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Forecasting Accounting Data: A Multiple Time-series Analysis

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ABSTRACT

This paper examines the relative forecasting performance of multivariate time-series analysis. One hundred consecutive monthly observations for three accounting series were obtained from a manufacturing division of a large corporation. Regression, univariate time-series, transfer-function, and multiple time-series models were identified, estimated, and used to forecast each accounting series. The multiple time-series model yielded the smallest forecast variances.

KEY WORDS Forecasting Multiple time-series analysis Accounting data

Accounting data are typically observed over uniform time periods and frequently exhibit autocorrelation, cross-correlations with other series, trend, and seasonality. Since these dynamic characteristics present many interesting analysis and forecasting problems, the stochastic properties of accounting data have been widely studied. A number of different methodologies have been proposed for analysing accounting data: multiple regression models for cost estimation and control (Comiskey, 1966; Benston, 1966; Jensen, 1967); univariate time-series analysis for earnings forecasting (Foster, 1977; Griffin, 1977; and Brown and Rozeff, 1979), managerial forecasting and control (Mabert and Radcliff, 1974; Jacobs and Lorek, 1979) and auditing (Kinney, 1978); and transfer-function models for examining the unidirectional relationships among financial time series (Umstead, 1977; Kinney, 1978; Cramer and Miller, 1978; Hopwood, 1980).

Multiple regression, univariate time-series, and transfer-function analyses are all special cases of the generalized multiple time-series (hereafter, MTS) model. The objectives of this paper are to (1) to discuss MTS analysis and (2) examine the forecasting performance of MTS relative to the other common analysis methods. To accomplish these objectives, the paper is divided into several sections. First, necessary statistical background on MTS is provided. Secondly, some of the dynamic characteristics frequently exhibited by accounting time series are identified. Thirdly, MTS is applied to a set of cost accounting data, and the relative forecasting performance of MTS is analysed. A summary concludes the paper.

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MULTIVARIATE TIME-SERIES ANALYSIS

MTS is used to refer to multivariate autoregressive moving average (ARMA) models. The MTS model is:[1]

$$\mathbf{\Phi}(B)\mathbf{Z}_t = \boldsymbol{\theta}(B)\mathbf{a}_t$$

where $\mathbf{Z}_t' = (Z_{1t}, \ldots, Z_{Kt})$ is a vector of K time series at time t (assume that the means have been subtracted out),[2] $\mathbf{\Phi}(B) = \mathbf{I} - \boldsymbol{\phi}_1 B - \cdots - \boldsymbol{\phi}_p B^p$, is an autoregressive matrix polynomial with $\boldsymbol{\phi}_i, i = 1, \ldots, p$ being $K \times K$ autoregressive parameter matrices, $\boldsymbol{\theta}(B) = \mathbf{I} - \boldsymbol{\theta}_1 B - \cdots - \boldsymbol{\theta}_q B^q$ is a moving average matrix polynomial with $\boldsymbol{\theta}_i, i = 1, \ldots, q$, being $K \times K$ moving average parameter matrices, $\mathbf{a}_t' = (a_{1t}, \ldots, a_{Kt})$ is a $1 \times K$ vector of errors which are independent and identically distributed with mean the null vector and covariance matrix $\boldsymbol{\Sigma}$, and B is the backshift operator such that $B\mathbf{Z}_t = \mathbf{Z}_{t-1}$. The MTS model is stationary only if the zeros of $|\mathbf{\Phi}(B)|$ all lie outside the unit circle, and is invertible only if all zeros of $|\boldsymbol{\theta}(B)|$ lie outside the unit circle. In practice, non-stationarity can be allowed for by appropriately differencing some or all of the time series $Z_{it}, i = 1, \ldots, K$.

The MTS model is a multivariate generalization of the familiar univariate ARMA models discussed by Box and Jenkins (1970). It is also known (Tiao and Box, 1981) that the MTS model includes the class of transfer function models as special cases. Finally, if the $\boldsymbol{\phi}_i$s and $\boldsymbol{\theta}_i$s are all equal to the null matrix, this model reduces to the familiar multivariate normal distribution from which the multiple regression model is derived. Thus, the MTS model is also a generalization of the multiple regression model.

As in univariate time-series analysis, it is important to have a model-building strategy when attempting to analyse jointly a set of time series. The three stage strategy of identification, estimation, and diagnostic checking proposed by Box and Jenkins (1970) for univariate time-series analysis is also used in building MTS models.

MTS correlation functions

Let $\rho_{ii}(l)$ denote the theoretical autocorrelation between values of time series i and values of the same series lagged by l time periods, and let $\rho_{ij}(l)$ denote the theoretical cross-correlation between values of time series i and values of time-series j lagged by l time periods. Formally:

$$\rho_{ii}(l) = \frac{E[(Z_{it} - \mu_i)(Z_{i(t-l)} - \mu_i)]}{\mathrm{Var}\,(Z_{it})}$$

and

$$\rho_{ij}(l) = \frac{E[Z_{it} - \mu_i)(Z_{j(t-l)} - \mu_j)]}{\sqrt{[\mathrm{Var}\,(Z_{it})\,\mathrm{Var}\,(Z_{jt})]}}$$

where $E(.)$ denotes expected value, Var(.) denotes variance, $\mu_i = E(Z_{it})$, and Z_{it} denotes the value of the i^{th} series at time t.

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[1] Some references that discuss MTS models in more detail are Box *et al.* (1978), Granger and Newbold (1977), Hannan (1970), Jenkins (1979) and Tiao and Box (1981).

[2] Throughout the paper, the symbol ′ denotes the transpose of a vector or matrix and the symbol |.| denotes the determinant of a matrix.

