Model predictive control and trajectory optimization of large vehicle-manipulators

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Motivation

Automatizing mobile working machines

Vehicle + Manipulator

Complex tasks

Motivation

Automatizing mobile working machines

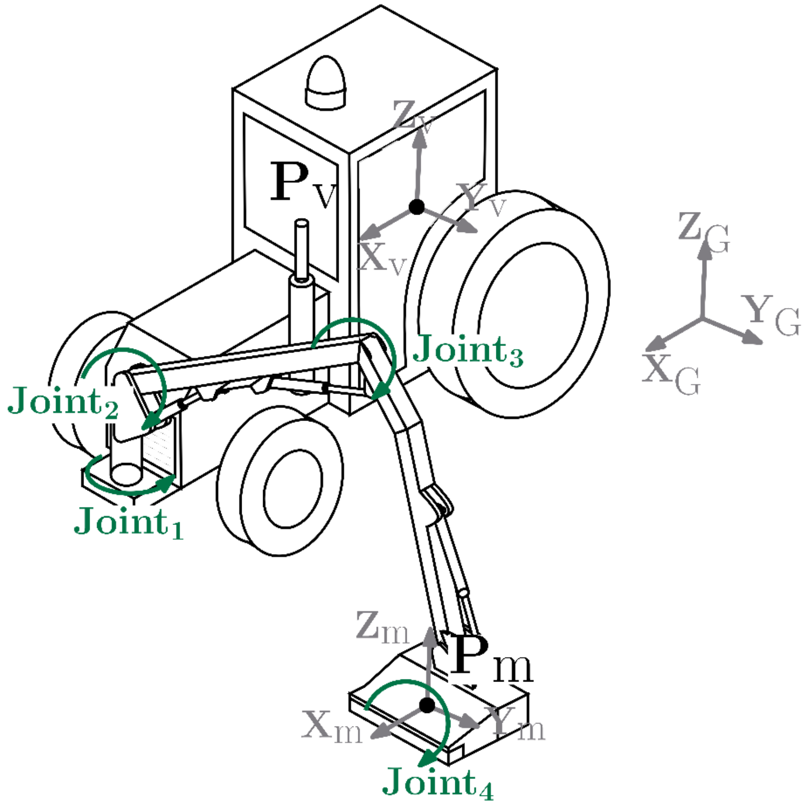
Vehicle + Manipulator

Complex tasks

(semi)-autonomic Functions

Relieving human worker

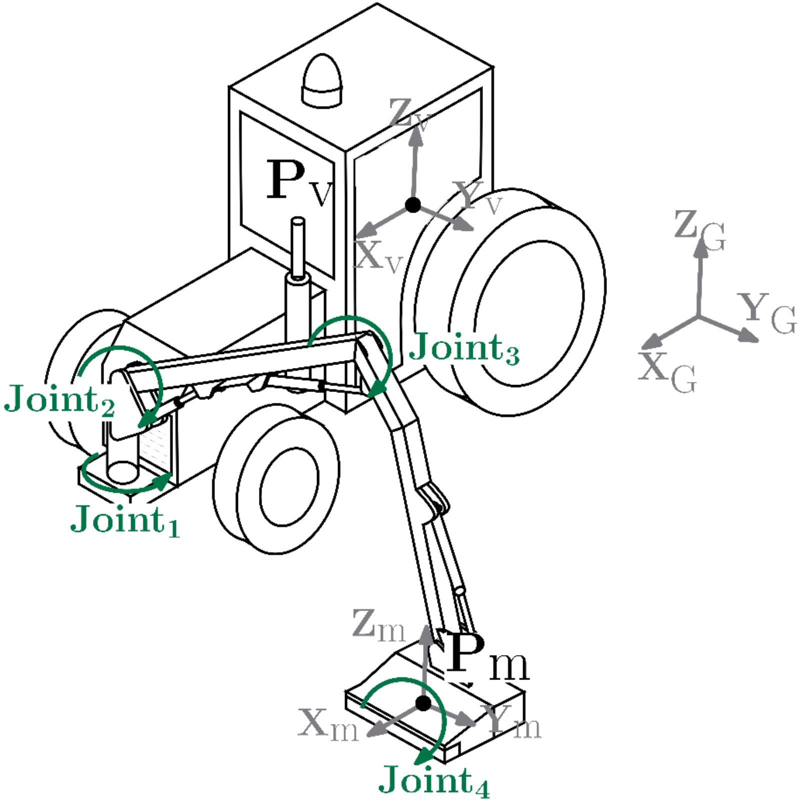
Goals of the research work



Goals of the research work

- Design of a general control model for a large vehicle-manipulator system

- Position control of the vehicle and the manipulator along their reference trajectories, with a model predictive control (MPC)



Outline of the presentation

State of the art

Simulation environment

Control model concept for vehicle-manipulator systems

Simulation results

Conclusion and further research

State of the art

- **State of the art**
- Simulation environment
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State of the art

- Control of large hydraulic manipulators
  - Modelling and control methods for forestry cranes [Fodor 2015], [Hera 2015]

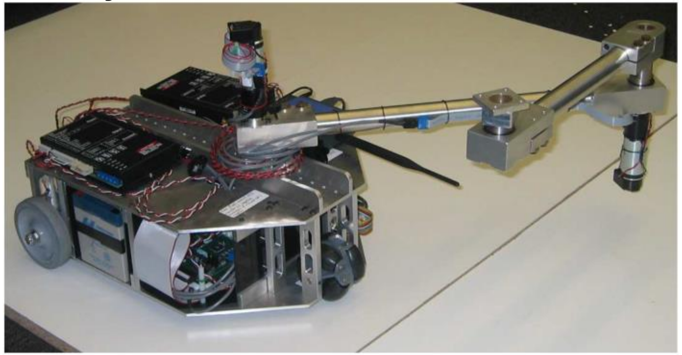
[Fodor 2015]

State of the art

- Control of large hydraulic manipulators
  - Modelling and control methods for forestry cranes [Fodor 2015], [Hera 2015]

[Fodor 2015]

- Small indoor
  - Small mobile manipulators [Mashali 2014], [White 2009]
  - Control methods for dual trajectories



[White 2009]

State of the art

- Control of large hydraulic manipulators
  - Modelling and control methods for forestry cranes [Fodor 2015], [Hera 2015]
- Small indoor
  - Small mobile manipulators [Mashali 2014], [White 2009]
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There is no control model for large vehicle-manipulators

Simulation environment

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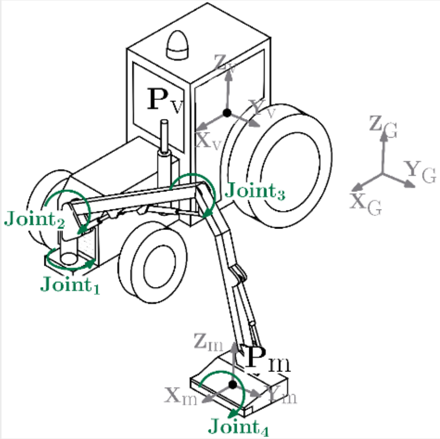
Simulation environment

Control model

Simulation model

Simulation model consists of two subsystems: vehicle + robotic arm

Vehicle model:



Simulation environment

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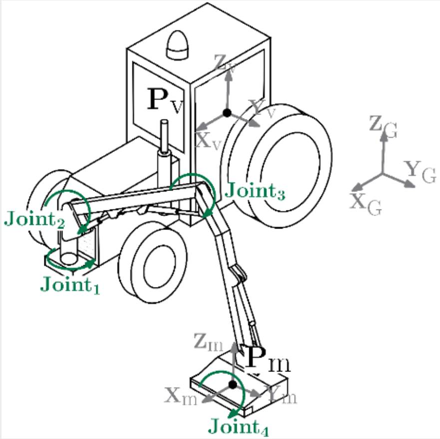
Vehicle model:

- 3D, nonlinear Model
- Equations of motion derived with Euler-Newton based on [Kovacs 2014]
- Inputs: Steering angle, driving torques



State of the Art | Simulation environment | Control concept | Results | Conclusion & further research

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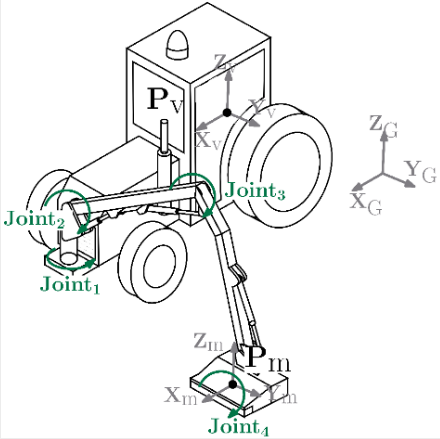
Modell of the manipulator:



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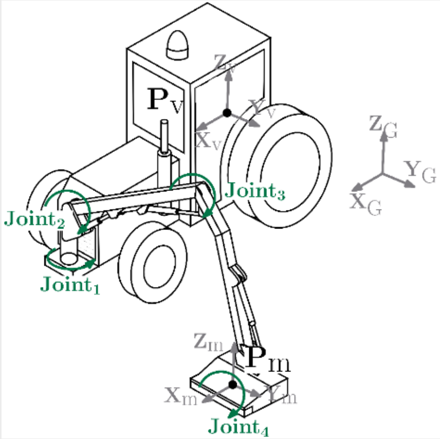
Modell of the manipulator:

- Large robotic arm,
  with hydraulic power unit based on [Ruderman 2017]
- Inputs: desired position and the orientation of the manipulator
- Computing the desired angles with inverse kinematic
- Joints are controlled with a PID-controller



Control concept

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Control model

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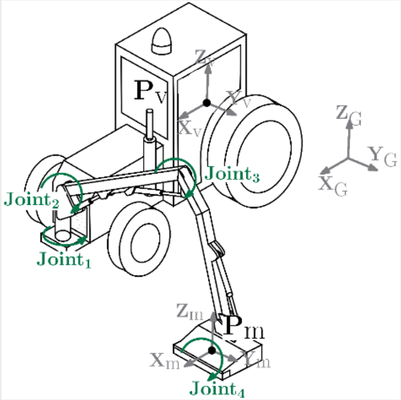
Simulation model

