MIE 1623 – Project 3: Surgical scheduling
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1.0 INTRODUCTION

The assignment's objective is to determine a schedule for 30 patients with given duration of surgeries. The hospital wants to figure out a surgery schedule for next week to fit all these patients in. There are two ORs in total for use, and each will run for from 8am – 3pm regularly from Monday to Friday. It costs 1400$ to open an OR for the day. Regular operating hours costs 500$ each hour. Any time after regular hours on weekdays and on weekends counts as overtime, which costs 655$ per extra hour. A surgery can also be cancelled instead, which costs 75$.

2.0 MODEL AND ASSUMPTIONS

The following Table 1 and Table 2 show the given data and parameters for the model setup.

Table 1: Given Data

Item	**Cost**
Cost of opening an OR	$1400
Hourly cost of running an OR	$500
Cost of a canceled surgery	$75
Hourly cost of overtime	$155*

Table 2: Model Parameters

Variables	Description
r_i	Number of OR rooms open on day i
h_j	Surgery duration for patient j
x_{ij}	Binary (1 if scheduled for that day for that patient, 0 if not)
n_j	Binary (1 if patient j surgery is cancelled, 0 if not)
o_i	Hours of OT on day i

Assumptions and Decisions:

- Unlimited staff, therefore, surgeries can be booked back-to-back.
- All cleaning and prep times are included in patient hours
- *Hourly cost of overtime is on top of the regular operating hour cost, making each hour of overtime cost $500 + $155 = $655
- Normal operating hours are 8am to 3pm on Monday through Friday
    - 7 hours a day, five days a week, 35 hour a week
- All hours are available in a day, and everything past 7 hours in a day per OR is considered overtime.
- All hours on the weekend are considered overtime

2.1 OBJECTIVE FUNCTION

Minimize the total cost of scheduled surgeries at Hospital Xena

$$\text{Min: } 1400 \sum_{i=1}^{7} r_i + 500 \sum_{j=1}^{30} \sum_{i=1}^{7} h_j x_{ij} + 75 \sum_{j=1}^{30} n_j + 655 \sum_{i=1}^{7} o_i$$

Sum of :

- Cost of all open ORs in the schedule
- Sum of sumproduct between binary variable and patient hours for all 30 patients and 7 days times cost of regular operating hours
- Cost of all cancelled surgeries
- Cost of all overtime hours

Decision variables:

- Number of OR to operate (2 or more)
- Binary: scheduled patient j on day i or cancelled bucket

2.2 CONSTRAINTS AND REQUIREMENTS

- Decision variables for patients should be binary
- The total duration of surgeries on a day cannot exceed the available time in available ORs on that day
- Each surgery can only be scheduled once, on a single day

$$\sum_{i=1}^{7} x_{ij} = 1 \; for \; each \; patient \; j$$

  The sum of scheduled and cancelled surgeries should equal 30

- Total hours of scheduled surgeries and surgeries cancelled should equal the total hours of all 30 patients (89.09)
- OR room should be an integer value (0,1 or 2 open on any given day), with the max cap of 2 OR rooms open on a given day
- If no OR rooms are open, no surgeries shall be scheduled for that day
- For cancellation constraints, the number of cancelled surgeries cannot exceed the allowable limit of 3 (for 90% service)
- All values should be greater than or equal to 0 (i.e. no negative hours or costs)

2.3 MODEL RESULTS

Initially, the model was simulated for 5 days a week (no weekends, or cancellations) to see if it was possible to run all surgeries on weekdays. The results in Figure 1 show that it is possible with overtime. Realistically though, the schedule is not plausible since it requires the staff to be on duty and continuously running both ORs on Friday for 24 hours. Variations of this schedule also resulted in two days of 48 hours being run, which is also not realistically scenario.