It is convenient to combine the autocorrelations and cross-correlations into a lag l correlation matrix, which for two time series is:

$$\rho(l)=\begin{bmatrix}\rho_{11}(l) & \rho_{12}(l)\\ \rho_{21}(l) & \rho_{22}(l)\end{bmatrix}$$

The set of correlation matrices for all lags is referred to as the *multivariate correlation function*. For stationary multivariate time-series models, the multivariate correlation function together with the variances and covariances at lag zero uniquely determine the multivariate time-series model. Therefore, estimates of the multivariate correlation function can be used in a manner analogous to the sample autocorrelation function in univariate time-series analysis (i.e. for tentative model identification). In practice, a familiarity with the autocorrelation patterns for various MTS models is required. These patterns are derived for a number of MTS models by Tiao and Box (1981).

MTS partial correlation functions

It is sometimes difficult to determine the order of a multivariate autoregressive model from the multivariate correlation function. In an attempt to resolve this problem, Tiao and Box (1981) developed the *multivariate partial correlation function*. Estimates of partial correlations are obtained by fitting successively higher order autoregressive models to the observed $\mathbf{Z}_t$s. For example, in a vector l^{th} order autoregressive model, $\mathbf{Z}_t=\boldsymbol{\phi}_{1l}\mathbf{Z}_{t-1}+\cdots-\boldsymbol{\phi}_{ll}\mathbf{Z}_{t-l}$, the parameter matrix estimates, $\hat{\boldsymbol{\phi}}_{1l},\ldots,\hat{\boldsymbol{\phi}}_{ll}$ can be obtained by least squares techniques for any lag l. The estimated sample partial correlation matrix, $\hat{\mathbf{p}}(l)$ for lag $l=1,\ldots,L$, is defined as the highest order autoregressive parameter matrix estimate for each lag (i.e. $\hat{\mathbf{p}}(l)=\hat{\boldsymbol{\phi}}_{ll}$ for $l=1,\ldots,L$). Tiao and Box (1981) have shown that the multivariate partial correlations cut off after lag p for the pth order multivariate autoregressive model. Thus, the values of $\hat{\mathbf{p}}(l)$ are helpful in tentatively identifying the order of a multivariate autoregressive models (analogous to the use of partial autocorrelations with univariate models).

Estimation and diagnostic checking of MTS models

Once the tentative MTS model has been identified, the results of Hillmer and Tiao (1979), together with maximum likelihood methods, can be used to estimate $\boldsymbol{\Phi}$, $\boldsymbol{\theta}$, $\boldsymbol{\Sigma}$ and their standard errors. Standard normal theory can be used to test the statistical significance of the parameter estimates.

MTS model building requires diagnostic checks to uncover possible model misspecifications. The standard diagnostic checks involve inspecting the standardized residual time series, and examining the cross-correlations of $\hat{\mathbf{a}}_t$. If the cross-correlation matrices, except $\hat{\rho}(0)$, are significantly different from the null matrix, the tentative MTS model may require some adjustments.

Limitations of MTS

Since the MTS model is more general than univariate, transfer-function, and multiple regression models, in theory, the MTS model is preferable to these other models. However, there are at least two potential limitations of MTS. First, applying MTS to actual data has been a recent development, so experience is quite limited as to how the method should be best applied to real world phenomena. Secondly, the model assumes that the cross-correlation structure among time series is relatively stable. If structural changes occur, the accuracy of the post sample forecasts could be reduced.

STATISTICAL PROPERTIES OF ACCOUNTING DATA

In order to assess the appropriateness of a forecasting methodology, the statistical properties of the data to be analysed should be identified. Although the theoretical statistical properties of accounting data have been derived for very few accounting settings (e.g. Sunder (1976) demonstrates that income, under both full cost and successful efforts accounting, is autocorrelated), numerous empirical studies have analysed the time-series properties of accounting data. Four basic conclusions emerge from these studies.[3] First, and almost without exception, earnings (Deschamps and Mehta, 1980; Brown and Rozeff, 1979; Griffin, 1977), sales (Foster, 1977; Mabert and Radcliffe, 1974), and costs (Foster, 1977) exhibit autocorrelation. Secondly, Watts (1975) and Griffin (1977), among others, find prevalent quarterly seasonality in the behaviour of quarterly earnings, and Kinney (1978) reports that monthly revenues exhibit a monthly seasonality. Thirdly, accounting data may exhibit a trend or drift (Deschamps and Mehta, 1980; Brown and Rozeff, 1979). Finally, cross-sectional correlation among accounting time series is likely to exist (Umstead, 1977; Cramer and Miller, 1978; Kinney, 1978; Hopwood, 1980).

Since prior empirical research provides overwhelming support for the conclusion that accounting data are likely to exhibit autocorrelation, seasonality, trend, and cross-correlations with other time series, researchers and practitioners who desire to forecast accounting data should use models which allow for these kinds of dynamic characteristics. MTS provides a class of models which, in theory, can efficiently analyse autocorrelation, seasonality, trend, and cross-correlations among time series. The potential increase in relative forecasting performance that can be obtained by using MTS to analyse the dynamic characteristics of actual accounting data is examined next.

AN APPLICATION TO ACTUAL ACCOUNTING DATA[4]

One hundred consecutive monthly observations for three series (dollar value produced, dollar cost of production, and tons of raw material processed) were obtained from the cost accounting records of one manufacturing division of a *Fortune* 200 company. The *dollar value produced* is the prospective divisional sales from production during the month. The *dollar cost of production* is the total cost allocated to the manufacturing division during the month. The *tons of material processed* is the amount of raw materials issued to production during the month. These three series were selected because the divisional managers experienced problems in forecasting these variables (especially dollar value produced and dollar cost of production). Since forecasts of these series are used in the divisional and corporate production planning and budgeting process, the inaccurate forecasts provided by the division resulted in serious problems regarding selection of work force levels, material management, and divisional performance evaluation.

