

- 2.1) Consider $x \cos \theta + y \sin \theta - \rho = 0$ with x and y as constants

$$x \cos \theta + y \sin \theta - \rho = 0 \rightarrow \rho = \sqrt{x^2 + y^2} \cos\left(\theta + \tan^{-1}\left(\frac{-y}{x}\right)\right)$$

The magnitude is $\sqrt{x^2 + y^2}$ and the phase is $\tan^{-1}\left(\frac{-y}{x}\right)$

- 2.2) This allows lines that point directly in the y direction. It also makes the parameter space smaller (one of two parameters has a limit, whereas x and y are both without limits).

$$x \cos \theta + y \sin \theta - \rho = 0 \rightarrow y \sin \theta = \rho - x \cos \theta \rightarrow y = \left(-\frac{\cos \theta}{\sin \theta}\right)x + \frac{\rho}{\sin \theta}$$

The slope is $\left(-\frac{\cos \theta}{\sin \theta}\right)$ and the intercept is $\frac{\rho}{\sin \theta}$

These both go to infinity as θ approaches 0 (line points in the y direction)

- 2.3) The maximum value of ρ occurs at the point farthest from the origin: $\rho = \sqrt{W^2 + H^2}$

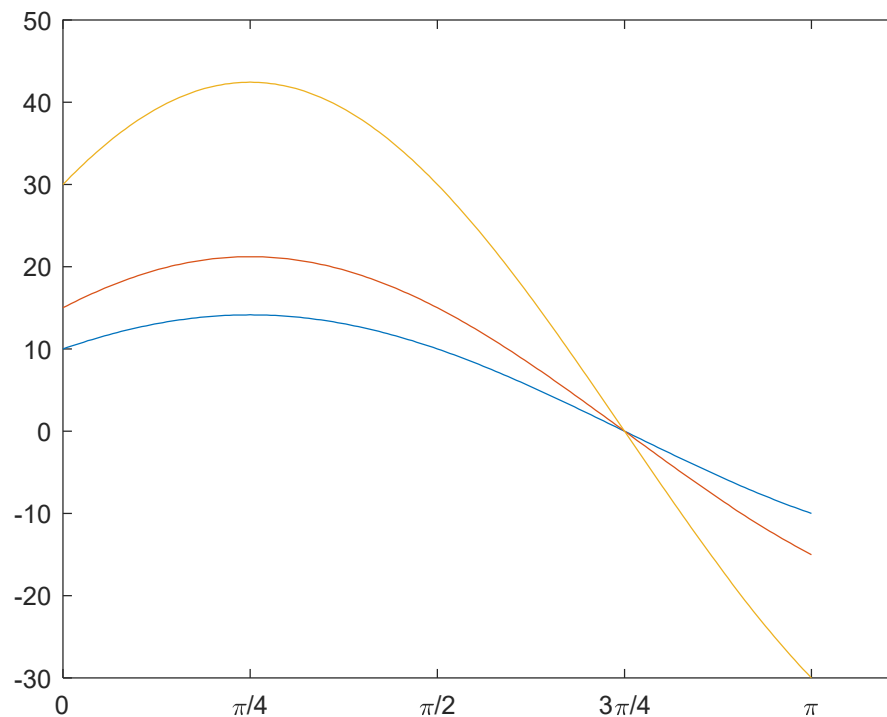
Because the points are only in one quadrant (all x and y are positive), θ only varies from 0° to 90°

- 2.4) $(x, y) = (10, 10) \rightarrow \rho = 10 \cos \theta + 10 \sin \theta \rightarrow \rho = 10\sqrt{2} \cos(\theta - 45^\circ)$

Same for the other points except for the larger magnitudes

The intersection point is at $\rho = 0$ and $\theta = \frac{3}{4}\pi$

This corresponds to a line with intercept 0 and slope 1



- 2.5) There are several functions that must loop over the whole parameter space. When the parameter space is $P \times \Theta$, then the method will be $O(|P \times \Theta|)$. If they are each split into n bins, that's $O(n^2)$. If you add a third parameter split into the same number of bins, that becomes $O(n^3)$. That's much slower, so every additional parameter makes the method more and more untenable.

One option would be to manage the sizes of the parameter spaces. For example, using a hough transform to find circles of unknown radius would have that $O(n^3)$ speed. If you could determine that the circle will be one of five sizes, that reduces the parameter space tremendously.

