MIE 1623 – Project 1: Facility Location

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1.0 PROBLEM DESCRIPTION

Your local regional health authority (RHA) is looking to improve access to CT scans for its inhabitants. The region has eight hospitals (where CT machines are located) and is divided into 19 census tracts, which are areas of roughly 40,000–80,000 residents. Some of these census tracts are better served than others in terms of CT scan accessibility. The RHA is instituting a major initiative to install new CT machines at some (or all) of its hospitals so that 90% of the population is within 45 minutes of a CT location. Your job is to determine the optimal number of new CT machines to install at each hospital.

2.0 QUESTIONS

2.1 Describe your model.

The given data is summarized in Table 1.

Table 1: Given Problem Data

Item	Information
Maximum number of exams per year per CT machine	19,743
Annual cost to operate a CT machine	$1,700,000
Cost to install a new CT machine	$1,250,000
Cost per exam	$62.26
Census Tract ID Demand	Average annual unmet demand for CT scans
Census Tract ID Travel Time	Travel time (minutes) from each census tract to each CT location site (hospital)

2.1.1 <u>What did you use for your objective function?</u>

The objective function is to minimize travel time [Z(x)] to allow for 90% of the population to be within 45 minutes of a CT location.

The equation for total travel time:

$$Z(x) = \sum_{i=1}^{19} \sum_{j=1}^{8} T_{ij} X_{ij}$$

where T_{ij} is the constant for the travel time from a census tract (19 census tracts) to a hospital (A to H denoted as 1 to 8 in this case) and X_{ij} is the number of CT scans to be performed at each hospital from each census tracts. Due to the goal of the RHA, travel time optimization was chosen over cost optimization, and thus the model assumes no budget constraint.

2.1.2 What are your variables?

The only factor that can be changed is the number of CT machines at each hospital. To achieve this, the decision variable X_{ij} was used to represent the unmet demand to service at each census tract by each hospital.

The decision variable X_{ij} is the number of CT scans made where:

i – the census tract region (1 to 19)

j – the hospital at which the CT machine is located (A to H denoted as 1 to 8 in this case)

The decision variables will directly indicate the number of CT machines required as the annual capacity of each CT machine is given as 19,743. Therefore, the number of CT machines needed is $\frac{X_{ij}}{19{,}743}$, which is then summed up for each hospital.

2.1.3 What are your constraints?

i. The unmet demand of each census tract should be covered.

The constraints used in this solution are to ensure that the average annual unmet demand for CT scans.

For example, census tract 1 requires 332 CT scans, therefore,

$$\sum_{j=1}^{8} Xij = 332$$

$$X_{11} + X_{12} + X_{13} + X_{14} + X_{15} + X_{16} + X_{17} + X_{18} = 332$$

This method applies to all census tract regions (1 to 19).

Note: The maximum number of exams per year per CT machine which is 19,743 is not a direct parameter in the model solver, however, it is used to determine the number of CT machines needed at each location.

ii. Non-negative constraints:

$$X_{ij} \geq 0$$

where, each hospital must fulfill 0 or more demand, and thus have either 0 CT machines or more.

iii. Decision variables should be integers.
There cannot be decimal values of tests done, therefore X_{ij} should be an integer. By extension, CT machines should also be integers numbers as there cannot be fractions of a machine. The results for CT machines were rounded up in the solution to ensure demand is met.

2.1.4 <u>How many binary and continuous variables and constraints does your model have?</u>

The model has $8 \times 19 = 152$ decision variables, no binary variables, no continuous variables, and 3 constraints for demand, non-negativity, and integer values.

2.2 Describe your solution.

The following Figure 1 shows the breakdown of unmet demand fulfilled by each hospital for each census tract as solved by Excel Solver. The parameters were set to minimize the SUMPRODUCT of the Time x Demand (from decision variable) matrix and constraints were set to ensure variables were nonnegative and each census tract demand was met.

