MIE 1623 - Project 5: Data envelopment analysis

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1.0 INTRODUCTION

The goal of this assignment was to determine which hospitals are most efficient based on the given data for Surgical and ED volume and Surgical and ED bed days.

2.0 MODEL AND ASSUMPTIONS

Given Data:

- Five enterprising hospitals
- All 5 have created a competition amongst themselves to see who is most efficient.
- Each hospital will be measured on performance in its emergency department (ED) and surgical unit.
- Table 1 shows a series of metrics for each hospital for the last fiscal year.
- Hospitals vary in size.

Table 1: Given Data

Hospital	Surgical volume	ED volume	Surgical bed-days	ED bed-days
Picard Hospital	9636	9360	32120	11700
Saint Guinan's Hospital	4505	4400	6007	6770
Crusher Cancer Centre	9332	9207	10369	46035
Worf Hospital	7917	7206	31668	14412
Hospital Q	7112	6961	11854	27844

Assumptions:

- To normalize on a “per bed day” scale, it was assumed that each bed day as an input resource was equivalent between the hospitals for surgical and ED bed days respectively.
- All other non given resources are assumed to be equal amongst the hospitals for comparison (e.g. surgeon availability, nurses etc.)

Model description (objective function, variables, constraints):

Linear Model Setup (Repeated 5 times, once for each chosen hospital):

$$output_k = \sum_{i=1}^{N} u_{ih} x_{ik}; \quad input_k = \sum_{j=1}^{M} v_{jh} y_{jk}$$

$$Productivity\ of\ hosptial\ k = \frac{\sum_{i=1}^{N} u_i x_{ik}}{\sum_{j=1}^{M} v_j y_{jk}}$$

Given,

$$x_{ik} = amount\ of\ output\ i\ produced\ by\ hospital\ k$$

$$y_{jk} = amount\ of\ input\ j\ given\ to\ hosptial\ k$$

$$u_i = weight\ (value) of\ output\ i$$

$$v_j = weight\ (value)\ of\ input\ j$$

Goal:

- Maximize productivity of chosen hospital (output/input)

Note: due to normalization, the inputs are all 1, therefore the goal simplifies to maximizing output

Decision Variables:

- Output Weights (u_{ih})

Constraints:

- Productivity must be <=1
- Output – Input must be <= 0

3.0 QUESTIONS

1. What analysis method did you use? Why?

The data analysis method used is Data Envelopment Analysis (DEA) to analyze the efficiency of each hospital. DEA was used because it measures the relative efficiency of each hospital by allowing them to pick the most optimal weights for their outputs and inputs and compare them to the other hospitals. This data analysis method is useful to identify efficient hospitals and then use them as a benchmark to allow for inefficient hospitals to increase their efficiency. The DEA method was repeated five times for each hospital with the aid of Excel Solver to choose the most optimal weights.

2. Of the metrics that were provided, which did you use? Did you use any metrics that were not explicitly provided by the hospitals?

All four metrics provided, which are Surgical volume, ED volume, Surgical bed-days, and ED bed-days were used. No metrics were used that were not explicitly provided by the hospitals however, the metrics were normalized for each hospital by dividing the number of Surgical volumes by Surgical bed-days and ED volume by ED bed-days to compare each hospital relatively. This ensures that the hospitals are compared on a “per bed day” basis and takes into account the different sizes of each hospital. If further comparison was needed, metrics such as bed day efficiency could be used, provided that more data on length of stay for surgery and ED patients is given.

3. Which hospital is most efficient?

After using DEA to calculate the efficiency of each hospital, three hospitals were considered the most efficient. These are on the Efficient Frontier and returned an output value of 1. As shown in Figure 1 and 2 below, Picard Hospital is considered the most efficient when considering ED Volume, Crusher Cancer Centre is considered the most efficient when considering Surgical Volume, and Saint Guinan's Hospital is considered the most efficient when considering a mix of both ED Volume and Surgical Volume.

4. How inefficient is each hospital, in a relative sense?

Table 2 and Figure 1 show the breakdown of the hospital efficiencies (output in this case since input is simplified to 1). This was achieved by solving for the optimal weights for each hospital respectively.

Table 2: Excel Solve Hospital Efficiencies

Hospital	Output
Picard Hospital	100%
Saint Guinan's Hospital	100%
Crusher Cancer Centre	100%
Worf Hospital	65%
Hospital Q	71%

Hospital	Surgical volume per bed day	ED volume per bed day	Input	Output	Output - Input
Picard Hospital	0.30	0.80	1	1	0
Saint Guinan's Hospital	0.75	0.65	1	0.81240768	-0.187592319
Crusher Cancer Centre	0.90	0.20	1	0.25	-0.75
Worf Hospital	0.25	0.50	1	0.625	-0.375
Hospital Q	0.60	0.25	1	0.3125	-0.6875
Weights	0.000	1.250			

Hospital	Surgical volume per bed day	ED volume per bed day	Input	Output	Output - Input
Picard Hospital	0.30	0.80	1	0.58630158	-0.413698417
Saint Guinan's Hospital	0.75	0.65	1	1	0
Crusher Cancer Centre	0.90	0.20	1	1	0
Worf Hospital	0.25	0.50	1	0.43109262	-0.568907376
Hospital Q	0.60	0.25	1	0.70688283	-0.293117169
Weights	1.034	0.345			