- Control model: kinematic single track and planar manipulator
- Controlling the system along two references → Frénet-Frame



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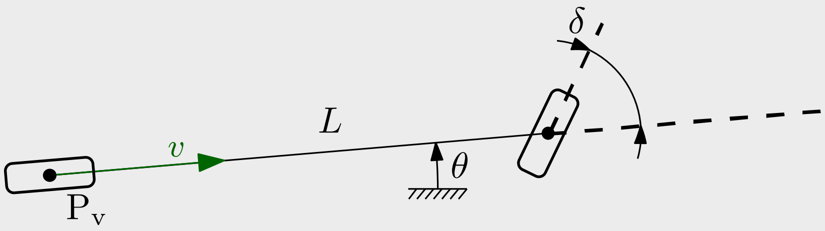

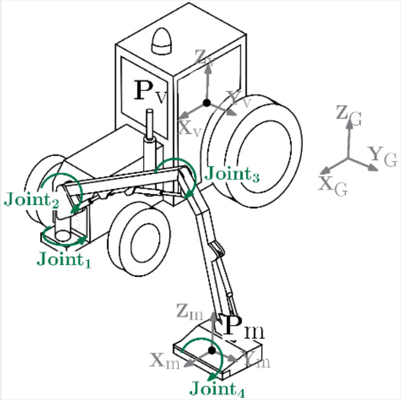

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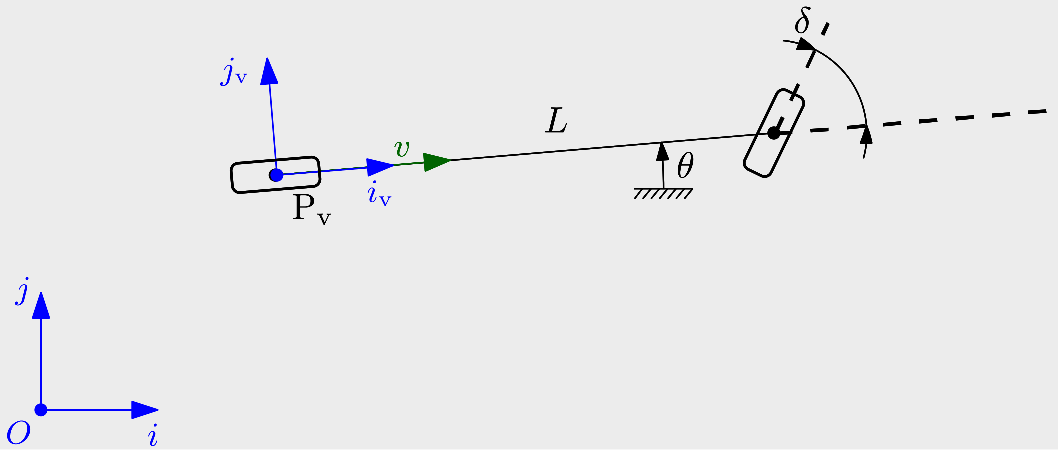

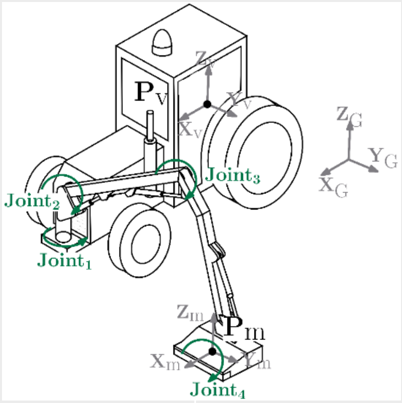

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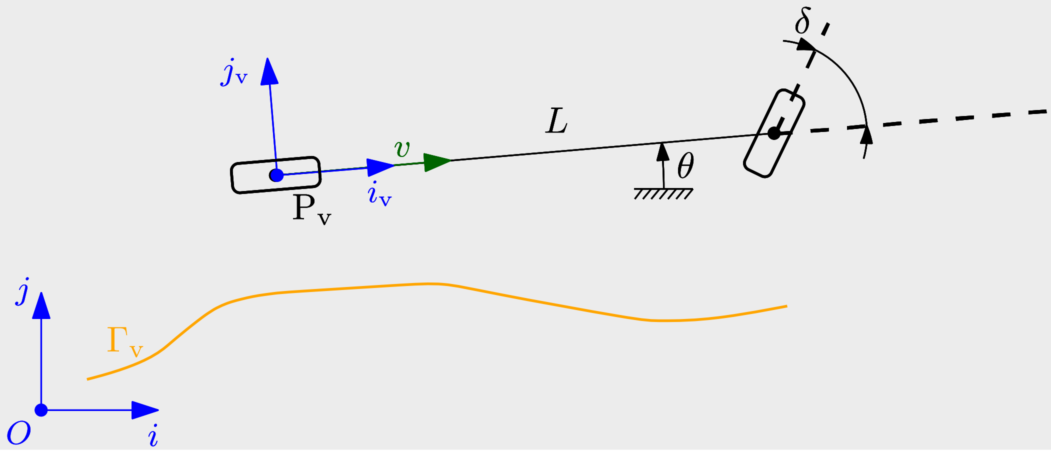

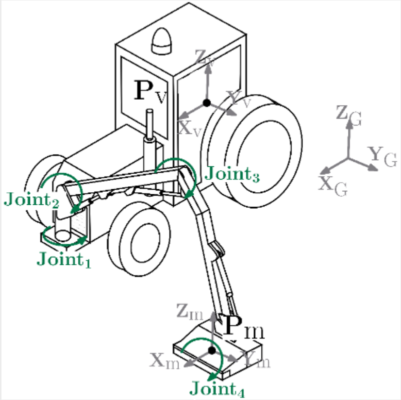

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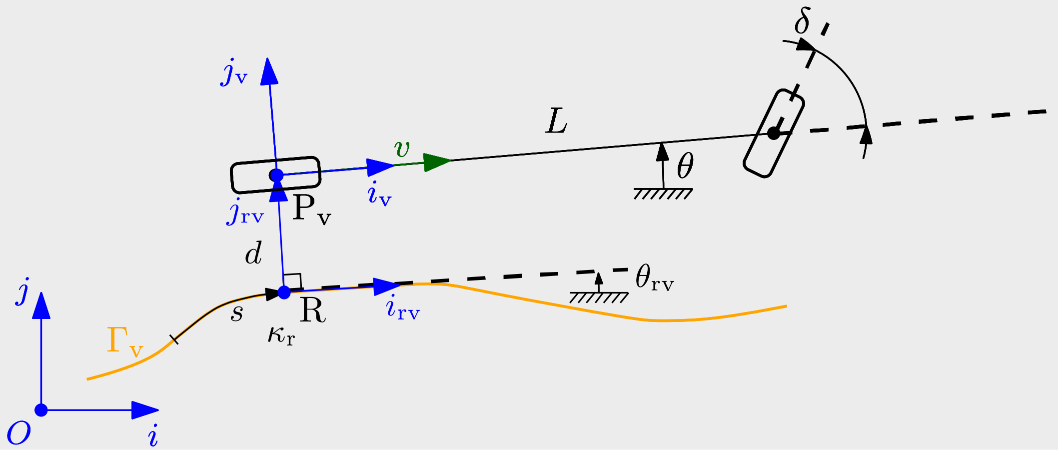

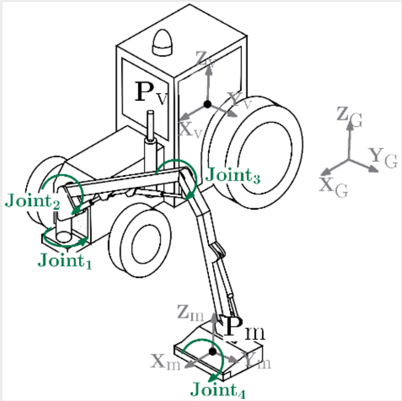

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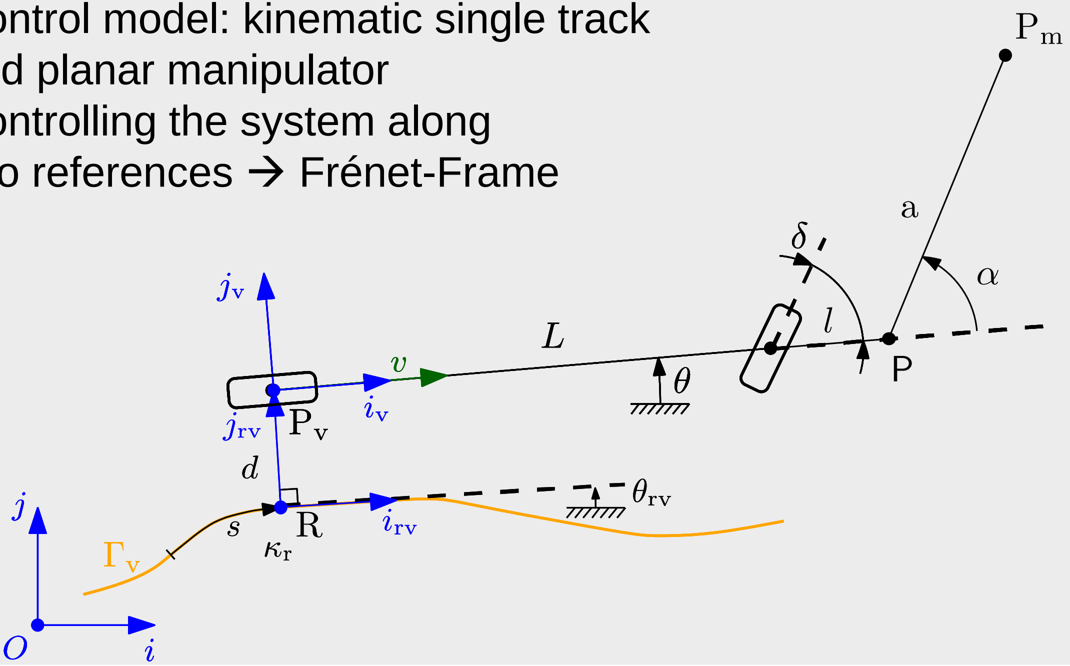