Patient ID	Monday	Tuesday	Wednesday	Thursday	Friday	Demand (HR)	Demand (Cap)	Sum
MAX OR ROOMS	2	2	2	2	2			
OR Rooms	0	0	1	2	1	4.00	10	
1	0	0	1	0	0	1.00	2.82	1
2	0	0	1	0	0	1.00	2.55	1
3	0	0	0	1	0	1.00	3.90	1
4	0	0	0	0	1	1.00	2.97	1
5	0	0	0	1	0	1.00	3.35	1
6	0	0	0	1	0	1.00	3.45	1
7	0	0	0	1	0	1.00	3.28	1
8	0	0	1	0	0	1.00	3.37	1
9	0	0	1	0	0	1.00	2.72	1
10	0	0	0	0	1	1.00	2.85	1
11	0	0	1	0	0	1.00	2.70	1
12	0	0	1	0	0	1.00	3.52	1
13	0	0	0	0	1	1.00	2.37	1
14	0	0	1	0	0	1.00	2.85	1
15	0	0	1	0	0	1.00	3.03	1
16	0	0	0	1	0	1.00	2.93	1
17	0	0	0	0	1	1.00	2.87	1
18	0	0	0	0	1	1.00	2.87	1
19	0	0	0	0	1	1.00	3.17	1
20	0	0	0	1	0	1.00	2.80	1
21	0	0	0	0	1	1.00	2.42	1
22	0	0	0	0	1	1.00	2.95	1
23	0	0	0	1	0	1.00	2.65	1
24	0	0	0	1	0	1.00	2.40	1
25	0	0	0	1	0	1.00	3.23	1
26	0	0	0	1	0	1.00	2.60	1
27	0	0	0	1	0	1.00	3.18	1
28	0	0	0	1	0	1.00	3.98	1
29	0	0	0	1	0	1.00	2.78	1
30	0	0	0	1	0	1.00	2.53	1
Total hrs per day	0	0	23.56	43.06	22.47	30	89.09	
Hours Cap in the Day	0	0	24	48	24			
Cost of opening an OR	0	0	1400	2800	1400			
Regular Hours	0	0	7	14	7			
Regular Hours Cost	$ -	$ -	$ 3,500.00	$ 7,000.00	$ 3,500.00			
OT Hours	0	0	16.56	29.06	15.47			
OT Hours Cost	$ -	$ -	$ 10,846.80	$ 19,034.30	$ 10,132.85			

Item	Cost ($)
Cost of opening an OR	1400
Hourly cost of running an OR	500
Cost of a canceled surgery	75
Hourly cost of overtime	155

Item	Cost ($)
Cost of opening an OR	$ 5,600
Hourly cost of running an OR	$ 14,000
Cost of a canceled surgery	
Hourly cost of overtime	$ 40,014
Total SUM	$ 59,614

Figure 1: Model with only weekdays

Due to Excel's restriction of 200 decision variables, the next model tried was inclusion of the weekend to see if it was most cost-effective to open ORs on the weekend to spread out the overtime. Figure 2 shows the model does not utilize the weekend at all, and thus weekends were not considered moving forwards. This aligns with the assumption that weekend hours are all overtime. As weekdays are able to accommodate all hours with overtime, and already have OR open, there is no extra additional cost to open an OR room as there would be on weekends.

