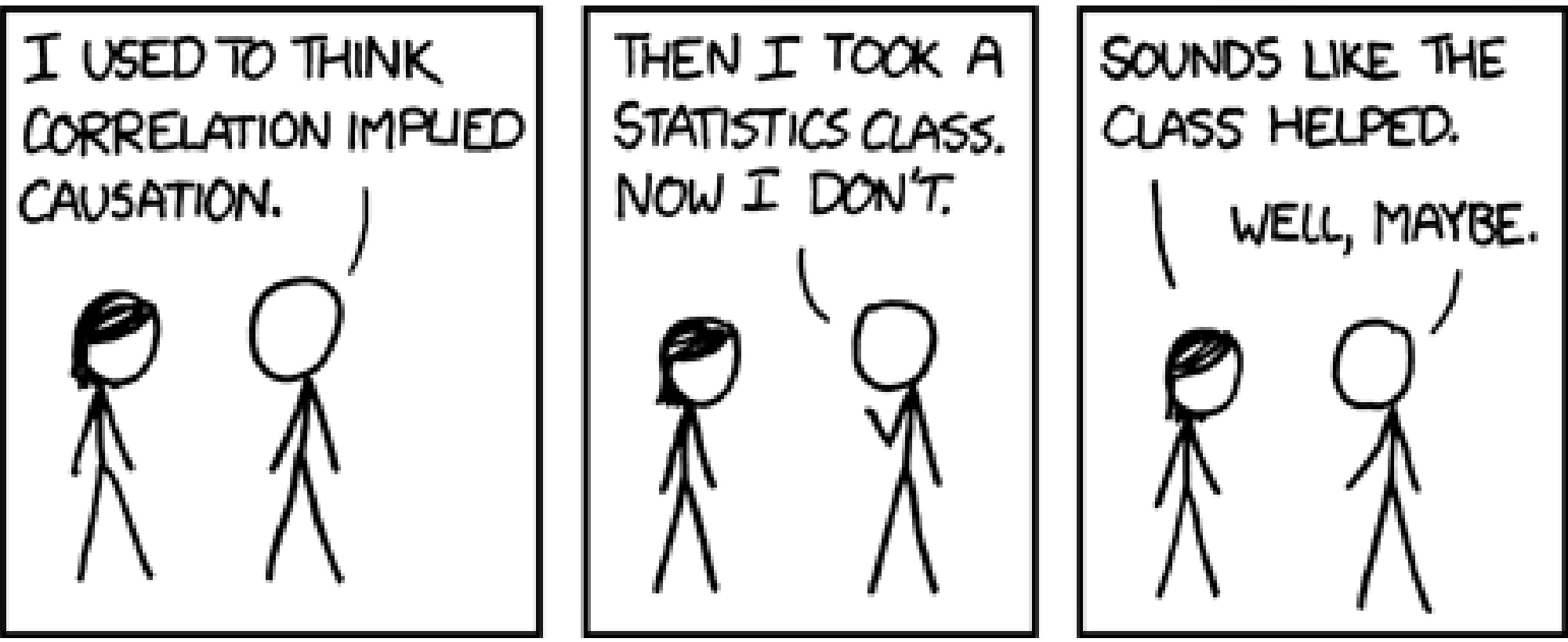
I USED TO THINK CORRELATION IMPLIED CAUSATION.
THEN I TOOK A STATISTICS CLASS. NOW I DON'T.
SOUNDS LIKE THE CLASS HELPED.
WELL, MAYBE.

Recitation 9: Treatment Effects

14.32 Fall 2023

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A simple causal model

We are interested in the effects of a treatment variable $D \in \{0, 1\}$ on an outcome of interest Y.

Examples:

- Y is a measure of health, D is access to health insurance
- Y is earnings, D is getting a college degree

We can think about this in terms of potential outcomes

$$Y(1) = \text{outcome if treated}$$

$$Y(0) = \text{outcome if not treated}$$

$$Y = DY(1) + (1 - D)Y(0)$$

We can only ever observe one of these values, the other is counterfactual

Individual and average treatment effects

The treatment effect for a single individual i is

$$Y_i(1) - Y_i(0)$$

This variable is the effect of treatment on the individual outcome; there is no hope of identifying this from data.
There is hope of identifying from data the average treatment effect (ATE):

$$\theta = \mathbb{E}\left[Y_i(1) - Y_i(0)\right]$$

The idea is to combine data for treated individuals with data for not treated individuals to identify the ATE.