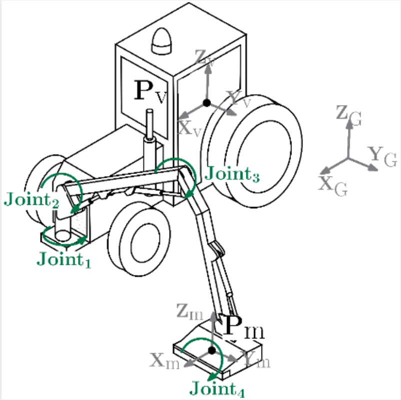

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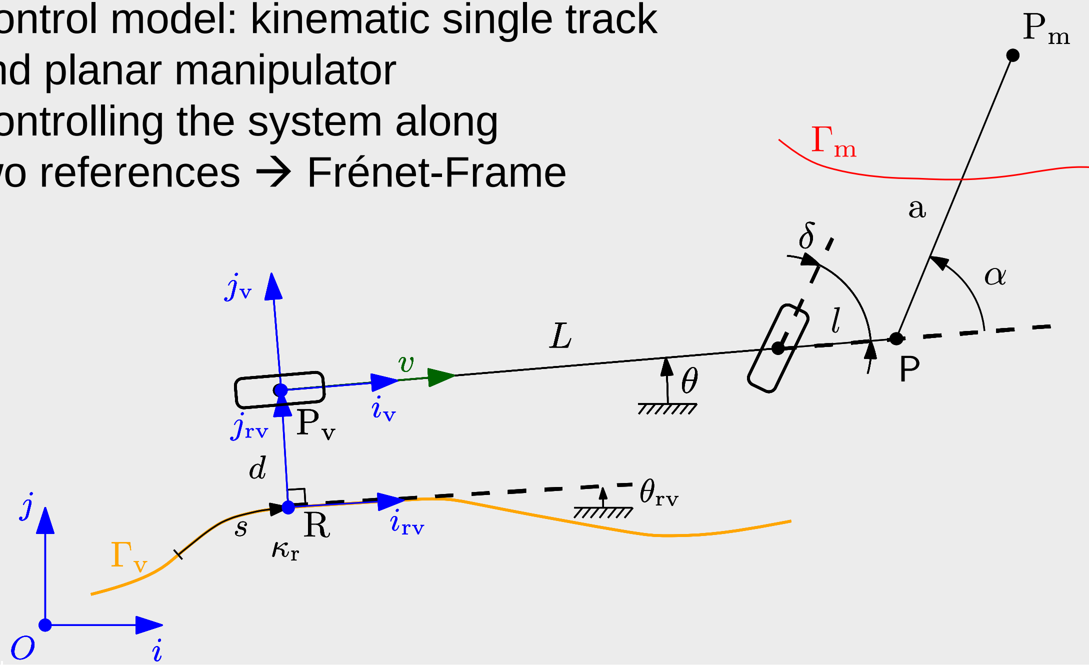
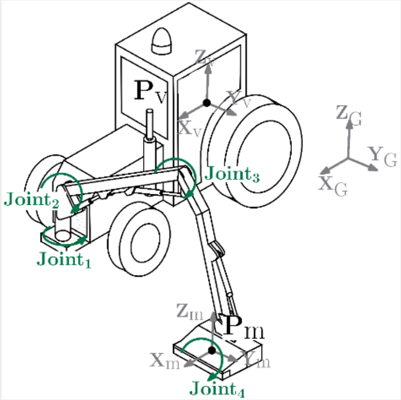


Simulation model



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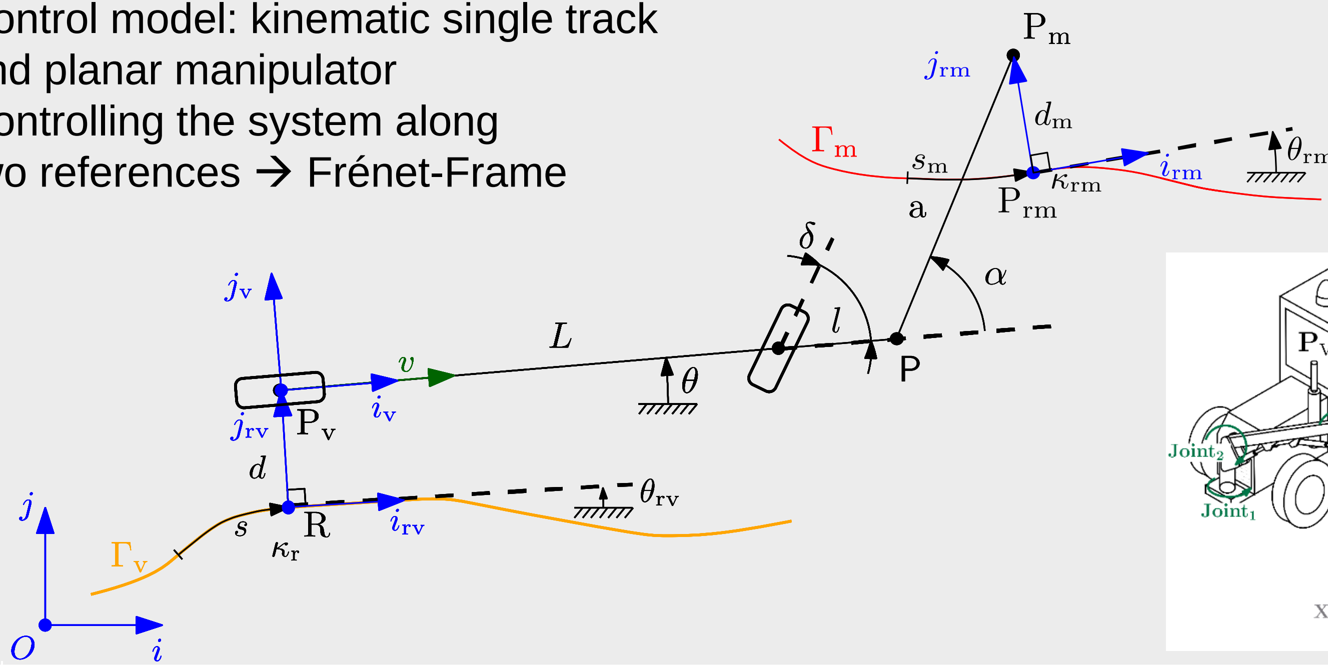
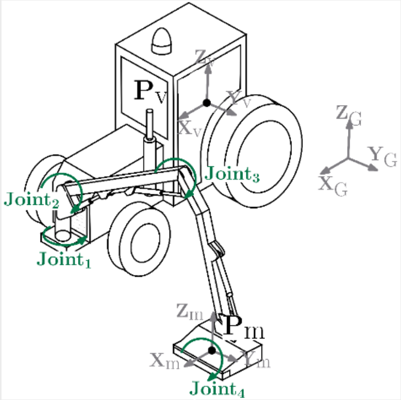
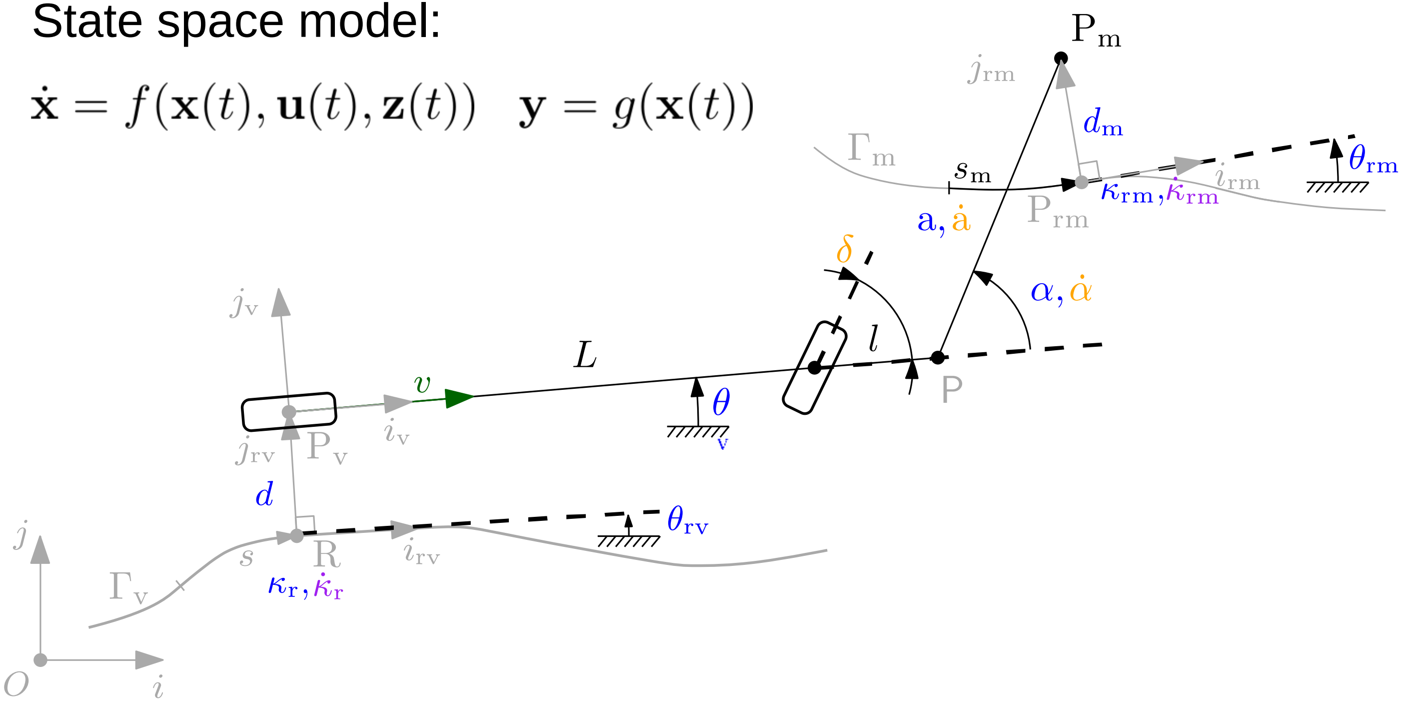