The three series are plotted in Figure 1. It is readily seen that for both dollar value produced and dollar cost of production, the variance of each series appears to be positively related to its level. This suggests violation of the constant variance assumption. Therefore, the analysis was performed on natural logs of the original series. The transformed data, appearing in Figure 2, are seen to have more constant variability. The remainder of this section describes an analysis of the first 88 observations (the last 12 observations were used as a 'holdout' sample). Hereafter, Z_{1t}, Z_{2t} and Z_{3t} refer to the natural log of dollar value produced, natural log of dollar cost of production, and tons of material processed, respectively.

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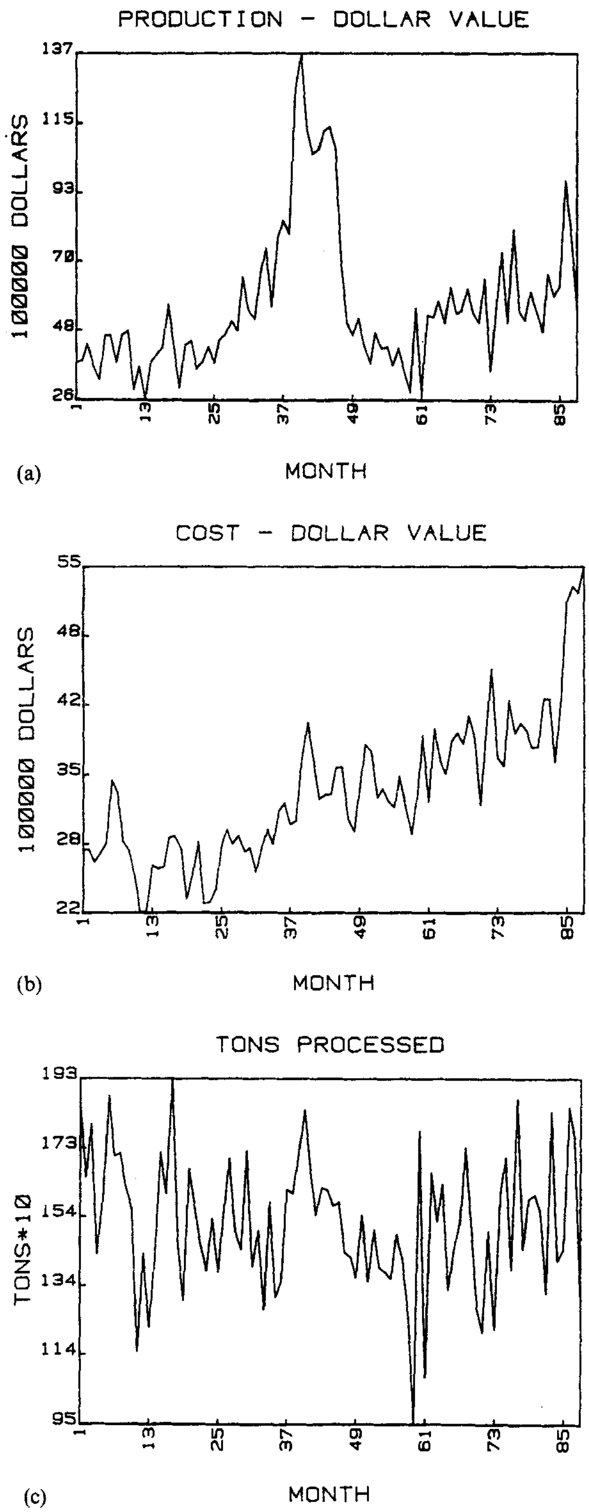
[3] See Foster (1978) for a comprehensive review of the literature on the time-series properties of accounting numbers.

[4] The analysis was conducted using the Wisconsin multiple time series (WMTS-1) program (Department of Statistics, University of Wisconsin—Madison, 1980).


PRODUCTION - DOLLAR VALUE
100000 DOLLARS
137
115
93
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48
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1
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73
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MONTH
COST - DOLLAR VALUE
100000 DOLLARS
55
48
42
35
28
22
1
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25
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73
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MONTH
TONS PROCESSED
TONS*10
193
173
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MONTH

(a) (b) (c)

