

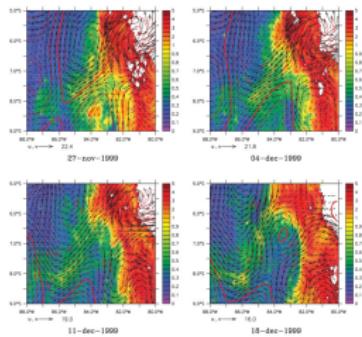
Empirical Orthogonal Functions and derived methods

B. Alglave and J. Thorson

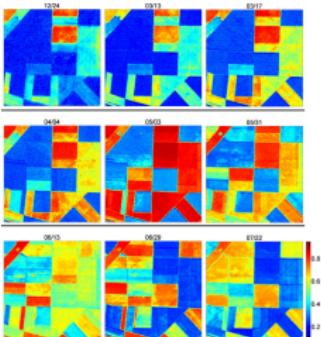
March 2024



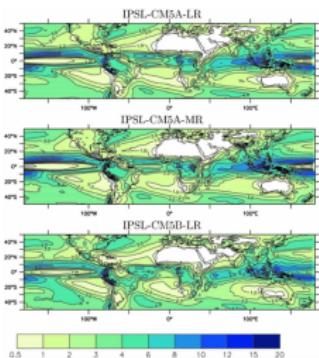
Oceanography



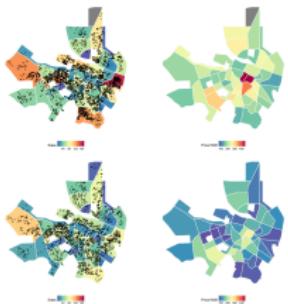
Agronomy



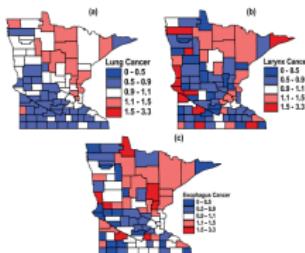
Climatology



Economics



Epidemiology



Ecology

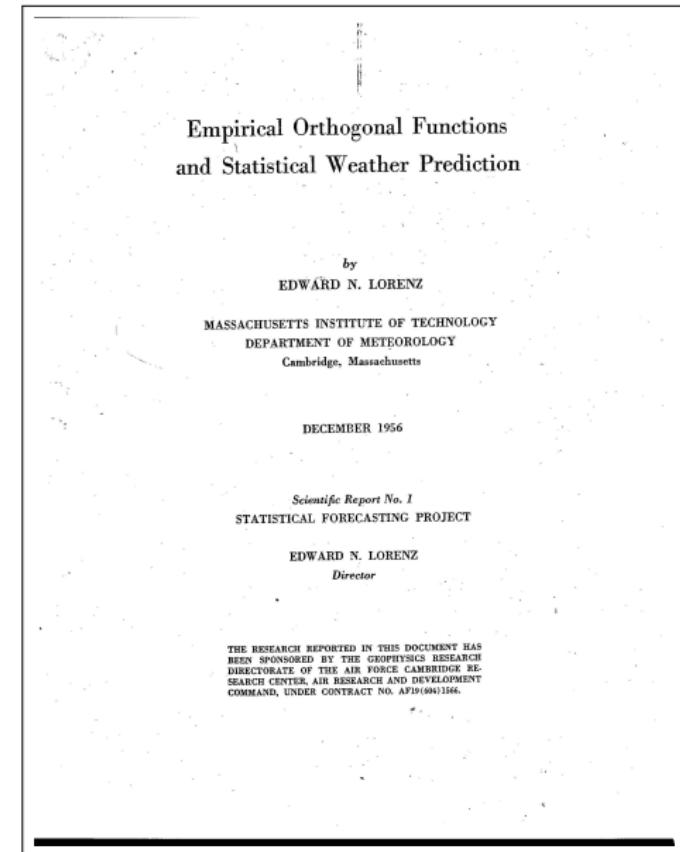
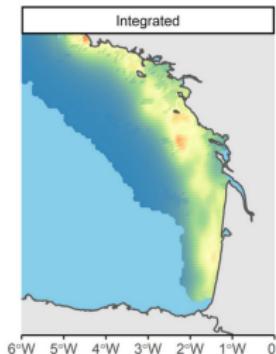


