Link Analysis Algorithms
Page Rank

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INF 553

With slide contributions from
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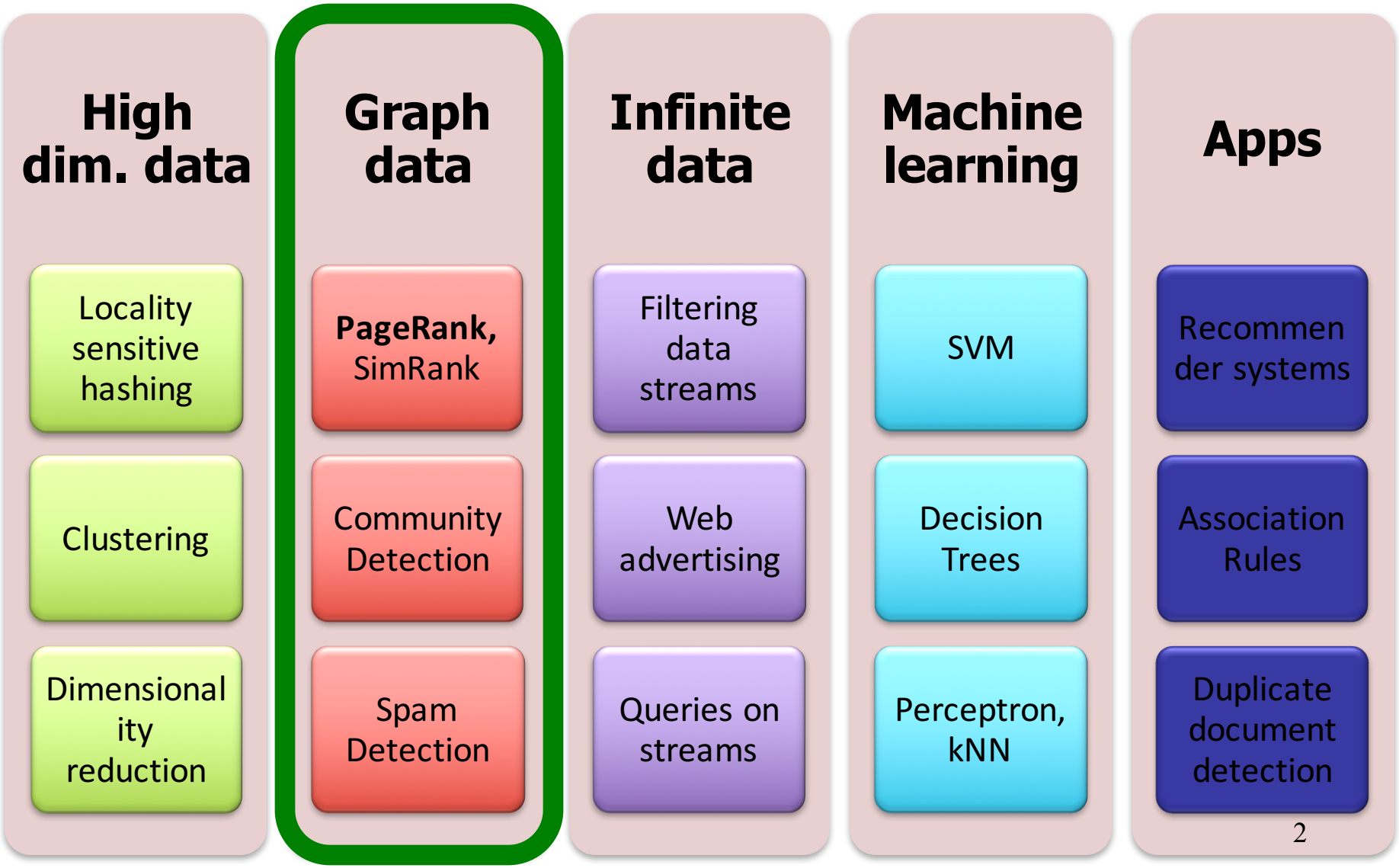
New Topic: Graph Data

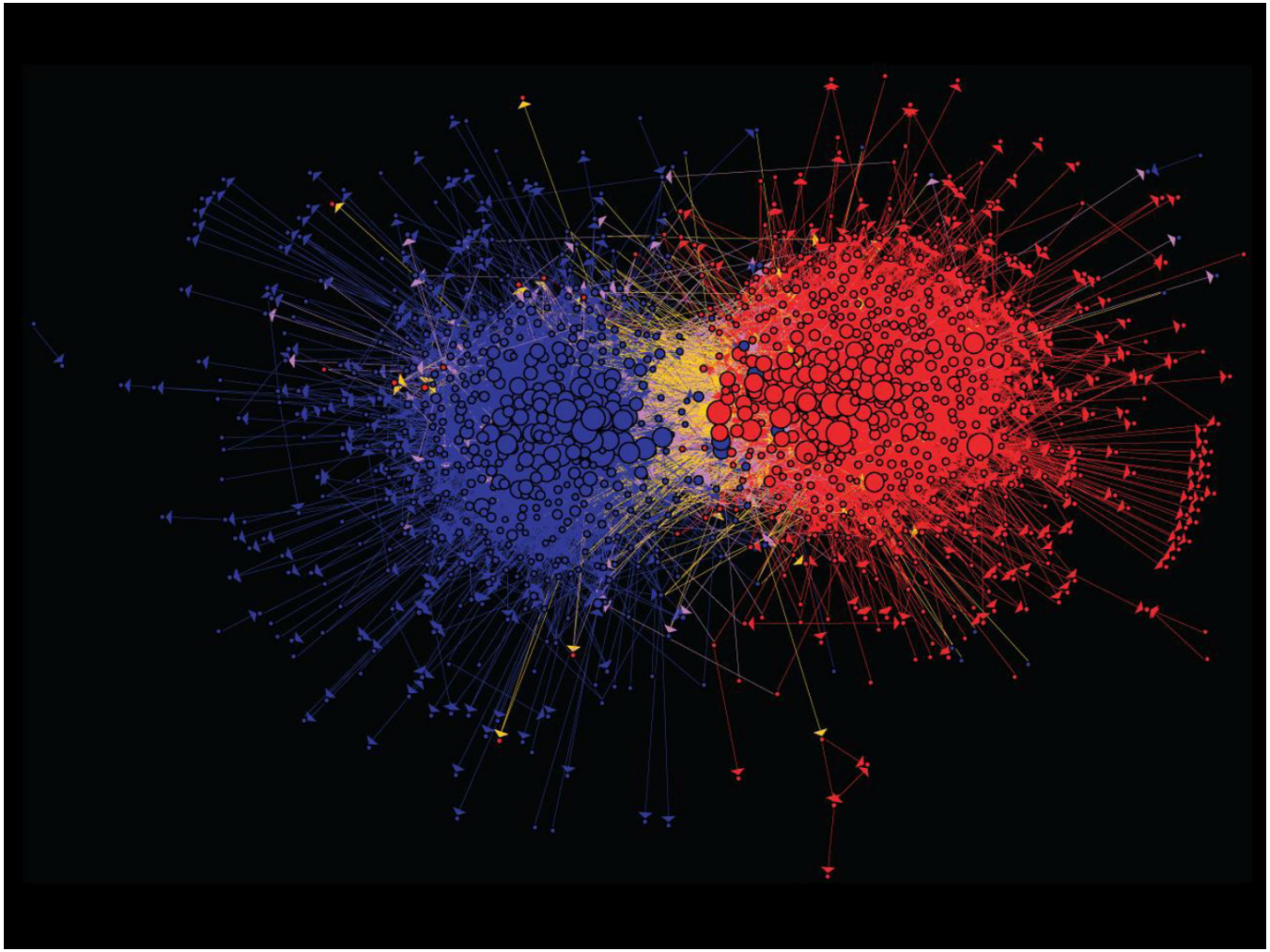


Graph Data: Social Networks

Facebook social graph

4-degrees of separation [Backstrom-Boldi-Rosa-Ugander-Vigna, 2011]

