Basel Alghanem
16-720
Homework 4

1.1 $p = M_1 P = K_1[R_1|t_1]P \rightarrow P = [R_1^T| - t_1]K_1^{-1}p$
$q = M_2 P \rightarrow P = [R_2^T| - t_2]K_2^{-1}q$
$[R_1^T| - t_1]K_1^{-1}p = [R_2^T| - t_2]K_2^{-1}q \rightarrow p = K_1[R_1|t_1][R_2^T| - t_2]K_2^{-1}q$
$K_1[R_1|t_1][R_2^T| - t_2]K_2^{-1}$ would be a 3x3 matrix

1.2 Since there's no translation, we can go to nonhomogenous coordinates for the 3D points
$x_1 = K_1 I X, x_2 = K_2 R X$
$X = K_1^{-1}x_1, X = R^T K_2^{-1} x_2$
$K_1^{-1}x_1 = R^T K_2^{-1} x_2 \rightarrow x_1 = K_1 R^T K_2^{-1} x_2$

1.3
1. It has DOF 8, because it has 9 values, but because the homography implies proportionality, the scale can be arbitrary, and one value (or the norm of h) can be set to an arbitrary value, reducing the DOF by 1.
2. Since the DOF is 8, it requires 8 values. Each pair of points is 2 values, so it requires 4 pairs of points.
3. $x_1 \equiv Hx_2$, and we want the form $Ah = 0$

$$\begin{bmatrix} x_{11} \\ x_{12} \\ 1 \end{bmatrix} \equiv \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{bmatrix} \begin{bmatrix} x_{21} \\ x_{22} \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} x_{11} \\ x_{12} \\ 1 \end{bmatrix} \equiv \begin{bmatrix} H_{11}x_{21} + H_{12}x_{22} + H_{13} \\ H_{21}x_{21} + H_{22}x_{22} + H_{23} \\ H_{31}x_{21} + H_{32}x_{22} + H_{33} \end{bmatrix}$$

$$x_{11} = \frac{H_{11}x_{21} + H_{12}x_{22} + H_{13}}{H_{31}x_{21} + H_{32}x_{22} + H_{33}}, x_{12} = \frac{H_{21}x_{21} + H_{22}x_{22} + H_{23}}{H_{31}x_{21} + H_{32}x_{22} + H_{33}}$$

$$x_{11}(H_{31}x_{21} + H_{32}x_{22} + H_{33}) = H_{11}x_{21} + H_{12}x_{22} + H_{13}$$
$$x_{12}(H_{31}x_{21} + H_{32}x_{22} + H_{33}) = H_{21}x_{21} + H_{22}x_{22} + H_{23}$$

$$x_{11}H_{31}x_{21} + x_{11}H_{32}x_{22} + x_{11}H_{33} - H_{11}x_{21} - H_{12}x_{22} - H_{13} = 0$$
$$x_{12}H_{31}x_{21} + x_{12}H_{32}x_{22} + x_{12}H_{33} - H_{21}x_{21} - H_{22}x_{22} - H_{23} = 0$$

$$Ah = \begin{bmatrix} -x_{21} & -x_{22} & -1 & 0 & 0 & 0 & x_{11}x_{21} & x_{11}x_{22} & x_{11} \\ 0 & 0 & 0 & -x_{21} & -x_{22} & -1 & x_{12}x_{21} & x_{12}x_{22} & x_{12} \end{bmatrix} \begin{bmatrix} H_{11} \\ H_{12} \\ H_{13} \\ H_{21} \\ H_{22} \\ H_{23} \\ H_{31} \\ H_{32} \\ H_{33} \end{bmatrix} = 0$$

1.4 Since the intrinsic matrices are the same, the K and K inverse cancel out, leaving:
$x_1 = R^T x_2$, where R is the rotation matrix corresponding to a rotation θ.
Let's then take an extra rotation, $x_2 = R^T x_3, x_1 = R^T x_2 = R^{T^2} x_3$
$R^{T^2} = (R^T)^2$, obviously, and R is the homography, so the squared homography corresponds to a double rotation

1.5 One simple counterexample is a non-planar scene. If the scene is 3D, then different features will be visible from different viewpoints.

1.6 This one is straightforward, because matrix multiplication is linear. Take $X_i = X_1 + (X_2 - X_1)i$ for $0 < i < 1$
Because matrix multiplication is linear, $x_i = PX_i = P(X_1 + (X_2 - X_1)i) = iPX_2 + (1 - i)PX_1 = ix_2 + (1 - i)x_1$
This demonstrates that the line in 3D is still a line in 2D

