MIE 1623 – Project 4: Family doctor practice queuing
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1.0 INTRODUCTION

The objective of this assignment is to build an Excel DSS that uses queuing theory to tell Dr. Soslow the optimal number of doctors that are needed in her practice to balance doctor salaries with wait times. Dr. Soslow should be able to change values in the spreadsheet according to different scenarios to see the resulting recommendation.

2.0 MODEL AND ASSUMPTIONS

Given Data:

- 3 Doctors.
- All 3 doctors share their patients.
- Each doctor earns $250,000/year.
- Each doctor works 40hrs per week with 44 work weeks per year
- Both the service times per doctor and the patient arrival rates are exponentially distributed, and that there are no significant differences between doctors or patients.
- Every minute a patient waits is valued at $5.

Table 1: Arrival and service data

Given	Value
Number of doctors	3
Patient arrival rate	30 patients/hour
Doctor service rate	5 patients/hour
Patients wait	$5/min
Doctor salary	$250,000/year
Hours worked per week	40hrs per week
# of work week	44 work weeks

Assumption:

- Even though doctors are salaried, assume that the salary can be converted into equivalent hourly cost by dividing by assumed hours per week and work week per year.
- Assume that there is no limit to the queue length.
- Ignore the fact that the clinic is only open for eight hours per day (i.e. 24/7).
- Assume exponential distribution for both service and interarrival time.
- Assume first-come, first-served.
- Assume no priority customers.
- Traffic Intensity U < 1 for valid solutions.

Arrival Rates:

- Arrival Rate (Average number of arrivals per unit time = 30 patients/hour): λ
- Interarrival Time (Average time between 2 consecutive arrivals): $\frac{1}{\lambda}$

Service Time:

Number of customers served per unit time (= 5 patients/hour * 3 doctors = 15 patients/hour): μ
The average time to service a customer (4mins per customer): $\frac{1}{\mu}$

Average time customer spends in the system:

$$W = \frac{1}{(\mu - \lambda)}$$

This metric includes both service time and queue time

Traffic intensity:

$$U = \frac{\lambda}{\mu}$$

$$U = \frac{30 \text{ patients/hour}}{(5 \text{ patients/hour } * \text{ 3 doctors})}$$
$$U = 2$$

(How fast do customers arrive?) / (How fast can you service them?)
If U > 1, then customers arrive faster than they are served, and the queue will grow to infinity (no steady state), therefore only U < 1 are valid solutions.

Average time the customer spends in queue:

$$W_q = \frac{U}{(\mu - \lambda)} = U \, x \, W$$

The utilization factor is 2 which means that the doctors cannot keep up with the number of patients arriving (i.e. customers arrive faster than they are served, and the queue will grow to infinity with no steady state). Therefore, the clinic must increase the number of doctors to decrease the utilization factor to be less than 1 in order for wait time improvements to be seen.

Model:

The Excel DSS allows the user to input any of the variables found in the grey cell. The model will calculate the appropriate wait times and costs and output the optimal number (lowest cost) of doctors needed in total. Variables that can be changed include Patient arrival per hour, doctor service rate per hour, cost of patient wait time per minute, doctor salary per year, hours worked per week and number of work weeks in a year. The output will also be graphed into doctor wages per hour, waiting costs per hour and total costs per hour for each doctor number scenario.

3.0 QUESTIONS

1. **What is the average wait time currently?**

Wait time:
The average wait time currently (with 3 doctors) is -0.133hrs (-0.133hrs * 60min/1hrs = -7.98min). This is not feasible because time cannot be negative. The reason for having a negative average wait time (-7.98 min) is due to the utilization factor being greater than 1. When the utilization factor is greater than 1 then patients arrive faster than they are served, and the queue will grow to infinity (no steady state).

Average wait time (sample):
For example, for a sample size of 10,000 patients arriving, the wait time is 172 hours, as shown in Figure 1. Assuming the projected patients arriving were infinity, the wait time would grow to infinity as well.