Empirical Orthogonal Functions

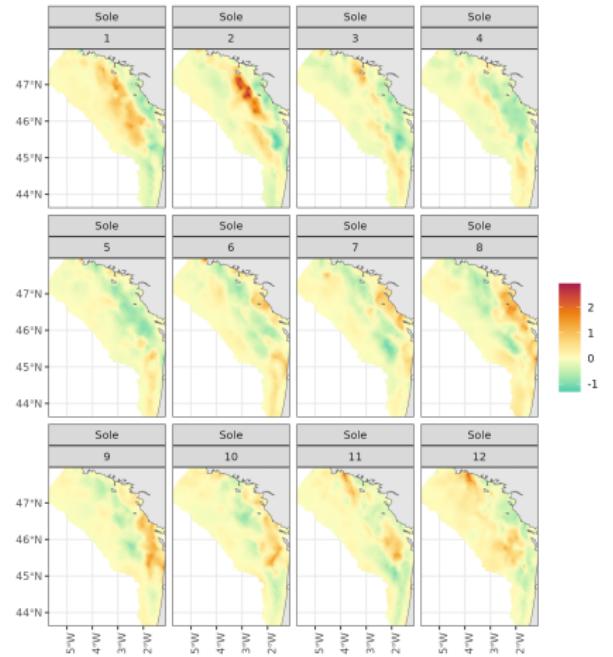
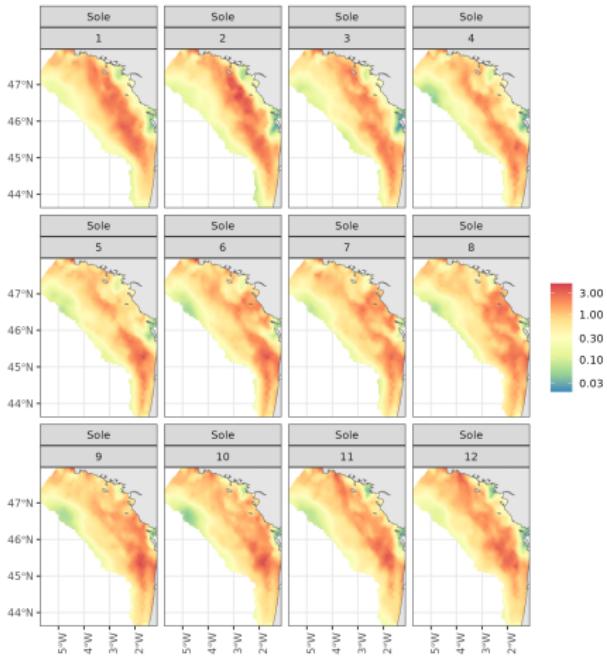
First introduced by Lorenz (1956)

- What is the **best representation** for a spatio-temporal field ?
 - How to perform **dimension-reduction** on a spatio-temporal field ?
 - How to make **projections**?
-

Presentation based on a fishery case study.
Demersal fisheries of the Bay of Biscay.
Monthly maps from 2008 to 2018 (132 maps)



Raw data



(left) Monthly spatial log-predictions $\log S(x, t)$ of the hierarchical model. (right) Monthly anomalies of the spatial predictions $S^*(x, t)$. Each panel corresponds to the average distribution of prediction anomalies for a month over the period 2008 - 2018.

Basics of EOF

The spatio-temporal field is decomposed so that:

$$S'(x, t) = \sum_{m=1}^r p_m(x) \cdot \alpha_m(t) + \epsilon_m(x, t)$$

with r the number of dimensions of the EOF ($r \leq \min(n, p)$), $p_m(x)$ the spatial term of EOF and $\alpha_m(t)$ the temporal term of EOF for dimension m .
 $\epsilon_m(x, t)$ is an error term.

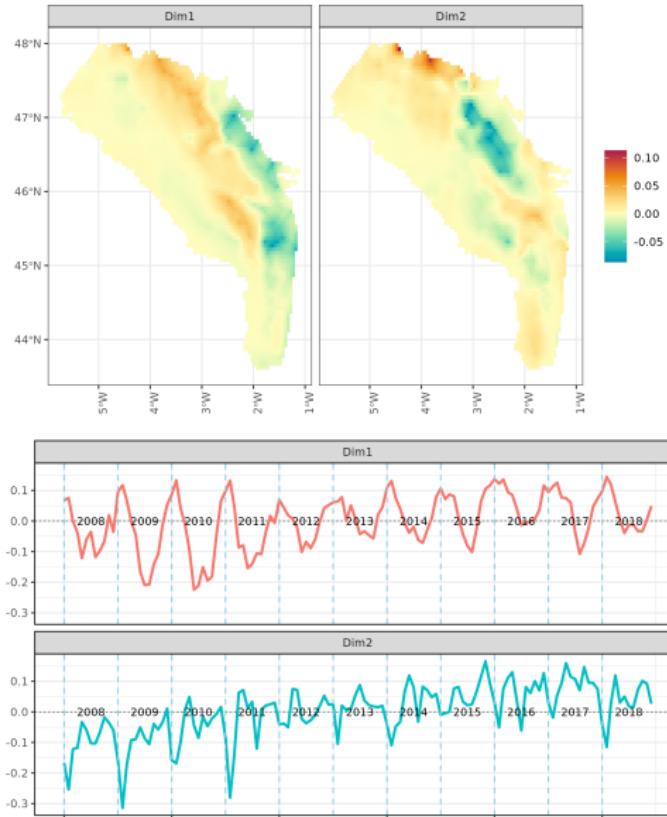
Constraints:

- minimize $E = \sum_m \sum_x \sum_y \epsilon_m(x, t)$
- spatial terms and temporal terms are orthogonal

$$\langle p_i(\cdot); p_j(\cdot) \rangle = 0 \quad i \neq j$$

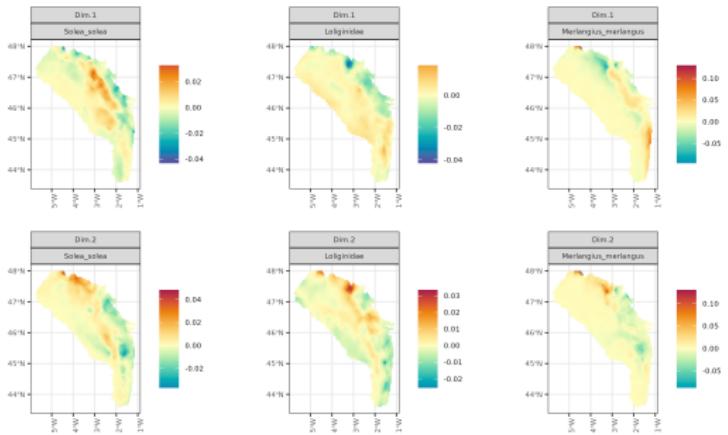
$$\langle \alpha_i(\cdot); \alpha_j(\cdot) \rangle = 0 \quad i \neq j$$

Illustration

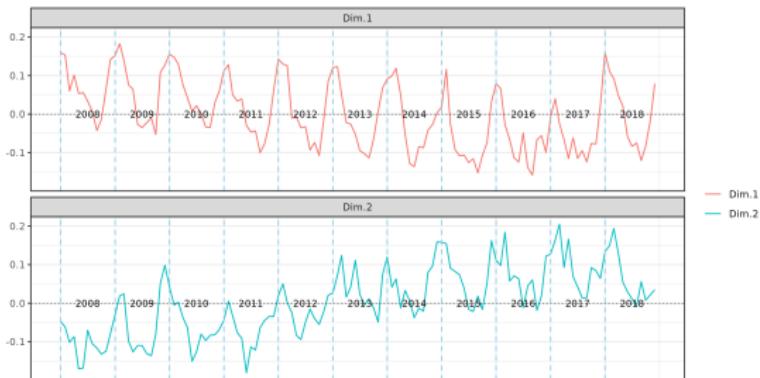


(Top) Spatial factors for the two first dimensions of the EOF. (Bottom) Loadings for the two first dimensions of the EOF. Blue dashed vertical lines corresponds to the month of January for each year.

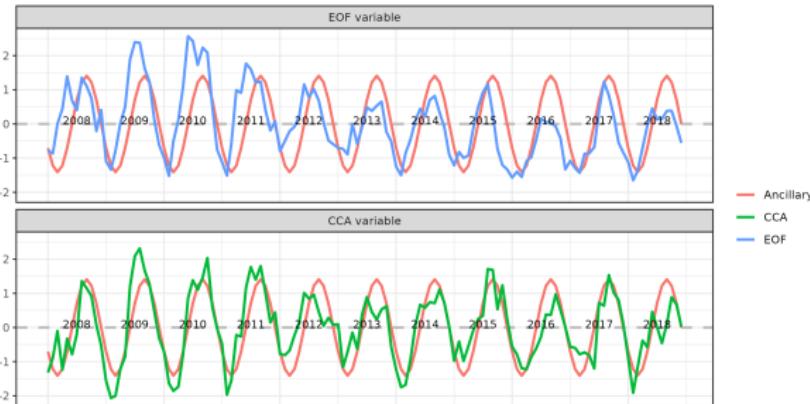
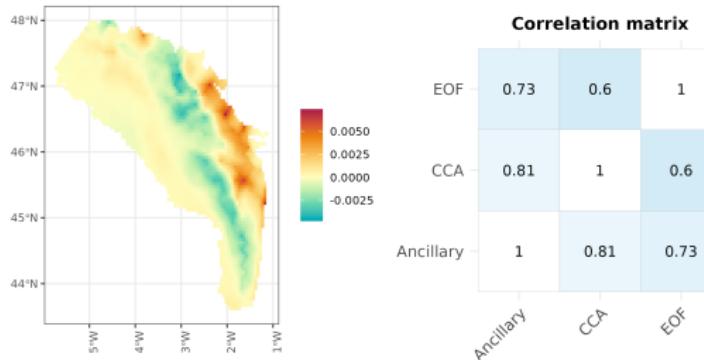
Multivariate EOF