Hospital	Surgical volume per bed day	ED volume per bed day	Input	Output	Output - Input
Picard Hospital	0.30	0.80	1	0.33333691	-0.666663095
Saint Guinan's Hospital	0.75	0.65	1	0.83329602	-0.16670398
Crusher Cancer Centre	0.90	0.20	1	1	0
Worf Hospital	0.25	0.50	1	0.27778075	-0.722219246
Hospital Q	0.60	0.25	1	0.66663632	-0.333363683
Weights	1.111	0.000			

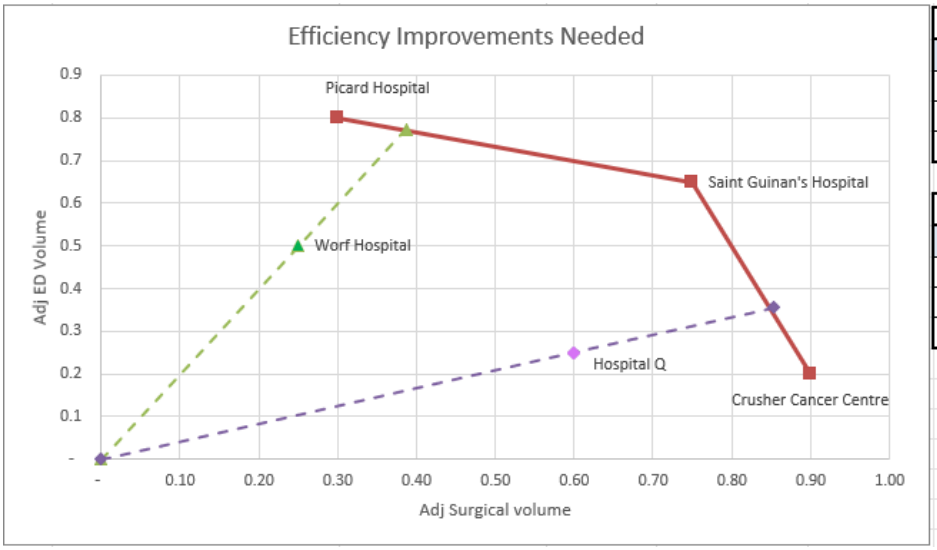
Hospital	Surgical volume per bed day	ED volume per bed day	Input	Output	Output - Input
Picard Hospital	0.30	0.80	1	1	0
Saint Guinan's Hospital	0.75	0.65	1	1	0
Crusher Cancer Centre	0.90	0.20	1	0.55571084	-0.444289165
Worf Hospital	0.25	0.50	1	0.64816018	-0.351839818
Hospital Q	0.60	0.25	1	0.50008497	-0.499915026
Weights	0.371	1.111			

Hospital	Surgical volume per bed day	ED volume per bed day	Input	Output	Output - Input
Picard Hospital	0.30	0.80	1	0.58630158	-0.413698417
Saint Guinan's Hospital	0.75	0.65	1	1	0
Crusher Cancer Centre	0.90	0.20	1	1	0
Worf Hospital	0.25	0.50	1	0.43109262	-0.568907376
Hospital Q	0.60	0.25	1	0.70688283	-0.293117169
Weights	1.034	0.345			

Figure 1: Weight Distribution and Output for each Linear Program

5. For each inefficient hospital, recommend a course of action to improve efficiency.

As seen in Figure 2, Worf Hospital and Hospital Q need to make improvements to reach the efficient frontier (green and purple line respectively). A line was generated from the origin to the Worf Hospital and Hospital Q, and extended to intersect the efficient frontier. ED and Surgery points used can be seen in Figure 2 on the right, with the second point being the hospital's current performance, and the third point being the hospital's potential efficient performance.



Worf	
ED	Surgery
0	0
0.5	0.25
0.77	0.39

Hospital Q	
ED	Surgery
0.00	0.00
0.25	0.60
0.36	0.85

Figure 2: Improvements to reach Efficient Frontier

For Worf, the closest benchmark is the frontier between Picard and Saint Guinan. Worf should aim to emulate Picard procedures and raise their surgical volumes by 0.14/surgical bed and their ER level by 0.27/surgical bed. Alternatively, they can allocate 11,368 less bed days for their surgical department to achieve a ratio of 0.39 surgeries per bed day and 5,054 less beds for their ER department assuming they cannot control the surgery and ED volumes predictably.

For Hospital Q, Crusher Cancer Centre is the closest benchmark. To reach the efficient frontier between Crusher Cancer Centre and Saint Guinan, Hospital Q should increase their ED level by 0.11/bed day and surgery by 0.25 per bed day. Alternatively, they can reduce assigned bed days by 8,508 for ED and 3,487 for surgery. Figure 3 depicts both results.

	Surgical volume	ED volume	Surgical bed-days	ED bed-days
Worf Hospital	7917	7206	31668	14412

Difference:	
ED/bd	Surgery/bd
0.27	0.14

	Surgery	ED
Bed Days Needed to Be Optimal	20,300	9,358
Difference	11,368	5,054

	Surgical volume	ED volume	Surgical bed-days	ED bed-days
Hospital Q	7112	6961	11854	27844

Difference:	
ED/bd	Surgery/bd
0.11	0.25

	Surgery	ED
Bed Days Needed to Be Optimal	8,367	19,336
Difference	3,487	8,508

Figure 3: Calculations for Worf Hospital and Hospital Q to increase efficiency to reach Efficient Frontier