Metric	Symbol	Value
Arrival Rate	λ	30
Average time between arrivals	1/λ	0.03
Service Rate (all doctors)	μ	15
Average Service Time	1/μ	0.07
Time Customer Spends in System	W = 1/(μ-λ)	- 0.07
Traffic Intensity	U = λ/μ	2.00
Average time waiting	Wq = UxW	- 0.13

Average Wait Time (Wq)	171.35
Average Time in System (W)	171.42
Percentage Waiting	0.01%
# of patients	10000

Figure 1: Average Wait Time Based on 10k Patient Sample

2. **What is the optimal number of doctors needed?**

The optimal number of doctors needed are 9 doctors (3 current doctors + 6 additional doctors). This is due to the hourly cost per doctor ($142.05/hrs) and hourly cost of patient wait time (5$/min *60min/hrs = $300/hrs). This was solved using both excel solver and enumerating different numbers of servers (doctors) until the optimal costs is achieved. As shown in the Figure 2 below and attached Excel file, a minimum of 1 doctor was calculated in this model and the number of doctors was increased by 1 each time. At six doctors, the queue is at a steady state. As the number of doctors increased, cost decreased up to the optimal solution, and then the cost increased.

Total Number of Doctors	Utilization Factor	Average Waiting Time in Queue (hrs)	Average # of Patients in System	Doctor Cost per hour	Waiting Cost per Hour	Total Cost / hour	Cost / year for doctor	Total Cost / year
1	6.000	-0.240	-1.200	$142.05	-$2,160.00	-$2,017.95	$250,000.00	-$3,551,600.00
2	3.000	-0.150	-1.500	$284.09	-$1,350.00	-$1,065.91	$500,000.00	-$1,876,000.00
3	2.000	-0.133	-2.000	$426.14	-$1,200.00	-$773.86	$750,000.00	-$1,362,000.00
4	1.500	-0.150	-3.000	$568.18	-$1,350.00	-$781.82	$1,000,000.00	-$1,376,000.00
5	1.200	-0.240	-6.000	$710.23	-$2,160.00	-$1,449.77	$1,250,000.00	-$2,551,600.00
6	1.000	#DIV/0!	#DIV/0!	$852.27	#DIV/0!	#DIV/0!	$1,500,000.00	#DIV/0!
7	0.857	0.171	6.000	$994.32	$1,542.86	$2,537.18	$1,750,000.00	$4,465,428.57
8	0.750	0.075	3.000	$1,136.36	$675.00	$1,811.36	$2,000,000.00	$3,188,000.00
9	0.667	0.044	2.000	$1,278.41	$400.00	$1,678.41	$2,250,000.00	$2,954,000.00
10	0.600	0.030	1.500	$1,420.45	$270.00	$1,690.45	$2,500,000.00	$2,975,200.00
11	0.545	0.022	1.200	$1,562.50	$196.36	$1,758.86	$2,750,000.00	$3,095,600.00
12	0.500	0.017	1.000	$1,704.55	$150.00	$1,854.55	$3,000,000.00	$3,264,000.00
13	0.462	0.013	0.857	$1,846.59	$118.68	$1,965.27	$3,250,000.00	$3,458,879.12
14	0.429	0.011	0.750	$1,988.64	$96.43	$2,085.06	$3,500,000.00	$3,669,714.29
15	0.400	0.009	0.667	$2,130.68	$80.00	$2,210.68	$3,750,000.00	$3,890,800.00
16	0.375	0.008	0.600	$2,272.73	$67.50	$2,340.23	$4,000,000.00	$4,118,800.00
17	0.353	0.006	0.545	$2,414.77	$57.75	$2,472.53	$4,250,000.00	$4,351,647.06
18	0.333	0.006	0.500	$2,556.82	$50.00	$2,606.82	$4,500,000.00	$4,588,000.00
19	0.316	0.005	0.462	$2,698.86	$43.72	$2,742.59	$4,750,000.00	$4,826,955.47
20	0.300	0.004	0.429	$2,840.91	$38.57	$2,879.48	$5,000,000.00	$5,067,885.71

Figure 2: Optimal Number of Doctors Needed

3. What is the average wait time with the optimal number of doctors?

The average wait time with the optimal number of doctors (9 doctors) is 0.044hrs which is 2.64mins. This is reflected in the 10k patient sample as well as seen in Figure 3.