Patient ID	Monday	Tuesday	Wednesday	Thursday	Friday	Weekend	Demand (HR)	Demand (Cap)	Sum
MAX OR ROOMS	2	2	2	2	2	4			
OR Rooms	0	0	1	2	1	0	4.00	10	
1	0	0	1	0	0	0	1.00	2.82	1
2	0	0	1	0	0	0	1.00	2.55	1
3	0	0	0	1	0	0	1.00	3.90	1
4	0	0	0	0	1	0	1.00	2.97	1
5	0	0	0	1	0	0	1.00	3.35	1
6	0	0	0	1	0	0	1.00	3.45	1
7	0	0	0	1	0	0	1.00	3.28	1
8	0	0	1	0	0	0	1.00	3.37	1
9	0	0	1	0	0	0	1.00	2.72	1
10	0	0	0	0	1	0	1.00	2.85	1
11	0	0	1	0	0	0	1.00	2.70	1
12	0	0	1	0	0	0	1.00	3.52	1
13	0	0	0	0	1	0	1.00	2.37	1
14	0	0	1	0	0	0	1.00	2.85	1
15	0	0	1	0	0	0	1.00	3.03	1
16	0	0	0	1	0	0	1.00	2.93	1
17	0	0	0	0	1	0	1.00	2.87	1
18	0	0	0	0	1	0	1.00	2.87	1
19	0	0	0	0	1	0	1.00	3.17	1
20	0	0	0	1	0	0	1.00	2.80	1
21	0	0	0	0	1	0	1.00	2.42	1
22	0	0	0	0	1	0	1.00	2.95	1
23	0	0	0	1	0	0	1.00	2.65	1
24	0	0	0	1	0	0	1.00	2.40	1
25	0	0	0	1	0	0	1.00	3.23	1
26	0	0	0	1	0	0	1.00	2.60	1
27	0	0	0	1	0	0	1.00	3.18	1
28	0	0	0	1	0	0	1.00	3.98	1
29	0	0	0	1	0	0	1.00	2.78	1
30	0	0	0	1	0	0	1.00	2.53	1
Total hrs per day	0	0	23.56	43.06	22.47	0	30	89.09	
Hours Cap in the Day	0	0	24	48	24	0			
Cost of opening an OR	0	0	1400	2800	1400	0			
Regular Hours	0	0	7	14	7	0			
Regular Hours Cost	$ -	$ -	$ 3,500.00	$ 7,000.00	$ 3,500.00	$ -			
OT Hours	0	0	16.56	29.06	15.47	0			
OT Hours Cost	$ -	$ -	$ 10,846.80	$ 19,034.30	$ 10,132.85	$ -			

Item	Cost ($)
Cost of opening an OR	1400
Hourly cost of running an OR	500
Cost of a canceled surgery	75
Hourly cost of overtime	155

Item	Cost ($)
Cost of opening an OR	$ 5,600
Hourly cost of running an OR	$ 14,000
Cost of a canceled surgery	
Hourly cost of overtime	$ 40,014
Total SUM	$ 59,614

Figure 2: Model with Weekend

For cancellations, in order to avoid the cheapest possible model, which is to cancel all 30 appointments as seen in Figure 3, two possibilities were considered. The first option would be that the hospital could have a service level, where they aim to conduct a certain percentage of surgeries out of the total scheduled ones. The service level for this option was set as 90%, meaning only 3 surgeries can be cancelled. The second option is that the hospital stipulates the number of working hours (e.g. mandated regular shifts), and cancels surgeries after the stated hours. Figure 4 shows the results of Option 1 (Capped Cancellations) and Figure 5 shows Option 2 (Capped Hours). As expected, the longest surgeries were cancelled first by the model.

Patient ID	Monday	Tuesday	Wedensday	Thursday	Friday	Cancellations	Demand (HR)	Demand (Cap)	Sum
MAX OR ROOMS	2	2	2	2	2	0			
OR Rooms	0	0	0	0	0	0	0.00	10	
1	0	0	0	0	0	1	0.00	2.82	1
2	0	0	0	0	0	1	0.00	2.55	1
3	0	0	0	0	0	1	0.00	3.90	1
4	0	0	0	0	0	1	0.00	2.97	1
5	0	0	0	0	0	1	0.00	3.35	1
6	0	0	0	0	0	1	0.00	3.45	1
7	0	0	0	0	0	1	0.00	3.28	1
8	0	0	0	0	0	1	0.00	3.37	1
9	0	0	0	0	0	1	0.00	2.72	1
10	0	0	0	0	0	1	0.00	2.85	1
11	0	0	0	0	0	1	0.00	2.70	1
12	0	0	0	0	0	1	0.00	3.52	1
13	0	0	0	0	0	1	0.00	2.37	1
14	0	0	0	0	0	1	0.00	2.85	1
15	0	0	0	0	0	1	0.00	3.03	1
16	0	0	0	0	0	1	0.00	2.93	1
17	0	0	0	0	0	1	0.00	2.87	1
18	0	0	0	0	0	1	0.00	2.87	1
19	0	0	0	0	0	1	0.00	3.17	1
20	0	0	0	0	0	1	0.00	2.80	1
21	0	0	0	0	0	1	0.00	2.42	1
22	0	0	0	0	0	1	0.00	2.95	1
23	0	0	0	0	0	1	0.00	2.65	1
24	0	0	0	0	0	1	0.00	2.40	1
25	0	0	0	0	0	1	0.00	3.23	1
26	0	0	0	0	0	1	0.00	2.60	1
27	0	0	0	0	0	1	0.00	3.18	1
28	0	0	0	0	0	1	0.00	3.98	1
29	0	0	0	0	0	1	0.00	2.78	1
30	0	0	0	0	0	1	0.00	2.53	1
Total hrs per day	0	0	0	0	0	30	0	89.09	
Hours Cap in the Day	0	0	0	0	0	0			
Cost of opening an OR	0	0	0	0	0	0			
Regular Hours	0	0	0	0	0	0			
Regular Hours Cost	$ -	$ -	$ -	$ -	$ -	$ -			
OT Hours	0	0	0	0	0	0			
OT Hours Cost	$ -	$ -	$ -	$ -	$ -	$ -			