Figure 1

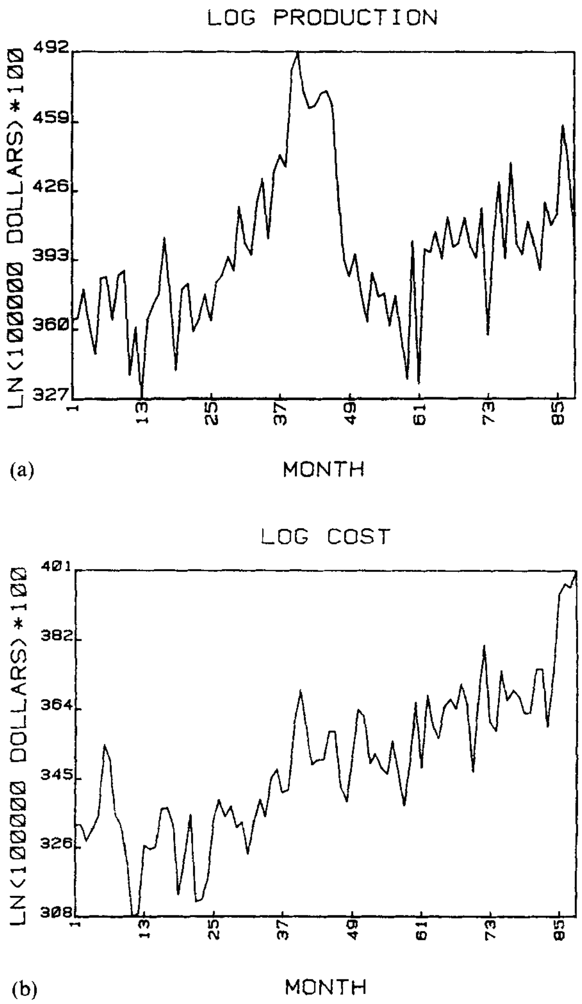

Figure 2

Univariate models

The first step is to construct a univariate time-series model for each of the three series. The sample autocorrelation and partial autocorrelation functions of Z_{1t} are given in Table 1. Since the sample autocorrelation function dies out slowly, the first difference of Z_{1t} was taken. The sample autocorrelation and partial autocorrelation functions of $(1 - B)Z_{1t}$ are also given in Table 1. The autocorrelations cut off after lag 1. Consequently, the model was tentatively identified as:

$$(1 - B)Z_{1t} = (1 - \theta_1 B)a_{1t} \tag{1}$$

for the dollar value produced series.

Estimates of the parameters in equation (1) are given in Table 2. Since the θ_1 parameter was statistically significant and an examination of the residuals revealed no large values or significant autocorrelations, equation (1) appeared to describe adequately the dollar value produced series.

The sample autocorrelation function of Z_{2t} is also presented in Table 1. Since the sample autocorrelations for this series were still quite large after 10 lags, it appeared that the series was

Table 1. Values of sample autocorrelations and partial autocorrelations for the accounting series

Dollar value produced (Z_{1t})										
Lag	1	2	3	4	5	6	7	8	9	10
Autocorrelations	0.70	0.66	0.64	0.51	0.44	0.44	0.32	0.22	0.20	0.11
Standard error	0.11	0.15	0.19	0.21	0.23	0.24	0.24	0.25	0.25	0.25
Partial autocorrelation	0.70	0.34	0.21	−0.10	−0.08	0.10	−0.11	−0.15	0.00	−0.04
Standard error	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11
First difference of dollar value produced [$(1-B)Z_{2t}$]										
Lag	1	2	3	4	5	6	7	8	9	10
Autocorrelations	−0.43	−0.04	0.26	−0.18	−0.08	0.20	−0.07	−0.08	0.10	−0.04
Standard error	0.11	0.13	0.13	0.13	0.14	0.14	0.14	0.14	0.14	0.14
Partial autocorrelation	−0.43	−0.28	0.16	0.01	−0.16	0.04	0.09	−0.02	−0.05	−0.02
Standard error	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11
Dollar cost of production (Z_{2t})										
Lag	1	2	3	4	5	6	7	8	9	10
Autocorrelations	0.90	0.83	0.81	0.78	0.74	0.72	0.69	0.65	0.64	0.60
Standard error	0.11	0.16	0.19	0.22	0.24	0.25	0.26	0.27	0.28	0.29
Partial autocorrelation	0.90	0.07	0.22	−0.03	−0.04	0.25	−0.02	0.05	0.21	−0.10
Standard error	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11
First difference of dollar cost of production [$(1-B)Z_{2t}$]										
Lag	1	2	3	4	5	6	7	8	9	10
Autocorrelations	−0.17	−0.31	0.10	0.04	−0.14	0.05	0.02	0.19	−0.13	0.10
Standard error	0.11	0.11	0.12	0.12	0.12	0.12	0.12	0.12	0.13	0.13
Partial autocorrelation	−0.17	−0.38	−0.05	−0.09	−0.13	−0.05	−0.04	−0.27	0.05	−0.04
Standard error	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11
Tons of material processed (Z_{3t})										
Lag	1	2	3	4	5	6	7	8	9	10
Autocorrelations	0.06	0.18	0.08	0.00	−0.11	0.07	−0.04	−0.02	−0.02	0.03
Standard error	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11
Partial autocorrelation	0.06	0.18	0.06	−0.04	−0.14	0.09	0.01	−0.03	−0.03	0.03
Standard error	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11

Table 2. Estimated parameter values for the univariate models

Model	Parameter	Estimate	Standard error
(1)	θ_1	0.473	0.094
	σ_a^2	0.05189	
(2)	θ_1	0.088	0.100
	θ_2	0.507	0.100
	σ_a^2	0.00928	
(3)	θ_2	0.5132	0.100
	σ_a^2	0.00893	

non-stationary. The sample autocorrelations of $(1-B)Z_{2t}$ cut off after lag 2. Thus, the model was tentatively identified as:

$$(1-B)Z_{2t} = (1-\theta_1 B - \theta_2 B^2)a_{2t} \tag{2}$$

for the dollar cost of production series.

The parameter estimates for this model also appear in Table 2. The residuals revealed that the $\hat{a}_{2t}$ at time 84 was over four standard errors away from zero. Moreover, the estimated parameter θ_1 was not statistically significant. Thus, equation (2) did not appear to adequately describe Z_{2t}.

An attempt was made to find a reason for the apparent outlier, but divisional managers had no explanation for the result. Since an extreme observation can adversely influence the parameter estimates, an adjusted value for Z_{2t} at time 84 was calculated and the equation (2) parameters were re-estimated.[5] Nevertheless, the estimate of θ_1 remained statistically insignificant. Finally, θ_1 was constrained to be zero and the parameters were re-estimated. Based upon the sample autocorrelations and examination of the residuals, the model:

$$(1-B)Z_{2t} = (1-\theta_2 B^2)a_{2t} \tag{3}$$

appeared to adequately represent the dollar cost of production series.

