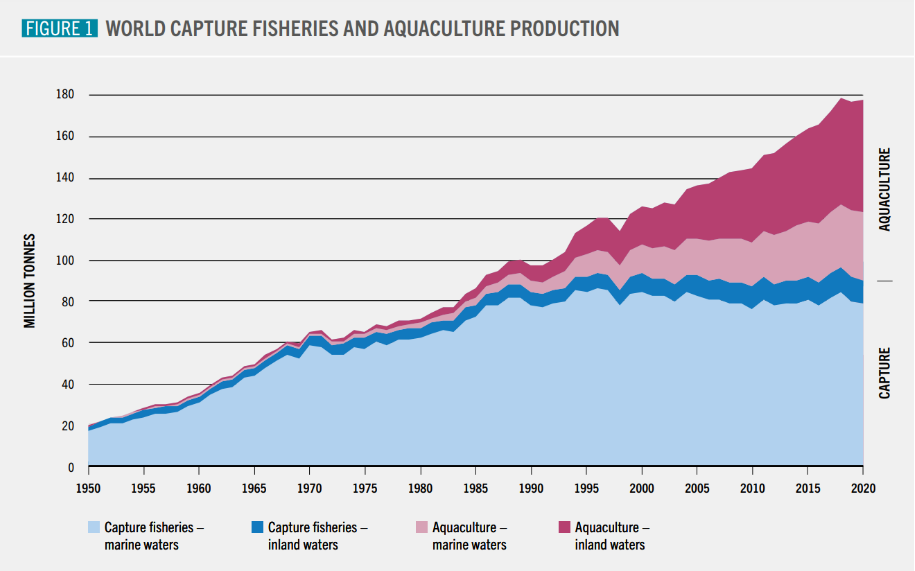
Statistics, fisheries management and spatial modelling.

Towards spatio-temporal stock assessment methods.

Based on a collaboration between: Maxime Olmos, James Thorson, André Punt, Cole Monnahan, *Baptiste Alglave*, Cody Szuwalski.

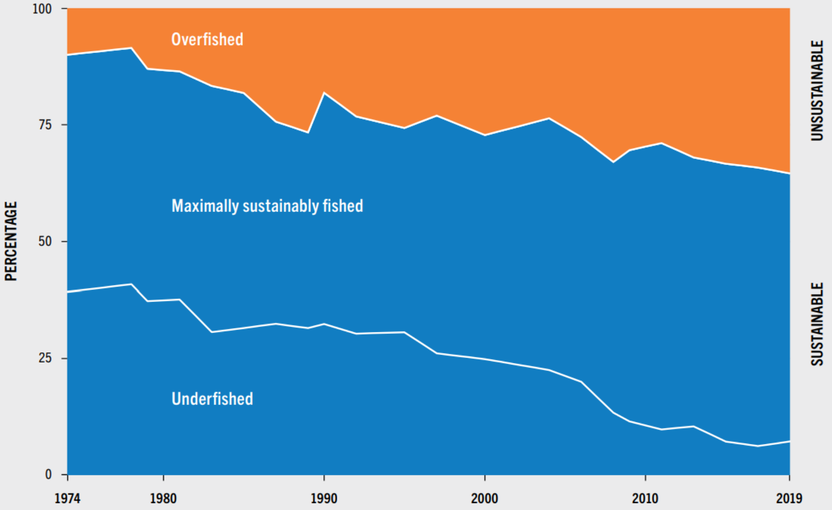
December 2023

Some graphics as introduction



NOTES: Excluding aquatic mammals, crocodiles, alligators, caimans and algae. Data expressed in live weight equivalent.
SOURCE: FAO.

FIGURE 23 GLOBAL TRENDS IN THE STATE OF THE WORLD'S MARINE FISHERY STOCKS, 1974–2019



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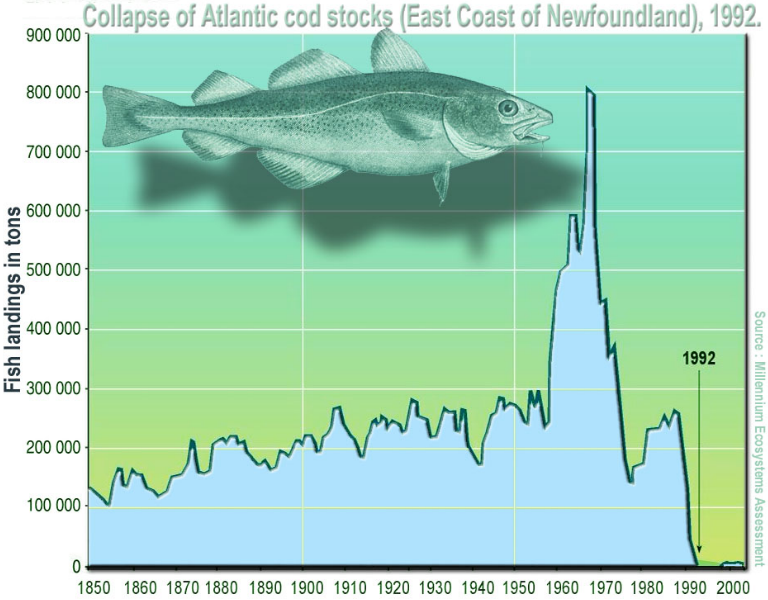
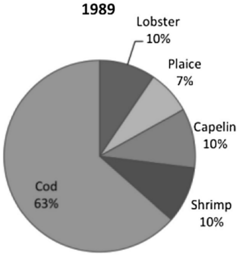
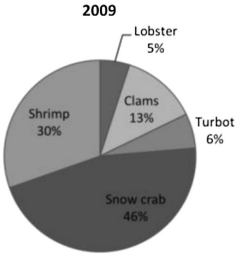
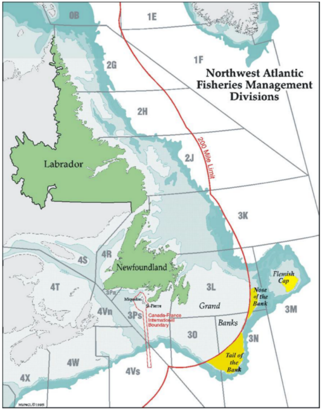