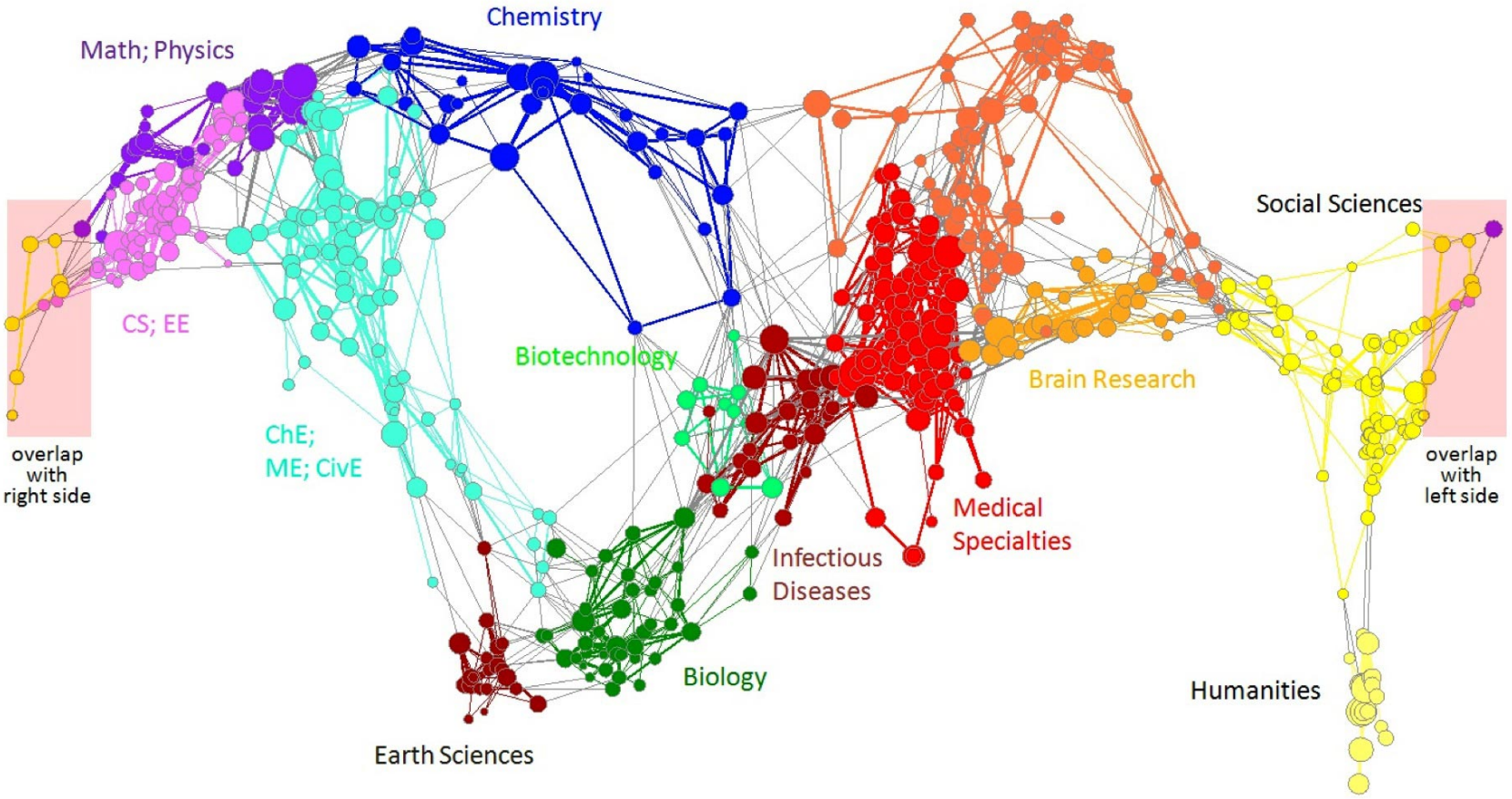
Graph Data: Media Networks



Connections between political blogs

Polarization of the network [Adamic-Glance, 2005]

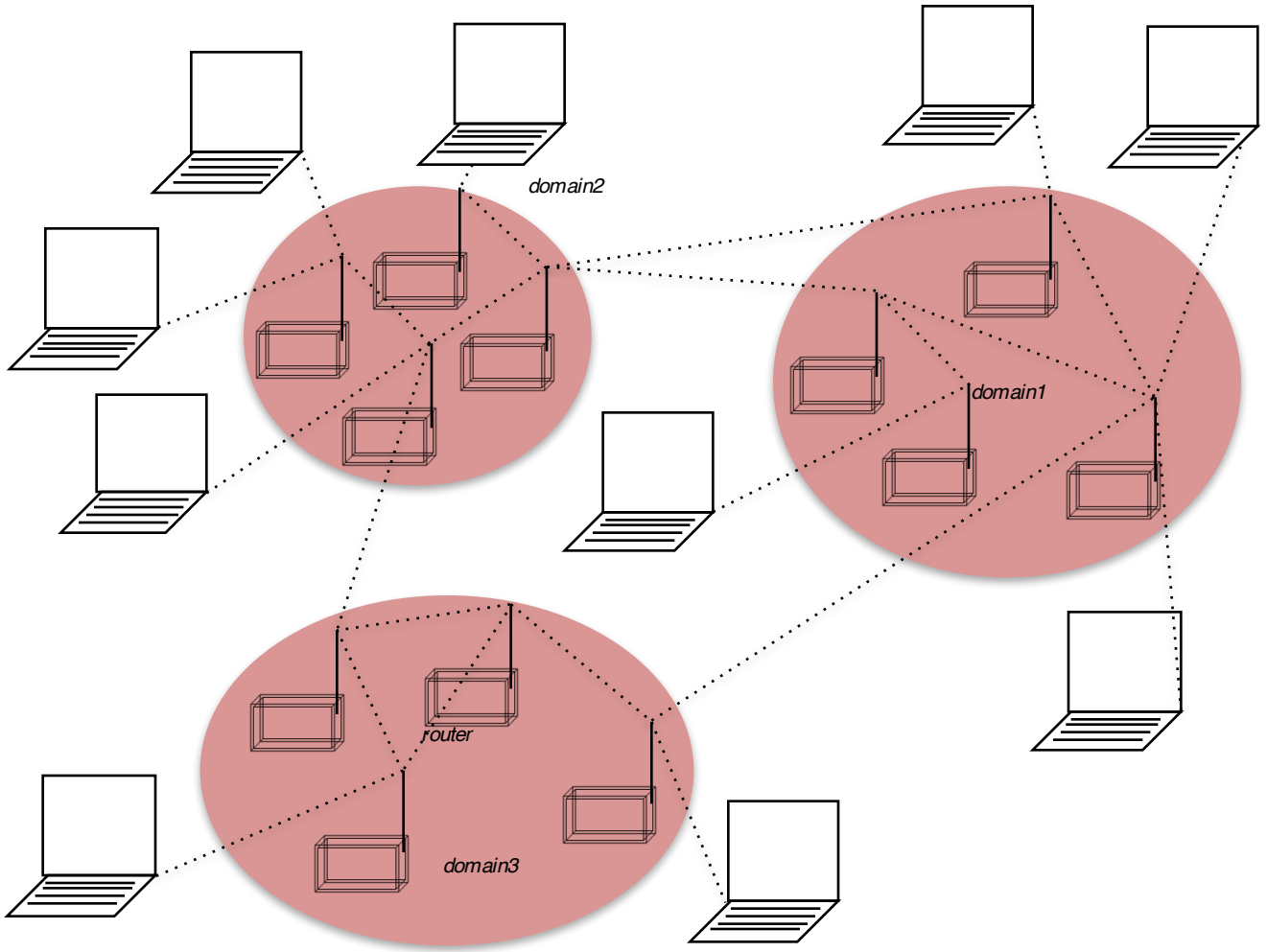
Graph Data: Information Nets



Citation networks and Maps of science

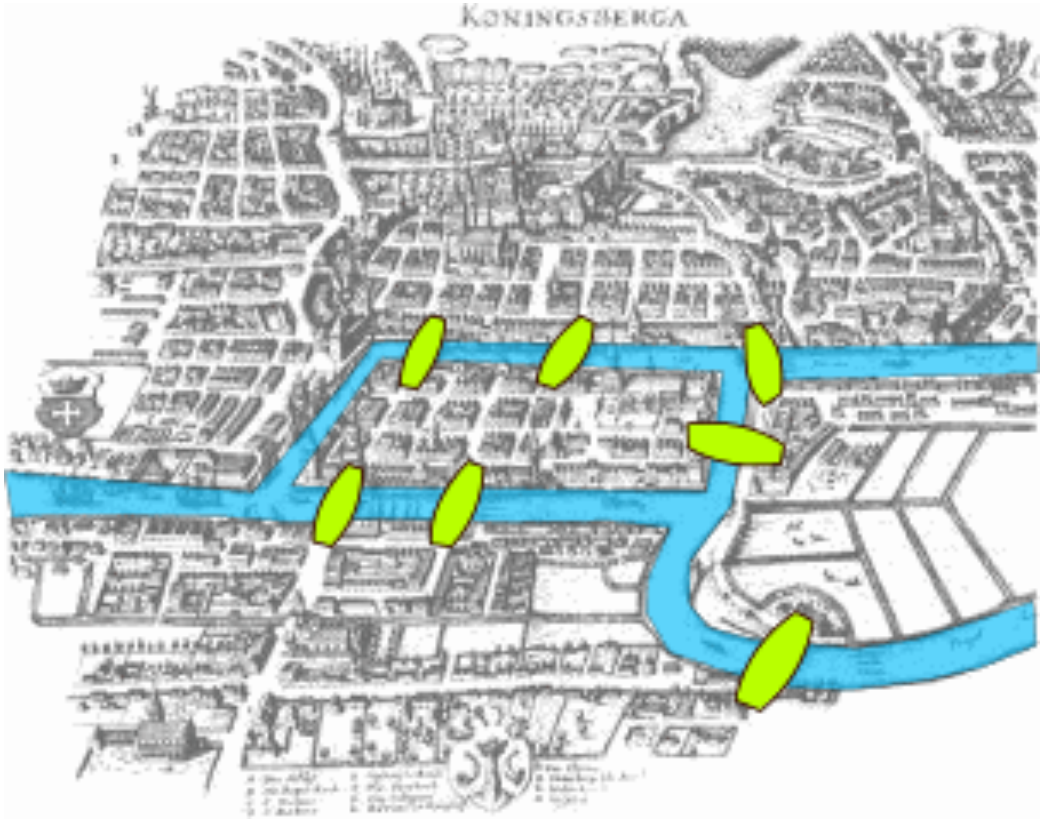
[Börner et al., 2012]