The sample autocorrelations and partial autocorrelations for the time series, Z_{3t}, are also given in Table 1. Since there were no significant autocorrelations or partial autocorrelations for this series, the tons of material processed series appeared to be a random, or white noise, series.

A multivariate model

Analysis of the univariate time series suggested that Z_{1t} and Z_{2t} were non-stationary or close to it. In univariate time-series analysis, non-stationarity can frequently be dealt with by differencing. However, overdifferencing will lead to models that are not invertible. As an example of how this may occur in jointly modelling two or more series, suppose there are two series U_{1t} and U_{2t} such that $U_{1t} = U_{1(t-1)} + a_{1t} - \alpha a_{1(t-1)}$ and $U_{2t} = \beta U_{1t} + a_{2t}$. Since both series are non-stationary, one might conclude that first differences, $W_{it} = (1-B)U_{it}$ for $i = 1, 2$, should be jointly analysed for both series. It is easy to show that the bivariate model for the differenced series is:

$$\begin{bmatrix} W_{1t} \\ W_{2t} \end{bmatrix} = \begin{bmatrix} a_{1t} \\ a_{2t} \end{bmatrix} - \begin{bmatrix} \alpha & 0 \\ \beta(\alpha-1) & 1 \end{bmatrix} \begin{bmatrix} a_{1(t-1)} \\ a_{2(t-1)} \end{bmatrix}$$

However, $|\theta(B)| = (1-\alpha\beta)(1-B)$. That is, a non-invertible model is created by differencing both

[5] The outlier was corrected by using an intervention model (Box and Tiao, 1975) with the intervention being a 'pulse' at time 84. The adjusted series for dollar cost of production was used for estimating and forecasting the univariate, MTS, transfer-function and regression models.

series. The non-invertibility problem occurs because there is only *one* source of non-stationarity for both univariate series.

At the specification stage of MTS model building, the non-stationarity in a set of univariate time series may be caused by non-stationarity of a smaller number of latent variables. For example, dollar value produced and dollar cost of production may be non-stationary because of relationships to a non-stationary latent variable, such as the general level of production activity. Unfortunately, it is not known at the specification stage of the model building whether this is the cause of non-stationarity.

One way to detect a non-stationary latent variable is to difference the time series to the same degree as in the univariate analysis, model the differenced series, and search for unit roots in the moving average operator. If this method is used, it is important to use an exact maximum likelihood estimation algorithm (see Hillmer and Tiao, 1979). A second approach is to use the canonical analysis of Box and Tiao (1977) on the undifferenced data to determine the number of non-stationary components. The approach that this paper adopts is to fit a model to the undifferenced data and then search for near unit roots in the final aggressive operator. This approach is preferred because it avoids overdifferencing and allows the data to suggest the necessary degree of differencing.[6]

The estimated multivariate partial correlation matrices for the undifferenced data (Table 3) suggest that at least a multivariate first order autoregressive model is necessary. In addition, the multivariate correlation matrices for the *residuals* of a first order autoregressive model were statistically significant at lags one and two. This suggests that a second order moving average component is also required. Therefore, a tentative model for characterizing the three series simultaneously is:

$$\begin{bmatrix} Z_{1t} \\ Z_{2t} \\ Z_{3t} \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{bmatrix} \begin{bmatrix} Z_{1(t-1)} \\ Z_{2(t-1)} \\ Z_{3(t-1)} \end{bmatrix} + \begin{bmatrix} a_{1t} \\ a_{2t} \\ a_{3t} \end{bmatrix} - \begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{13} \\ \theta_{21} & \theta_{22} & \theta_{23} \\ \theta_{31} & \theta_{32} & \theta_{33} \end{bmatrix} \begin{bmatrix} a_{1(t-1)} \\ a_{2(t-1)} \\ a_{3(t-1)} \end{bmatrix} - \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ \beta_{21} & \beta_{22} & \beta_{23} \\ \beta_{31} & \beta_{32} & \beta_{33} \end{bmatrix} \begin{bmatrix} a_{1(t-2)} \\ a_{2(t-2)} \\ a_{3(t-2)} \end{bmatrix} \tag{4}$$

where the ϕs are first order autoregressive parameters, the θs are first order moving average parameters, and the βs are second order moving average parameters.

Model (4) has 27 autoregressive or moving average parameters to be estimated from 264 data values (3 series $\times 88$ observations per series). Although it is possible to estimate all these parameters, it is likely that the interpretability of the model and the forecasting performance can be improved if a relatively parsimonious structure can be identified before estimation. In our particular example, accounting theory provides no guidance in determining which parameters should theoretically be constrained to equal zero. However, the results of the univariate analysis suggested several potential simplifications to model (4).

Since only the univariate model for Z_{2t} required a second order moving average component, it is reasonable to assume that the components of the second order moving average matrix, except β_{22},

[6] It is shown later that the data suggest a need to difference both Z_{1t} and Z_{2t}. Thus, the analysis could have been performed in terms of the differences of these series. However, since this will not always be the case, it is prudent to allow for general behaviour.