Multivariate EOF. (Top) Factor maps for each species and dimensions. (Bottom) Loadings for the two first dimensions.

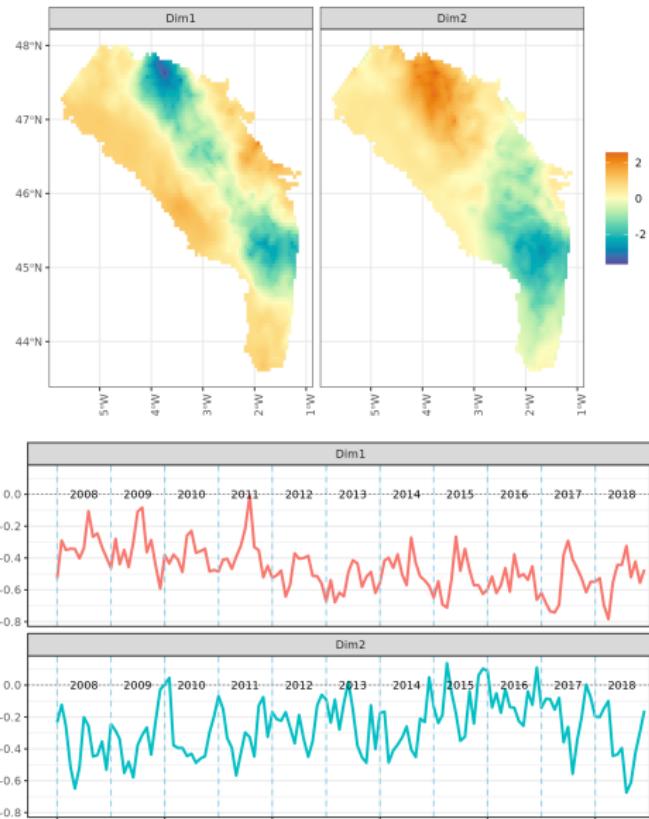


Constraining the EOF with an ancillary variable



Results of the canonical correlation analysis. (Top left) Canonical vectors that maximise correlation between EOF and ancillary variable. (Top right) Correlation matrix between the first time series of the EOF, the EOF time series in the CCA and the ancillary variable. (Bottom) Comparison of the EOF variables with the ancillary variable and the CCA variables with the ancillary variable. These time series are standardized.

Spatially orthogonal EOF factors



(Top) Spatial factors obtained by EOM for the two first dimensions.
(Bottom) Temporal loadings for the first two dimensions.

Conclusion

Some bibliographic metrics

sources	articles
JOURNAL OF CLIMATE	391
CLIMATE DYNAMICS	271
JOURNAL OF GEOPHYSICAL RESEARCH: OCEANS	257
INTERNATIONAL JOURNAL OF CLIMATOLOGY	256
GEOPHYSICAL RESEARCH LETTERS	142
JOURNAL OF THE ATMOSPHERIC SCIENCES	126
MONTHLY WEATHER REVIEW	103
REMOTE SENSING	99
JOURNAL OF GEOPHYSICAL RESEARCH ATMOSPHERES	97
THEORETICAL AND APPLIED CLIMATOLOGY	85
PROCEEDINGS OF SPIE - THE INTERNATIONAL SOCIETY FOR OPTICAL ENGINEERING	75
ATMOSPHERE	72
CONTINENTAL SHELF RESEARCH	72
JOURNAL OF PHYSICAL OCEANOGRAPHY	72
QUARTERLY JOURNAL OF THE ROYAL METEOROLOGICAL SOCIETY	68

➡ But progressively coming to ecology
e.g. Thorson et al. (2020), Bez et al. (2023)

Thank you for your attention!



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