Collapse of Atlantic cod stocks (East Coast of Newfoundland), 1992.
Fish landings in tons
900 000
800 000
700 000
600 000
500 000
400 000
300 000
200 000
100 000
0
1850
1860
1870
1880
1890
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
2000
1992
Source : Millennium Ecosystems Assessment

Landed value of fish in Newfoundland and Labrador, 1989 - 2009.







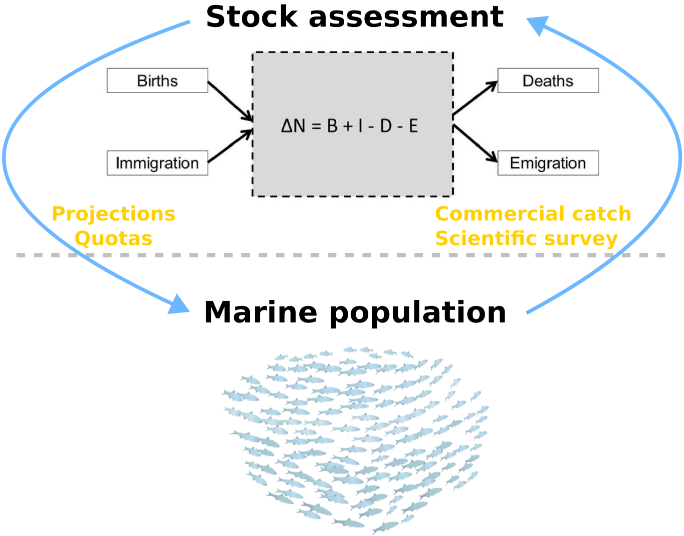
Mather, C. (2013). From cod to shellfish and back again? The new resource geography and Newfoundland's fish economy. Applied Geography, 45, 402-409.

How to assess and manage fish ressources ?

➠ Stock assessment models

How to assess and manage fish ressources ?

➠ **Stock assessment models**



How to build a stock assessment model?

Plan

1. Introduction
2. Basics of stock assessment models
3. The hierarchical formalism: state-space modelling
4. Towards spatio-temporal modelling of marine ressources
5. Conclusion

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1 Introduction

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3 The hierarchical formalism: state-space modelling

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5 Conclusion

Surplus production model

Logistic model

(Continuous formulation)

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = r \cdot N(t) \left(1 - \frac{N}{K}\right)$$

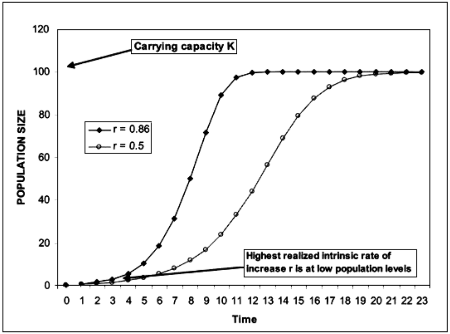
(Discrete formulation)

$$N(t+1) = N(t) + r \cdot N(t) \left(1 - \frac{N(t)}{K}\right)$$

Schaefer model

$$N(t+1) = N(t) + r \cdot N(t) \left(1 - \frac{N(t)}{K}\right) - C(t)$$

with $C(t) = q \cdot e \cdot N(t)$



N: abundance, C: catch, K: carrying capacity, r: intrinsic rate of population increase, q: catchability, e: effort
$B, K, r, t, q, f \in \mathbb{R}^+$

The middle term is known as the surplus production (denoted SP_t).

If $SP(t) > C(t)$, population size increases

If $SP(t) = C(t)$, population size is steady

If $SP(t) < C(t)$, population size decreases

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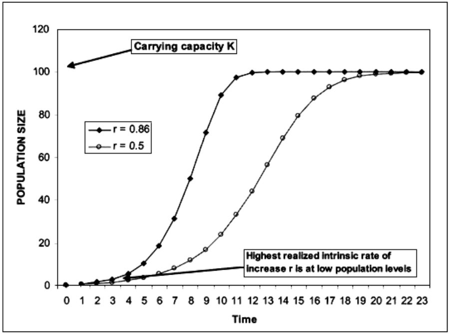

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Virtual population analyis

Catch is disaggregated into age-groups.

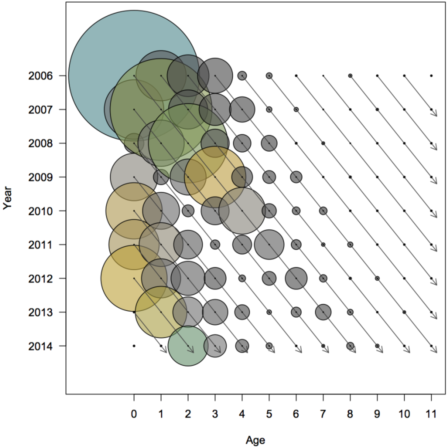
Cohort is the key concept.
A cohort comprises all the individuals (fish in this case) that were born in the same year.

