

# Taxation Through Coercion

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## 1 | Introduction

Taxation has a long history, with the earliest instances dating back to ancient Mesopotamia around 3000BC, where the Sumerians enforced a tax system to support the ruling class and finance public infrastructure[1]. Similarly Ancient Egyptians implemented a tax system to sustain the Pharaoh and other public services[1], as did every other civilization throughout history. As civilizations evolved, so did the role and reach of taxation, however one aspect of taxation has remained constant. From the Sumerians providing to their rulers, to the Egyptians supporting the Pharaoh, and to the serfs and lesser lords paying tax to the King or Emperor, the state apparatus has had a sustainable, preferential treatment through taxation. From the past to today, politicians and bureaucrats have used and abused taxation as a means to sustain their position and livelihood, as well as increase their power and personal wealth. In the present day, the bureaucratic machinery of government, funded by taxation, continues this tradition, by funding public salaries, thereby maintaining a system in which public officials are directly reliant on the extracted wealth of their citizenry.

French economist and legislator Frédéric Bastiat, one of the early pioneers of classical liberalism, challenged the purpose of taxation, by suggesting that the state, empowered by a perverted notion of what the law is about, could conduct activities that, if performed by a criminal, would be considered illegal and immoral.

*See if the law takes from some persons what belongs to them, and gives it to other persons to whom it does not belong. See if the law benefits one citizen at the expense of another by doing what the citizen himself cannot do without committing a crime.*  
Frédéric Bastiat[2]

This line of thought was developed further by several philosophers, writers, and ideologues, most notably the Austrian School, including Ludwig von Mises, Frederik Hayek, and Murray Rothbard amongst others, who all contributed significantly to the classical liberal and libertarian movements and their collective work forms a critical backbone to the discourse around taxation, authority, and coercion.

Following Bastiat and these other pivotal thinkers, Michael Huemer presents the same idea, albeit in a different way. In the introduction of *The Problem with Political Authority*, he tells a story about an individual, who acts in ways typically reserved for governments. He imposes rules on his neighbors, forcefully extracts 'protection money' from them, and punishes those who fail to comply. Outside of the governmental context, these actions would undoubtedly be considered criminal, but, when carried out by the government, these same actions are not only accepted but often seen as essential. Huemer uses this thought experiment to illuminate the disparity in moral judgment we apply to state versus individual actions.

*What does this story have to do with political philosophy? In the story, you behaved like a rudimentary government. Though you did not take on all the functions of a typical, modern state, you assumed two of its most central roles: you punished people who violated others' rights or disobeyed your commands, and you collected non-voluntary contributions to finance your activities. In the case of the government, these activities are referred to as the criminal justice system and the tax system. In your case, they are referred to as kidnapping and extortion.*

Michael Huemer[3]

Building on these ideas, this project uses the paper "Analysing Tax Evasion Dynamics via the Ising Model"[4] to show how governments require the use of force to curb tax evasion and ensure compliance, which validates the notions set by Bastiat and Huemer.

## 2 | The Working Model

Tax evasion is a complex issue that has been studied from various perspectives, including economic, legal, and statistical. The Ising model is a mathematical model frequently used in statistical physics and related fields to study systems with interacting particles or agents, but has also been applied to various topics outside of physics, where we observe dual behaviour<sup>1</sup>, such as social networks, economics, and decision-making.

The Ising model constitutes a simplified model of magnetism in materials and magnetic phase transitions. In the model, we have spins  $S_i$  that can either take the value  $-1$  or  $+1$  with each spin living on each site of an arbitrary N-lattice. Typically, the interaction of spins is between nearest neighbors, with the equation dictating the behaviour being

$$\mathcal{H} = - \sum_{\langle i,j \rangle} J_{i,j} S_i S_j \quad (2.1)$$

where  $J_{i,j}$  is the coupling constant, or the influence one has on it's neighbours, which is assumed to be constant ( $J_{i,j} = J$  for all neighbours).

In the context of tax evasion, we can interpret the Ising model as a representation of individual taxpayers in a society, where each taxpayer is a spin  $S_i$  that can take one of two values,  $+1$  for honest taxpayers who fully comply with tax laws, and  $-1$  for tax evaders who do not comply. The interaction between taxpayers, represented by the coupling constant  $J_{i,j}$ , can be thought of as the influence that individuals exert on each other within their social networks.