Individual and average treatment effects

We can directly observe $\mathbb{E}[Y \mid D = 1]$ and $\mathbb{E}[Y \mid D = 0]$.

Given that $Y = DY(1) + (1 - D)Y(0)$, we know

$$\mathbb{E}[Y \mid D = 1] = \mathbb{E}[Y(1) \mid D = 1]$$
$$\mathbb{E}[Y \mid D = 0] = \mathbb{E}[Y(0) \mid D = 0]$$

It follows that (see board for derivation)

$$\mathbb{E}[Y \mid D = 1] - \mathbb{E}[Y \mid D = 0] = \underbrace{\mathbb{E}[Y(1) - Y(0) \mid D = 1]}_{\text{average treatment effect on treated}} + \underbrace{\mathbb{E}[Y(0) \mid D = 1] - \mathbb{E}[Y(0) \mid D = 0]}_{\text{selection bias}}$$

Example: Randomized Controlled Trial (RCT)

In a randomized controlled trial (RCT), D is randomly assigned, so $Y(1)$ and $Y(0)$ are independent of D.

This implies that selection bias must be zero! Then

$$\mathbb{E}[Y \mid D=1] - \mathbb{E}[Y \mid D=0] = \mathbb{E}[Y(1) - Y(0)]$$

A simple estimator of θ is $\hat{\theta} = \bar{Y}_1 - \bar{Y}_0$.

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We have made a strong assumption here, however:

We have assumed that everyone follows the treatment that we give them.

Differing responses to treatment

Let's switch notation and call Z a dummy for whether a person was assigned to treatment, and X a dummy for whether someone was actually treated. We have four possibilities:

- Always takers (A): $X = 1$ independent of Z
- Never takers (N): $X = 0$ independent of Z
- Compliers (C): $X = Z$
- Defiers (F): $X = 1 - Z$

Some comments

- Note that observing X and Z is not sufficient to identify the types!
- OLS will have omitted variable bias (do you see why?)
- In general, we *assume away* defiers (F).

Treatment assignment as an instrument

You should convince yourself that, in the absence of defiers and with sufficient compliers, treatment assignment is a good instrument.