For a single cohort:

$$N(t+1) = N(t) - C(t) - D(t)$$
$$\text{with } D(t) = N(t) \cdot (1 - S)$$

$$\Rightarrow N(t) = (N(t+1) + C(t))/S$$

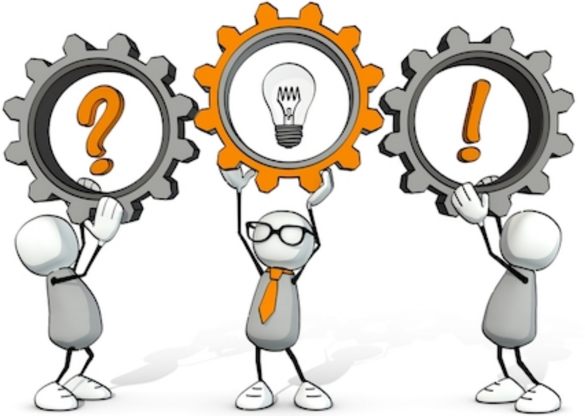
$N(t)$: the abundance at the beginning of time t
$C(t)$: the catch during time t
$D(t)$: the amount of removals due to natural death during time t
S: the survival rate



Let's assume there are **no more fish left of the oldest age** (all caught or died)
➠ calculate the numbers last year with only catch and mortality as input values
➠ start from the oldest ages and move backwards to the youngest

In both cases

- criticism of the model
- criticism of the fitting/estimation procedure



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Modelling hidden processes of population dynamics

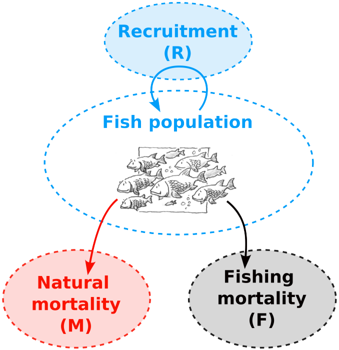
Population dynamics

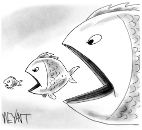
$$N(a+1, t+1) = N(a,t) \cdot \exp\left(-F(a,t) - M(a,t)\right)$$

Explotation dynamics
(Baranov equation)

$$C(a,t) = \frac{F(a,t)}{F(a,t)+M(a,t)} \cdot N(a,t) \cdot [1 - \exp(-(F(a,t)+M(a,t)))]$$

And there is also **Recruitment**, but that's a whole different issue...





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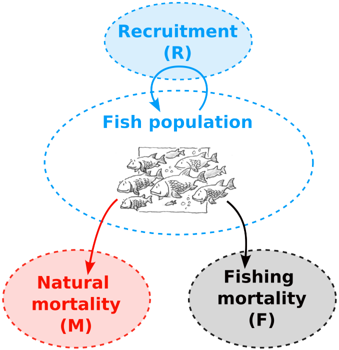

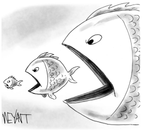

The available data

Scientific survey data $(N_{obs}(a,t))$

Commercial catch data $(C_{obs}(a,t))$

Hierarchical modelling

Parameter layer

Latent layer

Observation layer

Hierarchical modelling

Parameter layer

Population dynamics

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Latent layer

Exploitation dynamics

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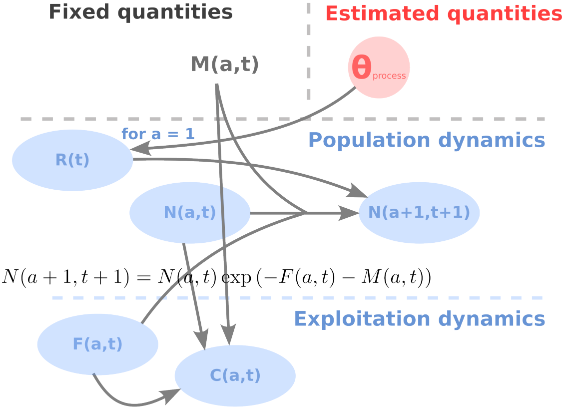
Observation layer

Hierarchical modelling



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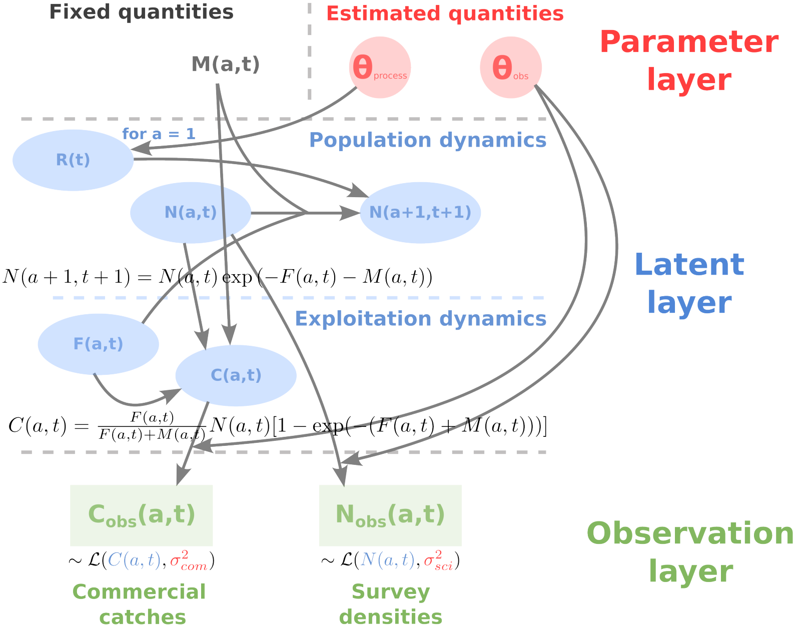