Control model

State space model:

$$\dot{\mathbf{x}} = f(\mathbf{x}(t), \mathbf{u}(t), \mathbf{z}(t)) \quad \mathbf{y} = g(\mathbf{x}(t))$$



Control model

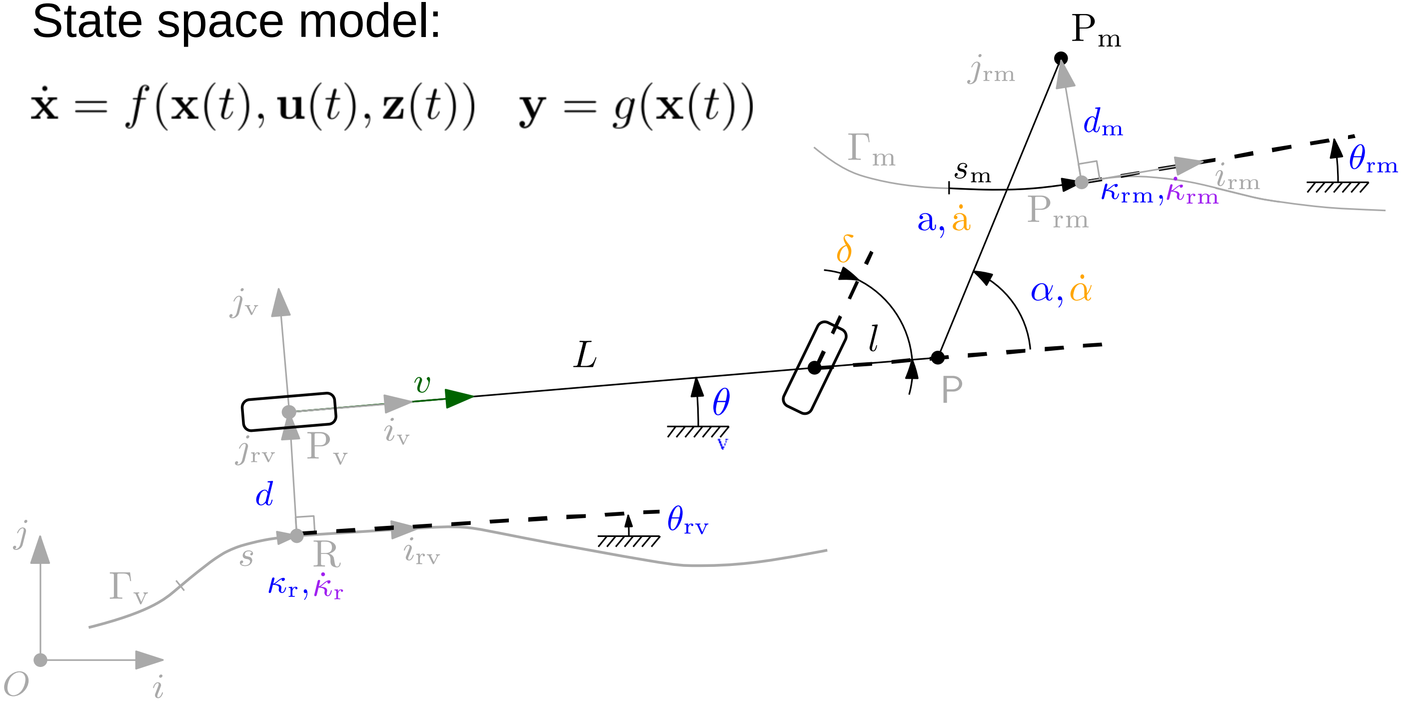
State space model:

$$\dot{\mathbf{x}} = f(\mathbf{x}(t), \mathbf{u}(t), \mathbf{z}(t)) \quad \mathbf{y} = g(\mathbf{x}(t))$$

$$\kappa_{\mathrm{v}} = \frac{\tan(\delta)}{L}$$

$$\Delta\theta_{\mathrm{m}} = \theta_{\mathrm{v}} - \theta_{\mathrm{rm}}$$

$$\Delta\theta_{\mathrm{v}} = \theta_{\mathrm{v}} - \theta_{\mathrm{rv}}$$



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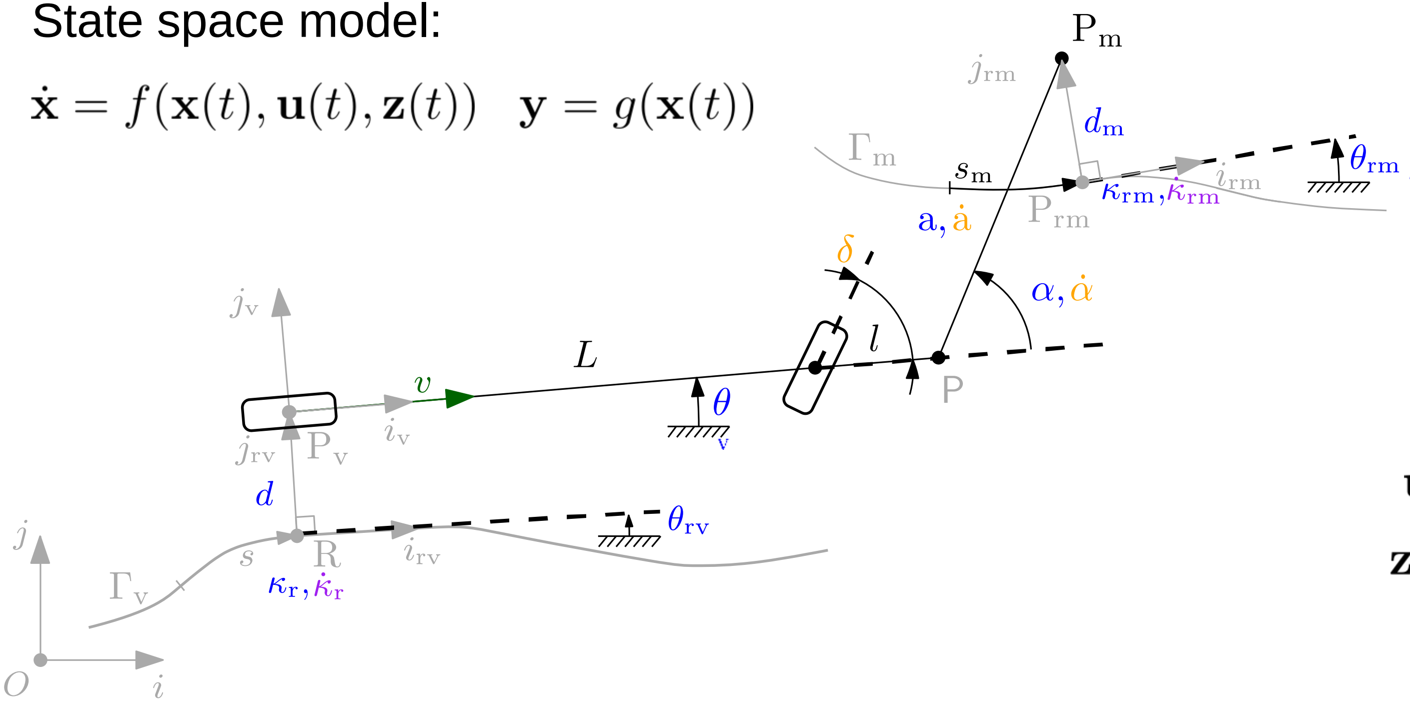

$$\kappa_v = \frac{\tan(\delta)}{L}$$

$$\Delta\theta_m = \theta_v - \theta_{rm}$$

$$\Delta\theta_v = \theta_v - \theta_{rv}$$

$$\mathbf{u}(t) = [\kappa_v \; \dot{a} \; \dot{\alpha}]^T$$

$$\mathbf{z}(t) = [\dot{\kappa}_{rv} \; \dot{\kappa}_{rm}]^T$$

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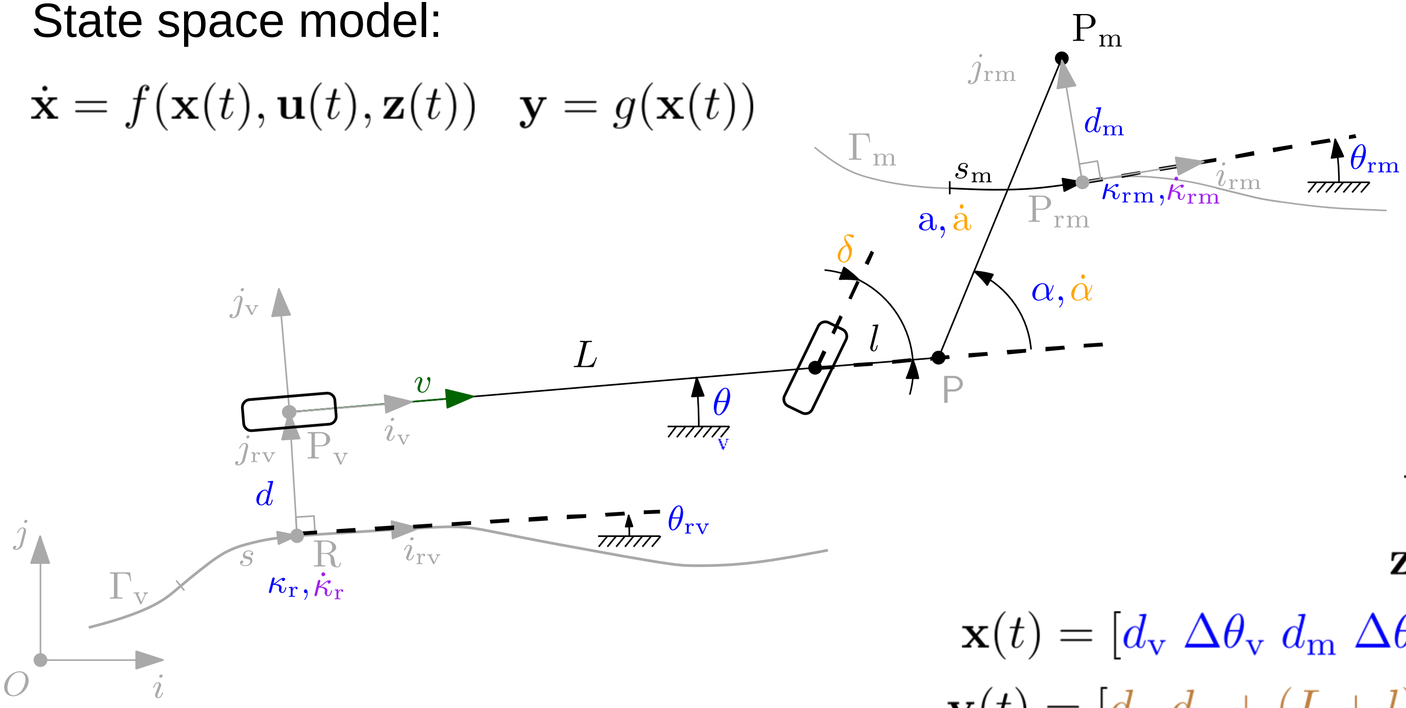