The total energy or Hamiltonian of the system,  $\mathcal{H}$ , represents the overall state of tax compliance in the society. A lower energy state, where most spins align in the same direction, could represent a society with high tax compliance or high tax evasion, depending on the direction of alignment. Conversely, a higher energy state, where spins are more randomly aligned, could represent a society with mixed levels of tax compliance.

The model also introduces the temperature  $T$ , which in the model stands for the individuality one feels<sup>2</sup>, and measures the extent to which individuals behave randomly rather than copy societal norms. This means that for low temperatures, each individual will base their actions according to their neighbours, while in higher temperatures they will tend to randomly flip between the two states.

In addition, the model incorporates the probability of an efficient audit. If tax evasion is detected, the individual must remain honest for a number of periods to be specified. This introduces a temporal aspect to the model, where the past behavior of individuals can influence their future behavior, albeit in a very limited and constricted way.

This approach allows us to study the dynamics of tax evasion in a society as a collective phenomenon, influenced by both individual decision-making and social interactions. It provides a framework for understanding how changes in policy measures such as audit rates and penalty levels can affect overall levels of tax compliance.

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<sup>1</sup>for simplicity, we assume that you either evade taxation or pay it

<sup>2</sup>I felt like this wording fit better than the original choice, which was "social temperature"

### 3 | Model Dynamics

By using the 2d Ising model we can already draw some conclusions. Based on the phase-change diagram of the Ising Model[5, 4] we know that a phase change occurs at about  $T_C \approx 2.2$ , which means that for  $T > T_c$  we have half of the spins pointing to one direction, and the other half point in the other. Accordingly, for  $T < T_c$ , all spins point towards the same direction. This means that if we reduce individuality enough, we have a compliant population<sup>1</sup>.

For the implementation of the model, tax evasion is simulated for various temperature levels and different levels of punishment. The severity of punishment is represented by  $k$ , which denotes the number of periods a detected tax evader must remain honest.

The model is simulated over 300 time steps, with the probability of an audit sequentially increased, from 0 to 1. For a given probability of an audit, the dynamics of tax evasion (measured as a portion of the entire population) is depicted over the range of these time steps.