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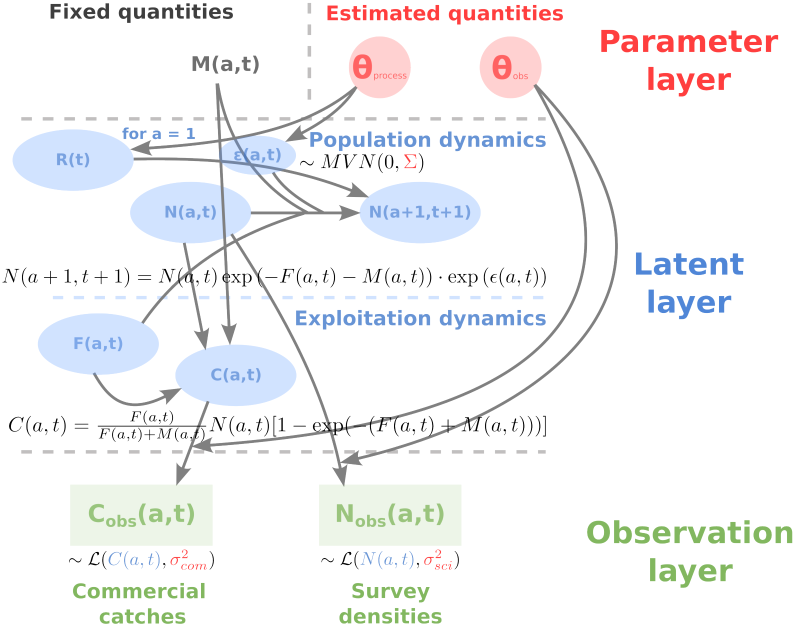

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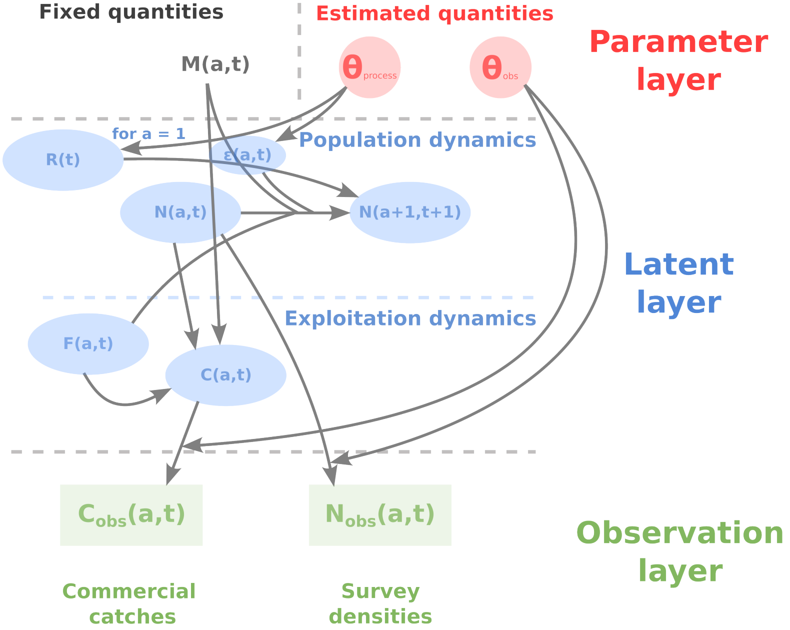

Model inference

Let's denote Y the observations and θ the parameters.

Frequentist approach

Search for the parameter values $\hat{\theta}$ that maximize the likelihood $P(Y|\theta)$:

➠ Search for $\underset{\theta}{\operatorname{argmax}}\,(P(Y|\theta))$

Bayesian approach

Compute the posterior distribution of the parameters θ knowing the data Y through the Bayes rule:

$$P(\theta|Y) = \frac{P(Y|\theta)P(\theta)}{P(Y)}$$

In the Bayesian approach, **Markov Chain Monte Carlo** (MCMC) methods were very popular in the past decades.

➠ **Provide a generic modelling and inference framework for fitting any kind of model**

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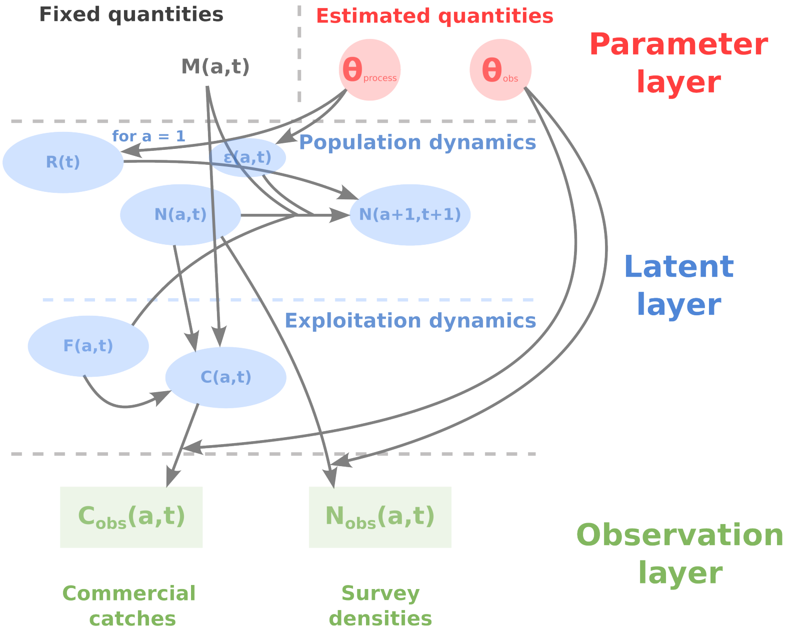