Table 3. Multivariate partial autocorrelation matrices for the accounting series

Lag	Estimate			Standard error			Test statistic*
1	0.850	0.095	−0.543	0.088	0.144	0.148	
	0.006	0.914	−0.114	0.039	0.064	0.065	227.1
	−0.004	0.104	0.028	0.073	0.121	0.124	
2	0.308	−0.330	−0.030	0.165	0.275	0.164	
	0.116	−0.019	−0.167	0.073	0.121	0.072	16.5
	−0.026	−0.217	0.236	0.139	0.232	0.138	

* The test statistic is a test for statistical significance of the multivariate partial autocorrelation matrix. It has an approximate chi-squared distribution with 9 degrees of freedom. The critical value for $\alpha = 0.05$ and 9 degrees of freedom is 16.9.

may be set equal to zero. The univariate analysis also indicated that Z_{3t} was a random series, suggesting that each element in the bottom row of each matrix in (4) could be set equal to zero. The parameters in the bottom row of the matrices may, of course, be non-zero in such a way that Z_{3t} is random. One method of checking the reasonableness of setting $\phi_{31} = \phi_{32} = \phi_{33} = \theta_{31} = \theta_{32} = \theta_{33} = 0$ is to estimate these parameters and then test the null hypothesis that they equal zero.[7] This test was conducted by first estimating model (4) with $\theta_{31} = \theta_{32} = \theta_{33} = 0$, and re-estimating (4) with $\phi_{31} = \phi_{32} = \phi_{33} = 0$. The estimates for the parameters of interest and the associated likelihood ratio test statistics for these two models are reported in Table 4. In both cases, the parameter estimates were nearly uncorrelated and the parameter estimates were statistically insignificant (smaller than twice their standard error). Further, in both cases the likelihood ratio test statistics were smaller than the critical chi-square value. Thus, the hypothesis that these parameters are zero cannot be rejected, and ϕ_{31}, ϕ_{32}, ϕ_{33}, θ_{31}, θ_{32} and θ_{33} were set equal to zero in the model (4).

After incorporating the above restrictions, the remaining parameters of (4) were re-estimated, and insignificant parameters were set equal to zero. Parameter estimates for ϕ_{11} and ϕ_{22} were very close to unity, so these parameters sere set equal to one when the model was re-estimated. The resulting model for the three series was:

$$\begin{bmatrix} Z_{1t} \\ Z_{2t} \\ Z_{3t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \phi_{23} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} Z_{1(t-1)} \\ Z_{2(t-1)} \\ Z_{3(t-1)} \end{bmatrix} + \begin{bmatrix} a_{1t} \\ a_{2t} \\ a_{3t} \end{bmatrix} - \begin{bmatrix} \theta_{11} & 0 & \theta_{13} \\ 0 & \theta_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_{1(t-1)} \\ a_{2(t-1)} \\ a_{3(t-1)} \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & \beta_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_{1(t-2)} \\ a_{2(t-2)} \\ a_{3(t-2)} \end{bmatrix} \tag{5}$$

Parameter estimates for model (5) are reported in Table 5. The diagnostic checks for this model revealed no inadequacies. In addition, the likelihood ratio test statistic for the parameter restrictions leading to model (5) from the model in which $\phi_{31} = \phi_{32} = \phi_{33} = \theta_{31} = \theta_{32} = \theta_{33} = 0$ equalled 13.11 (the critical chi-square value for $\alpha = 0.05$ and eight degrees of freedom is 15.5). Thus, the parameter restrictions in (5) appeared to be consistent with the data.

[7] One potential problem in conducting this test is that if the true model for Z_{3t} is a white noise series and the model $(1 - \phi_{33}B)Z_t = (1 - \theta_{33}B)a_t$ is estimated, the parameter estimates $\hat{\phi}_{33}$ and $\hat{\theta}_{33}$ can be highly correlated. Therefore, the cancellation of the autoregressive and moving average polynomial may be difficult to realize and both parameter sets, ϕ_{31}, ϕ_{32}, ϕ_{33} and θ_{31}, θ_{32}, θ_{33}, should not be included in the model at the same time.

Table 4. Statistical tests for the restrictions on model (4)

Parameter	Estimate	Standard error
Model 1: $\theta_{31}=\theta_{32}=\theta_{33}=0$		
ϕ_{31}	−0.0438	0.0676
ϕ_{32}	0.1671	0.1125
ϕ_{33}	0.0283	0.0862
Likelihood ratio test statistic* (H_0: $\phi_{31}=\phi_{32}=\phi_{33}=0$) = 2·16		
Model 2: $\phi_{31}=\phi_{32}=\phi_{33}=0$		
θ_{31}	−0.0265	0.1027
θ_{32}	0.0249	0.1971
θ_{33}	0.0064	0.1088
Likelihood ratio test statistic* (H_0: $\theta_{31}=\theta_{32}=\theta_{33}=0$) = 0.27		

* The likelihood ratio test statistic has an approximate chi-squared distribution with 3 degrees of freedom. The critical value for $\alpha=0.05$ and 3 degrees of freedom is 7.81.

Table 5. Parameter estimates for the multivariate model (5)

Parameter	Estimate	Standard error
ϕ_{23}	0.0039	0.0016
θ_{11}	0.3415	0.0967
θ_{13}	0.3916	0.1104
θ_{22}	0.2070	0.0858
β_{22}	0.5380	0.0871

Estimated error covariance matrix, $\hat{\Sigma}=\begin{bmatrix}0.04814 & 0.00846 & 0.03056\\ 0.00846 & 0.00793 & 0.00551\\ 0.03056 & 0.00551 & 0.03534\end{bmatrix}$

Substitution of the parameter estimates and completion of the matrix multiplication in (5) yields:

$$\begin{aligned}
Z_{1t} &= Z_{1(t-1)} + a_{1t} - 0.341\,a_{1(t-1)} - 0.392\,a_{3(t-1)}\\
Z_{2t} &= Z_{2(t-1)} + 0.004\,Z_{3(t-1)} + a_{2t} - 0.207\,a_{2(t-1)} - 0.538\,a_{2(t-2)}\\
Z_{3t} &= a_{3t}
\end{aligned} \tag{6}$$