Metric	Symbol	Value
Arrival Rate	λ	30
Average time between arrivals	$1/\lambda$	0.03
Service Rate (all doctors)	μ	45
Average Service Time	$1/\mu$	0.02
Time Customer Spends in System	$W = 1/(\mu-\lambda)$	0.07
Traffic Intensity	$U = \lambda/\mu$	0.67
Average time waiting	$Wq = UxW$	0.04
Average Wait Time (min)	0.04	
Average Time in System (min)	0.06	
Percentage Waiting	33.98%	
# of patients	10000	

Figure 3: Wait time with 9 doctors with 10k patient sample

4. How much can the wait cost of a patient go up or down before more/fewer doctors are needed?

As shown In Table 1 below, the number of doctors needed differs depending on the wait cost of a patient. The wait cost needs to below or equal to $2 in order to have only eight doctors needed. In contrast, once the wait cost goes up to $6, an extra doctor is needed, for a total number of ten.

Table 1: Number of doctors needed depending on the cost of wait time per patient.

Scenario #	Cost of Wait Time ($/min)	Number of Doctors needed
Original	5	9
1	0	7
2	2	8
3	6	10
4	10	11
5	16	12
6	23	13

5. **For every scenario you test, plot the wage costs, hourly costs, and total costs per number of servers.**

Figure 4, 5 and 6 depict the wage cost, hourly cost and total cost for the optimal number of doctors for each wage cost time tested. Figures 7, 8, 9 show the individual enumerated scenarios for each wait time cost for all number of doctors, Figure 10 shows the total cost for $6/min wait time, and Figure 11 shows total cost for $2/min. The full list of scenarios and graphs can be found in the attached excel file in the tabs.

