Basel Alghanem
16-720
Homework 5

1.1 $x'^T F x = 0$ multiplies out to be $xx'f_1 + xy'f_2 + xf_3 + yx'f_4 + yy'f_5 + yf_6 + x'f_7 + y'f_8 + f_9 = 0$

$x = y = x' = y' = 0 \Rightarrow f_9 = 0$

1.2

$$t = \begin{bmatrix} t_x \\ 0 \\ 0 \end{bmatrix}, t_\times = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E = Rt_\times = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix}$$

$$l' = Ex = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -t_x x_3 \\ t_x x_2 \end{bmatrix}$$

Since the line has no x component, that means it's parallel to the x-axis

1.3 The second answer is pretty obvious, given in the lecture slides:

$E = R_{rel} t_{rel_\times}$ and $F = K^T E K = K^T R_{rel} t_{rel_\times} K$

For the first answer, compounding translations are additive, so $t_{rel} = t_2 - t_1$

Compounding rotations are multiplicative, so $R_{rel} = \frac{R_2}{R_1} = R_1^{-1} R_2$

1.4 Suppose we construct a reference frame such that the mirror is on the plane $x = 0$

One point on the object is X in homogenous 3D coordinates and with image coordinates $x = PX = K[R|t]X$

Let's also assume that the camera is not rotated relative to the frame, so $R = I$, and its translation $t = \begin{bmatrix} t_1 \\ 0 \\ 0 \end{bmatrix}$

The image of the object in the mirror is the same as the image of another object placed symmetric about the mirror plane. Let's call the same point on the new object $\bar{X} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} X$

The image of the new point on the original camera is $\bar{x} = P\bar{X} = K[I|t] \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} X$

$$0 = \bar{x}^T E x \Rightarrow 0 = \left[K[I|t] \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} X \right]^T E[K[I|t]X] = \left[[I|t] \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} X \right]^T K^T E K [I|t] X$$

$$= \left[[I|t] \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} X \right]^T F[I|t]X = \begin{bmatrix} -X_1 + t_1 \\ X_2 \\ X_3 \end{bmatrix}^T F \begin{bmatrix} X_1 + t_1 \\ X_2 \\ X_3 \end{bmatrix} = 0$$

Since the object can have any number of points equidistant from the mirror (same X_1), this should hold for all X_2 and X_3. Still, I don't see that this would restrict F to being skew-symmetric. Must have gone wrong somewhere along the way?

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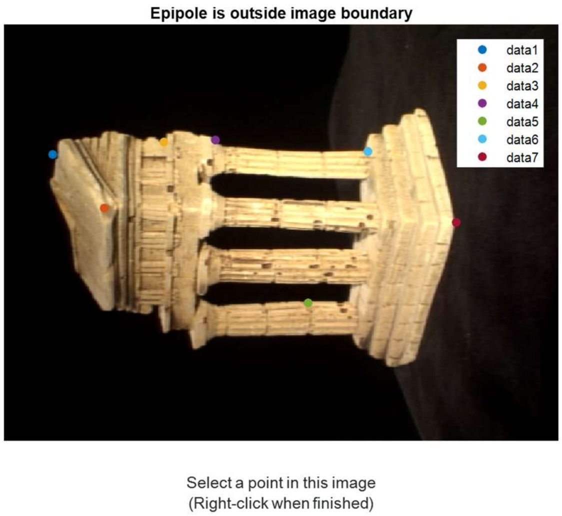
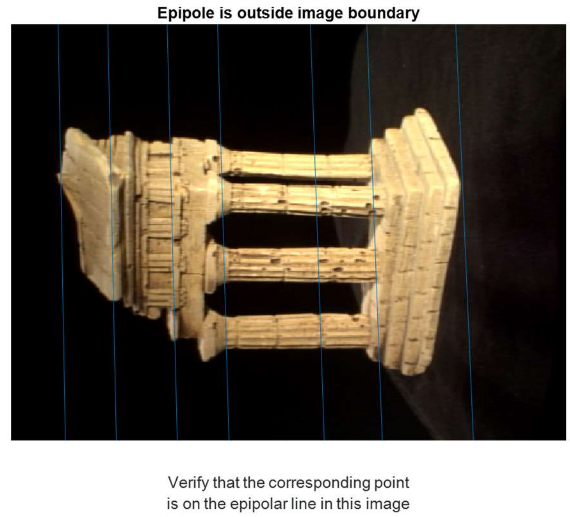
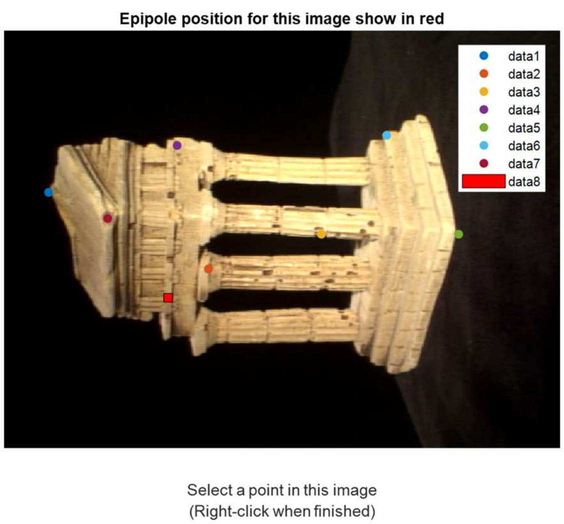
2.1