The corporate managers supplying these data indicated that the tons of material processed during month $T+1$ were essentially known in month T. Therefore, the equations (6) can be rewritten to reflect the effect of current values of Z_{3t} on Z_{1t} and Z_{2t}. Using properties of the multivariate normal distribution (Anderson, 1958), given $a_{3t}(Z_{3t})$, the vector (a_{1t}, a_{2t}) has a bivariate normal distribution with mean vector, $(0.865\,Z_{3t}, 0.156\,Z_{3t})$, and covariance matrix:

$$\begin{bmatrix}0.02171 & 0.00370\\ 0.00370 & 0.00707\end{bmatrix}$$

For a given Z_{3t}, $a_{1t} = 0.865\,Z_{3t} + a^*_{1t}$, where Z_{3t} and a^*_{1t} are independent, and $a_{2t} = 0.156\,Z_{3t} + a^*_{2t}$, where Z_{3t} and a^*_{2t} are independent. Substituting these expressions into (6) yields:

$$Z_{1t} = Z_{1(t-1)} + a^*_{1t} - 0.341\,a^*_{1(t-1)} + 0.865\,Z_{3t} - 0.687\,Z_{3t(t-1)} \tag{7}$$

$$Z_{2t} = Z_{2(t-1)} + a^*_{2t} - 0.207\,a^*_{2(t-1)} - 0.538\,a^*_{2(t-2)} + 0.156\,Z_{3t} - 0.028\,Z_{3(t-1)} - 0.084\,Z_{3(t-2)} \tag{8}$$

The following observations can be made regarding equations (7) and (8): (1) except for a correlation of 0.299 between a^*_{1t} and a^*_{2t}, the dollar value produced and dollar cost production did not interact,[8] (2) Z_{1t} depended upon its own past and upon the current and past values of Z_{3t}, and (3) Z_{2t} depended upon its own past and upon current and past values of Z_{3t}.

Equations (7) and (8) provide several intuitive results regarding the production and accounting processes of this manufacturing division. Both Z_{1t} and Z_{2t} were related to their immediate past, suggesting that if tons processed were held constant, periods of high levels of dollar value (or cost) produced would be followed by continued high levels. Each series was also positively related to the tons of material processed at a lag of zero. Therefore, as expected, larger quantities of material processed in a month gave rise to larger value produced and larger cost of production. More interesting, both series were negatively related to tons processed at other lags. Corporate and divisional managers suggested that this result occurred because of the characteristics in their product markets. In particular, their product markets were non-seasonal, and the potential market in any short-run time period was basically fixed. If the division processed a 'large' number of tons in a given month, not only did dollar value (and cost) produced increase, but the level of production needed to satisfy market demand in subsequent periods was also lower. Therefore, next month's production would be cut back, and a negative relationship between dollar value produced and dollar cost of production with lagged values of tons processed would occur.

Transfer-function models

The MTS model (5) has parameter matrices which are upper triangular, implying that unidirectional relationships can adequately describe the three accounting series. In particular, there is no interaction between Z_{1t} and Z_{2t}, and Z_{3t} is a leading indicator of both Z_{1t} and Z_{2t}. The results suggest that MTS models for Z_{1t} and Z_{2t} are equivalent to transfer functions with one input, Z_{3t}. Therefore, (7) and (8) are simply transfer-function models as discussed by Box and Jenkins (1970).

Given these results from the multivariate analysis, the following transfer-function fits were obtained for Z_{1t} and Z_{2t} (standard errors in parentheses):

$$Z_{1t} = Z_{1(t-1)} + a_{1t} - \underset{(0.102)}{0.350}\,a_{1(t-1)} + \underset{(0.076)}{0.813}\,Z_{3t} - \underset{(0.074)}{0.773}\,Z_{3(t-1)} \tag{9}$$

$$Z_{2t} = Z_{2(t-1)} + a_{2t} - \underset{(0.080)}{0.125}\,a_{2(t-1)} - \underset{(0.080)}{0.748}\,a_{2(t-2)} + \underset{(0.047)}{0.215}\,Z_{3t} - \underset{(0.026)}{0.059}\,Z_{3(t-1)} - \underset{(0.047)}{0.137}\,Z_{3(t-2)} \tag{10}$$

Obviously, the only differences between (9) and (7) and (10) and (8) involve the parameter estimates. It has been argued by Nelson (1976) and demonstrated empirically by Moriarty and

[8] That is, the past values of Z_{2t} do not appear in the equation for Z_{1t} and past values of Z_{1t} do not appear in the equation for Z_{2t}. The correlation between a^*_{1t} and a^*_{2t} is $\mathrm{Cov}(a^*_{1t}, a^*_{2t})/\sqrt{[\mathrm{Var}(a^*_{1t})\,\mathrm{Var}(a^*_{2t})]} = 0.00370/\sqrt{[(0.02171)(0.00707)]} = 0.299$.

Salamon (1980) that joint estimation of contemporaneously correlated time series can produce more efficient parameter estimates and better forecasts than individual analysis. Since MTS simultaneously analyses all three series, the parameter estimates in (7) and (8) are more efficient than those in (9) and (10). Therefore, the forecasting performance of MTS was expected to be the same or superior to that of transfer function methods.