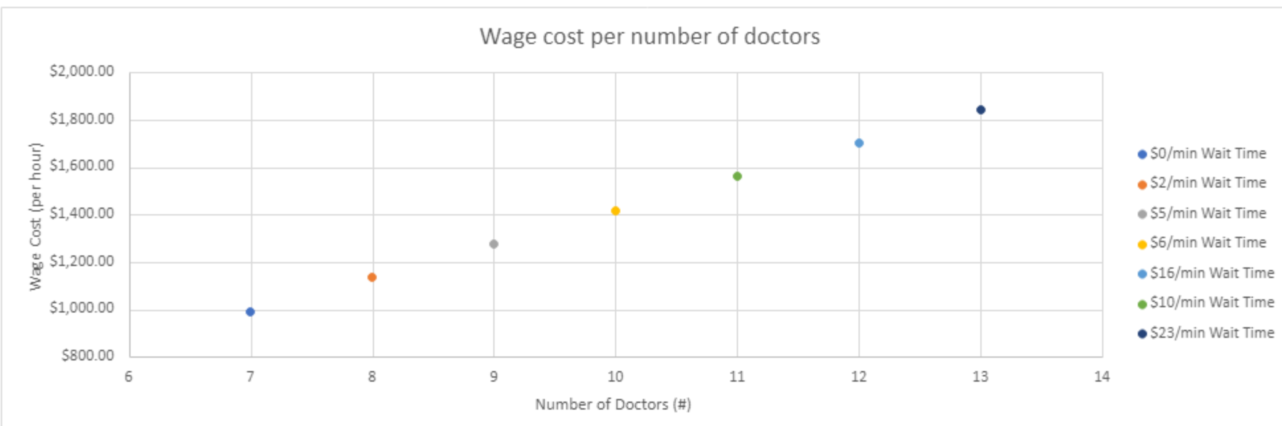

Figure 4: Wage Cost per Optimal Number of Doctors per Wait Time Cost

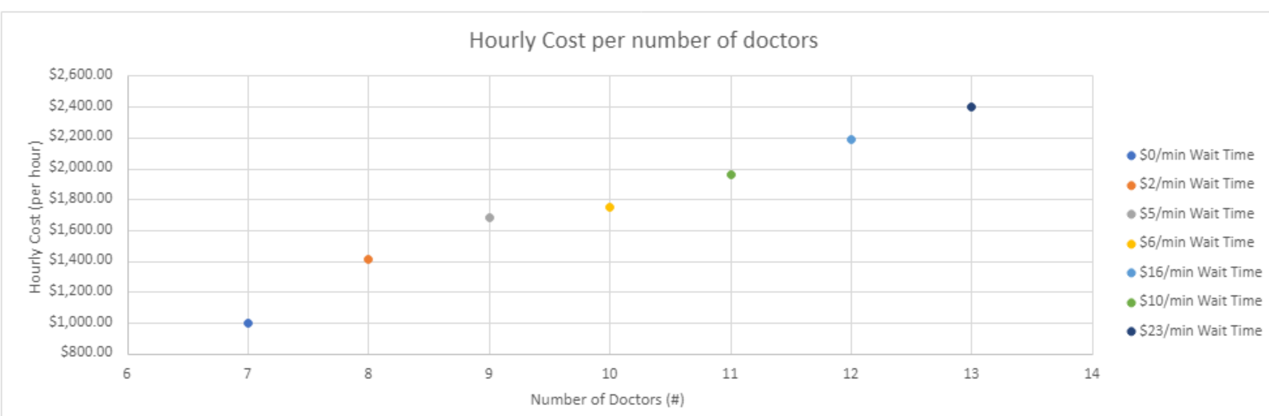

Figure 5: Hourly Cost per Optimal Number of Doctors per Wait Time Cost

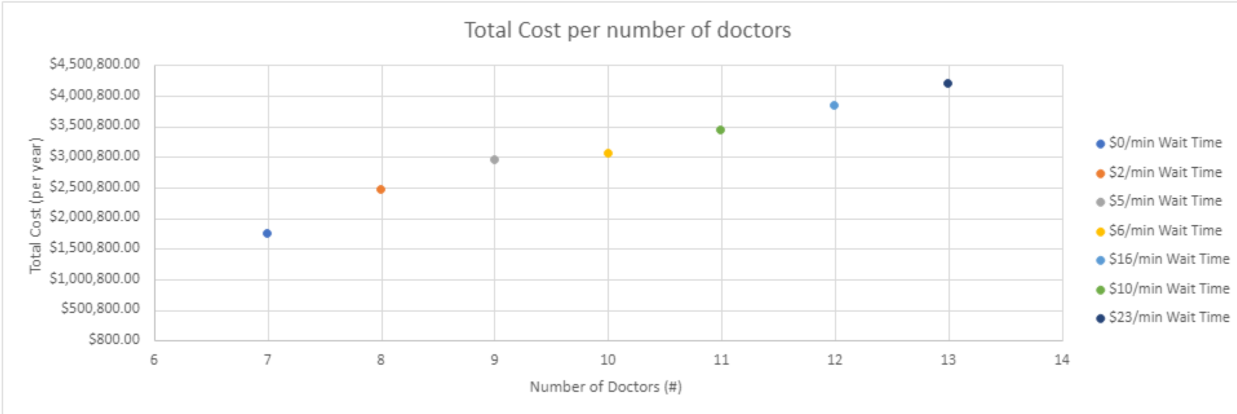

Figure 6: Total Cost per Optimal Number of Doctors per Wait Time Cost