Item	Cost ($)
Cost of opening an OR	1400
Hourly cost of running an OR	500
Cost of a canceled surgery	75
Hourly cost of overtime	155

Acceptable Service Level	0%
Actual Service Level	0%
Allowable Cancelled Surgeries	30
Actual Cancelled Surgeries	30
Cancelled Time	89.09

Item	Cost ($)
Cost of opening an OR	$ -
Hourly cost of running an OR	$ -
Cost of a canceled surgery	$ 2,250
Hourly cost of overtime	$ -
Total SUM	$ 2,250

Check sum of time	89.09

Figure 3: Cheapest possible model - cancel all surgeries.

Patient ID	Monday	Tuesday	Wedensday	Thursday	Friday	Cancellations	Demand (HR)	Demand (Cap)	Sum
MAX OR ROOMS	2	2	2	2	2	0			
OR Rooms	1	1	1	0	1	0	4.00	10	
1	0	0	0	0	1	0	1.00	2.82	1
2	0	0	0	0	1	0	1.00	2.55	1
3	0	0	0	0	0	1	0.00	3.90	1
4	1	0	0	0	0	0	1.00	2.97	1
5	0	0	0	0	0	1	0.00	3.35	1
6	0	0	0	0	1	0	1.00	3.45	1
7	1	0	0	0	0	0	1.00	3.28	1
8	0	0	0	0	1	0	1.00	3.37	1
9	0	0	0	0	1	0	1.00	2.72	1
10	0	0	0	0	1	0	1.00	2.85	1
11	0	0	0	0	1	0	1.00	2.70	1
12	0	1	0	0	0	0	1.00	3.52	1
13	0	0	0	0	1	0	1.00	2.37	1
14	0	0	1	0	0	0	1.00	2.85	1
15	0	0	1	0	0	0	1.00	3.03	1
16	0	0	1	0	0	0	1.00	2.93	1
17	0	0	1	0	0	0	1.00	2.87	1
18	0	0	1	0	0	0	1.00	2.87	1
19	1	0	0	0	0	0	1.00	3.17	1
20	0	0	1	0	0	0	1.00	2.80	1
21	1	0	0	0	0	0	1.00	2.42	1
22	1	0	0	0	0	0	1.00	2.95	1
23	0	1	0	0	0	0	1.00	2.65	1
24	0	0	1	0	0	0	1.00	2.40	1
25	0	0	1	0	0	0	1.00	3.23	1
26	0	1	0	0	0	0	1.00	2.60	1
27	0	1	0	0	0	0	1.00	3.18	1
28	0	0	0	0	0	1	0.00	3.98	1
29	1	0	0	0	0	0	1.00	2.78	1
30	0	1	0	0	0	0	1.00	2.53	1
Total hrs per day	17.57	14.48	22.98	0	22.83	3	27	89.09	
Hours Cap in the Day	24	24	24	0	24	0			
Cost of opening an OR	1400	1400	1400	0	1400	0			
Regular Hours	7	7	7	0	7	0			
Regular Hours Cost	$ 3,500.00	$ 3,500.00	$ 3,500.00	$ -	$ 3,500.00	$ -			
OT Hours	10.57	7.48	15.98	0	15.83	0			
OT Hours Cost	$ 6,923.35	$ 4,899.40	$ 10,466.90	$ -	$ 10,368.65	$ -			

Item	Cost ($)
Cost of opening an OR	1400
Hourly cost of running an OR	500
Cost of a canceled surgery	75
Hourly cost of overtime	155

