

If  $L = \{0^m 1^n \mid n = m^2, m, n \in \mathbb{N}\}$  is a context-free language then  $L$  has a “pumping length”  $P$  such that any string  $S$  may be divided into 5 pieces,  $S = uvxyz$  where  $|S| \geq P$  [0] and following conditions must be true:

$uv^i xy^i z$  is in  $L$  for every  $i \geq 0$  [1],

$|vy| \geq 1$  [2],

$|vxy| \leq P$  [3]

[2], [3] indicates that  $P \geq 1$  with [0] which indicates  $m \geq 1$  then  $|S| \geq 2$

**case 1:**  $vxy$  contains only either of the letters  $\{0, 1\}$

**case 1a:**  $vxy$  contains only 0s

This indicates  $|z| \geq 1$  and for all  $i > 1$  [1] fails as the number of 1s stays *constant* while the number of 0s increase

**case 1b:**  $vxy$  contains only 1s

Similar to 1a  $|u| \geq 1$  and for all  $i > 1$  [1] fails as the number of 0s stays constant while the number of 1s increase

**case 2:**  $vxy$  contains both 0s and 1s

**case 2ab:**  $v$  or  $y$  contains both 0s and 1s

This case fails as it produces some 1s before 0s when pumped using [1] which then fails [1] as eg. 0000010101.... Fails to be in  $L$

case 2c:  $v$  contains only 0s and  $y$  contains only 1s

This case fails since the number of 1s depends on the number of 0s *exponentially* whereas the pumping production of [1] 0s and 1s are done *linearly*. This will cause [1] to fail again for all  $i > 1$  as both  $v$  and  $y$  are being pumped up based on  $i$ , whereas  $y$  needs to be pumped up by  $i^2$  to satisfy the language grammar.

Since all the cases have at least one outlier for some  $P$  and/ or  $i$ ,  $L$  is not a context-free language.