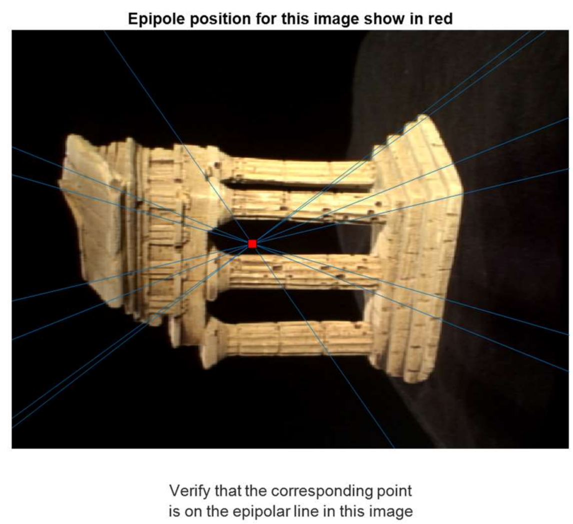
$$F = \begin{bmatrix} -1.3e\text{-}9 & -1.3e\text{-}7 & 0.011 \\ -5.9e\text{-}8 & 3.6e\text{-}8 & -1.7e\text{-}5 \\ -0.011 & 3.0e\text{-}5 & -0.0042 \end{bmatrix}$$





2.2





$$F_1 = \begin{bmatrix} 7.4e\text{-}9 & 3.3e\text{-}7 & -1.0e\text{-}3 \\ -1.8e\text{-}7 & -1.1e\text{-}8 & 9.3e\text{-}5 \\ 9.5e\text{-}4 & -1.1e\text{-}4 & 0.0065 \end{bmatrix}$$

$$F_2 = \begin{bmatrix} -1.2e\text{-}7 & 7.4e\text{-}7 & -2.1e\text{-}4 \\ -1.3e\text{-}6 & -6.0e\text{-}7 & 4.3e\text{-}4 \\ 3.6e\text{-}4 & -5.8e\text{-}5 & -0.050 \end{bmatrix}$$

$$F_3 = \begin{bmatrix} -1.5e\text{-}7 & 8.2e\text{-}7 & -5.1e\text{-}5 \\ -1.5e\text{-}6 & -7.1e\text{-}7 & 5.0e\text{-}4 \\ 2.4e\text{-}4 & -4.8e\text{-}5 & -0.061 \end{bmatrix}$$

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3.1 $$E = \begin{bmatrix} -0.0030 & -0.30 & 1.7 \\ -0.14 & 0.0083 & -0.051 \\ -1.7 & -0.013 & -0.0013 \end{bmatrix}$$