$$\kappa_v = \frac{\tan(\delta)}{L}$$

$$\Delta\theta_m = \theta_v - \theta_{rm}$$

$$\Delta\theta_v = \theta_v - \theta_{rv}$$

$$\mathbf{u}(t) = [\kappa_v \ \dot{\mathrm{a}} \ \dot{\alpha}]^T$$

$$\mathbf{z}(t) = [\dot{\kappa}_{rv} \ \dot{\kappa}_{rm}]^T$$

$$\mathbf{x}(t) = [d_v \ \Delta\theta_v \ d_m \ \Delta\theta_m \ \mathrm{a} \ \alpha \ \kappa_{rv} \ \kappa_{rm}]^T$$

$$\mathbf{y}(t) = [d_v \ d_v + (L+l)sin(\Delta\theta) \ d_m \ \mathrm{a}]^T$$

Control model

State space model:

$$\dot{\mathbf{x}} = f(\mathbf{x}(t), \mathbf{u}(t), \mathbf{z}(t))$$

Control model

State space model:

$$\dot{\mathbf{x}} = f(\mathbf{x}(t), \mathbf{u}(t), \mathbf{z}(t))$$

Linearizing around the Equilibrium:

$$\mathbf{x} = [0 \;\; 0 \;\; 0 \;\; 0 \;\; \mathrm{a}_r \;\; \alpha_r \;\; 0 \;\; 0]^T$$

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$\rightarrow$ a_r : Ref. length of the robotic arm
α_r : Ref. angle of the robotic arm

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$\rightarrow$ a_r : Ref. length of the robotic arm
α_r : Ref. angle of the robotic arm

$\rightarrow$ Linear state space model:

$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\Delta\,\mathbf{x}(t) + \mathbf{B}(t)\Delta\,\mathbf{u}(t) + \mathbf{Z}\Delta\,\mathbf{z}(t)$$

$$\mathbf{y}(t) = \mathbf{C}\Delta\,\mathbf{x}(t)$$

MPC design

Continuous state space model:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}_c\,\boldsymbol{x} + \boldsymbol{B}_c\,\boldsymbol{u} + \boldsymbol{E}_c\,\boldsymbol{z}$$

$$\boldsymbol{y} = \boldsymbol{C}_c\,\boldsymbol{x}$$

Based on [Borrelli 2011]

MPC design

Continuous state space model:

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$$\boldsymbol{y} = \boldsymbol{C}_c\,\boldsymbol{x}$$

Time discrete system model:

Time discretization with the sampling T

$$\boldsymbol{x}_{k+1} = \boldsymbol{A}\,\boldsymbol{x}_k + \boldsymbol{B}\,\boldsymbol{u}_k + \boldsymbol{E}\,\boldsymbol{z}_k$$

$$\boldsymbol{y}_k \quad = \boldsymbol{C}\,\boldsymbol{x}_k$$

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Predictions:

Over N steps horizon

→ *batch*-Formula

vector sequences

$$\mathbf{x} = \left[\boldsymbol{x}_1^{\mathrm{T}}, \dots, \boldsymbol{x}_N^{\mathrm{T}}\right]^{\mathrm{T}}$$
$$\mathbf{u} = \left[\boldsymbol{u}_0^{\mathrm{T}}, \dots, \boldsymbol{u}_{N-1}^{\mathrm{T}}\right]^{\mathrm{T}}$$
$$\mathbf{z} = \left[\boldsymbol{z}_0^{\mathrm{T}}, \dots, \boldsymbol{z}_{N-1}^{\mathrm{T}}\right]^{\mathrm{T}}$$
$$\mathbf{y} = \left[\boldsymbol{y}_1^{\mathrm{T}}, \dots, \boldsymbol{y}_N^{\mathrm{T}}\right]^{\mathrm{T}}$$

Transformation

$$\mathbf{x} = \boldsymbol{\mathcal{A}}\,\boldsymbol{x}_0 + \boldsymbol{\mathcal{B}}\,\mathbf{u} + \boldsymbol{\mathcal{E}}\,\mathbf{z}$$
$$\mathbf{y} = \boldsymbol{\mathcal{C}}\,(\boldsymbol{\mathcal{A}}\,\boldsymbol{x}_0 + \boldsymbol{\mathcal{B}}\,\mathbf{u} + \boldsymbol{\mathcal{E}}\,\mathbf{z})$$

Based on [Borrelli 2011]

MPC design

Cost function:

$$J(\boldsymbol{x},\boldsymbol{u}) = \sum_{k=1}^{N} \boldsymbol{x}_k^{\mathrm{T}} \boldsymbol{Q} \boldsymbol{x}_k + \sum_{k=0}^{N-1} \boldsymbol{u}_k^{\mathrm{T}} \boldsymbol{R} \boldsymbol{u}_k$$

$$\boldsymbol{Q} = \operatorname{diag}(G_d, G_{\Delta\theta}, G_{dM}, 0, G_{\Delta\alpha}, 0, 0)$$
$$\boldsymbol{R} = \operatorname{diag}(G_{\boldsymbol{\kappa}_v}, G_{\dot{\boldsymbol{a}}}, G_{\dot{\boldsymbol{\alpha}}})$$

Based on [Borrelli 2011]

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Constraints:

Constraints on the inputs:

$$\mathbf{u}_{\min} = [\boldsymbol{u}_{\min}^{\mathrm{T}}, \dots, \boldsymbol{u}_{\min}^{\mathrm{T}}]^{\mathrm{T}}$$
$$\mathbf{u}_{\max} = [\boldsymbol{u}_{\max}^{\mathrm{T}}, \dots, \boldsymbol{u}_{\max}^{\mathrm{T}}]^{\mathrm{T}}$$

and the outputs:

$$\mathbf{y}_{\min} = [\boldsymbol{y}_{\min}^{\mathrm{T}}, \dots, \boldsymbol{y}_{\min}^{\mathrm{T}}]^{\mathrm{T}}$$
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Based on [Borrelli 2011]

LQ-Optimization

→ Implementation as a quadratic programming with constraints:

$$J(\boldsymbol{x}_0, \mathbf{z}, \mathbf{u}) = \mathbf{u}^{\mathrm{T}} \boldsymbol{H}\, \mathbf{u} + 2\left(\boldsymbol{x}_0^{\mathrm{T}} \boldsymbol{F} + \mathbf{z}^{\mathrm{T}} \boldsymbol{G}\right)\mathbf{u}$$

s.t. $\mathbf{u}_{\mathrm{min}} \leq \mathbf{u} \leq \mathbf{u}_{\mathrm{max}}$

$\boldsymbol{y}_{\boldsymbol{min}} \leq \boldsymbol{y} \leq \boldsymbol{y}_{\boldsymbol{max}}$

MPC design

Cost function:

$$J(\boldsymbol{x},\boldsymbol{u}) = \sum_{k=1}^{N} \boldsymbol{x}_k^{\mathrm{T}} \boldsymbol{Q} \boldsymbol{x}_k + \sum_{k=0}^{N-1} \boldsymbol{u}_k^{\mathrm{T}} \boldsymbol{R} \boldsymbol{u}_k$$

$$\boldsymbol{Q} = \operatorname{diag}(G_d, G_{\Delta\theta}, G_{dM}, 0, G_{\Delta\alpha}, 0, 0)$$
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$$\mathbf{y}_{\min} = [\boldsymbol{y}_{\min}^{\mathrm{T}}, \dots, \boldsymbol{y}_{\min}^{\mathrm{T}}]^{\mathrm{T}}$$
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Based on [Borrelli 2011]