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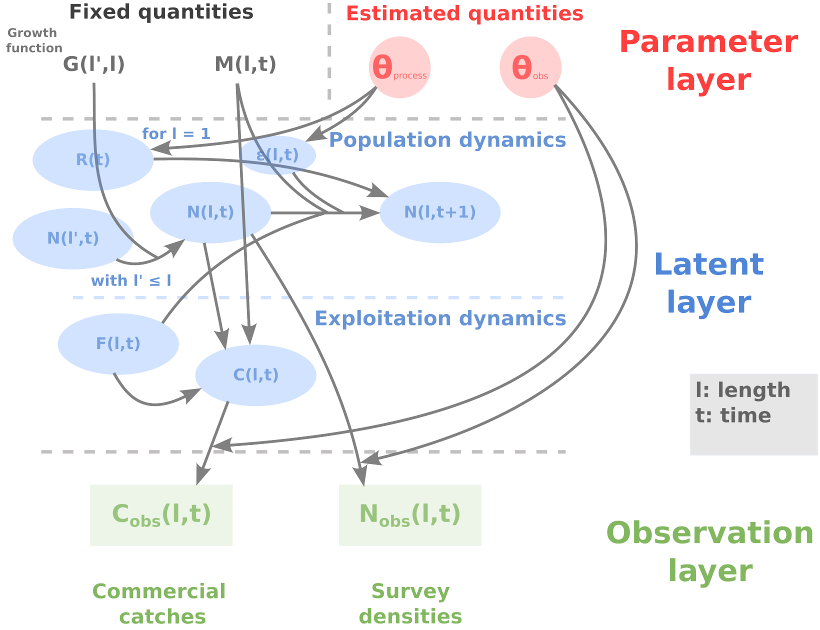
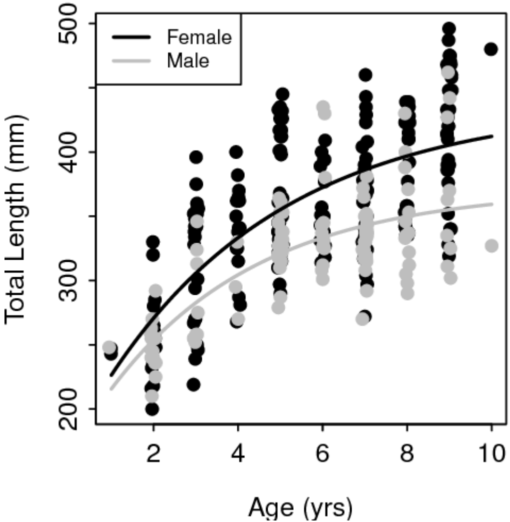


Complexifying the hidden process

Von Bertalanffy growth curve



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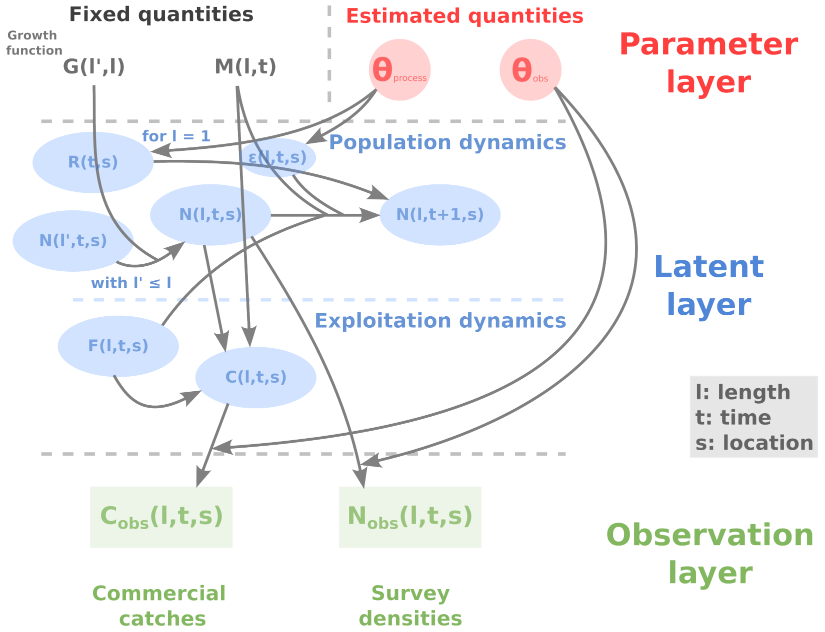
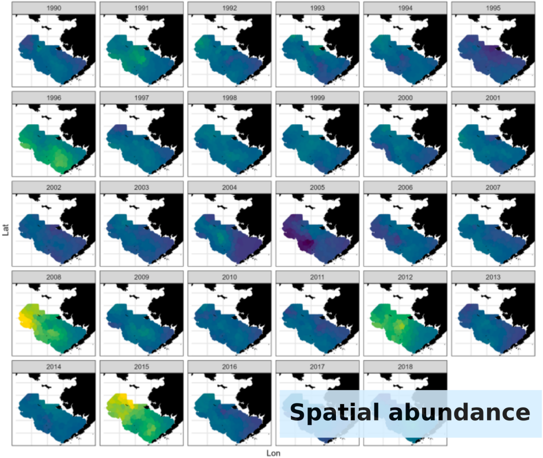