3.2 Pulled directly from Hartley and Zisserman

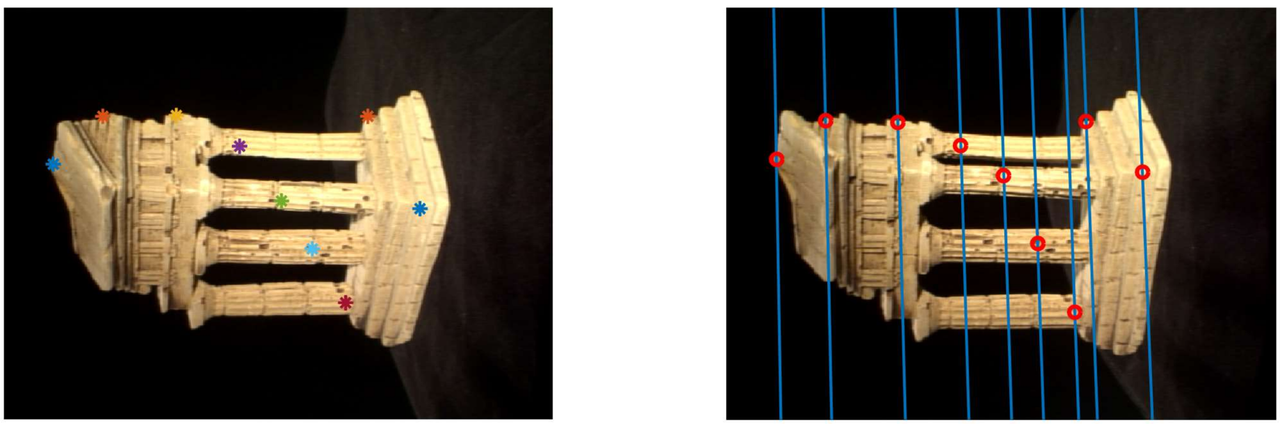
For $\boldsymbol{x_i} = \begin{bmatrix} x_i \\ y_i \end{bmatrix}$, $\boldsymbol{x'_i} = \begin{bmatrix} x'_i \\ y'_i \end{bmatrix}$, $C_1 = \begin{bmatrix} C_1^1 \\ C_1^2 \\ C_1^3 \end{bmatrix} = \begin{bmatrix} C_1^{11} & C_1^{12} & C_1^{13} & C_1^{14} \\ C_1^{21} & C_1^{22} & C_1^{23} & C_1^{24} \\ C_1^{31} & C_1^{32} & C_1^{33} & C_1^{34} \end{bmatrix}$, $C_2 = \begin{bmatrix} C_2^1 \\ C_2^2 \\ C_2^3 \end{bmatrix} = \begin{bmatrix} C_2^{11} & C_2^{12} & C_2^{13} & C_2^{14} \\ C_2^{21} & C_2^{22} & C_2^{23} & C_2^{24} \\ C_2^{31} & C_2^{32} & C_2^{33} & C_2^{34} \end{bmatrix}$

$$A_i = \begin{bmatrix} x_i C_1^3 - C_1^1 \\ y_i C_1^3 - C_1^2 \\ x'_i C_2^3 - C_2^1 \\ y'_i C_2^3 - C_2^2 \end{bmatrix}$$

3.3 $$C_2 = \begin{bmatrix} 1520 & -22 & 305 & -22 \\ -58 & 1408 & 635 & -1506 \\ 0.0013 & -0.26 & 0.97 & 0.083 \end{bmatrix}$$

$\text{total error} = 93.1$ or $\text{average error} = 0.85$

4.1



Select a point in this image
(Right-click when finished)