Graph Data: Communication Nets



Internet

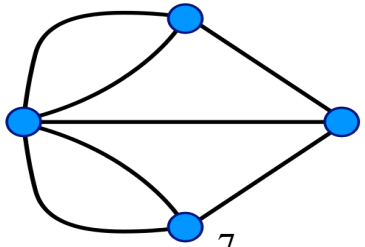
Graph Data: Technological Networks



Seven Bridges of Königsberg

[Euler, 1735]

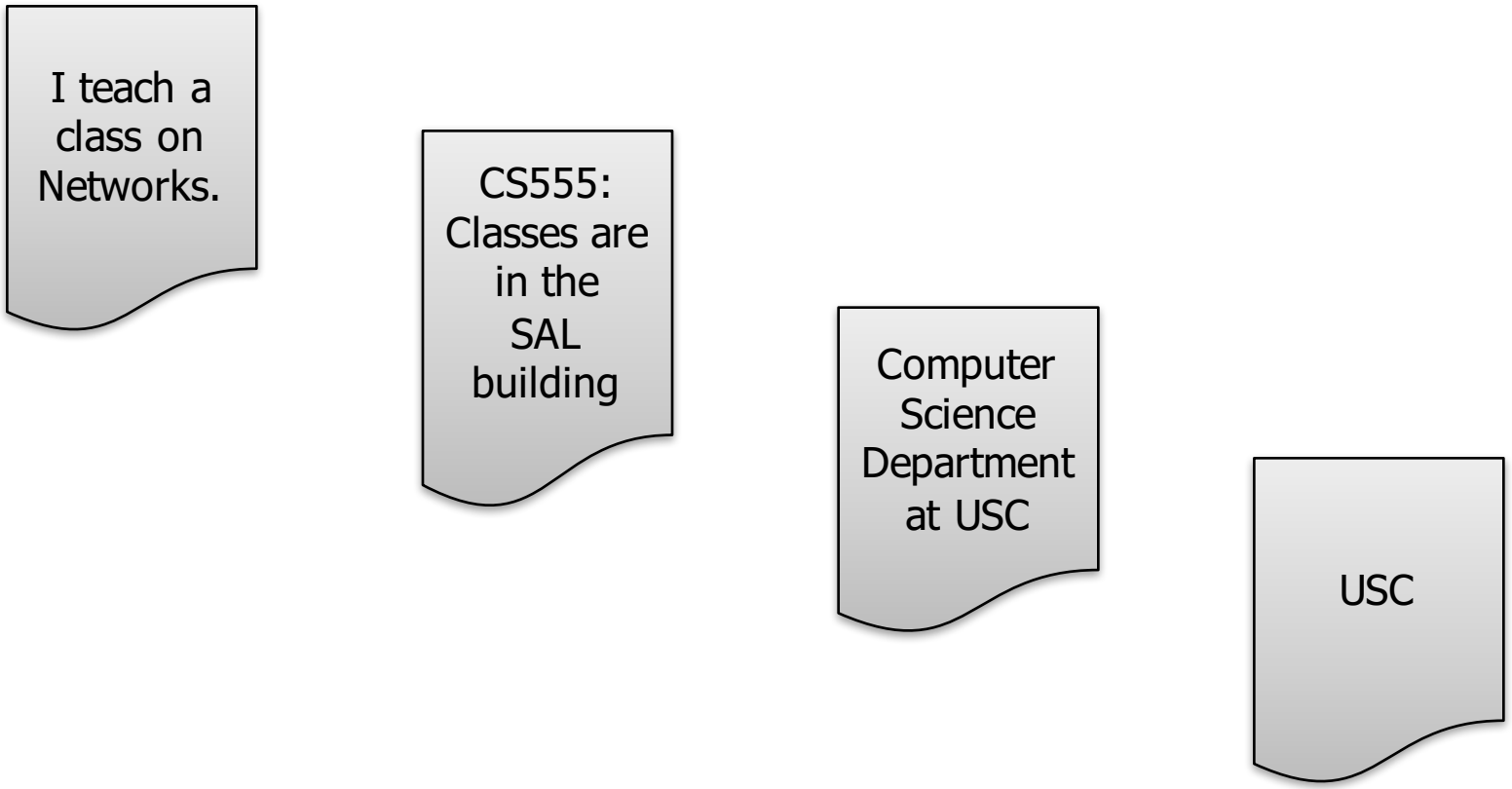
Return to the starting point by traveling each link of the graph once and only once.