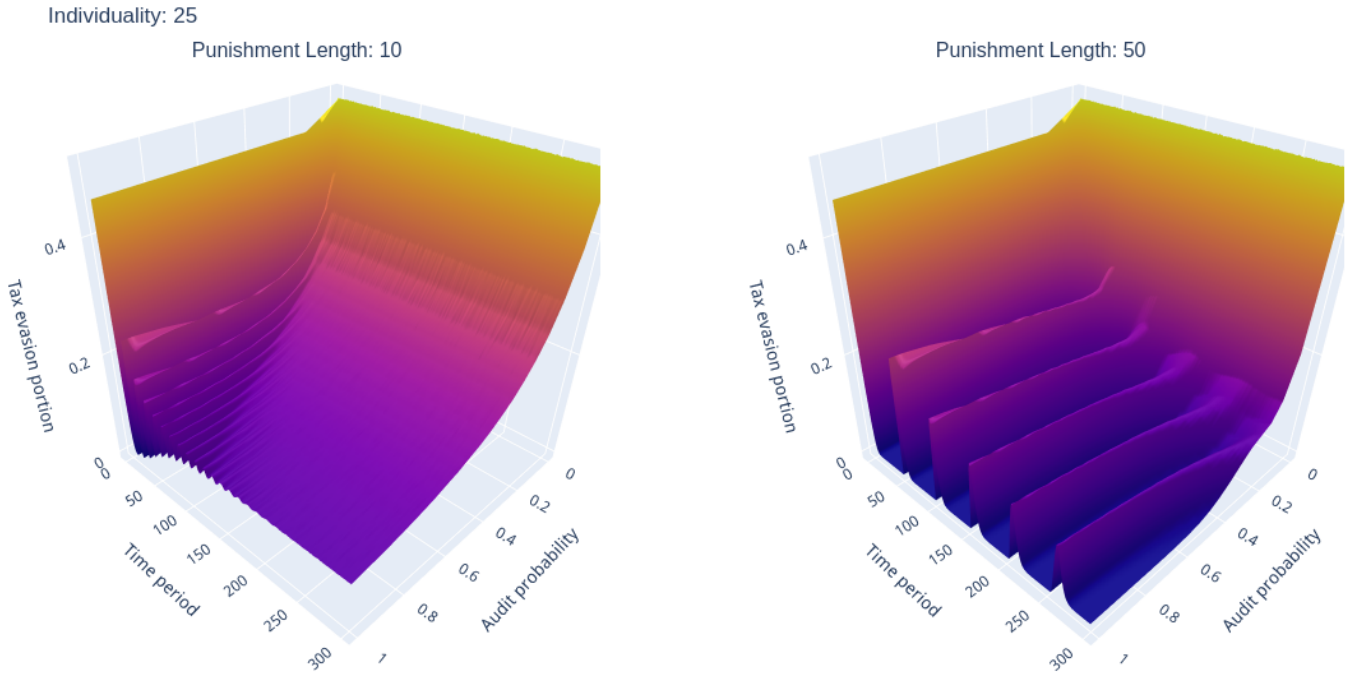


Figure 3.1: Tax evasion dynamics for  $T=25$ , for two different lengths of punishment.

At  $T = 25$ , if the penalty is high enough (e.g.,  $k = 50$ ), tax evasion can be reduced to 0 in the short run, given that the probability of an audit is sufficiently high (Fig. 3.1). In the case of a penalty duration of 50 periods and a probability of an audit of 0.9, within only a few periods each individual eventually is compelled to remain honest. This happens because spins flip relatively often at this temperature, which is far above the critical level (Fig. 3.2).

<sup>1</sup>If we chose to use higher dimensions it is very hard to calculate the critical point (if it exists), and the exact calculations are still an open problem for  $d = 3$ .