Verify that the corresponding point
is on the epipolar line in this image

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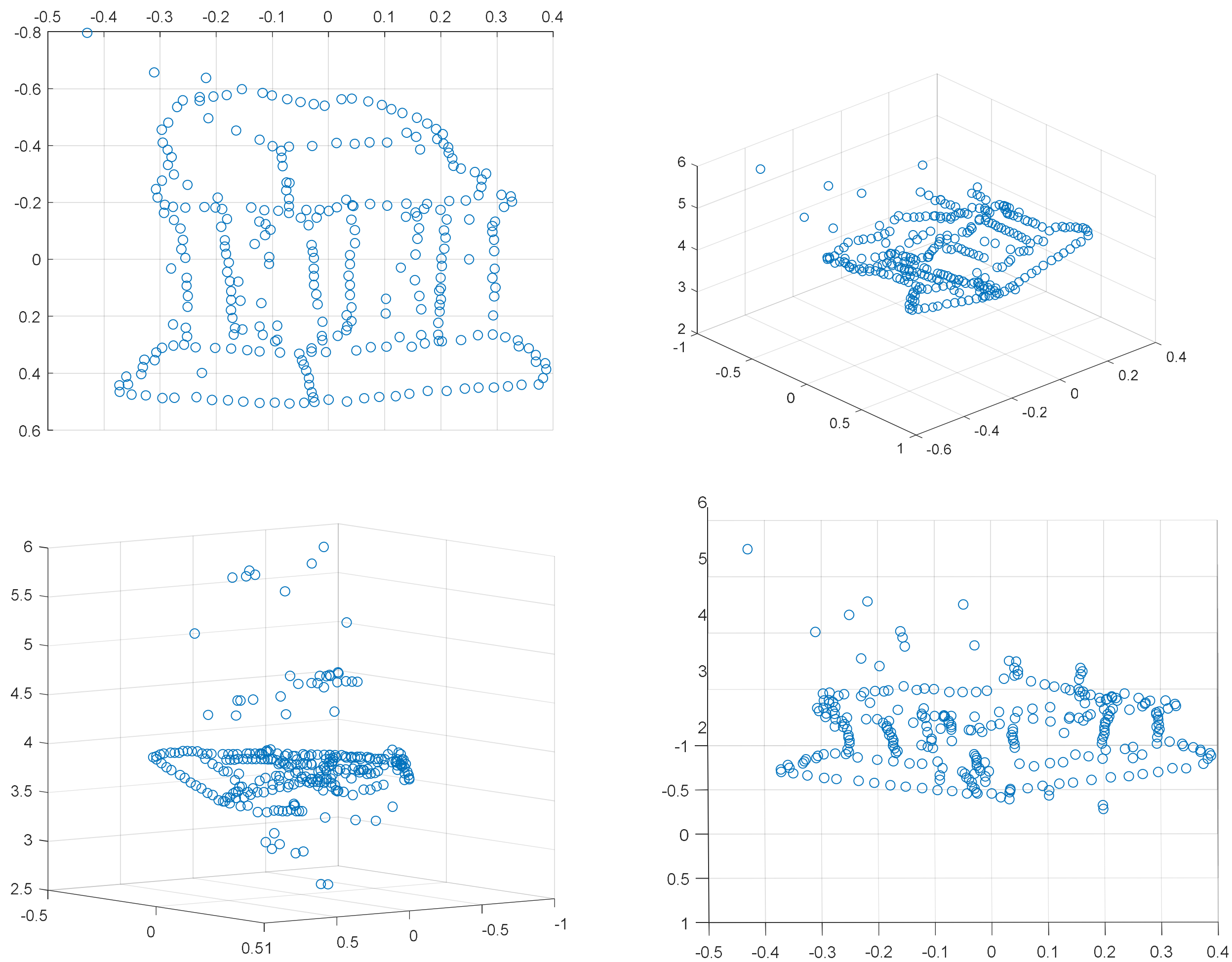
4.2



5.1 I used the original calculation of F as my error metric: $err_i = x_i'^T F x_i$. I counted all points with error below a threshold as inliers. Additionally, I took random sets of 15 points instead of 8 points. More points means that it's more likely there will be points from all around the image, but 15 is still small enough that many of the 10000 iterations will be all inliers and no outliers.

Plugging all the points in eightpoint came out with a trash result, but using ransac wasn't colossally better. For example, in both cases, the result had an epipole in the image, whereas the noiseless correspondence results in the epipole being outside the image.