Complexifying the hidden process

Modelling population demography in space



Cao, J., Thorson, J. T., Punt, A. E., & Szuwalski, C. (2020). A novel spatiotemporal stock assessment framework to better address fine-scale species distributions: development and simulation testing. Fish and Fisheries, 21(2), 350-367.

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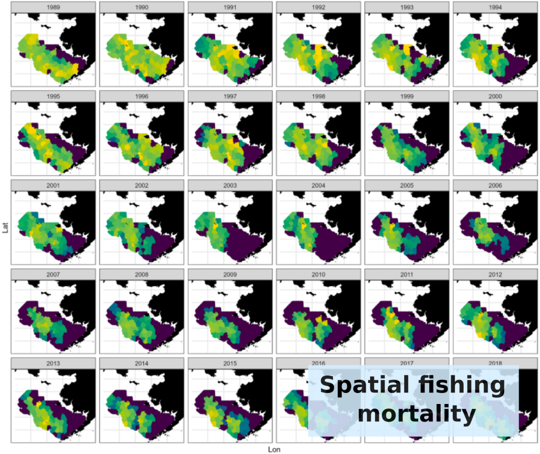

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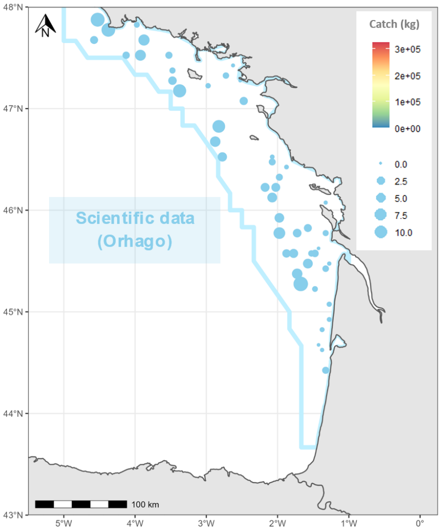
Including the spatial component add extra complexity !



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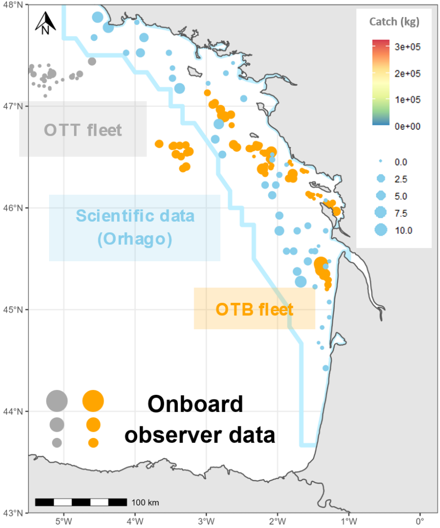

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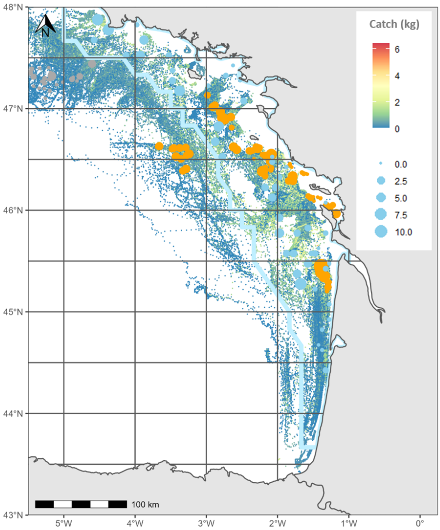

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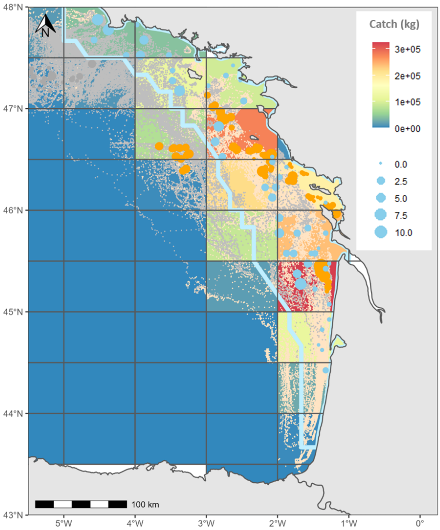

Limits of the standard methods of inference

Standard methods are no more efficient.

The keystone is the likelihood:

$$L_M(\theta) = P(Y|\theta) = \int_{\mathbb{R}^q} P(Y, \delta|\theta) d\delta$$

Y are the observations, θ are the parameters, δ are the latent variables.

In a spatio-temporal context q can be very high

➠ require efficient numerical methods (1) to bypass the integration step, (2) to reduce the dimensionality of δ and (3) to derive efficiently the likelihood.