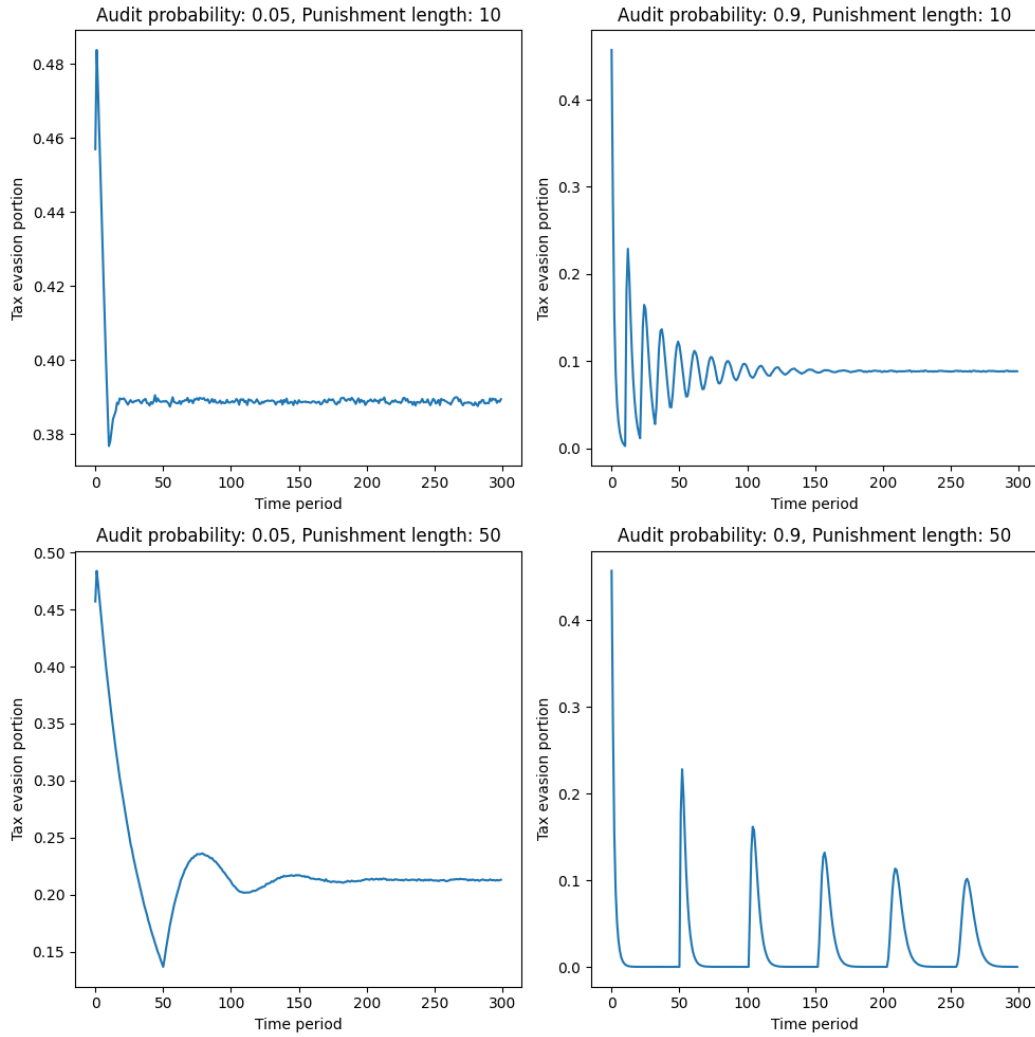


Figure 3.2: Tax evasion dynamics for  $T=25$ , for two different audit probabilities and two different punishment lengths.

However, if the punishment is set equal to 10 periods, tax evasion only approaches zero percent and finally reaches equilibrium at a tax evasion rate of around 10% of the total population, in the case of an audit probability of 0.9. Because the length of punishment now is too small to reach full compliance, the peaks are somewhat greater than at  $k = 50$  for the same audit probability. This suggests that at high temperatures, a sufficiently long punishment period is crucial for achieving full compliance.

Looking at other temperatures above the critical temperature  $T_c \approx 2.2$ , the adjustment towards the equilibrium in the baseline model (i.e., 50% non-compliance) occurs more slowly. As individuals become tax evaders more slowly, the tax evasion problem is less pronounced from the beginning compared to higher temperatures. However, because spins flip less frequently at lower temperatures, the same enforcement mechanisms may work less efficiently in the short run than at higher temperatures (Fig. 3.3).

When considering the time series with  $k = 50$  and  $p_a = 0.9$  at  $T = 2.5$ , we see that evasion cannot be reduced to zero percent anymore (Fig. 3.3). If the temperature is at  $T = 25$ , everybody becomes an evader within only a few periods, so that the enforcement mechanism quickly catches the entire population, given that the probability of an audit is sufficiently high. Yet, for the considered low temperature of  $T = 2.5$ , the enforcement mechanism does not encompass every person anymore, because it takes longer for all individuals to once take on the type of a tax evader.

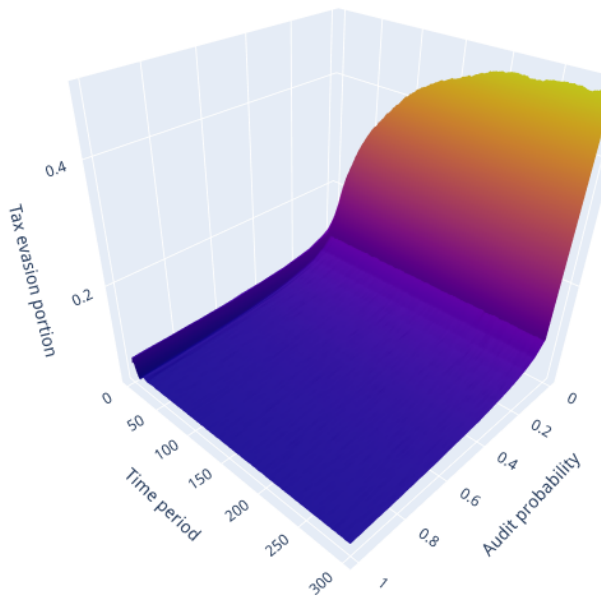
If we consider  $T = 2$  which is below the critical temperature  $T_c$ , we see a different behaviour. With such low individuality, people rarely decide to become tax evaders, since on average the people around them are compliant, and so societal pressure becomes too much. This means that the equilibrium levels for tax evasion are much lower compared to higher temperatures, so assuming a small punishment length of 10, and an audit probability of 0.05, more than 96% of the population comply with the taxation (Fig. 3.4).

This high level of compliance is due to the strong influence of the surrounding compliant society, which creates a sort of persistent societal norm. The model suggests that in such a society, even a low audit level can maintain a high level of compliance. However, it's important to note that this doesn't mean tax evasion is completely eradicated. There will always be a small percentage of individuals who, despite the societal pressure and the risk of being audited, will choose to evade taxes (Fig. 3.4).