Decision variables										
CID / CT loc	A	B	C	D	E	F	G	H	Total	Demand
1	332	-	-	-	-	-	-	-	**332**	332
2	-	14,076	-	-	-	-	-	-	**14,076**	14,076
3	-	-	12,343	-	-	-	-	-	**12,343**	12,343
4	-	-	-	18,034	-	-	-	-	**18,034**	18,034
5	-	-	-	-	15,367	-	-	-	**15,367**	15,367
6	-	-	-	-	-	10,654	-	-	**10,654**	10,654
7	-	-	-	-	-	-	17,976	-	**17,976**	17,976
8	-	-	-	-	-	-	-	8,745	**8,745**	8,745
9	-	-	8,243	-	-	-	-	-	**8,243**	8,243
10	-	-	-	-	-	-	-	9,667	**9,667**	9,667
11	6,505	-	-	-	-	-	-	-	**6,505**	6,505
12	4,026	-	-	-	-	-	-	-	**4,026**	4,026
13	-	-	3,593	-	-	-	-	-	**3,593**	3,593
14	-	-	-	3,327	-	-	-	-	**3,327**	3,327
15	-	-	-	-	-	-	3,296	-	**3,296**	3,296
16	-	-	-	3,199	-	-	-	-	**3,199**	3,199
17	-	-	-	-	-	2,406	-	-	**2,406**	2,406
18	-	-	-	2,118	-	-	-	-	**2,118**	2,118
19	-	-	-	-	-	-	1,914	-	**1,914**	1,914
Totals	**10,863**	**14,076**	**24,179**	**26,678**	**15,367**	**13,060**	**23,186**	**18,412**	**145,821**	**145,821**
Machines Needed	1.00	1.00	2.00	2.00	1.00	1.00	2.00	1.00		

Figure 1: Initial Solution from Excel Solver on unmet demand breakdown to fulfill for each census tract by each hospital

The solution was further optimized by shifting the demand from census tract 9 fulfilled at hospital C to hospital D to reduce the machine count needed at hospital C by 1. This was done since both hospitals are 24 minutes travel distance from CID 9. This is seen in Figure 2.

Decision variables										
CID / CT loc	A	B	C	D	E	F	G	H	Total	Demand
1	332	-	-	-	-	-	-	-	**332**	332
2	-	14,076	-	-	-	-	-	-	**14,076**	14,076
3	-	-	12,343	-	-	-	-	-	**12,343**	12,343
4	-	-	-	18,034	-	-	-	-	**18,034**	18,034
5	-	-	-	-	15,367	-	-	-	**15,367**	15,367
6	-	-	-	-	-	10,654	-	-	**10,654**	10,654
7	-	-	-	-	-	-	17,976	-	**17,976**	17,976
8	-	-	-	-	-	-	-	8,745	**8,745**	8,745
9	-	-	-	8,243	-	-	-	-	**8,243**	8,243
10	-	-	-	-	-	-	-	9,667	**9,667**	9,667
11	6,505	-	-	-	-	-	-	-	**6,505**	6,505
12	4,026	-	-	-	-	-	-	-	**4,026**	4,026
13	-	-	3,593	-	-	-	-	-	**3,593**	3,593
14	-	-	-	3,327	-	-	-	-	**3,327**	3,327
15	-	-	-	-	-	-	3,296	-	**3,296**	3,296
16	-	-	-	3,199	-	-	-	-	**3,199**	3,199
17	-	-	-	-	-	2,406	-	-	**2,406**	2,406
18	-	-	-	2,118	-	-	-	-	**2,118**	2,118
19	-	-	-	-	-	-	1,914	-	**1,914**	1,914
Totals	**10,863**	**14,076**	**15,936**	**34,921**	**15,367**	**13,060**	**23,186**	**18,412**	**145,821**	**145,821**
Machines Needed	1.00	1.00	1.00	2.00	1.00	1.00	2.00	1.00		

Figure 2: Optimized Solution from Excel Solver on unmet demand breakdown to fulfill for each census tract by each hospital.

Optimization can be taken one step further if the demand for one CID can be split between multiple hospitals. In Figure 3, CID 7 has its demand split between hospital E and G, which reduces the total number of CT machines needed by 1. However, given the varying factors that may come with splitting up demand, such as having to make appointments or turning away patients for CT scans if capacity is reached earlier than expected, the rest of the assignment will assume that each CID is serviced by one hospital, with the data in Figure 2.

Decision variables										
CID / CT loc	A	B	C	D	E	F	G	H	Total	Demand
1	332	-	-	-	-	-	-	-	**332**	332
2	-	14,076	-	-	-	-	-	-	**14,076**	14,076
3	-	-	12,343	-	-	-	-	-	**12,343**	12,343
4	-	-	-	18,034	-	-	-	-	**18,034**	18,034
5	-	-	-	-	15,367	-	-	-	**15,367**	15,367
6	-	-	-	-	-	10,654	-	-	**10,654**	10,654
7	-	-	-	-	4,376	-	13,600	-	**17,976**	17,976
8	-	-	-	-	-	-	-	8,745	**8,745**	8,745
9	-	-	-	8,243	-	-	-	-	**8,243**	8,243
10	-	-	-	-	-	-	-	9,667	**9,667**	9,667
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14	-	-	-	3,327	-	-	-	-	**3,327**	3,327
15	-	-	-	-	-	-	3,296	-	**3,296**	3,296
16	-	-	-	3,199	-	-	-	-	**3,199**	3,199
17	-	-	-	-	-	2,406	-	-	**2,406**	2,406
18	-	-	-	2,118	-	-	-	-	**2,118**	2,118
19	-	-	-	-	-	-	1,914	-	**1,914**	1,914
Totals	**10,863**	**14,076**	**15,936**	**34,921**	**19,743**	**13,060**	**18,810**	**18,412**	**145,821**	**145,821**
Machines Needed	1.00	1.00	1.00	2.00	1.00	1.00	1.00	1.00		