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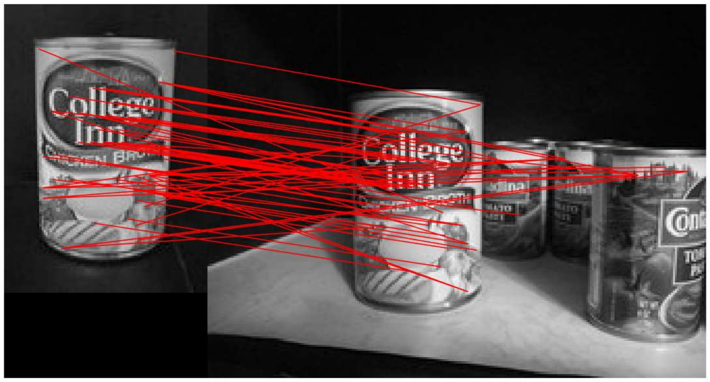
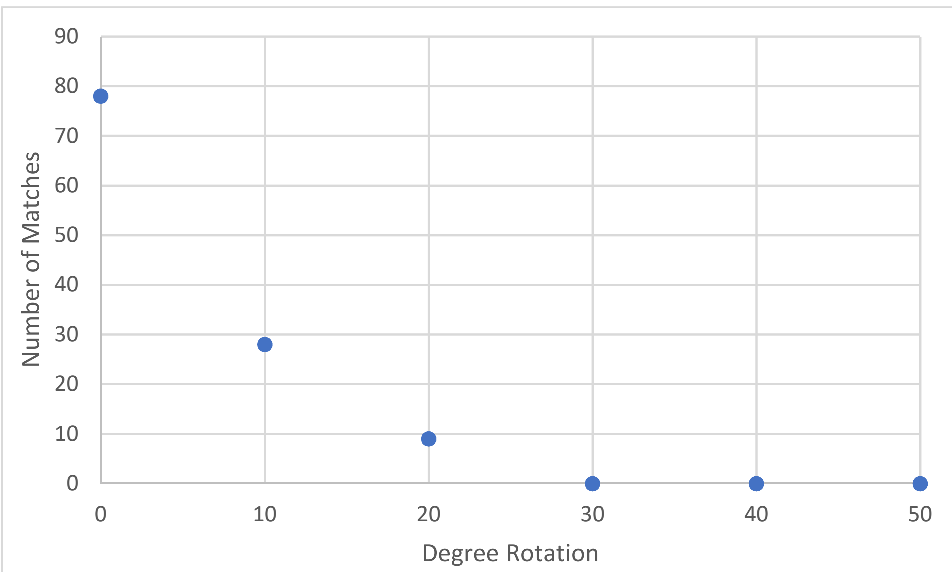
2.4.1



2.5



3.4 Had to resize the hp_cover image so that it warps properly

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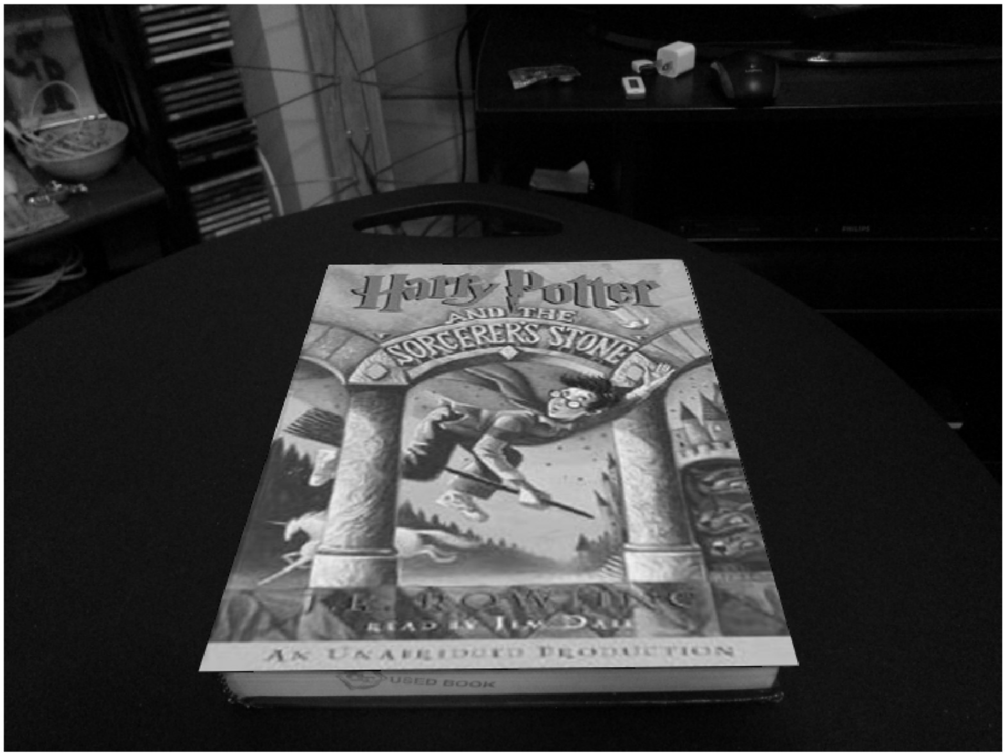
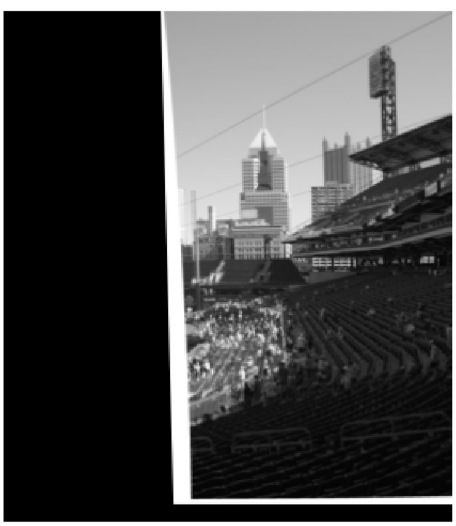


$$H = \begin{bmatrix} 0.752 & -0.350 & 238.8 \\ 0.000 & 0.222 & 192.3 \\ 0.000 & -0.001 & 1 \end{bmatrix}$$

4.1




$$H = \begin{bmatrix} 1.009 & -0.007 & -101.9 \\ 0.012 & 1.019 & 6.849 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$$

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4.3

$$H = \begin{bmatrix} 0.636 & -0.072 & 367.6 \\ -0.080 & 0.829 & -7.980 \\ 0.000 & 0.000 & 1 \end{bmatrix}$$