Simulation results

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Results

Scenario 1: References Γ_{v} and Γ_{M}

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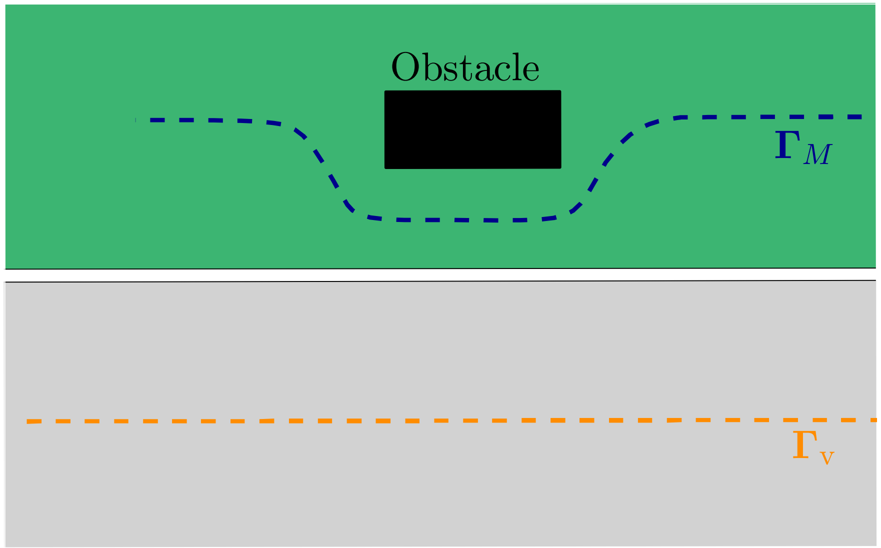
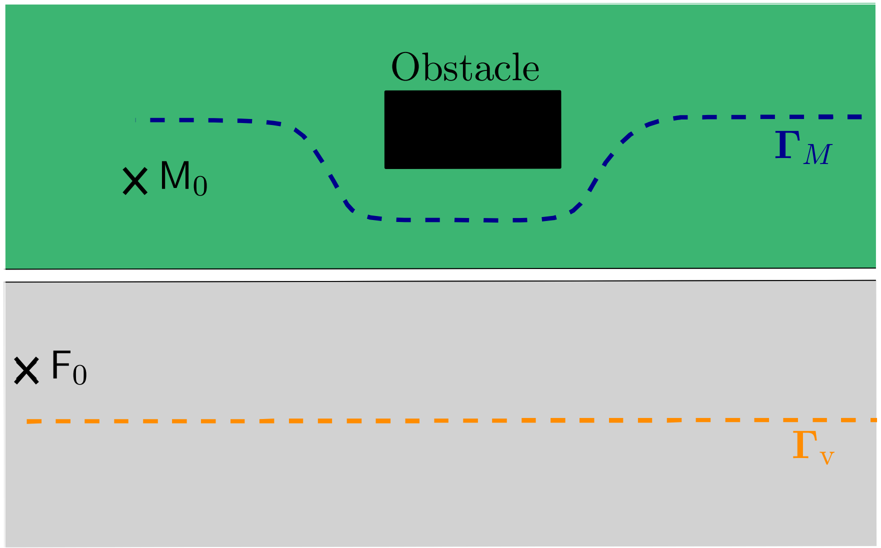


Starting value
$d_0 = 0{,}25\ \mathrm{m}$
$\theta_0 = 0{,}1\ \mathrm{rad}$
$a_0 = 2{,}1\ \mathrm{m}$
$\alpha_0 = 1{,}1\ \mathrm{rad}$

Results

Scenario 1: References Γ_v and Γ_M



Reference angle

$$\alpha_r = \frac{3\,\pi}{8}$$

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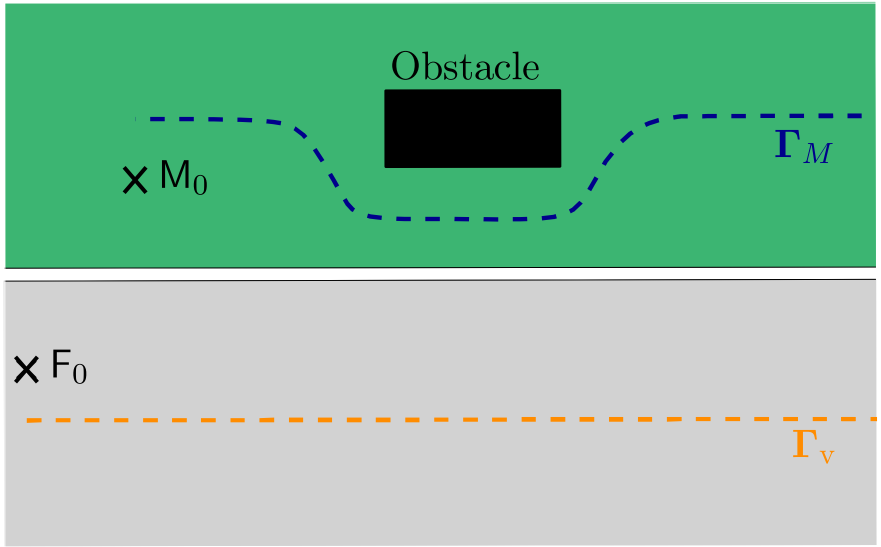

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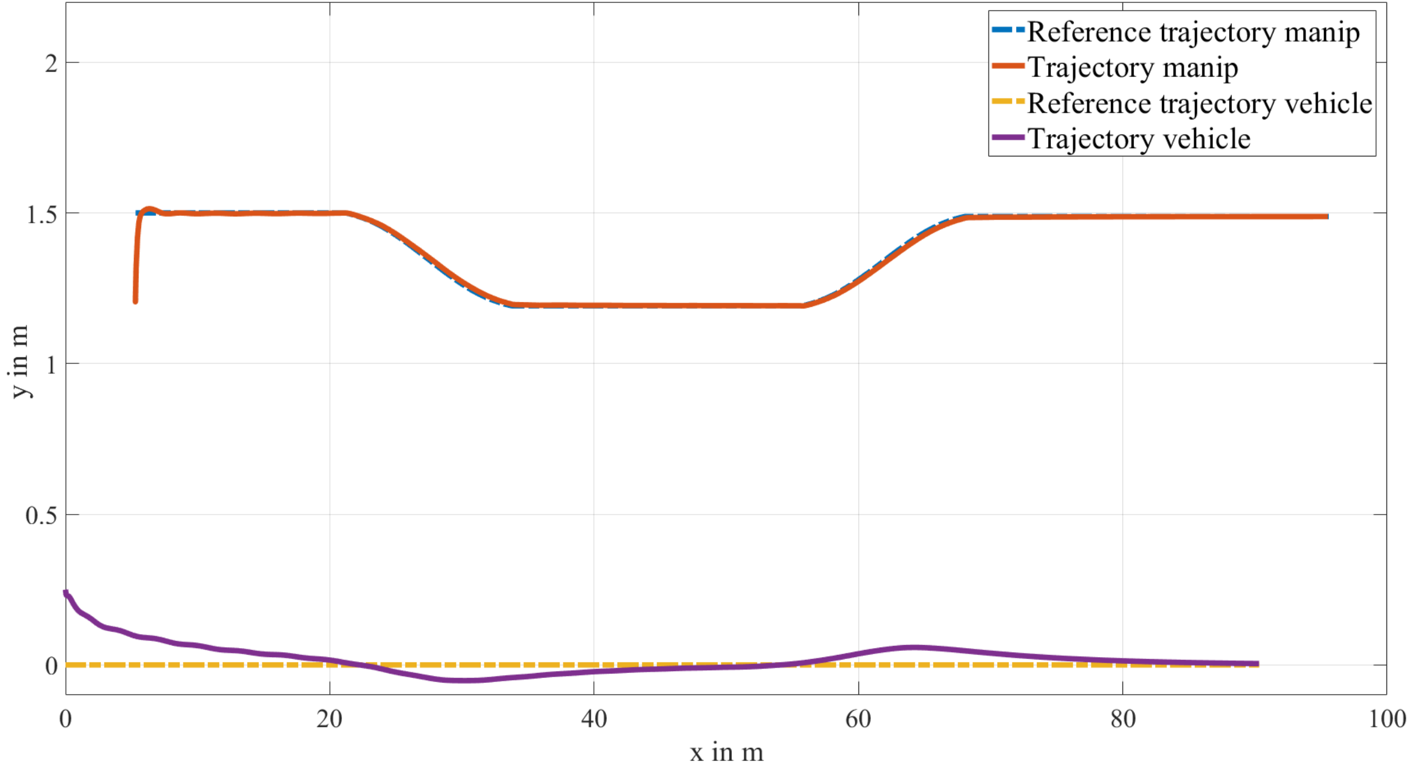
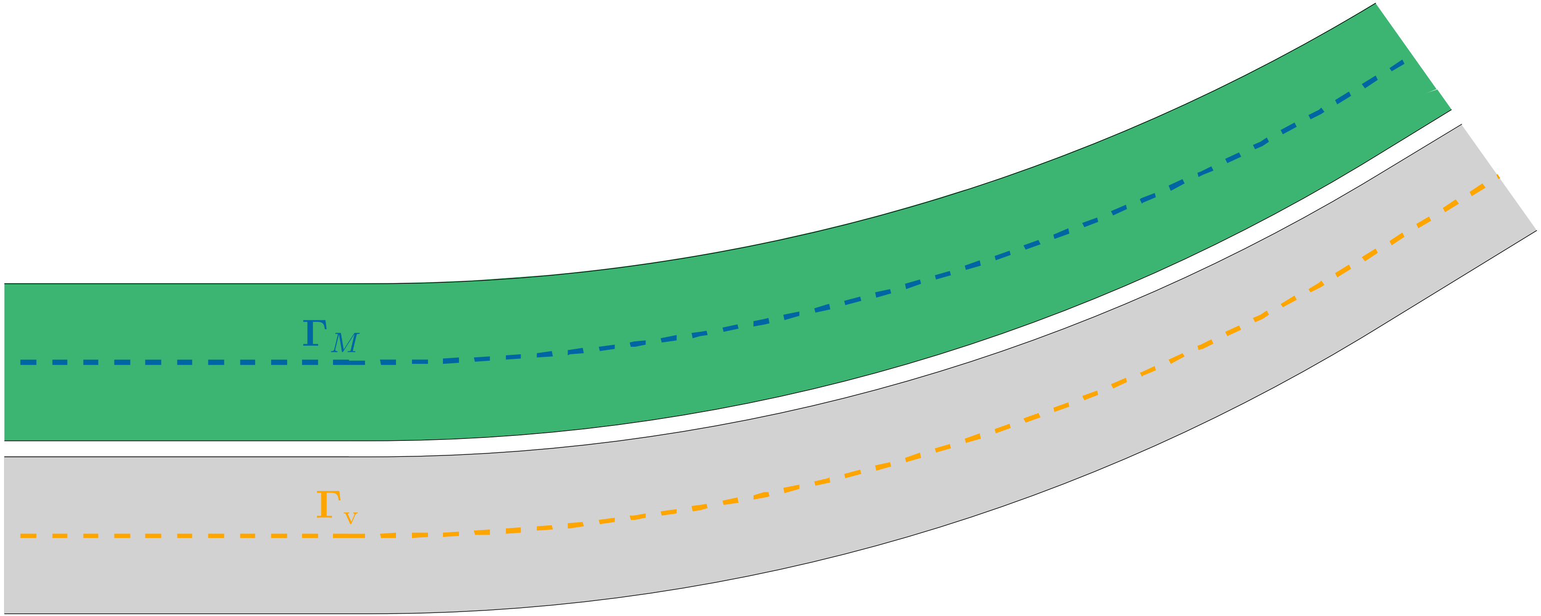
Results