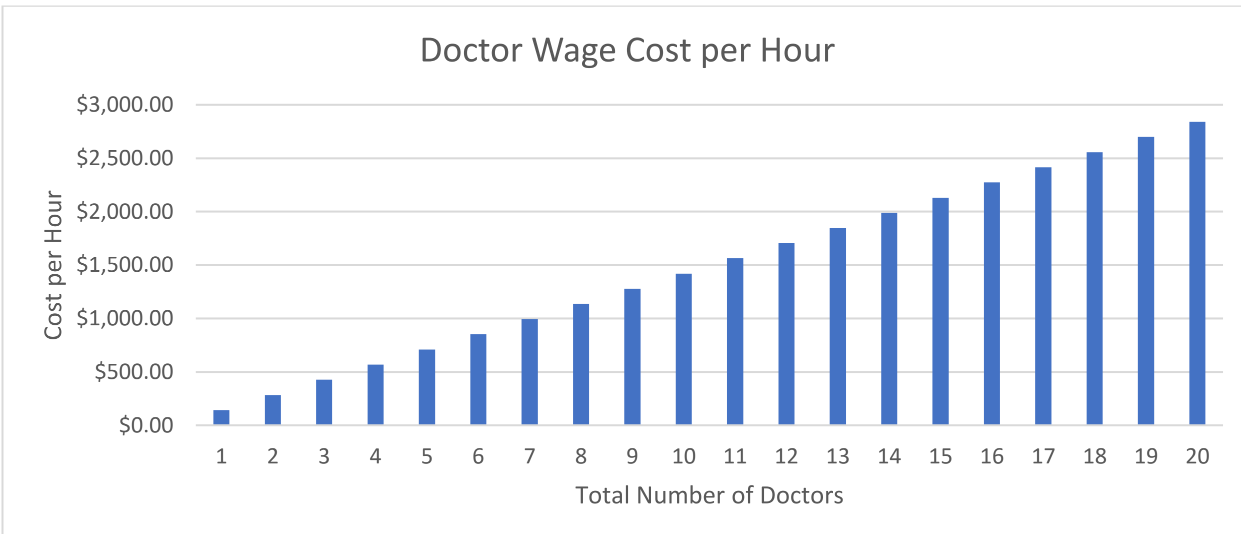

Figure 7: Wage Cost for number of doctors

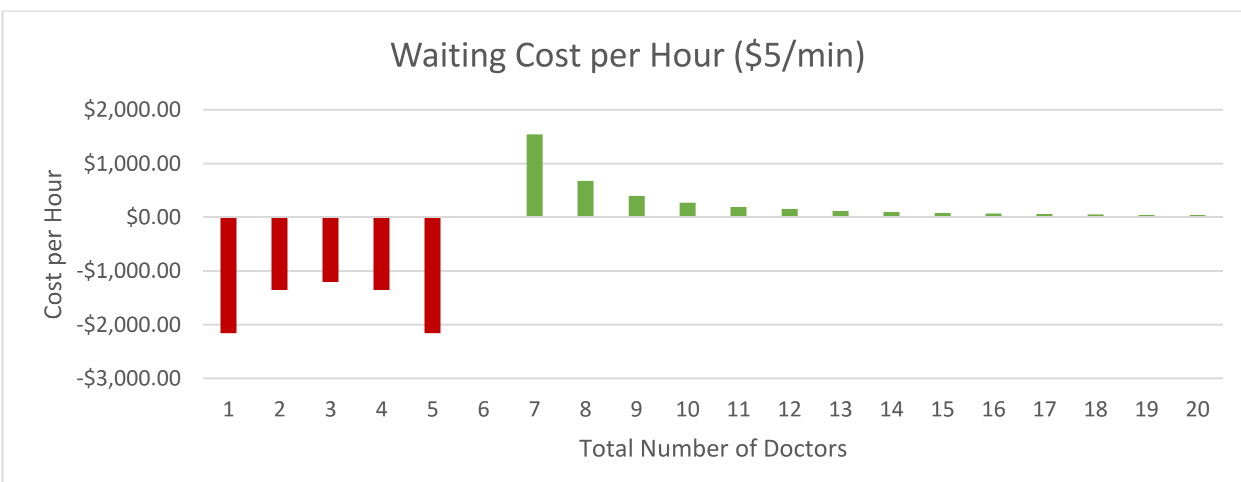

Figure 8: Wait Cost per Hour for Wait Cost of $5/min

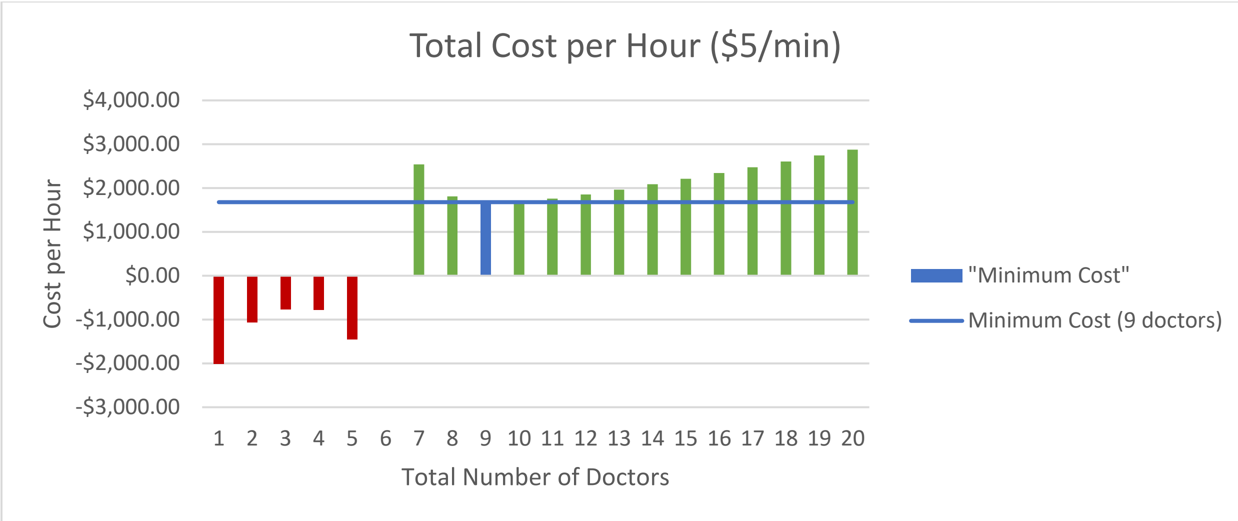

Figure 9: Total Costs per Hour for Wait Time Cost of $5/min

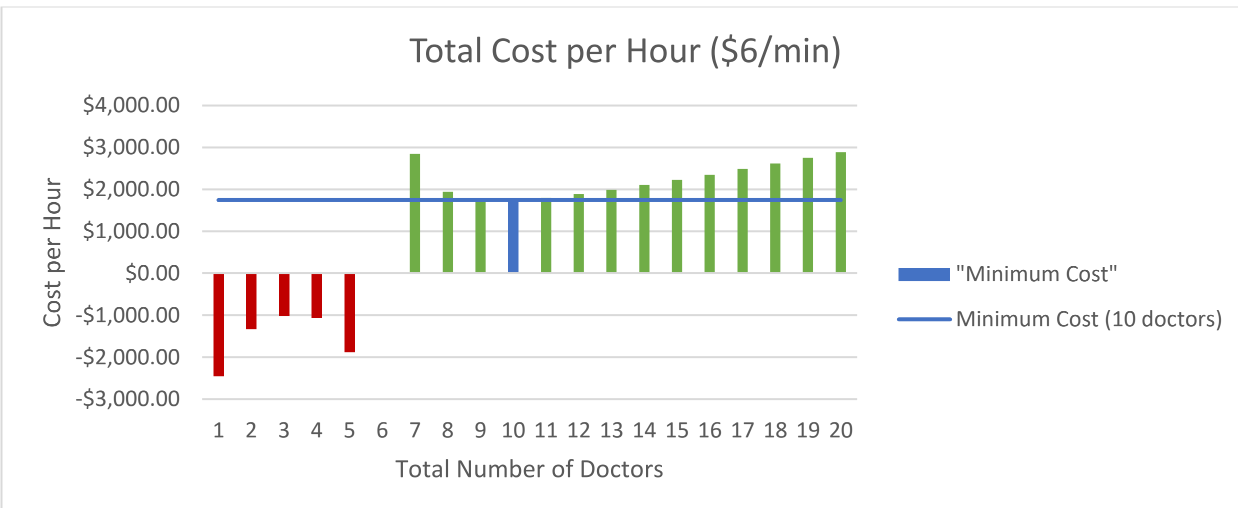