Total Cost									
Cost	A	B	C	D	E	F	G	H	Total
Machines Needed	1.00	1.00	1.00	2.00	1.00	1.00	1.00	1.00	9
New Machine Fixed Cost	1,250,000	1,250,000	1,250,000	2,500,000	1,250,000	1,250,000	1,250,000	1,250,000	11,250,000
								Total Fixed Cost:	11,250,000
Annual Operating Cost	1,700,000	1,700,000	1,700,000	3,400,000	1,700,000	1,700,000	1,700,000	1,700,000	15,300,000
Cost per Exam	676,330	876,372	992,175	2,174,181	1,229,199	813,116	1,171,111	1,146,331	9,078,815
								Total Annual Operating Cost:	24,378,815

Hospital:	A	B	C	D	E	F	G	H
Machine Use Potential	55%	71%	81%	88%	100%	66%	95%	93%

Figure 3: Further Optimization with Split Demand

2.2.1 Does your solution meet the target that 90% of the population is within 45 minutes of a CT machine? If not, how close did you get?

Figure 4 displays the census tracts that are within 45min of a hospital (highlighted in green) and the census tracts that only have travel times greater than 45min to a hospital (highlighted in red). Assuming that each census tract has the same population number, only 84% of the population would be within 45min of a hospital. Furthermore, even if the calculations were based on the unmet CT scan demands at each area (using the unmet demands as the population number), only 89% of the population would be within 45min of a hospital. Therefore, the solution does not meet the target that 90% of the population is within 45 minutes of a CT machine.

Distance								
CID / CT lo	A	B	C	D	E	F	G	H
1	10	48	64	80	152	152	168	232
2	48	4	40	32	104	104	120	192
3	64	40	8	40	96	128	96	184
4	80	32	40	12	72	80	80	152
5	152	104	96	72	4	32	8	112
6	152	104	128	80	32	7	64	144
7	168	120	96	80	8	64	4	96
8	232	192	184	152	112	144	96	15
9	80	56	24	24	88	128	80	144
10	224	176	160	136	80	112	64	56
11	40	104	64	104	160	192	160	248
12	32	64	40	72	136	176	128	192
13	88	64	24	64	120	152	104	160
14	136	80	72	48	72	104	64	128
15	184	136	112	96	24	80	16	80
16	112	64	88	48	56	56	64	160
17	176	128	136	96	48	32	56	72
18	112	64	56	32	40	72	48	152
19	152	104	96	72	32	88	24	104

Figure 4: The census tracts that are within 45min of a hospital (highlighted in green) and the census tracts that are greater than 45min to a hospital (highlighted in red).

2.2.2 What is the 90th-percentile travel time?

Based on Figure 5 below with the travel time for each census tract, the 90th percentile travel time is 48 minutes. This was solved using Excel functions.

Actual Time Taken								
CID / CT loc	A	B	C	D	E	F	G	H
1	10	-	-	-	-	-	-	-
2	-	4	-	-	-	-	-	-
3	-	-	8	-	-	-	-	-
4	-	-	-	12	-	-	-	-
5	-	-	-	-	4	-	-	-
6	-	-	-	-	-	7	-	-
7	-	-	-	-	-	-	4	-
8	-	-	-	-	-	-	-	15
9	-	24	24	-	-	-	-	-
10	-	-	-	-	-	-	-	56
11	40	-	-	-	-	-	-	-
12	32	-	-	-	-	-	-	-
13	-	-	24	-	-	-	-	-
14	-	-	-	48	-	-	-	-
15	-	-	-	-	-	-	16	-
16	-	-	-	48	-	-	-	-
17	-	-	-	-	-	32	-	-
18	-	-	-	32	-	-	-	-
19	-	-	-	-	-	-	24	-

Figure 5: Time Distribution per hospital per CID

2.2.3 What percent of the population is within 45 minutes of a CT machine?

As shown in Figure 6 below, by assuming that the population of each census tract is the unmet demand for CT scans, the percentage of the population that is within 45 minutes of a CT machine is 89%.