The Problem

Web as a Graph

- **Web as a directed graph:**
  - **Nodes: Webpages**
  - **Edges: Hyperlinks**



Web as a Graph

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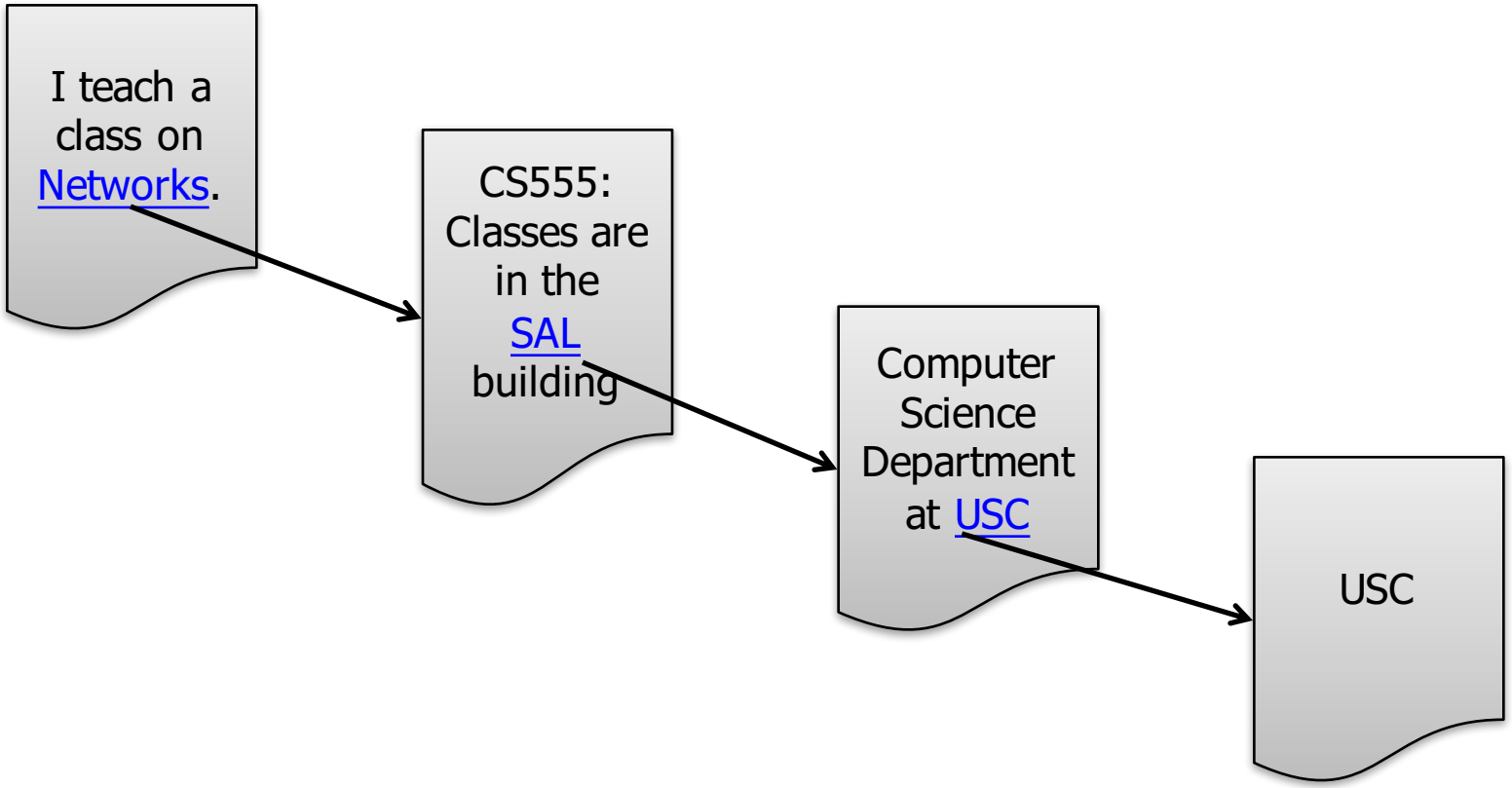
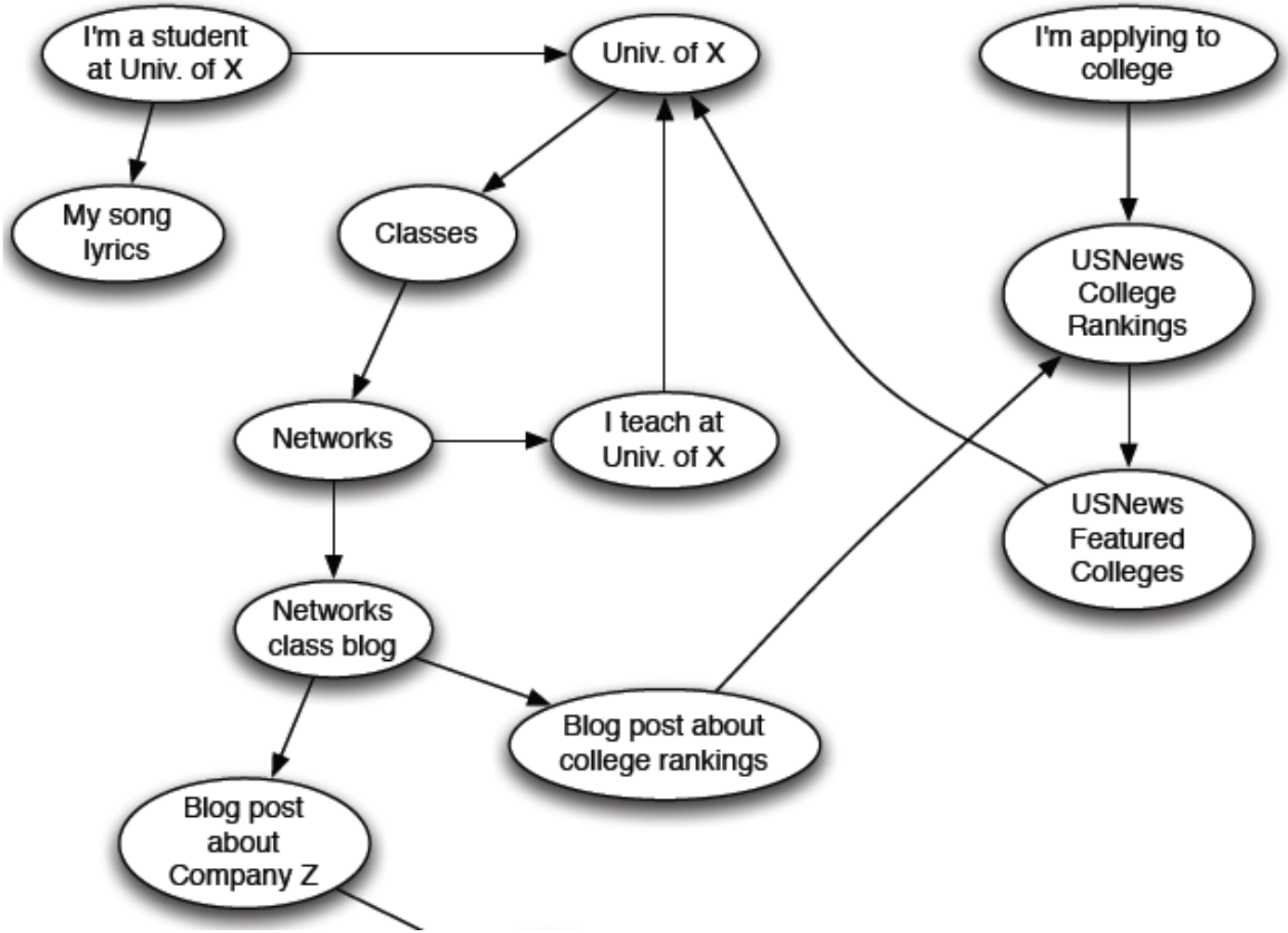
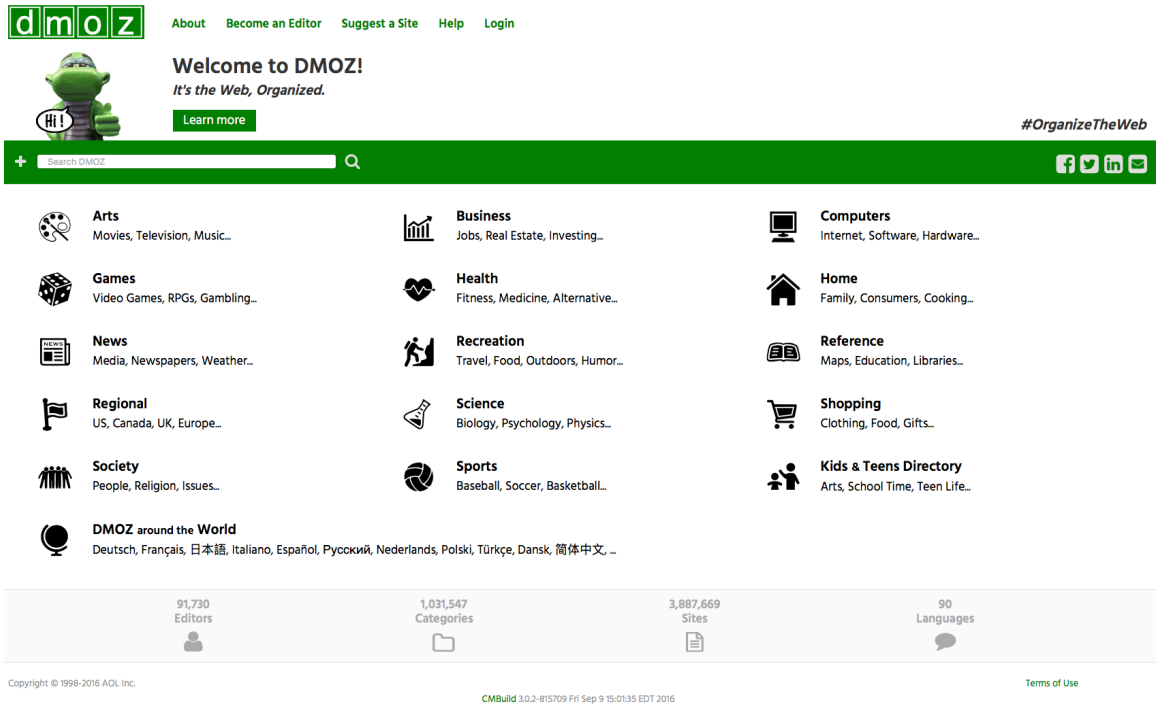


Web as a Directed Graph



Broad Question

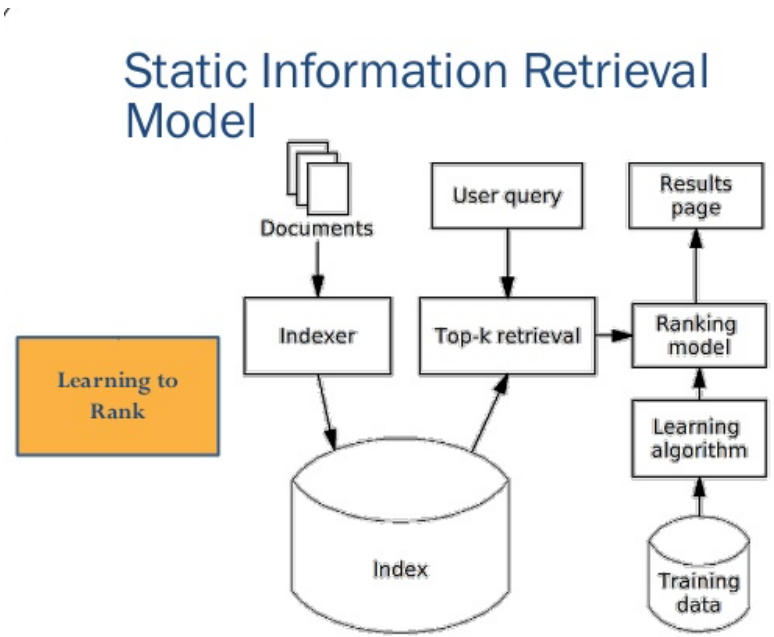
- **How to organize the Web?**
- **First try:** Human curated
  Web directories
  - Yahoo, DMOZ, LookSmart



Broad Question (Cont'd)

- **How to organize the Web?**
- **Second try: Web Search**
  - **Information Retrieval** investigates: Find relevant docs in a small and trusted set
    - Newspaper articles, patents, etc.

But: Web is **huge**, full of untrusted documents, random things, web spam, etc.