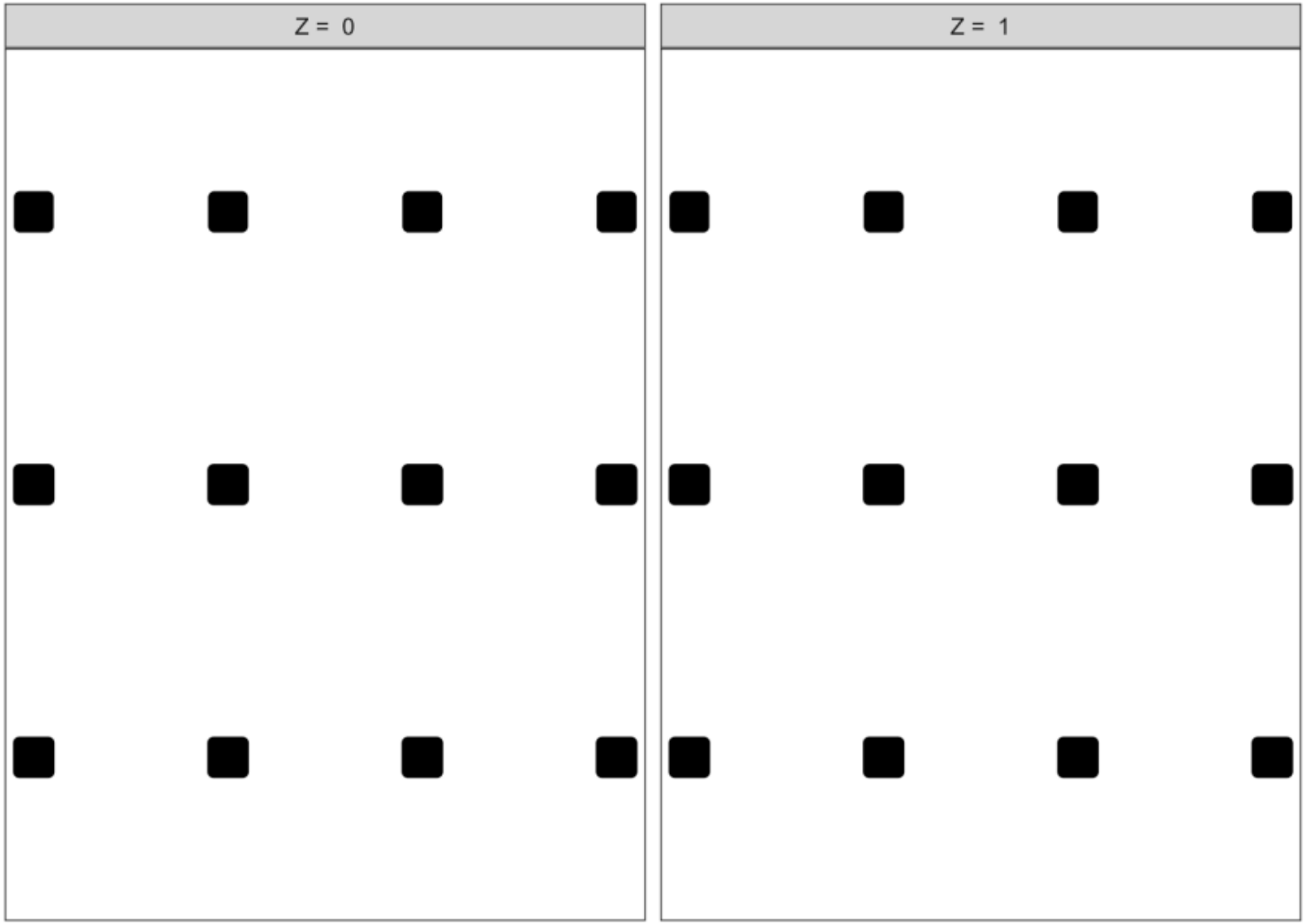
- Relevance: $\text{Cov}(X, Z) > 0$ due to the compliers
- Exogeneity: $\text{Cov}(Z, \varepsilon) = 0$ since Z is randomly assigned

Under two additional assumptions (independence and monotonicity), our IV estimator can identify a *local average treatment effect* (LATE)

- This holds for continuous treatments and instruments as well, but we focus on the binary case here