Regression models

Simple regression analysis is a rather naïve statistical method for forecasting Z_{1t} and Z_{2t}, conditional on Z_{3t}. Since Z_{1t} and Z_{2t} are uncorrelated, simple linear regressions of dollar value produced and dollar cost of production were estimated with current tons of material processed (Z_{3t}) as an independent variable. However, these regressions are not properly specified because of autocorrelation in the data. Results of the regression models are presented only to indicate how forecasts can improve when more appropriate time series methods are used. The parameter estimates for the two simple linear regressions (standard errors in parentheses) were:

$$Z_{1t} = \underset{(0.273)}{2.67} + \underset{(0.179)}{0.854}\, Z_{3t} \tag{11}$$

and

$$Z_{2t} = \underset{(0.171)}{3.26} + \underset{(0.111)}{0.148}\, Z_{3t} \tag{12}$$

Forecasting performance

The forecasting equation ($\hat{Z}_{it}$), expected forecasting performance, and actual forecasting performance of the four models (simple regression, univariate time series, transfer function and MTS) are summarized in Table 6. For the series Z_{1t}, 79 per cent, 58 per cent, and 5 per cent

Table 6. Accuracy of alternative forecasting models

Forecasting dollar value produced (Z_{1t})			
Method	Model	Mean squared forecast error Expected	Actual
Regression	$\hat{Z}_{1T} = 2.67 + 0.854\, Z_{3(T+1)}$	0.10227	0.05700
Univariate time series	$\hat{Z}_{1T} = Z_{1T} - 0.473\, a_{1T}$	0.05189	0.07860
Transfer function	$\hat{Z}_{1T} = Z_{1T} - 0.350\, a_{1T} + 0.813\, Z_{3(T+1)} - 0.773\, Z_{3T}$	0.02297	0.04070
MTS	$\hat{Z}_{1T} = Z_{1T} - 0.341\, a^*_{1T} + 0.865\, Z_{3(T+1)} - 0.687\, Z_{3T}$	0.02171	0.04113

Forecasting dollar cost of production (Z_{2t})			
Method	Model	Mean squared forecast error Expected	Actual
Regression	$\hat{Z}_{2T} = 3.26 + 0.148\, Z_{3(T+1)}$	0.03968	0.23000
Univariate time series	$\hat{Z}_{2T} = Z_{2T} - 0.513\, a_{2(T-1)}$	0.00839	0.00672
Transfer function	$\hat{Z}_{2T} = Z_{2T} - 0.125\, a_{2T} - 0.748\, a_{2(T-1)} + 0.215\, Z_{3(T+1)} - 0.059\, Z_{3T} - 0.137\, Z_{3(T-1)}$	0.00699	0.00570
MTS	$\hat{Z}_{2T} = Z_{2T} - 0.207\, a^*_{2T} - 0.538\, a^*_{2(T-1)} + 0.156\, Z_{3(T+1)} - 0.028\, Z_{3T} - 0.084\, Z_{3(T-1)}$	0.00707	0.00343

increases in accuracy were expected when using the MTS model rather than the simple regression, univariate, and transfer-function models, respectively. For the series Z_{2t}, 82 per cent and 16 per cent increase and 1 per cent decreases in forecast accuracy were expected when using the MTS model rather than the simple regression, univariate, and transfer-function models, respectively.

In order to compare expected forecast accuracy with actual forecast accuracy, one-step-ahead forecasts for the holdout sample (observations 89 to 100) were computed, and the actual one-step-ahead post sample forecast errors were calculated. The actual mean squared forecast errors for the post sample period are reported in Table 6. For the Z_{1t} series, there were 28 per cent and 48 per cent increases, and a 1 per cent decrease in forecast accuracy when using the MTS model rather than the simple regression, univariate, and transfer-function models, respectively. For Z_{2t}, there were 98 per cent, 49 per cent, and 49 per cent increases in forecast accuracy when using the MTS model rather than the simple regression, univariate, and transfer-function models, respectively. Although the usefulness of a decrease in forecast variance of the magnitude found in this example should be related to issues such as the specific managerial decision being considered, the loss function of the decision maker, and the additional cost to implement the MTS analysis (Demski and Feltham, 1972), these results generally suggest that (1) MTS models allow efficient analysis of the dynamic properties typically observed in accounting data and (2) MTS models can lead to superior forecasting performance.[9]

SUMMARY

This study analysed the relative forecasting performance of regression, univariate time-series, transfer-function, and multivariate time-series analyses in the context of a set of cost accounting data. Regression, univariate time-series, and transfer-function analyses are reasonably well established in the accounting literature. However, the theory and computational aspects of MTS analysis have only recently been developed. MTS combines the advantages of regression, univariate time-series, and transfer-function analyses, and thus has the potential, in theory, to yield smaller forecast variances than any of these methods. This theoretical conclusion was generally supported by the empirical results.

The discussions and applications in this study also illustrate four additional advantages of MTS over traditional transfer function methods.

(1) The application of MTS is analogous to the logic that is used to construct univariate time-series models. The difficulties involved in identifying a transfer function from the cross-correlations between one original series and a prewhitened series are avoided.
(2) The small number of transfer-function applications in accounting have considered only two series. Although there is no theoretical reason which restricts transfer-function analysis to only two series, an analysis of more than two series is cumbersome. As demonstrated in the example, MTS analysis can be readily applied to three series simultaneously and, in theory, MTS can be generalized to any number of series.
(3) Since MTS analyses all time series simultaneously, the MTS parameter estimation is more efficient than transfer-function estimation.

[9] MTS may be especially helpful in forecasting divisional profitability and selecting optimal production levels. For example, cost–volume–profit analysis (e.g. Adar *et al.*, 1977) requires the expected value and variance of profit. Since profit is a linear function of two correlated series (revenues and costs), an analysis of the autocorrelations *and* cross-correlation of revenues and costs via MTS is likely to provide a more accurate assessment of the expected value and variance of profit than a simple analysis of the autocorrelations of the profit series.

(4) Transfer functions do not allow *each* series to be a function of both its past *and* the past of the other series (i.e. a non-recursive form). Although the example in this study did not require modelling a non-recursive structure, MTS can identify and estimate these models.

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