Reference trajectory manip
Trajectory manip
Reference trajectory vehicle
Trajectory vehicle
y in m
2
1.5
1
0.5
0
x in m
0
20
40
60
80
100

Results

Scenario 2: References Γ_v and Γ_M



Results

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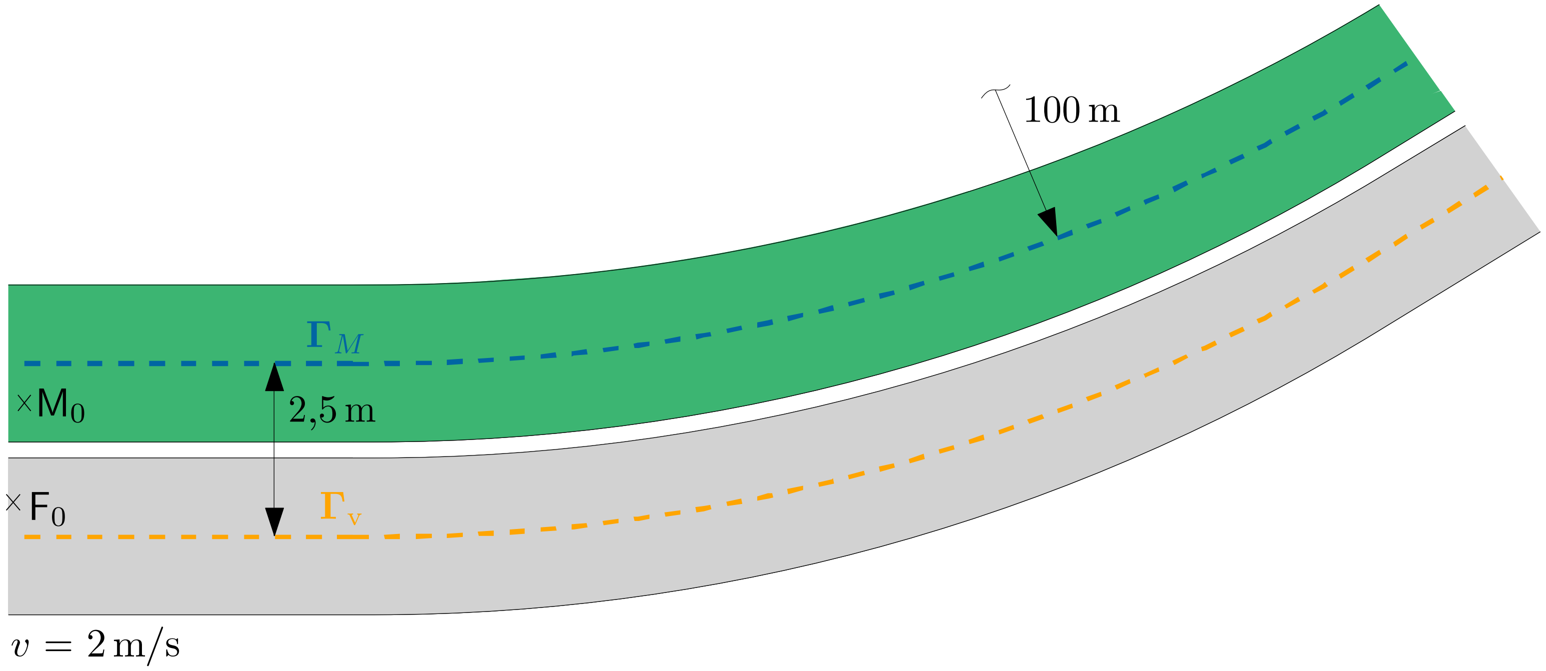

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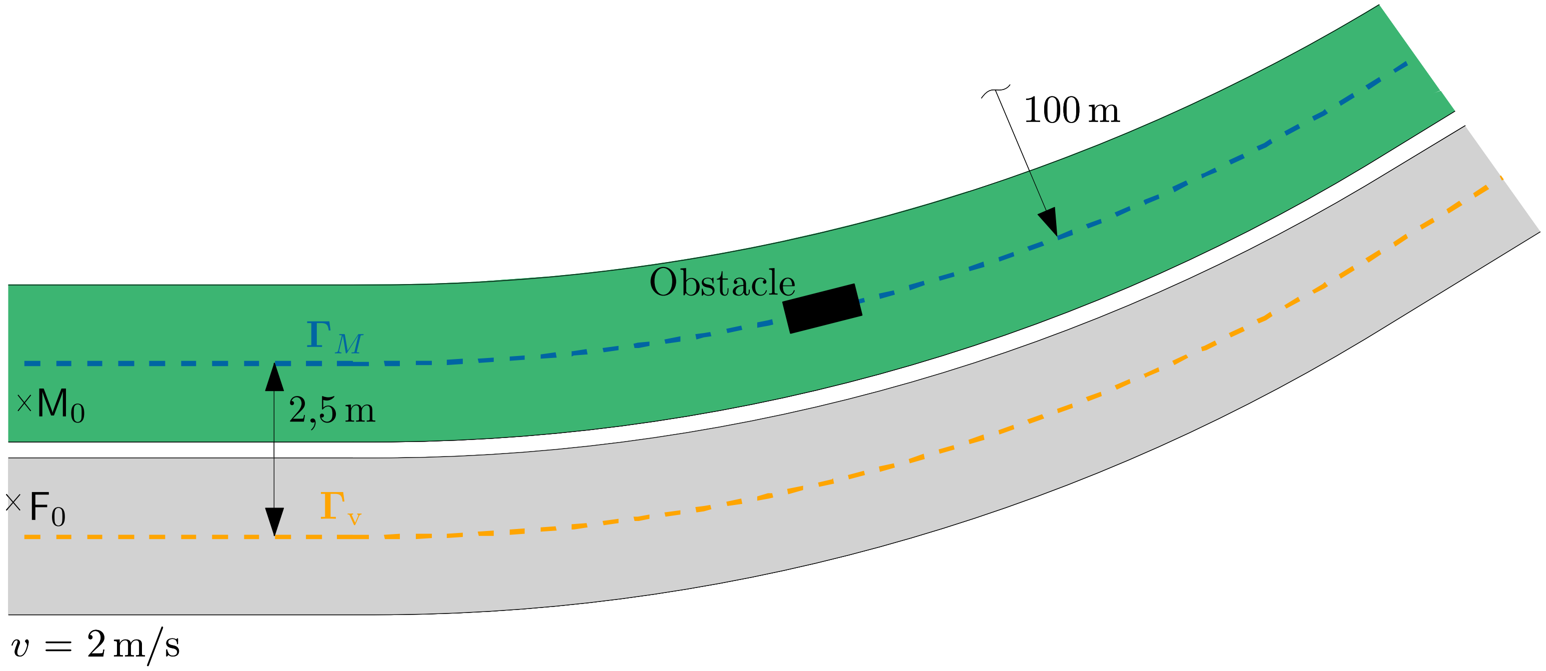
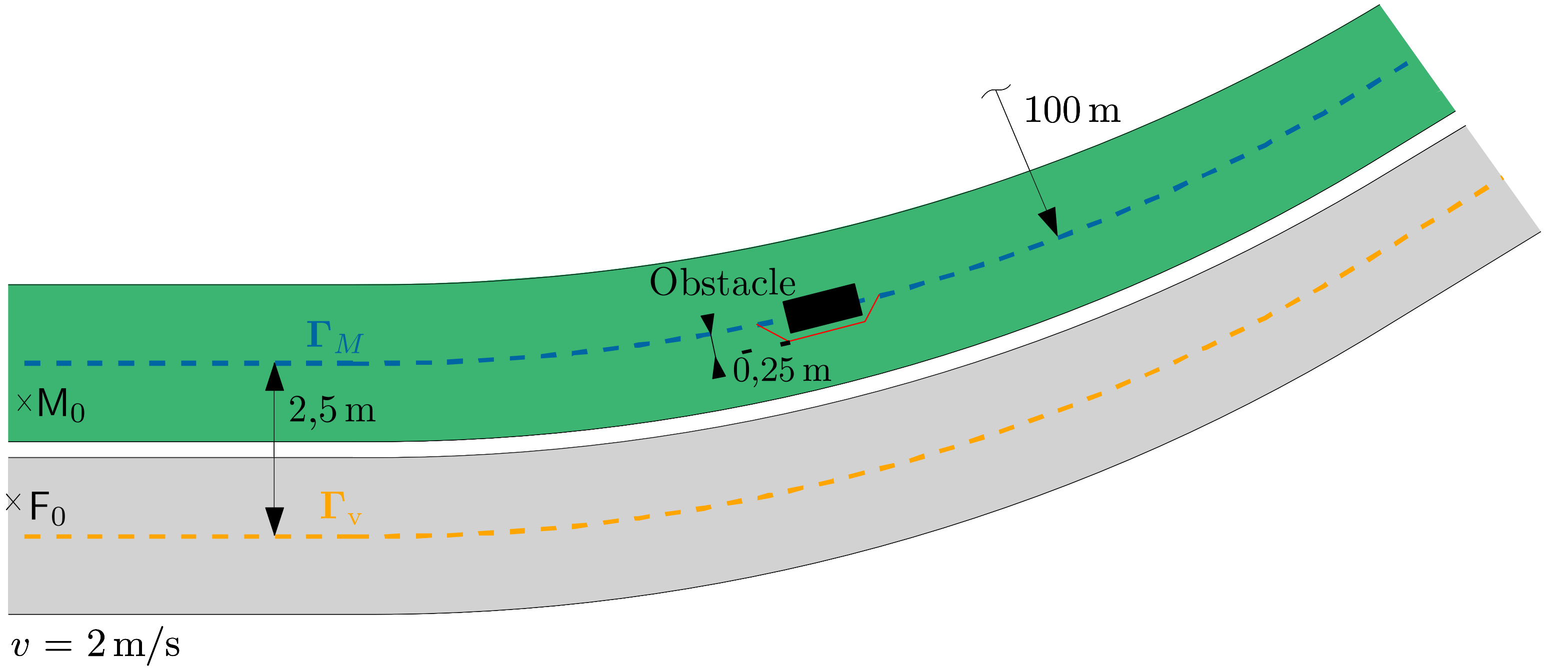
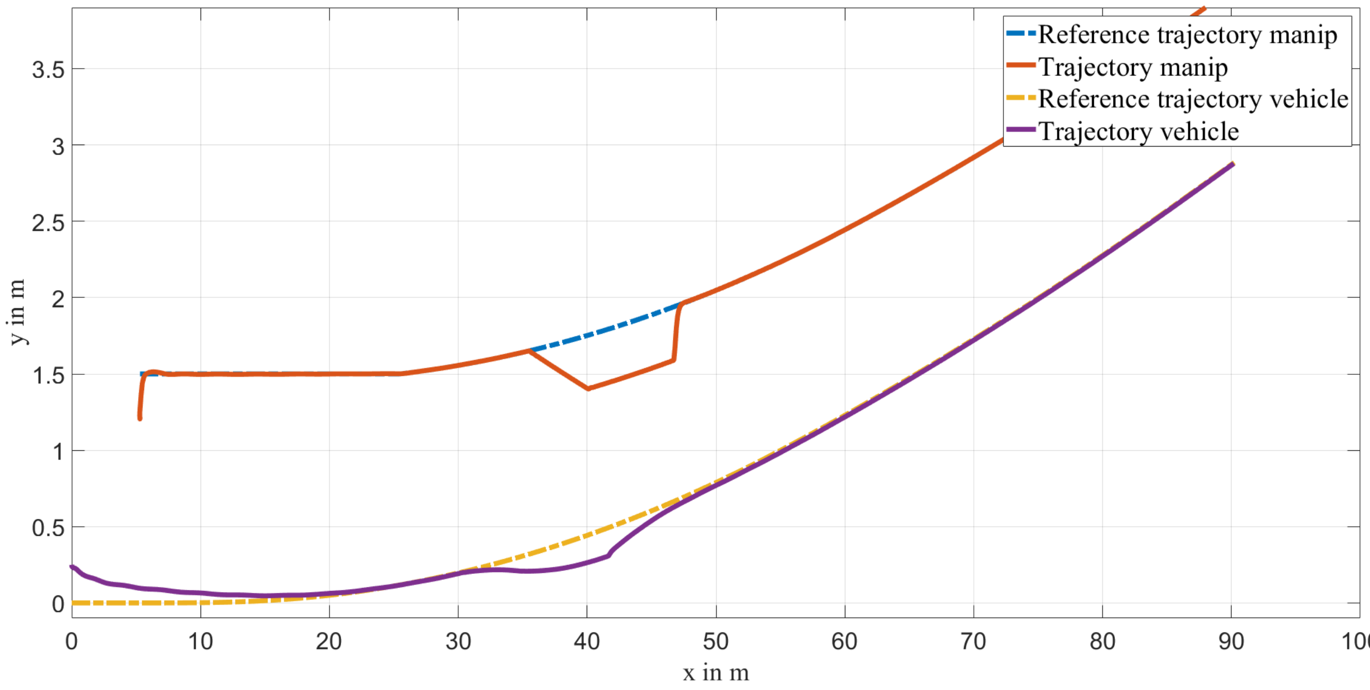