Actual Time Taken												
CID / CT loc	A	B	C	D	E	F	G	H	45 Minutes	Actual Time	<= 45?	Population
1	14,940	-	-	-	-	-	-	-	14,940	3,320	TRUE	332
2	-	633,420	-	-	-	-	-	-	633,420	56,304	TRUE	14,076
3	-	-	555,435	-	-	-	-	-	555,435	98,744	TRUE	12,343
4	-	-	-	811,530	-	-	-	-	811,530	216,408	TRUE	18,034
5	-	-	-	-	691,515	-	-	-	691,515	61,468	TRUE	15,367
6	-	-	-	-	-	479,430	-	-	479,430	74,578	TRUE	10,654
7	-	-	-	-	-	-	808,920	-	808,920	71,904	TRUE	17,976
8	-	-	-	-	-	-	-	393,525	393,525	131,175	TRUE	8,745
9	-	-	-	370,935	-	-	-	-	370,935	197,832	TRUE	8,243
10	-	-	-	-	-	-	-	435,015	435,015	541,352	FALSE	9,667
11	292,725	-	-	-	-	-	-	-	292,725	260,200	TRUE	6,505
12	181,170	-	-	-	-	-	-	-	181,170	128,832	TRUE	4,026
13	-	-	161,685	-	-	-	-	-	161,685	86,232	TRUE	3,593
14	-	-	-	149,715	-	-	-	-	149,715	159,696	FALSE	3,327
15	-	-	-	-	-	-	148,320	-	148,320	52,736	TRUE	3,296
16	-	-	-	143,955	-	-	-	-	143,955	153,552	FALSE	3,199
17	-	-	-	-	-	108,270	-	-	108,270	76,992	TRUE	2,406
18	-	-	-	95,310	-	-	-	-	95,310	67,776	TRUE	2,118
19	-	-	-	-	-	-	86,130	-	86,130	45,936	TRUE	1,914
											Total Population:	145,821
											Population Further than 45	16,193
											Goal:	89%

Figure 6: Percent of the population that is within 45mins of a CT machine.

2.2.4 What is the total cost?

Figure 7 displays the total cost breakdown based on how many CT machines are required at each hospital. The total fixed cost for new machines is $12,500,000 based on the amount of CT machines that should be built at each hospital. The annual operating cost for these machines and annual exams is $26,078,815.

Total Cost									
Cost	A	B	C	D	E	F	G	H	Total
Machines Needed	1.00	1.00	1.00	2.00	1.00	1.00	2.00	1.00	10
New Machine Fixed Cost	1,250,000	1,250,000	1,250,000	2,500,000	1,250,000	1,250,000	2,500,000	1,250,000	12,500,000
								Total Fixed Cost:	12,500,000
Annual Operating Cost	1,700,000	1,700,000	1,700,000	3,400,000	1,700,000	1,700,000	3,400,000	1,700,000	17,000,000
Cost per Exam	676,330	876,372	992,175	2,174,181	956,749	813,116	1,443,560	1,146,331	9,078,815
								Total Annual Operating Cost:	26,078,815

Figure 7: Total cost breakdown

2.3 How many new CTs should be built at each hospital?

Based on Figure 7 above, the number of CT machines that should be built at each hospital is broken down in Table 2 below.

Table 2: Number of new CT machines that should be built at each hospital.

Hospitals	**Number of CT Machine**
A	1
B	1
C	1
D	2
E	1
F	1
G	2
H	1
Total	**10**

2.2.6 What is the utilization of each CT machine?

The results of all hospital utilization are summarized in Table 3 below. As shown, certain machines are under-utilized (<60%), such as hospital A and G.

Table 3: Utilization of each CT machine based on unmet demand.

Hospitals	Machine Utilization
A	55%
B	71%
C	81%
D	88%
E	78%
F	66%
G	59%
H	93%

2.3 Is the RHA's goal realistically achievable?

The goal is not achievable since the minimum travelling time from CID 10, 14 and 16 to any hospital are 56, 48, and 48 minutes respectively. Therefore, no matter how many CT scanners hospitals get, the amount of population that needs to travel for more than 45 minutes is still 16,193. The best RHA can achieve is to have 89% of the population be within 45 minutes of travelling distance.

Realistically, many factors go into the travel times given for each CID. While the current travel times as given cannot achieve 90%, minor reductions in CID 10, 14 and 16 travel times can result in the 90% goal being achieved. For instance, direct shuttle bus implementation, or mobile CT scanners could help reduce travel times from these CIDs.