Early Web Search

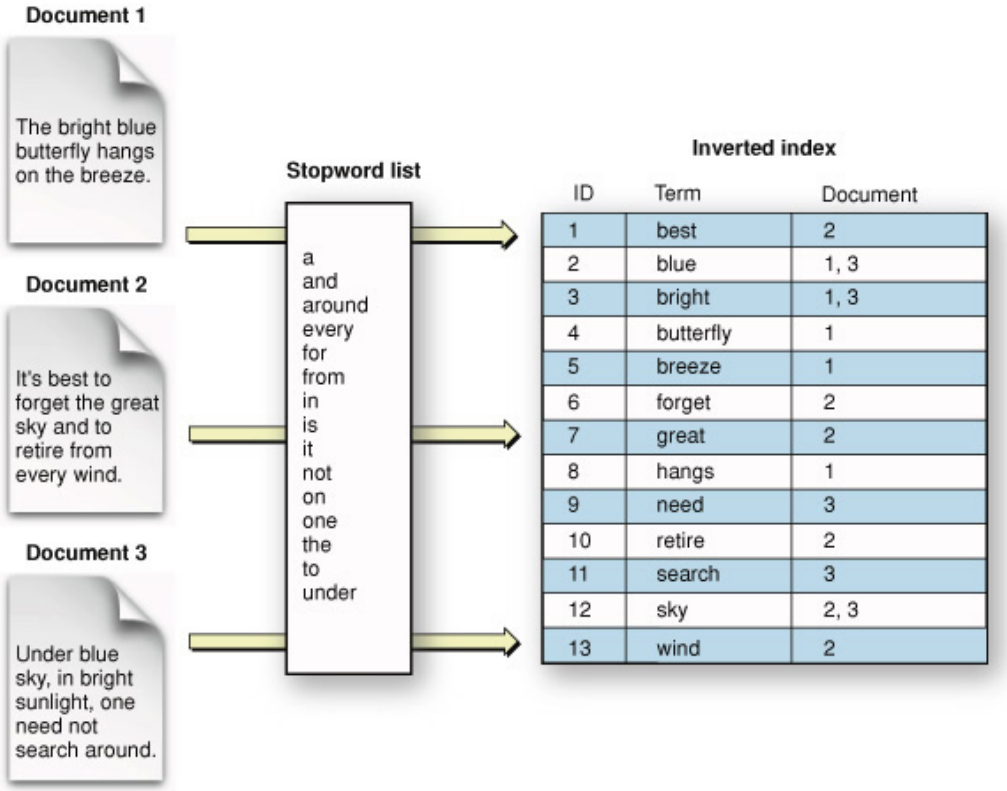
- Keywords extracted from web pages
  - E.g., title, content
  - Used to build inverted index

```
"a":      {2}
"banana": {2}
"is":     {0, 1, 2}
"it":     {0, 1, 2}
"what":   {0, 1}
```

- Queries are matched with web pages
  - Via lookup in the inverted index
  - Pages ranked by occurrences of query keywords

Inverted Index

- Problem: susceptible to term spam



https://developer.apple.com/library/mac/documentation/UserExperience/Conceptual/SearchKitConcepts/searchKit_basics/searchKit_basics.html

Term Spam

- Disguise a page as something it is not about
  - E.g., adding thousands of keyword “movies”
  - Actual content may be some advertisement
  - Fool search engine to return it for query “movies”
- May even fade spam words into background
- Spam pages may be based on top-ranked pages

Web Search: Two Challenges

Two challenges of web search:

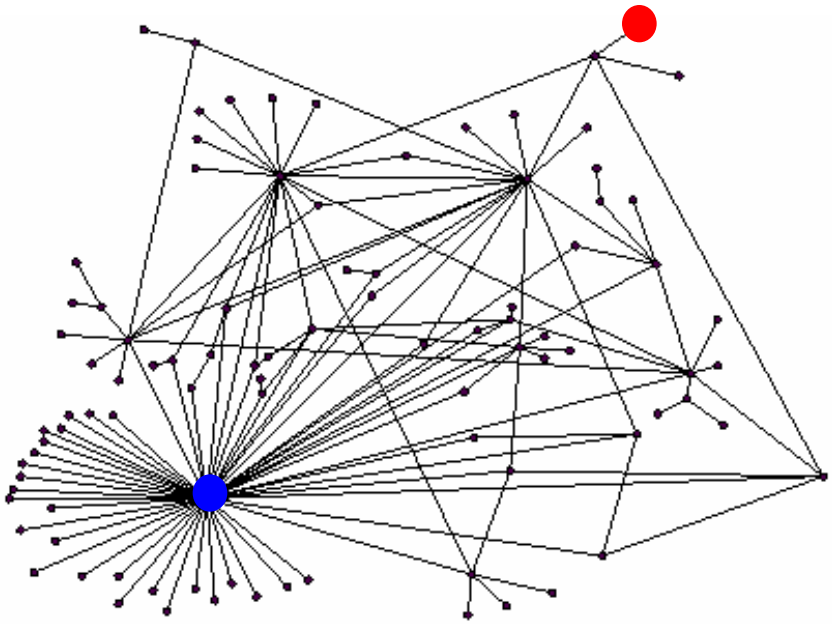
- **(1) Web contains many sources of information Who to "trust"?**
  - **Trick:** Trustworthy pages may point to each other!
- **(2) What is the "best" answer to query "newspaper"?**
  - No single right answer
  - **Trick:** Pages that actually know about newspapers might all be pointing to many newspapers.

Ranking Nodes on the Graph

- **All web pages are not equally "important"**

  www.joe-schmoe.com vs. www.usc.edu

- There is large diversity in the web-graph node connectivity
- **Let's rank the pages by the link structure!**



Link Analysis Algorithms

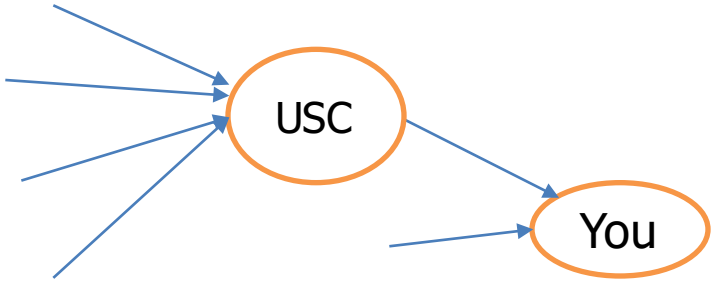
- We will cover the following **Link Analysis approaches** for computing **importance** of nodes in a graph:
  - Page Rank
  - Topic-Specific (Personalized) Page Rank
  - Web Spam Detection Algorithms.

PageRank:
The “Flow” Formulation

PageRank: Combating Term Spam

- Key idea: rank pages by linkage too
  - How many pages point to a page
  - How important these pages are

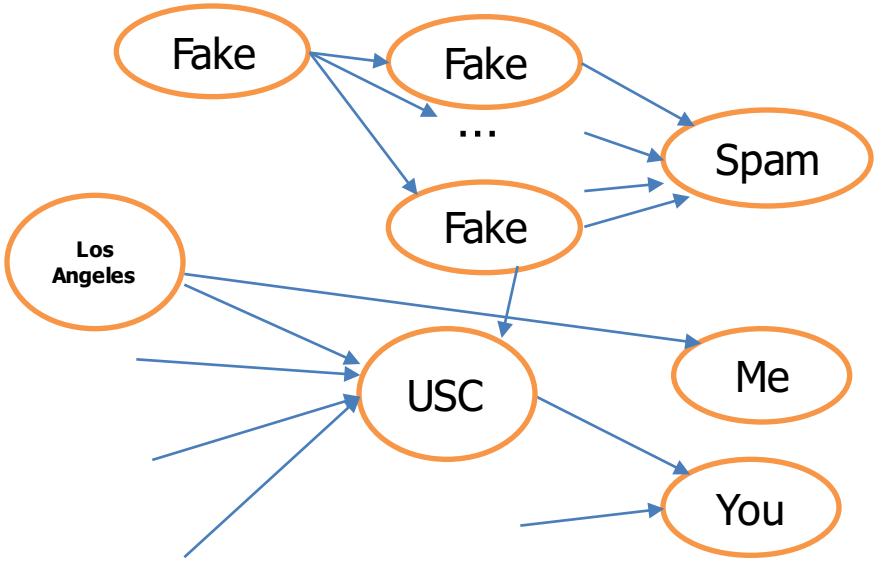
  => PageRank



- USC.edu can be important
  - because many pages point to it
- Your home page can be important
  - If it is pointed to by USC ☺

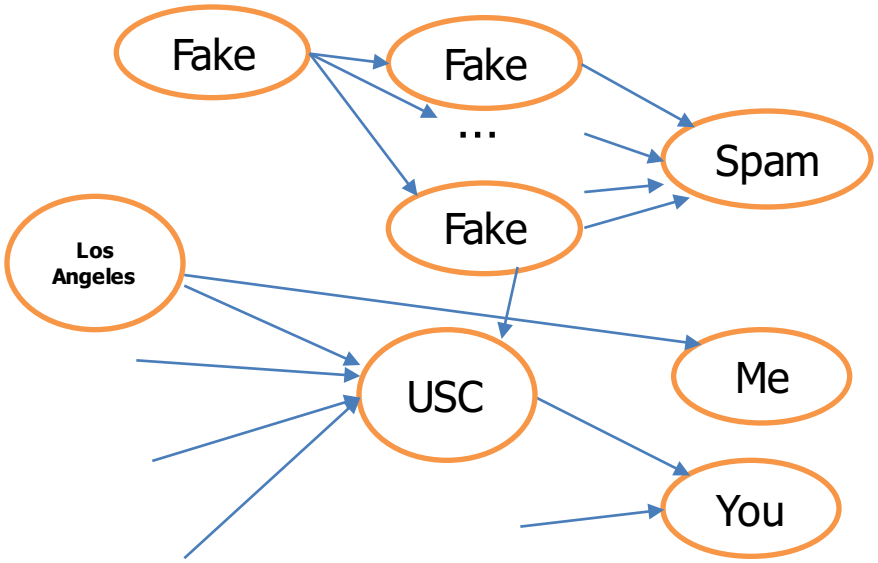
Random Surfer Model

- Random surfer of web
  - starts from any page
  - follows its outgoing links randomly
- Page is important if it attracts a large # of surfers