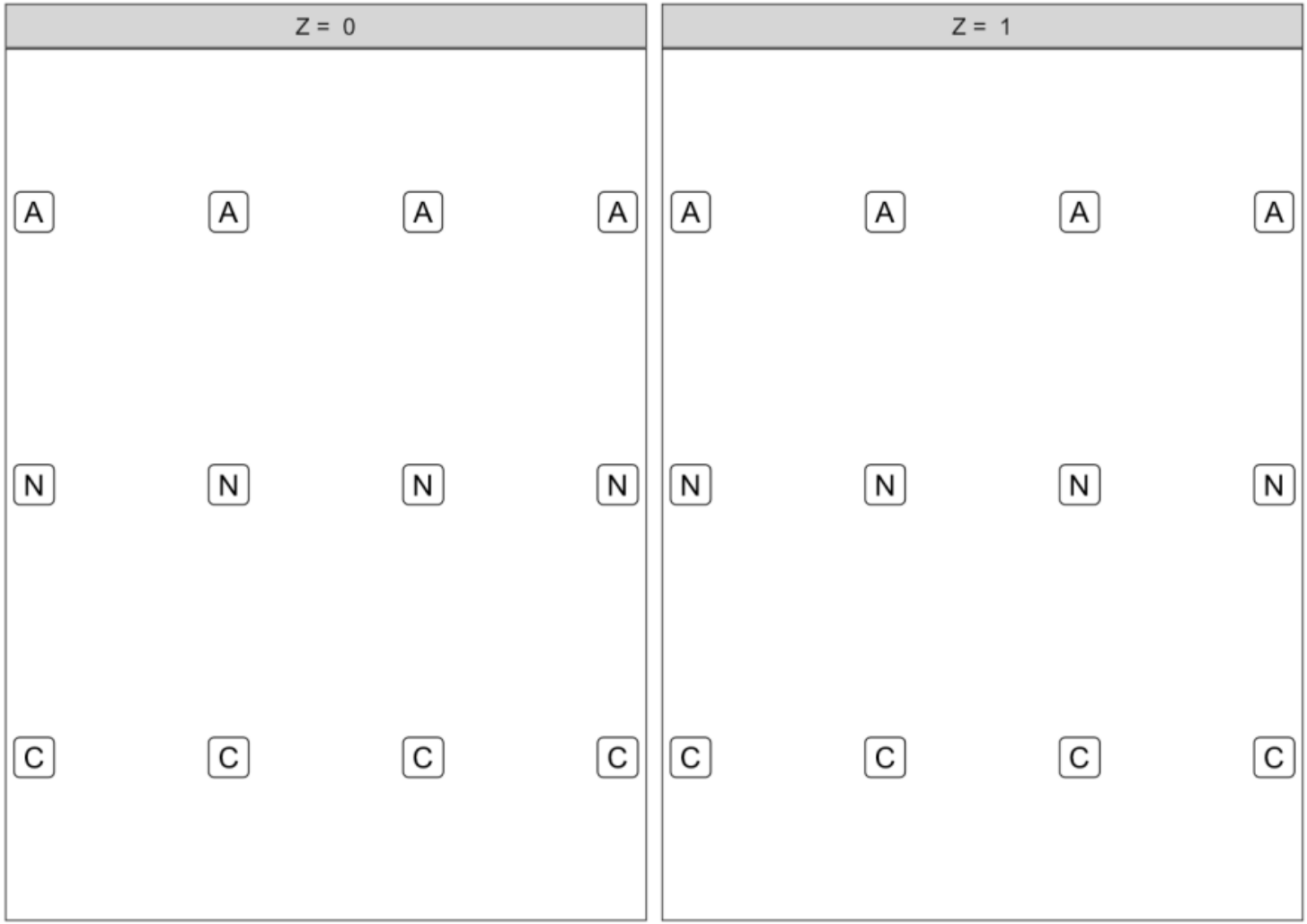
Visual representation

Suppose we have 24 units in our study and we randomly assign half of them to be treated. Visually:



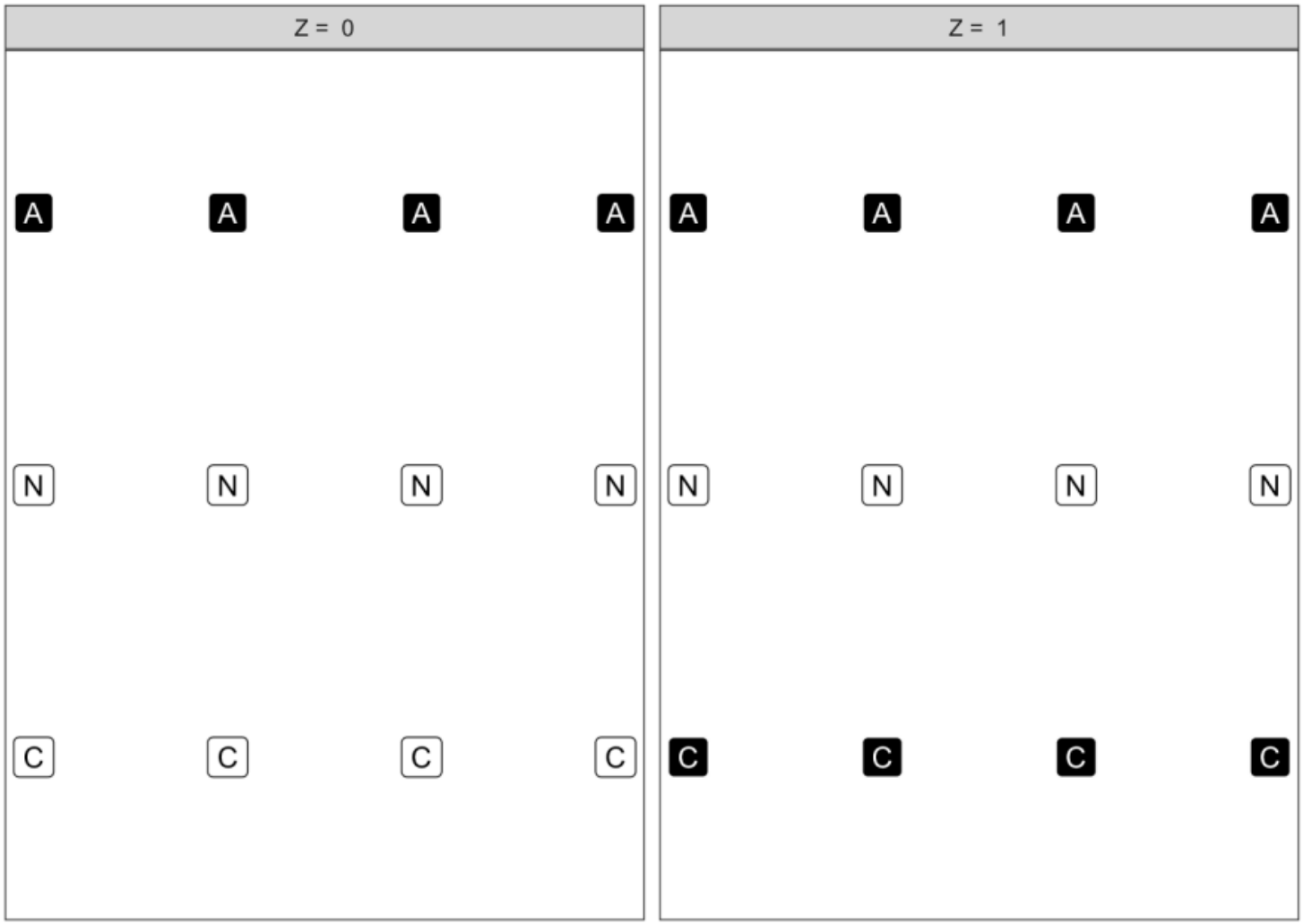
Visual representation

Since we are assigning the treatment randomly, we should expect the distributions of A, N and C types to be identical in both groups:



Visual representation

Who actually gets treated? What happens if we simply compare the averages between these two groups?