Acceptable Service Level	90%
Actual Service Level	90%
Allowable Cancelled Surgeries	3
Actual Cancelled Surgeries	3
Cancelled Time	11.23

Item	Cost ($)
Cost of opening an OR	$ 5,600
Hourly cost of running an OR	$ 14,000
Cost of a canceled surgery	$ 225
Hourly cost of overtime	$ 32,658
Total SUM	$ 52,483

Check sum of time	89.09

Figure 4: Option 1 - 90% Service Level Model

The Hour Cap prioritizes the regular operating hours of the OR rooms. Both OR rooms are open and run a minimum 14 hours during weekdays. This approach balances the trade-off between cost and serviceability, by ensuring that surgeries are conducted only during the operating hours of both OR's. In this approach, surgeries that are scheduled to start during the operating hours can run over time, but no surgeries are scheduled after operating hours. This is because running one OR over time, even if it is more financially favourable, it is not realistic in real life situation.

Patient	Surgery	Monday	Tuesday	Wednesday	Thursday	Friday	SUM
1	169	1	0	0	0	0	1
2	153	0	0	0	0	1	1
3	234	0	0	1	0	0	1
4	178	1	0	0	0	0	1
5	201	0	0	0	0	1	1
6	207	0	0	0	0	0	0
7	197	0	0	0	0	0	0
8	202	0	1	0	0	0	1
9	163	0	0	0	0	1	1
10	171	1	0	0	0	0	1
11	162	0	0	0	0	0	0
12	211	0	0	0	0	1	1
13	142	0	0	0	0	0	0
14	171	0	0	1	0	0	1
15	182	0	0	1	0	0	1
16	176	0	0	1	0	0	1
17	172	0	0	0	1	0	1
18	172	0	0	1	0	0	1
19	190	0	0	0	1	0	1
20	168	0	1	0	0	0	1
21	145	0	0	0	1	0	1
22	177	0	0	0	1	0	1
23	159	0	0	0	1	0	1
24	144	1	0	0	0	0	1
25	194	0	1	0	0	0	1
26	156	1	0	0	0	0	1
27	191	0	1	0	0	0	1
28	239	0	1	0	0	0	1
29	167	0	0	0	0	1	1
30	152	1	0	0	0	0	1
	Total Time	970	994	935	843	895	
	Total Time(hr)	16.166667	16.566667	15.583333	14.05	14.916667	
	number of cancellat	4					

	Monday	Tuesday	Wednesday	Thursday	Friday
OR1	1	1	1	1	1
OR2	1	1	1	1	1
OVERTIME	130	154	95	3	55

Given Variables	Value
Cost of Opening an OR	$ 1,400
Hourly Cost of Runinng an OR	$ 500
Cost of cancelled surgery	$ 75
Hourly Cost of overtime	$ 655

scenario 1: no cancellation	
Total Cost of Opening an OR	$ 14,000
OR running cost	$ 38,642
cancelled surgery cost	$ 300
OR over time	$ 4,771
Total cost	$ 57,712

Figure 5: Option 2 – Hour Cap model

3.0 QUESTIONS

1. How many variables and constraints does your model have? Distinguish between discrete and continuous variables.

There are 186 decision variables – 180 discrete binary for patient and 6 discrete integers for OR rooms.

There are 8 listed range constraints across a matrix of 5 days + 1 cancellation bucket and 30 patients. 30 constraints for limiting one surgery per patient, 180 constraints for binary, 6 constraints for limiting the hours cap in a day, 6 integer constraints are for the OR room number (one for each bucket) and general non negativity constraints.

2. How long does it take your model to find a solution?

The following Table 3 shows the solving time using Excel Solver sensitivity report.

Table 3: Solving Time of Schedules

Model	**Solution Time**
Weekday only (no cancellation)	34.219 Seconds
Weekend Included (no cancellation)	8.07 Seconds
Cancellation Cap	240.719 Seconds
Hours Cap	55 Seconds

3. What is the total cost of next week's schedule?

The following Table 4 shows the cost of the schedules for next week.