PageRank

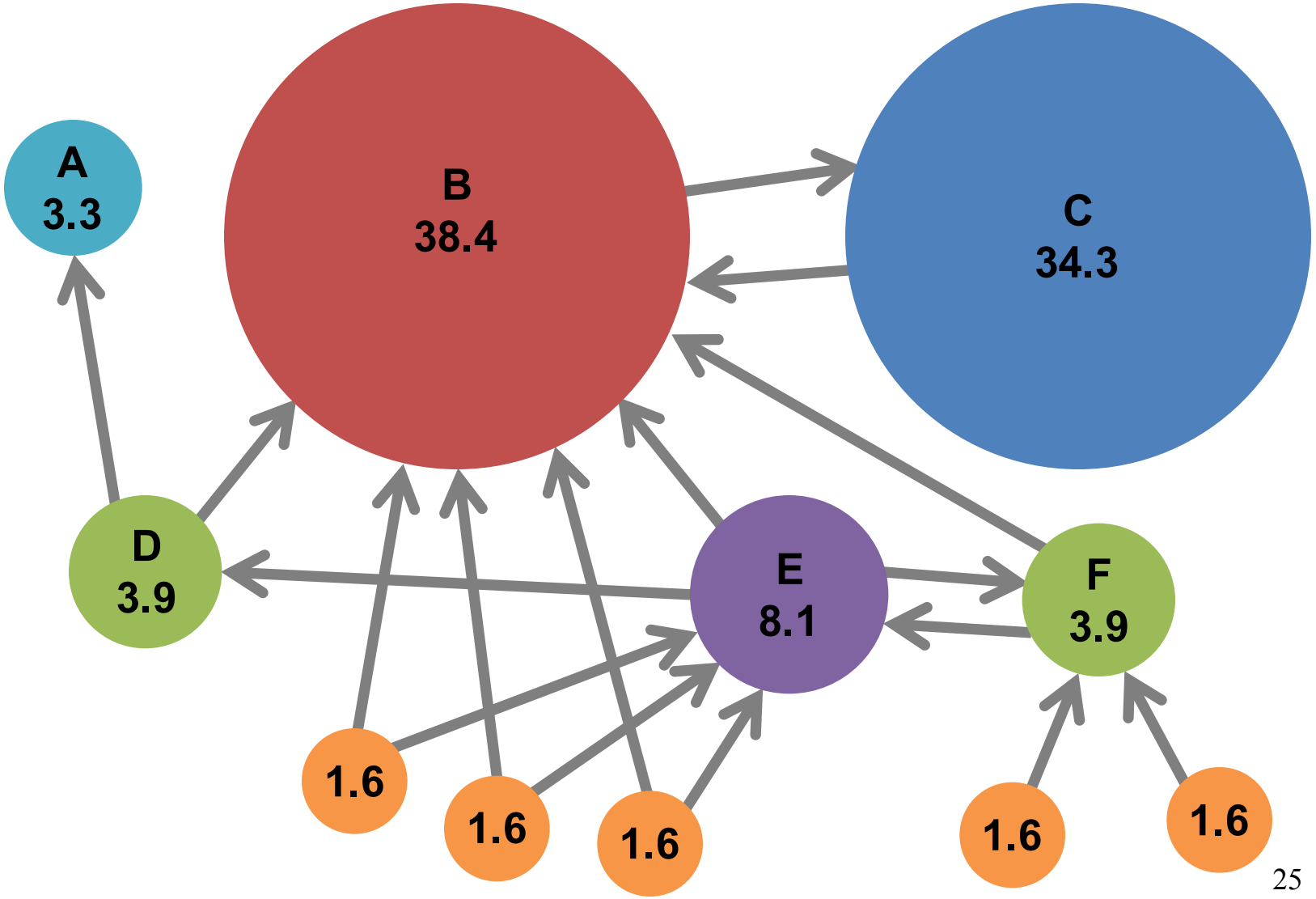
- Probability that a random surfer lands on the page



Intuition

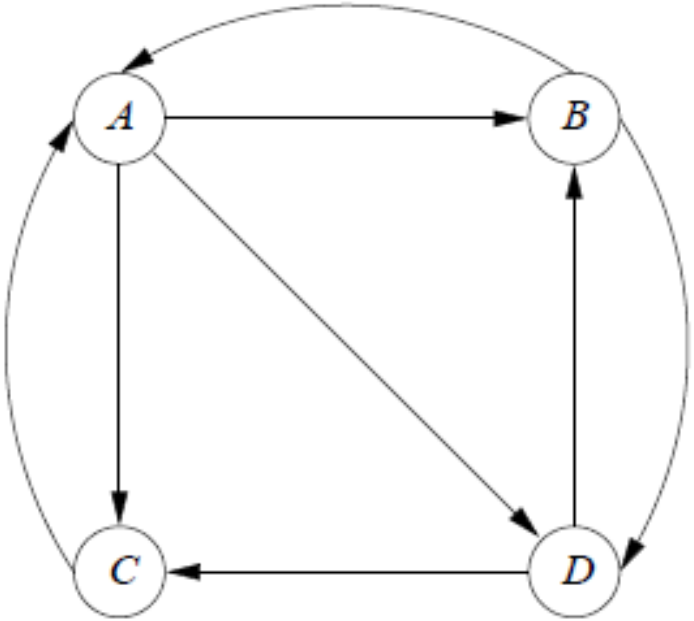
- If a page is important, then
  - many other pages may directly/indirectly link to it
  - random surfer can easily find it
- Spam pages are less connected
  - So less chance to attract random surfer
- Random surfer model more robust than manual approach
  - A collective voting scheme.

Example: PageRank Scores



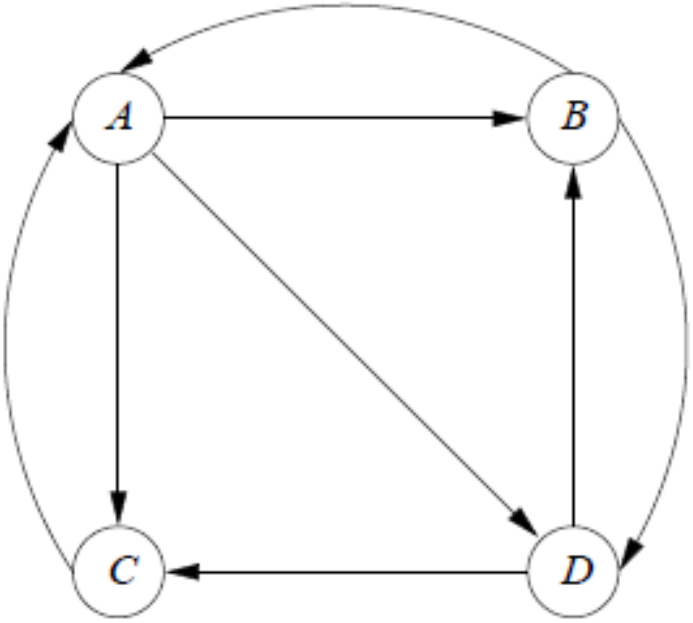
Assumption: A Strongly Connected Web Graph

- Nodes = pages
- Edges = hyperlinks between pages



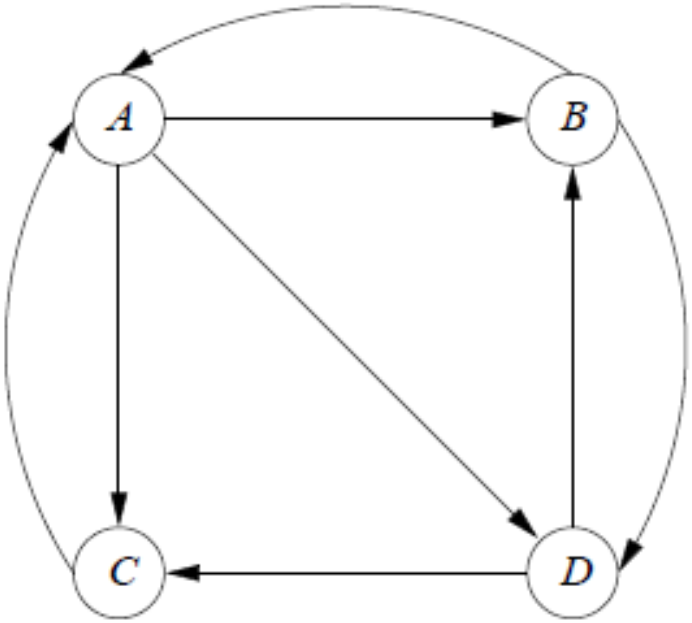
Model: Random Surfer on the Graph

- Can start at any node, say A
  - Can next go to B, C, or D, each with 1/3 prob.
  - If at B, can go to A and D, each with 1/2 prob.
  - So on…