Figure 10: Total Costs per Hour for Wait Time Cost of $6/min

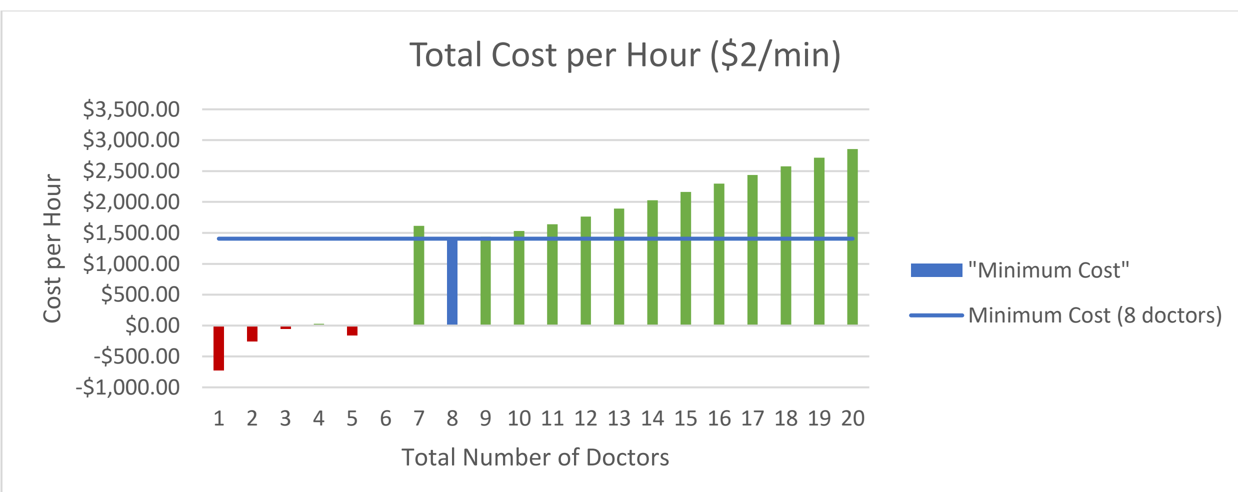

Figure 11: Total Costs per Hour for Wait Time Cost of $2/min

<u>M/M/1 Formulas:</u>

The utilization rate:

$$U = \frac{\lambda}{\mu}$$

Pr(system is idle):

$$P_0 = 1 - U$$

Pr(n customers in system):

$$P_0 = (1 - U)U^n$$

Avg # customers in system:

$$L = \frac{\lambda}{(\mu - \lambda)} = \lambda * W$$

Avg # customers in queue:

$$L_q = \frac{U * \lambda}{(\mu - \lambda)}$$

Avg wait time in system:

$$W = \frac{1}{(\mu - \lambda)} = W_q + \frac{1}{\mu}$$

Avg wait time in queue:

$$W_q = \frac{U}{(\mu - \lambda)} = U \: x \: W$$