Three methods to enhance computing:

- Laplace approximation (= approximation of the likelihood)
- SPDE approach (= approximation of the Gaussian random effect)
- Automatic differentiation (= efficient derivation technics)

R package: `R-INLA` for bayesian inference (Rue et al., 2017) and `TMB` for maximum likelihood inference (Kristensen et al., 2015).

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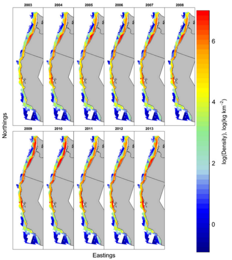
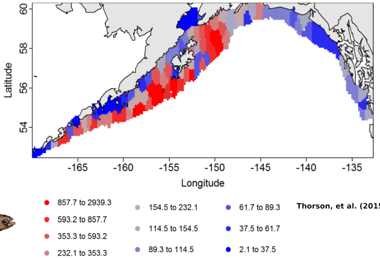
Some examples

Estimated density of big skate in the US waters of the California Current



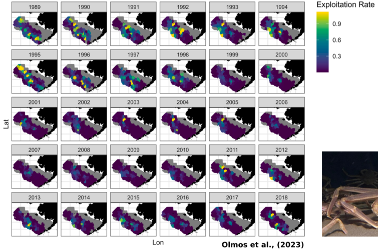
Thorson, et al. (2017)

Estimates of spatial variation in recruitment rex sole in the Gulf of Alaska

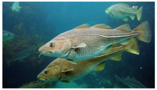


Thorson, et al. (2015)

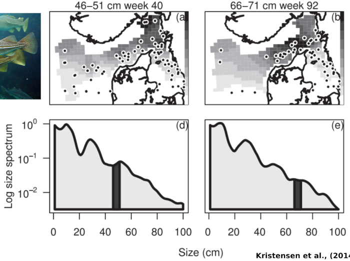
Spatiotemporal variation in exploitation rate for snow crab of Alaska



Olmos et al., (2023)



Estimated spatial distribution and demography of a cohort of individuals over 3 years for cod



Kristensen et al., (2014)

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Take home message

A short history of stock assessment:

- **1950 - 1980:** development of the basis of stock assessment methods

  Surplus production model, virtual population analysis.

- **1980 - 2010:** bringing these methods into statistics

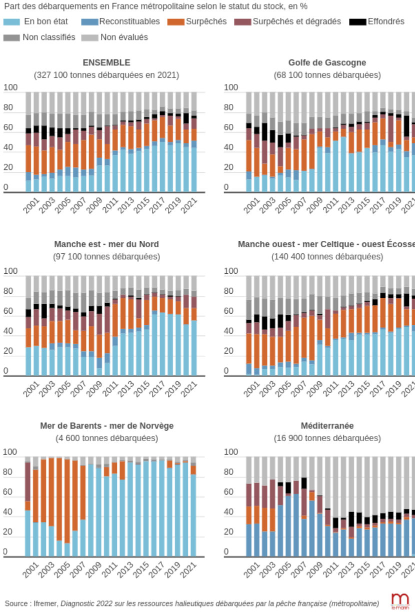
  State space modelling, maximum likelihood, bayesian inference, integrated approach.

- **2010 - today:** towards spatio-temporal modelling of population dynamics (among other things)

  Spatio-temporal modelling, Laplace approximation, SPDE approach, automatic differentiation.

Does this mean that old methods are outdated ?

➠ Of course not, all these methods are **complementary**.



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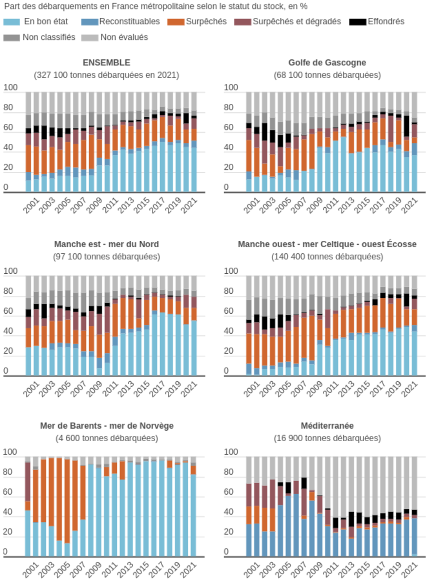

Source : Ifremer, *Diagnostic 2022 sur les ressources halieutiques débarquées par la pêche française (métropolitaine)*

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A short history of stock assessment:

- **1950 - 1980:** development of the basis of stock assessment methods

  Surplus production model, virtual population analysis.

- **1980 - 2010:** bringing these methods into statistics

  State space modelling, maximum likelihood, bayesian inference, integrated approach.

- **2010 - today:** towards spatio-temporal modelling of population dynamics (among other things)

  Spatio-temporal modelling, Laplace approximation, SPDE approach, automatic differentiation.

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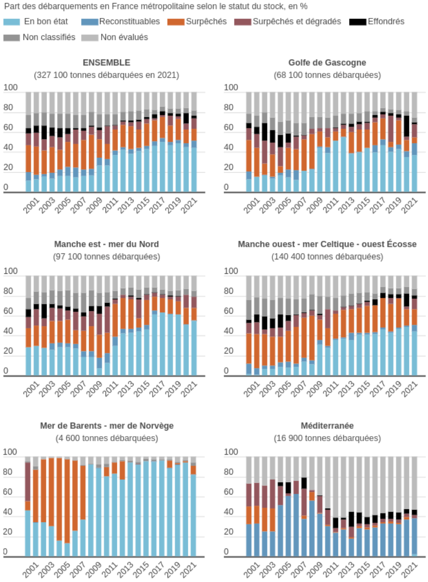
Does this mean that old methods are outdated ?