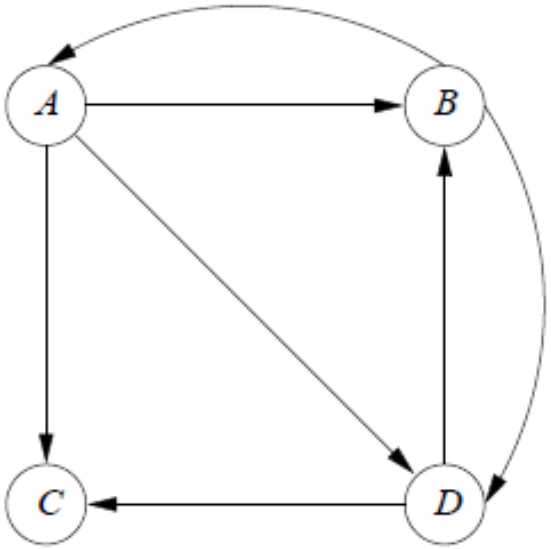
Random Surfer Property: Memoryless

- Where to go from node X is not affected by how the surfer got to X



Extreme Case: Dead End

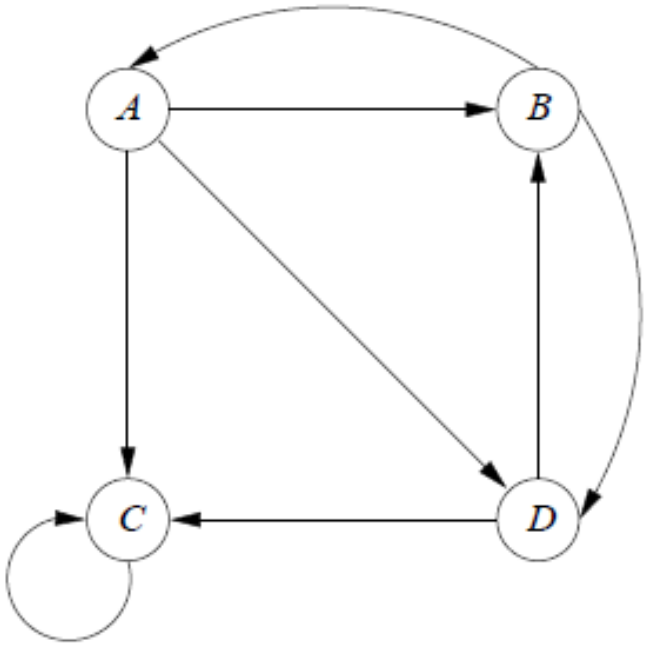
- Dead end: a page with no edges out
  - Absorb PageRanks
  - PageRank → 0 for any page that can reach the dead end (including the dead end itself)



Dead end

Extreme Case Spider Trap

- Group of pages with no edges going out of group
  - Absorb all PageRanks (rank of C → 1, others → 0)
  - Surfer can never leave, once trapped
  - Can have > 1 nodes



Spider trap

PageRank: Formulation Details

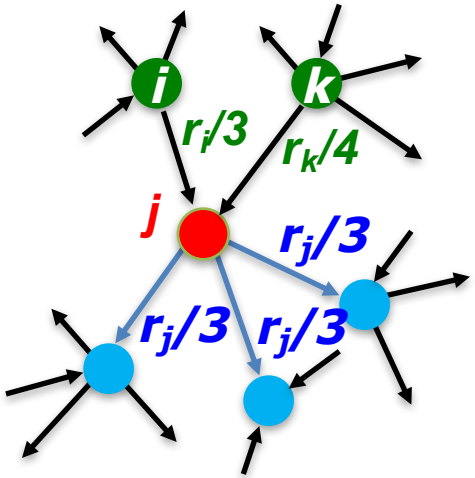
PageRank: Links as Votes

- **Idea: Links as votes**
  - **Page is more important if it has more links**
    - In-coming links? Out-going links?
- **Think of in-links as votes:**
  - **Eg:**
  - www.USC.edu ~ has 23,400 in-links
  - www.joe-schmoe.com has 1 in-link
- **Are all in-links are equal?**
  - **Links from important pages count more**
  - Recursive question!

Simple Recursive Formulation

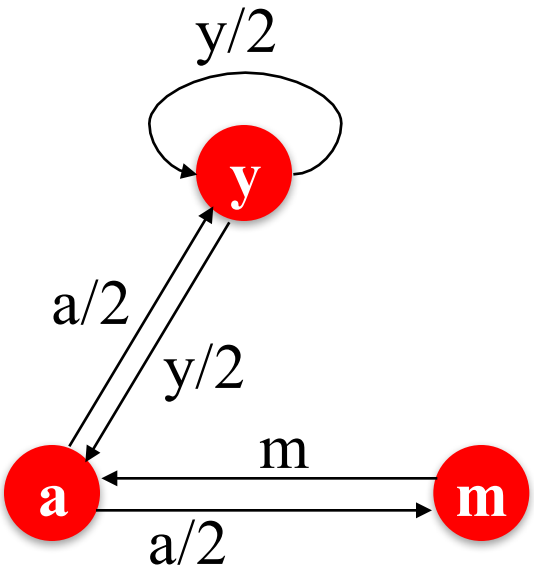
- Each link's vote is proportional to the **importance** of its source page
- If page ***j*** with importance r_j has ***n*** out-links, each link gets r_j / n votes
- Page ***j***'s own importance is the sum of the **votes on its in-links**

$$r_j = r_i/3 + r_k/4$$



PageRank: The "Flow" Model

- A "vote" from an important page is worth more
- A page is important if it is pointed to by other important pages
- Define a "rank" r_j for page j



$$r_j = \sum_{i \to j} \frac{r_i}{d_i}$$

d_i = out-degree of node i

"Flow" equations:

$$r_y = r_y/2 + r_a/2$$
$$r_a = r_y/2 + r_m$$
$$r_m = r_a/2$$

Solving the flow equations

"Flow" equations:

$$r_y = r_y/2 + r_a/2$$
$$r_a = r_y/2 + r_m$$
$$r_m = r_a/2$$

- **3 equations, 3 unknowns, no constants**
  - No unique solution
  - All solutions equivalent modulo scale factor
- **Additional constraint forces uniqueness**
  - $r_y + r_m + r_a = 1$
  - **Solution:** $r_y = \frac{2}{5}$, $r_a = \frac{2}{5}$, $r_m = \frac{1}{5}$
- **Gaussian elimination method works for small examples, but we need a better method for large web-size graphs**
- **We need a new formulation!**

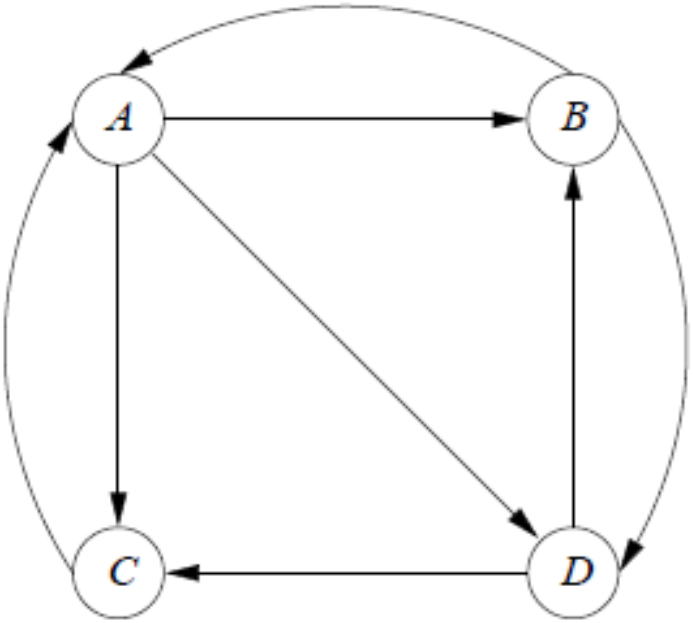
PageRank: Matrix formulation

- **Stochastic Transition (or adjacency) Matrix M**
- **Suppose page j has n outlinks**
  - If outlink j -> i, then $\mathbf{M_{ij}=1/n}$
  - Else $\mathbf{M_{ij}=0}$
- **M** is a **column stochastic matrix**
  - Columns sum to 1

Transition Matrix

- M[i,j] = prob. of going from node j to node i
  - If j has k outgoing edges, prob. for each edge = 1/k

$$M = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1/2 & 1 & 0 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{bmatrix} \end{array}$$