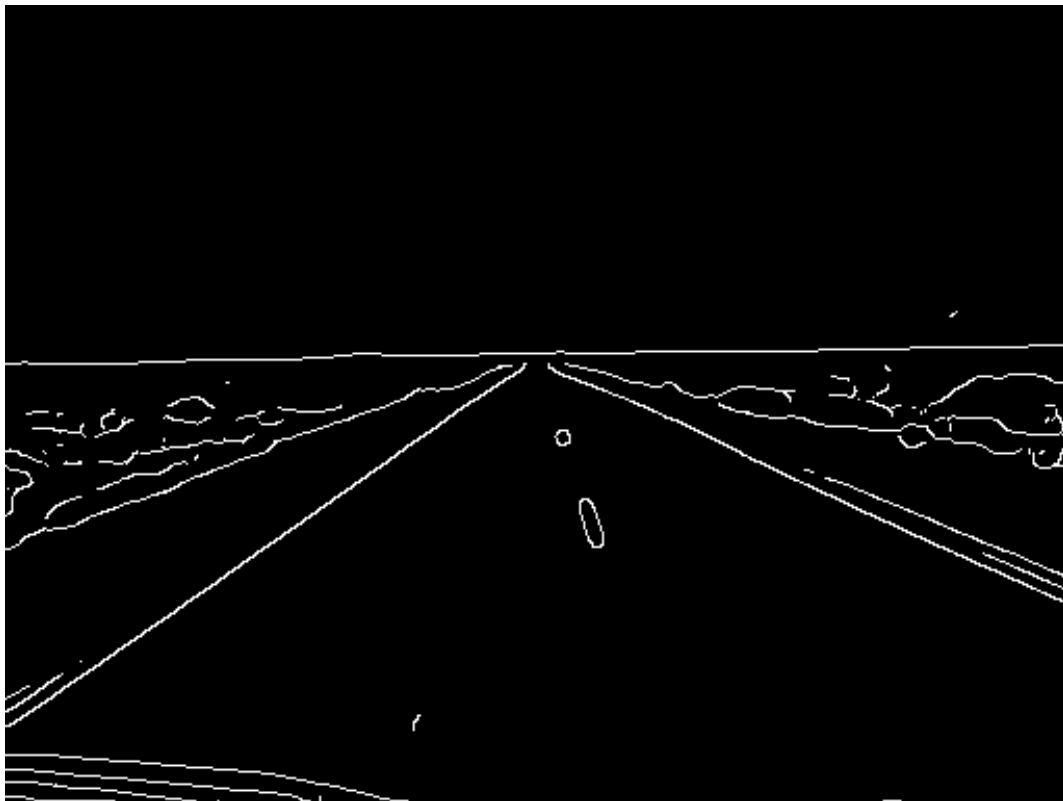
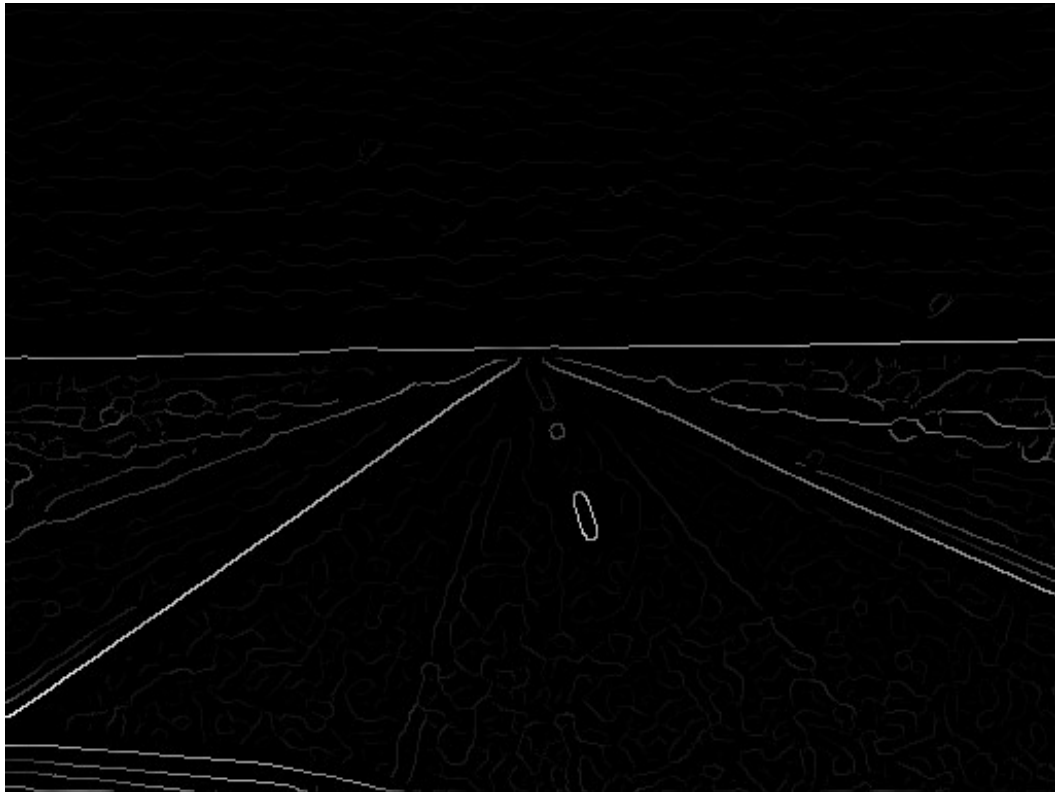
- 3.2) I'm quite sure the way I did this was a hack, and it wasn't any faster than using two for loops (confirmed through testing). It probably wasn't the intended method, even if it technically satisfies the constraint (one for loop). What I did was iterate over the full matrix index, then convert the matrix index into row and column indices.
- 4) There's no single set of parameters that works perfectly for all images. The combination of sigma and threshold is a fine balance because they have complementary behaviors. A high-sigma gaussian blur will reduce the magnitude of edge differences, reducing the edges that will be higher than the boundary. No gaussian filter – threshold pair was great for all images. Image 9 had clouds that were hard to remove, whereas Image 6 had many low magnitude edges that needed to be caught.

There was also a balance between eliminating colinear edges and removing overlapping edges. This is especially clear in the difficulty of avoiding overlapping edges in Image 1. Decreasing resolution and expanding the non-maximal suppression filter were each methods of eliminating redundant edges, but neither worked perfectly. Decreasing resolution especially hurt edge detection quality. My final submission had a 9x9 NMS filter instead of the instructed 3x3 NMS filter.

Some images simply had more lines than others. For example, there are relatively few lines in Image 4. One approach is to increase the initial edge magnitude threshold, implement a threshold in the hough space for which maximal points can be considered lines, and increase the number of allowable lines. The first increased threshold is needed to eliminate fine lines that, once they've made it past that threshold, will represent very strong lines in the hough space. However, the drawbacks of this approach are too big, in that it will eliminate important lines in some images with inherently low edge magnitudes.

The intermediate images for Image 3 are included below (with a particular set of parameters).

Basel Alghanem
16-720
Homework 1



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16-720
Homework 1