Putting the Wald estimator to work

We saw in Problem Set 6 that we can express our IV estimator as

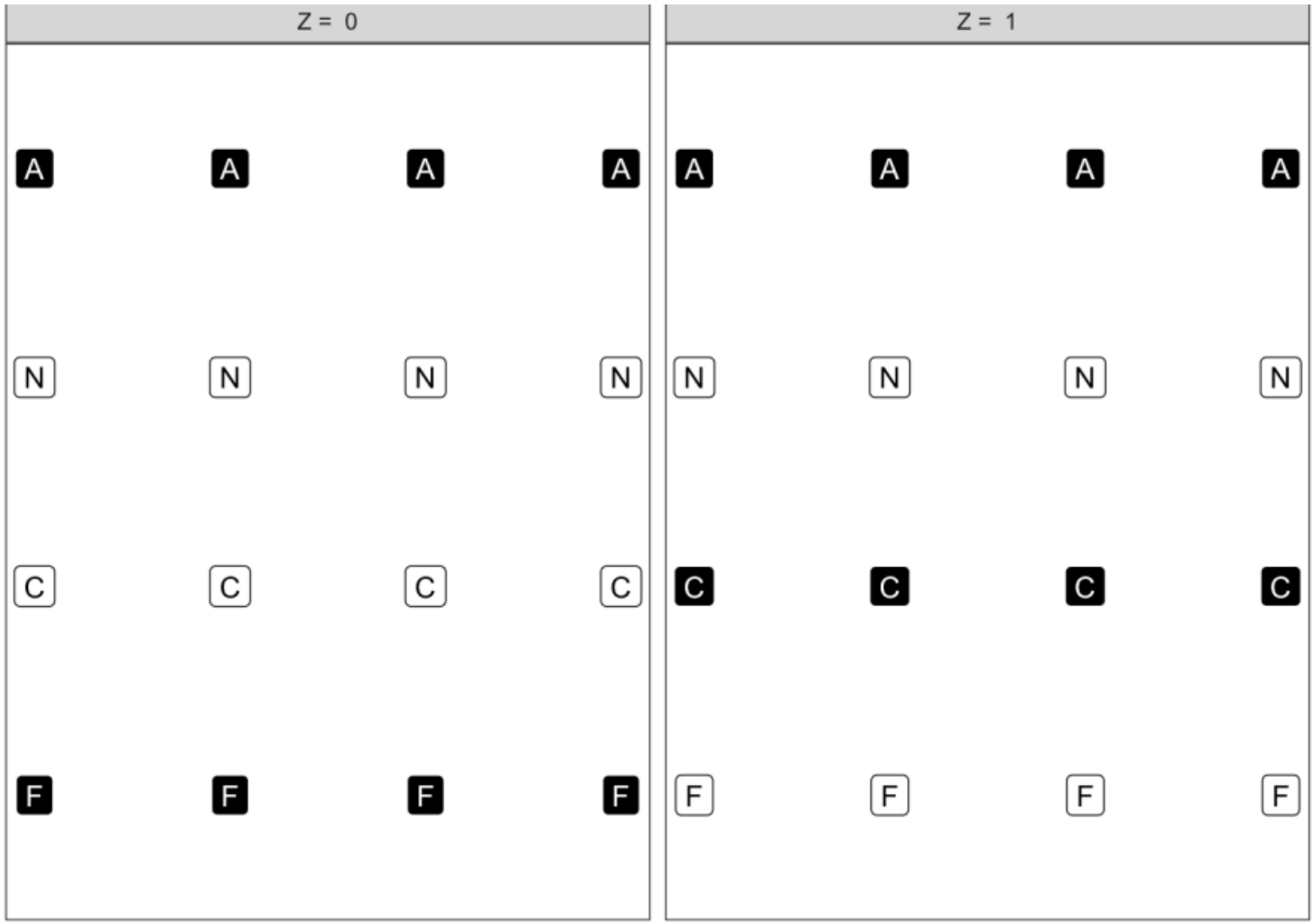
$$\begin{aligned}\hat{\beta}_{IV} &= \frac{\mathbb{E}\left[Y \mid Z=1\right] - \mathbb{E}\left[Y \mid Z=0\right]}{\mathbb{E}\left[X \mid Z=1\right] - \mathbb{E}\left[X \mid Z=0\right]} \\ &\underset{\text{see board!}}{=} \frac{\mathbb{P}\left(C\right)\mathbb{E}\left[Y_C\left(1\right) - Y_C\left(0\right)\right]}{\mathbb{P}\left(C\right)} \\ &= \mathbb{E}\left[Y_C\left(1\right) - Y_C\left(0\right)\right]\end{aligned}$$

Thus we obtain the treatment effect *only for the compliers*, those who are actually affected by assignment. We call this the LATE.

- See Lecture Notes 22 for a more detailed discussion with continuous variables.

Adding the defiers back in

Let's think about the same problem when there are defiers. What will our IV estimator pick up?



Adding the defiers back in

With defiers, our IV estimator is

$$\begin{aligned}\hat{\beta}_{IV} &= \frac{\mathbb{E}\left[Y \mid Z=1\right] - \mathbb{E}\left[Y \mid Z=0\right]}{\mathbb{E}\left[X \mid Z=1\right] - \mathbb{E}\left[X \mid Z=0\right]} \\ &= \frac{\mathbb{P}\left(C\right)\mathbb{E}\left[Y_C\left(1\right) - Y_C\left(0\right)\right] - \mathbb{P}\left(F\right)\mathbb{E}\left[Y_F\left(1\right) - Y_F\left(0\right)\right]}{\mathbb{P}\left(C\right) - \mathbb{P}\left(F\right)}\end{aligned}$$

Defiers make things messy! When there are many defiers or when treatment effects for defiers are large, it's hard to interpret IV estimators. Note that this problem goes away under homogenous treatment effects ($Y_C\left(1\right) - Y_C\left(0\right) = Y_F\left(1\right) - Y_F\left(0\right)$)