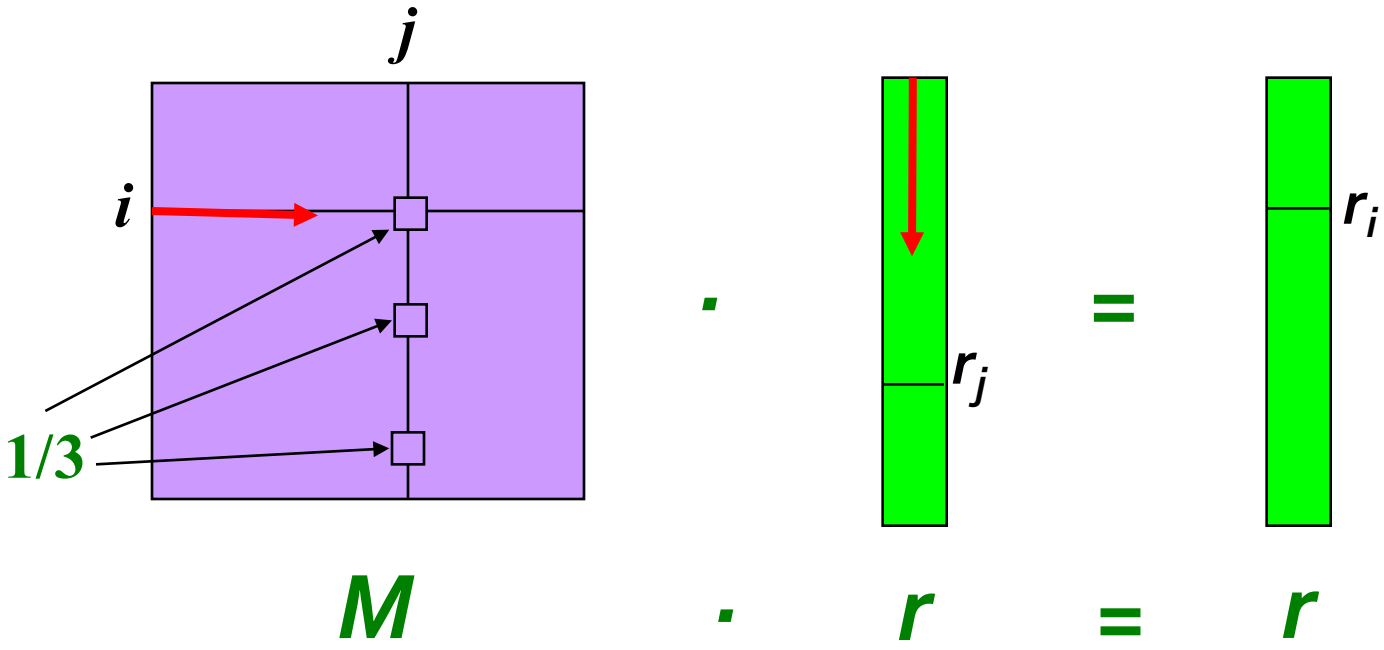


PageRank: Matrix formulation (Cont'd)

- **Stochastic Transition (or adjacency) Matrix M**
- **Suppose page j has n outlinks**
  - If outlink j -> i, then $\mathbf{M_{ij}=1/n}$
  - Else $\mathbf{M_{ij}=0}$
- **M** is a **column stochastic matrix**
  - Columns sum to 1
- **Rank vector r** is a vector with one entry per web page
  - **r_i is the importance score of page I**
- The flow equations can be written as **r = Mr.**

Example

- **Flow equation in matrix form: Mr = r**
- Suppose page *j* links to 3 pages, including *i*



Stationary Distribution

- Limiting prob. distribution of random surfer
  - PageRanks are based on limiting distribution
  - the probability destruction will converge eventually
- Requirement for its existence
  - Graph is strongly connected: a node can reach any other node in the graph

  => Cannot have dead ends, spider traps.

Eigenvectors and Eigenvalues

- An **eigenvector** of a square matrix **A** is a non-zero vector **v** that, when the matrix multiplies **v**, yields the same as when some scalar multiplies **v**, the scalar multiplier often being denoted by λ
- That is:

$$\mathbf{Av} = \lambda\mathbf{v}$$

- The number λ is called the **eigenvalue** of **A** corresponding to **v.**

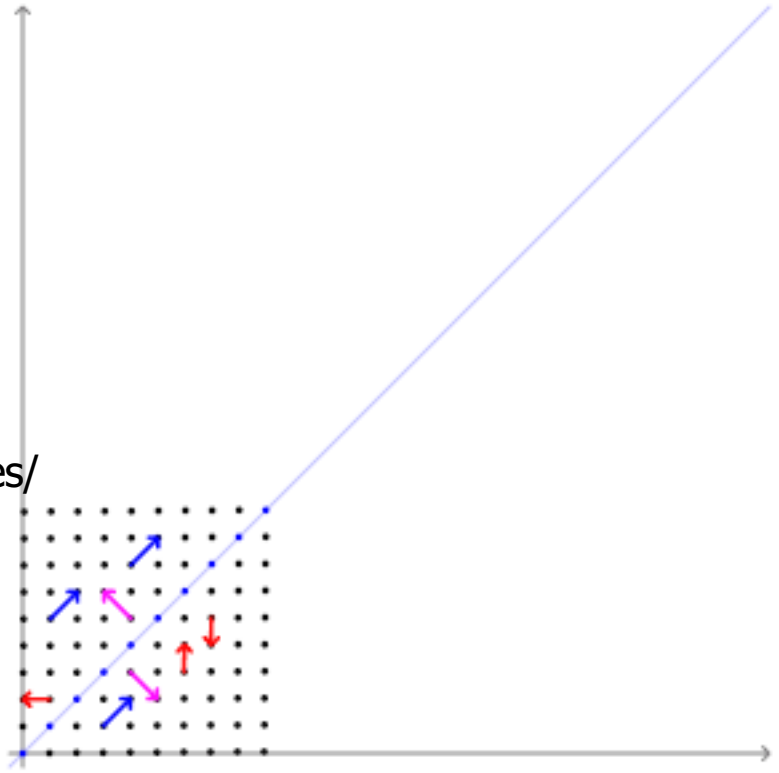
Eigenvalues and Eigenvectors Example

- The transformation matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ preserves the direction of vectors parallel to $\mathbf{v} = (1,-1)^T$ (in purple) and $\mathbf{w} = (1,1)^T$ (in blue). The vectors in red are not parallel to either eigenvector, so, their directions are changed by the transformation.

$$\boldsymbol{Ax = \lambda x}$$

http://setosa.io/ev/eigenvectors-and-eigenvalues/

https://en.wikipedia.org/wiki/Eigenvalues_and_eigenvectors



Eigenvector Formulation

- **The flow equations can be written**

$$\boldsymbol{M} \cdot \boldsymbol{r} = \boldsymbol{r}$$

limiting distribution

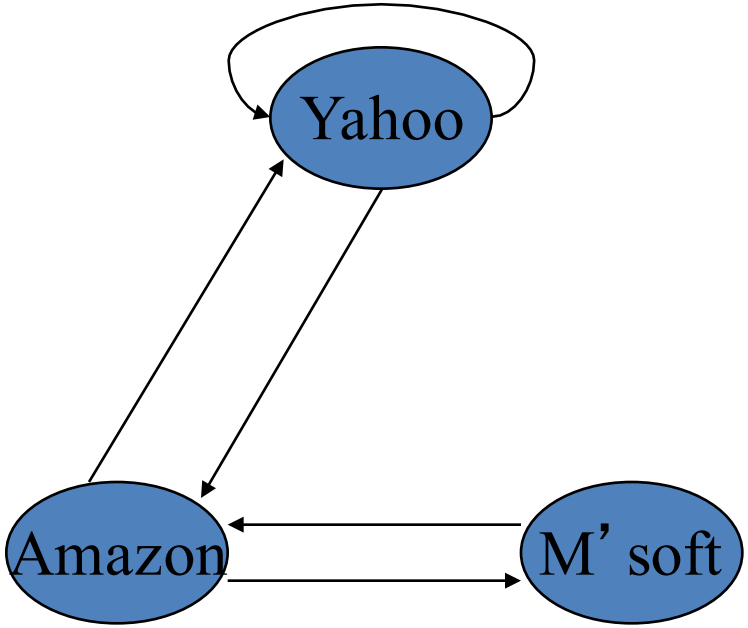
- So the **rank vector** ***r*** is an **eigenvector** of the stochastic web matrix ***M***
  - In fact, M's first or principal eigenvector, with corresponding eigenvalue ***1***
    - Largest eigenvalue of ***M*** is **1** since ***M*** is column stochastic (with non-negative entries)
      - *We know **r** is unit length and each column of **M** sums to one*

NOTE: ***x*** is an eigenvector with the corresponding eigenvalue **λ** if:

$$\boldsymbol{Ax} = \boldsymbol{\lambda x}$$

- **We can now efficiently solve for *r*!**
  - **1. Power Iteration:** https://en.wikipedia.org/wiki/Power_iteration
  - **2. Use the principal eigenvector.**

Example



$$r_y = r_y/2 + r_a/2$$
$$r_a = r_y/2 + r_m$$
$$r_m = r_a/2$$

	y	a	m
y	1/2	1/2	0
a	1/2	0	1
m	0	1/2	0

r = Mr

$$\begin{bmatrix} r_y \\ r_a \\ r_m \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1 \\ 0 & 1/2 & 0 \end{bmatrix} \begin{bmatrix} r_y \\ r_a \\ r_m \end{bmatrix}$$

Summary

- Web as a Directed Graph
  - **Nodes: Webpages**
  - **Edges: Hyperlinks**
- Early Web Search
  - Inverted Index
  - Term spam
- Link Analysis Algorithms
- Random Surfer Model
- PageRank: Links as Votes
- Eigenvalues and Eigenvectors.