➠ Of course not, all these methods are **complementary**.



Source : Ifremer, *Diagnostic 2022 sur les ressources halieutiques débarquées par la pêche française (métropolitaine)*

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Modelling hidden processes of population dynamics

Age-dependent only *(Continuous formulation)*

$$\frac{\mathrm{d}N(a)}{\mathrm{d}a} = -(F+M)N(a)$$
$$\Rightarrow N(a) = N(a_0)\exp\left(-(F+M)(a-a_0)\right), a_0 \leq a$$

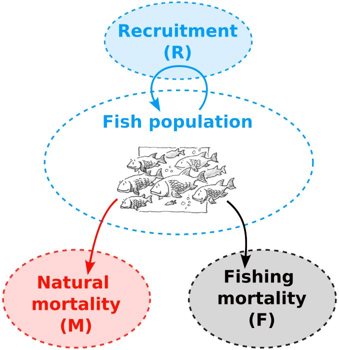
$$C(a,\Delta a) = \int_a^{a+\Delta a} F \cdot N(a)\mathrm{d}a \quad \textbf{(Baranov equation)}$$
$$= \frac{F}{F+M}N(a)[1-\exp(-(F+M)\Delta a)]$$

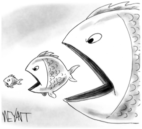
Usually, age is discretized and the catch is modelled between age a and $a+1$ ($\Delta a = 1$). Then, it gives:

$$C(a) = \frac{F}{F+M}N(a)[1-\exp(-(F+M))]$$

Age- and time-dependent *(Discrete formulation)*

$$N(a+1,t+1) = N(a,t)\exp\left(-F(a,t) - M(a,t)\right)$$
$$C(a,t) = \frac{F(a,t)}{F(a,t)+M(a,t)}N(a,t)[1-\exp(-(F(a,t)+M(a,t)))]$$





And there is also **Recruitment**, but that's a whole different issue...

Laplace approximation

Aim: approximate the integral where $f(x)$ is the log-likelihood of a Gaussian variable x:

$$I_n = \int_x \exp(nf(x))dx \quad \text{as } n \to \infty$$

Let's denote x_0, the point in which $f(x)$ has its maximum, then

$$I_n \approx \int_x \exp[n\,(f(x_0) + \frac{1}{2}(x - x_0)^2 f''(x_0))]dx$$

$$= \exp(nf(x_0))\sqrt{\frac{2\pi}{-nf''(x_0)}} = \widetilde{I_n}.$$

Application to likelihood:

If $f(\theta, \delta)$ is the joint log-likelihood, then $L_M(\theta)$ can be written as:

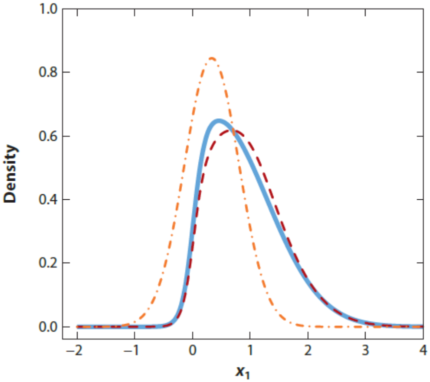
$$L_M(\theta) \approx L_M^*(\theta)$$

$$= (2\pi)^{q/2}|H(\theta)|^{-1/2}\exp[f(\theta, \widehat{\delta}_\theta)]$$

$H(\theta)$: the Hessian of the negative joint log-likelihood $-f$
q: the size of the latent effect
$\hat{\delta}_\theta = \underset{\delta}{\text{argmax}}(f(\theta, \delta))$

Conditions required:

- normality of the random effect δ
- some regularity conditions on f (*e.g.* only one maximum)



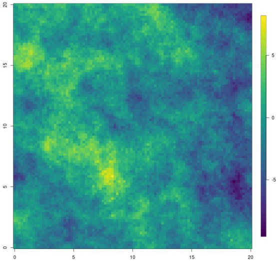
The true marginal (blue), the Laplace approximation (red) and the Gaussian approximation (orange).

SPDE approach

$$\delta|\theta \sim GF(\mu, \Sigma) \qquad \text{Latent Gaussian field}$$

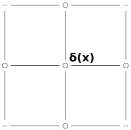
$$Y|\delta, \theta \sim \mathcal{L}(\delta|\theta) \qquad \text{Observation process}$$

- Y: observations.
- θ: parameters.
- δ: a spatial random effect (= Gaussian field)
- $\Sigma = Q^{-1}$: variance-covariance matrix of a spatial random effect. Q is the precision matrix.

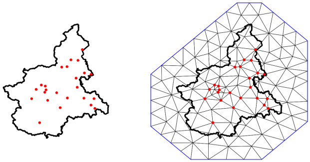


First set of results

A Gaussian field wih Matérn Covariance can be represented as a Gauss-Markov field ➠ sparse representation of the spatial effect



Second set of results



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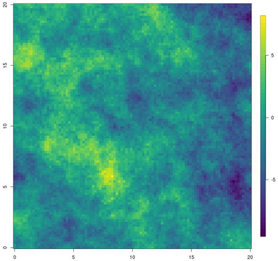

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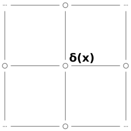

Second set of results