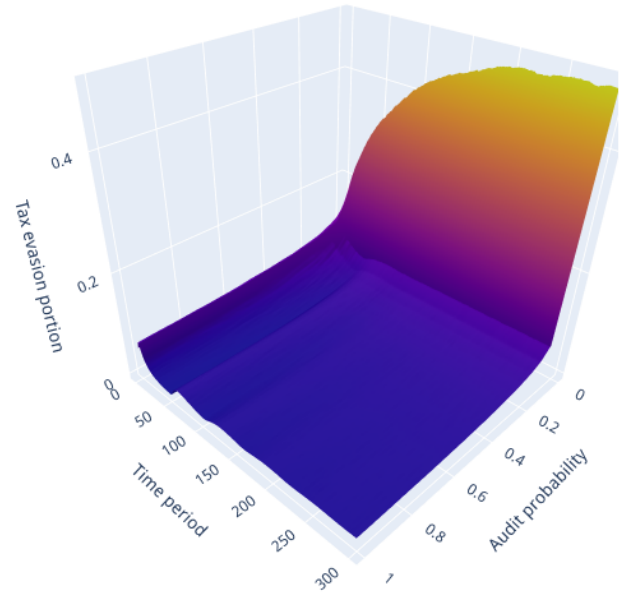
Furthermore, the model also implies that the effectiveness of enforcement mechanisms may vary depending on the social temperature. At lower temperatures, where individuals are more influenced by their surroundings, enforcement mechanisms may not need to be as stringent to achieve a high level of compliance. On the other hand, at higher temperatures, where individuals are more likely to make autonomous decisions, stronger enforcement mechanisms may be required to maintain the same level of compliance.

Individuality: 2.5

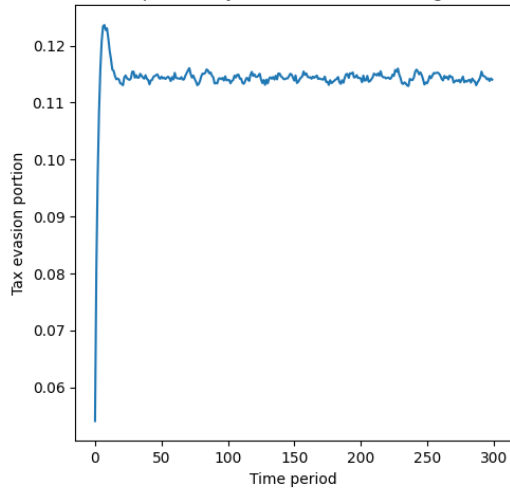
Punishment Length: 10



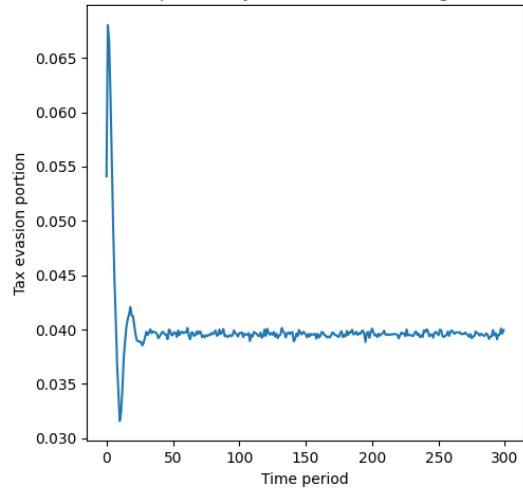
Punishment Length: 50



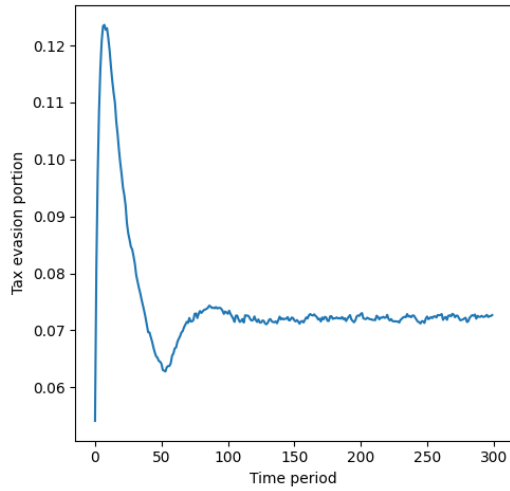
Audit probability: 0.05, Punishment length: 10