Table 4: Cost of Schedules

Model	**Schedule Cost**
Weekday only (no cancellation)	$59,614
Cancellation Cap (90% service)	$52,483
Hours Cap	$57,712

4. How many patients are canceled, and what are the total cancelation costs?

In the Cancellation Cap scenario, a total of 3 patients are cancelled, which was set for the hospital to have a service level of 90%. This would cost $225, and would cancel the three longest surgery times, 3.98 (patient 28), 3.90 (patient 3) and 3.35 (patient 5) hours respectively.

In the Hour Cap scenario, Patients 7, 8, 11, and 13 are cancelled because their scheduled surgeries are outside the operating hours of both OR rooms. The total cancellation cost for these four patients is $300.

5. How many hours are overtime, and what are the total overtime costs?

The following Table 5 show the overtime cost for each model.

Table 5: Overtime Costs

Model	OT Cost
Weekday only (no cancellation)	$40,014
Cancellation Cap (90% service)	$32,658
Hours Cap	$4,771

6. Are there any days in which an OR is not used?

Table 6 shows the days in which an OR is not used at all. Depending on how lenient the hospital is with working continuous 24 hours in both ORs, the number of free days can range between one to three (ie two full days of 48 hour surgery, or four days of around 15-22 hours of surgery).

Table 6: Days without open OR

Model	Days with no open ORs
Weekday only (no cancellation)	4 OR rooms total: If all surgeries are scheduled in 48 hours on two days, then three days out of five are not used. If not, then at most one day will be free.
Cancellation Cap (90% service)	One day is free
Hours Cap	All 2 OR rooms open on weekdays

7. Display the resulting schedule in a spreadsheet. You will be graded on clarity and aesthetics.

Tables 7 and 8 below show the final schedules for the two models, cancellation cap and hours cap respectively. The schedule can also be found in binary form in the attached excel spreadsheet.

Table 7: Cancellation Cap (90% service) Schedule

Monday	OR 1	Patient ID	Patient 4	Patient 7	Patient 19	Patient 21
		Operating Time (min)	178	197	190	145
		Patient ID	Patient 22	Patient 29		
		Operating Time (min)	177	167		
	OR 2	Closed				
Tuesday	OR 1	Patient ID	Patient 12	Patient 23	Patient 26	Patient 27
		Operating Time (min)	211	159	156	191
		Patient ID	Patient 30			
		Operating Time (min)	152			
	OR 2	Closed				
Wednesday	OR 1	Patient ID	Patient 14	Patient 15	Patient 16	Patient 17
		Operating Time (min)	171	182	176	172
		Patient ID	Patient 18	Patient 20	Patient 24	Patient 25
		Operating Time (min)	172	168	144	194
	OR 2	Closed				
Thursday	OR 1	Closed				
	OR 2	Closed				
Friday	OR 1	Patient ID	Patient 1	Patient 2	Patient 6	Patient 8
		Operating Time (min)	169	153	207	202
		Patient ID	Patient 9	Patient 10	Patient 11	Patient 13
		Operating Time (min)	163	171	162	142
	OR 2	Closed				

Table 8: Hours Cap Schedule

Monday	OR 1	Patient ID	Patient 1	Patient 4	Patient 10
		Operating Time (min)	169	178	171
	OR 2	Patient ID	Patient 24	Patient 26	Patient 30
		Operating Time (min)	144	156	152
Tuesday	OR 1	Patient ID	Patient 8	Patient 20	Patient 25
		Operating Time (min)	202	168	194
	OR 2	Patient ID	Patient 27	Patient 28	
		Operating Time (min)	191	239	
Wednesday	OR 1	Patient ID	Patient 3	Patient 15	
		Operating Time (min)	234	182	
	OR 2	Patient ID	Patient 14	Patient 16	Patient 18
		Operating Time (min)	171	176	172
Thursday	OR 1	Patient ID	Patient 17	Patient 21	Patient 23
		Operating Time (min)	172	145	159
	OR 2	Patient ID	Patient 19	Patient 22	
		Operating Time (min)	190	177	
Friday	OR 1	Patient ID	Patient 2	Patient 9	Patient 29
		Operating Time (min)	153	163	167
	OR 2	Patient ID	Patient	Patient 12	
		Operating Time (min)	201	211	