Audit probability: 0.9, Punishment length: 10



Audit probability: 0.05, Punishment length: 50



Audit probability: 0.9, Punishment length: 50

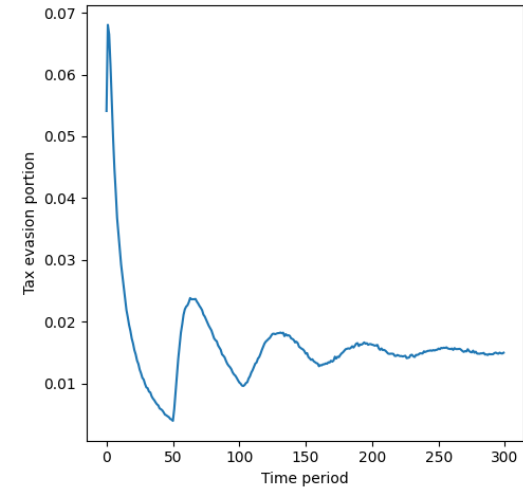
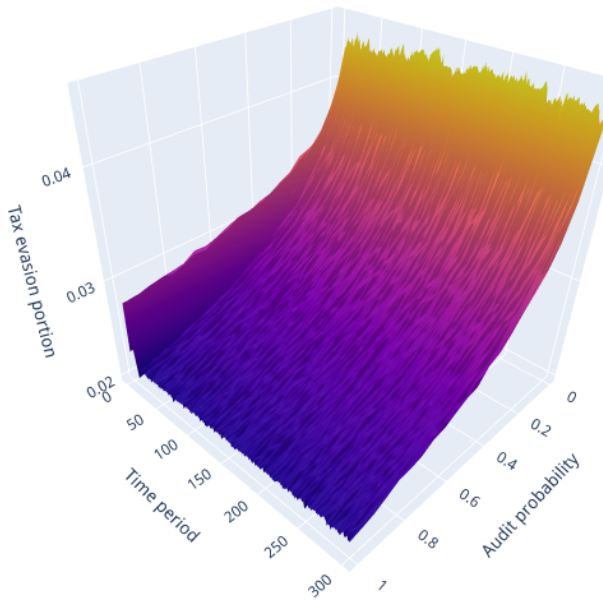


Figure 3.3: Tax evasion dynamics for  $T=2.5$ , and the corresponding graphs.

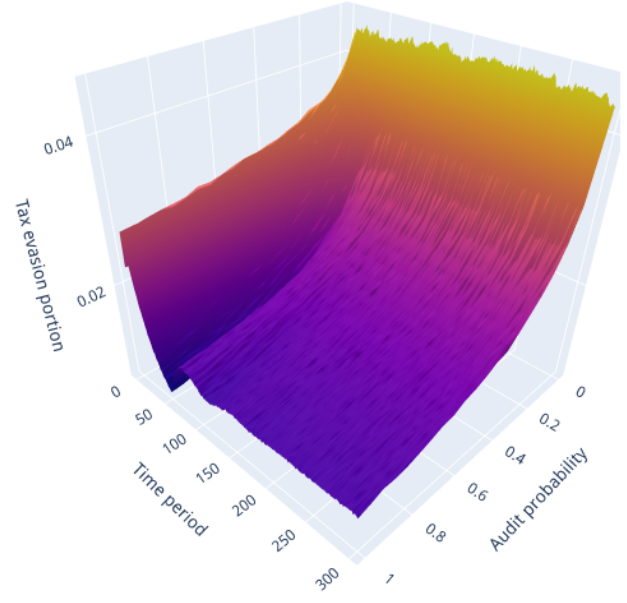


Individuality: 2

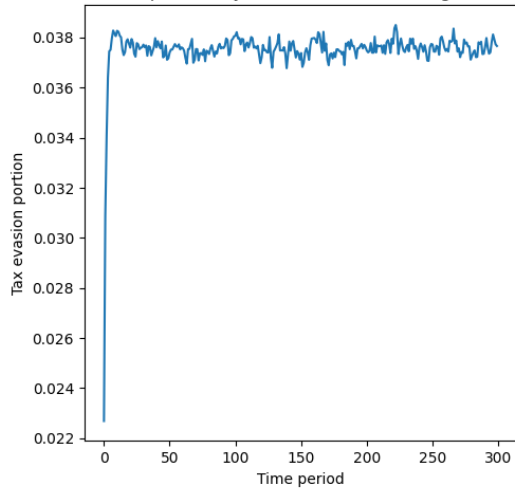
Punishment Length: 10



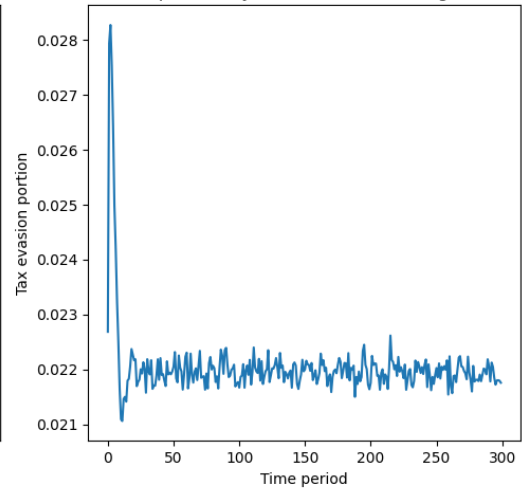
Punishment Length: 50



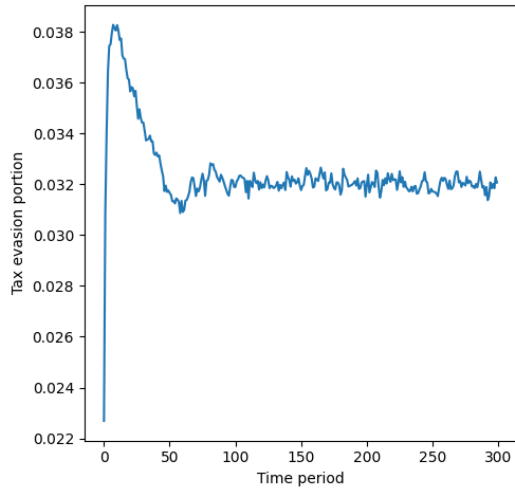
Audit probability: 0.05, Punishment length: 10



Audit probability: 0.9, Punishment length: 10



Audit probability: 0.05, Punishment length: 50



Audit probability: 0.9, Punishment length: 50

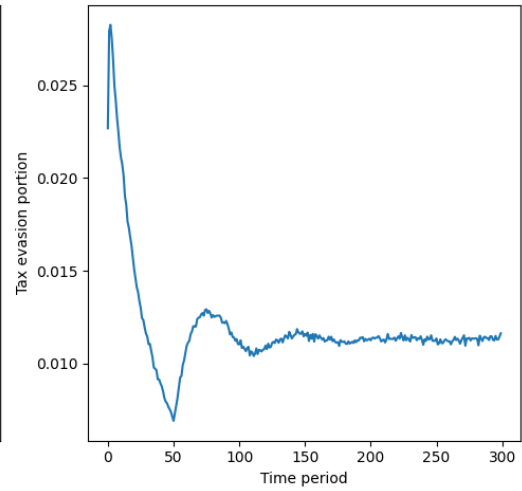


Figure 3.4: Tax evasion dynamics for  $T=2.5$ , and the corresponding graphs.

## 4 | Conclusion

Admittedly, this model is a very simplified version of reality. In the model, we have one mechanism by which tax evasion is possible, which is social pressure. Another issue is that one person either fully evades or fully pays taxes, which comes from the nature of the Ising model, which allows only for two states, which could be expanded upon through the use of the Heisenberg model, an extension of the Ising Model, where spins are represented as vectors on a unit circle, rather than real numbers. Another issue is that the model makes some large assumptions about the stability of the economic environment. One of the major assumptions is that the economic situation of this fictional country remains constant throughout the simulation, which does not align well with the dynamic behaviour of real world economies.

The paper by Zaklan et al.[4] takes the model a step further to make it more realistic by replacing the Ising Model with a Barabasi-Albert undirected graph network[6], but they only noticed a change in the value of the critical temperature  $T_c$ , and otherwise saw the same results[4].

Despite the model's simplifications and inherent underlying assumptions, which limit the ability to simulate complex real-world tax systems, those don't change the nature or essence of the discussion, and the findings of this project raise important moral questions. If the use of force, loss of individuality, or a combination of both is necessary to ensure tax compliance, the morality of such institutions is inevitably brought into question. This echoes the concerns raised by thinkers like Bastiat and Huemer about the inherent power dynamics of taxation.

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