Results

Scenario 2: References Γ_v and Γ_M



Results



Reference trajectory manip
Trajectory manip
Reference trajectory vehicle
Trajectory vehicle
y in m
x in m
3.5
3
2.5
2
1.5
1
0.5
0
0
10
20
30
40
50
60
70
80
90
100

Conclusion and further research

- Introduction
- State of the Art
- Control model concept for vehicle-manipulator systems
- Simulation results
- **Conclusion and further research**

Conclusion and further research work

Conclusion:

- Control model in Frenét-Frame for large vehicle-manipulator
- Implementation a MPC for position control
- Validation with simulations

Further research:

- Systematic method for the parameter tuning of the MPC
- Development of a control model for three-dimensional trajectories

Citations

- [Fodor 2015] S. Fodor, C. Vazquez, and L. Freidovich, "*Automation of slewing motions for forestry cranes*" in 2015 15th International Conference on Control, Automation and Systems (ICCAS). IEEE, 2015.
- [Hera 2015] P. L. Hera and D. O. Morales, "*Model-based development of control Systems for forestry cranes*" Journal of Control Science and Engineering, vol. 2015, pp. 1–15, 2015.
- [Mashali 2014] M. Mashali et al., "*Design, implementationand evaluation of a motion control scheme for mobile platforms with high uncertainties*" in 5th IEEE RAS/EMBS IEEE, 2014
- [White 2009] G. D. White et al. "*Experimental evaluation of dynamic redundancy resolution in a nonholonomic wheeled mobile manipulator*" IEEE/ASME Transactions on Mechatronics, vol. 14, no. 3, pp. 349–357, 2009
- [Kovacs 2014] R. Kovacs et al.: "*Modeling of commercial vehicles for vehicle dynamics control development*," in 2014 IEEE 9th IEEE International Symposium on Applied Computational Intelligence and Informatics (SACI). IEEE, 2014
- [Ruderman 2017] M. Ruderman, "*Full- and reduced-order model of hydraulic cylinder for motion control*" in IECON 2017 - 43rd Annual Conference of the IEEE Industrial Electronics Society, IEEE, 2017
- [Borrelli 2011] Borrelli et al.: Predictive Control for Linear and Hybrid Systems. Cambridge University Press, 2011.

Thank you for your attention!

Model predictive control and trajectory optimization of large vehicle-manipulators

Balint Varga, Selina Meier, Stefan Schwab, Sören Hohmann
18. March 2019

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CONTROL MODEL

FZI FORSCHUNGSZENTRUM INFORMATIK

Control model

- Differential equation of the vehicle
  - Position – Description of F

$$\boldsymbol{r}_{OF} = \boldsymbol{r}_{OR} + \boldsymbol{r}_{RF}$$
$$= \boldsymbol{r}_{OR} + d\boldsymbol{j}_R$$

$$\frac{\mathrm{d}\boldsymbol{r}_{OR}}{\mathrm{d}t} = \dot{s}\boldsymbol{i}_R \qquad \boldsymbol{j}_R = -\dot{\theta}_r \boldsymbol{i}_R = -\dot{s}\kappa_r(s)\boldsymbol{i}_R$$

$$\frac{\partial \boldsymbol{r}_{OF}}{\partial t} = \frac{\partial \boldsymbol{r}_{OR}}{\partial t} + \dot{d}\boldsymbol{j}_R - d\kappa_r(s)\dot{s}\boldsymbol{i}_R$$
$$= \dot{s}(1 - d\kappa_r(s))\boldsymbol{i}_R + \dot{d}\boldsymbol{j}_R$$

$$\frac{\partial \boldsymbol{r}_{OF}}{\partial t} = v\boldsymbol{i}_F$$
$$= v\cos\Delta\theta\boldsymbol{i}_R + v\sin\Delta\theta\boldsymbol{j}_R$$

$$\dot{s} = v\frac{\cos(\Delta\theta)}{1 - d\kappa_r}$$
$$\dot{d} = v\sin(\Delta\theta)$$

Control model

- Differential equation of the vehicle

$$\dot{s} = v\frac{\cos(\Delta\theta)}{1 - d\kappa_r} \qquad \xrightarrow[d\kappa_r \ll 1]{\Delta\theta = \theta - \theta_r \text{ klein}} \qquad \dot{s} \approx v$$

$$\dot{d} = v\sin(\Delta\theta)$$

  - Orientation $\Delta\theta = \theta - \theta_r$

$$\kappa_r(s) = \frac{\partial\theta_r}{\partial s} \longrightarrow \dot{s}\kappa_r(s) = \dot{\theta}_r$$

$$\Delta\dot{\theta} = \dot{\theta} - \dot{\theta}_r = v\frac{\tan(\delta)}{2L} - \dot{s}\kappa_r(s) \longrightarrow \Delta\dot{\theta} = v\left(\frac{\tan(\delta)}{2L} - \kappa_r\right)$$

$$\boxed{\begin{aligned} \dot{d} &= v\sin(\Delta\theta) \\ \Delta\dot{\theta} &= v\left(\frac{\tan(\delta)}{2L} - \kappa_r\right) \end{aligned}}$$

$$\tilde{\delta} = \frac{\tan(\delta)}{2L}$$

Control model

- Differential equation of the manipulator
  - Position – Description of M

$$\boldsymbol{r}_{OM} = \boldsymbol{r}_{OF} + (2L + l_P)\ \boldsymbol{i}_F + a\cos\alpha\ \boldsymbol{i}_F + a\sin\alpha\ \boldsymbol{j}_F$$

$$\rightarrow \frac{\partial \boldsymbol{r}_{OM}}{\partial t} = (v(t) + \dot{\alpha}\cos\alpha - a\sin\alpha\ (\dot{\alpha} + \dot{\theta}))\ \boldsymbol{i}_F$$

$$+ ((2L + l_P)\ \dot{\theta} + \dot{\alpha}\sin\alpha + a\cos\alpha\ (\dot{\alpha} + \dot{\theta}))\ \boldsymbol{j}_F$$

$$:= v_{xM}\ \boldsymbol{i}_F + v_{yM}\ \boldsymbol{j}_F$$

$$\rightarrow \frac{\partial \boldsymbol{r}_{OM}}{\partial t} = \boxed{(v_{xM}\cos\Delta\theta_M - v_{yM}\sin\Delta\theta_M)}\ \boldsymbol{i}_{RM} + \boxed{(v_{xM}\sin\Delta\theta_M + v_{yM}\cos\Delta\theta_M)}\ \boldsymbol{j}_{RM}$$

$$\text{Differenzwinkel}\ \Delta\theta_M = \theta - \theta_{rM}$$

$$\boldsymbol{r}_{OM} = \boldsymbol{r}_{OO_{RM}} + d_M\ \boldsymbol{j}_{RM}$$

$$\rightarrow \frac{\partial \boldsymbol{r}_{OM}}{\partial t} = \dot{s}_M(1 - d_M\kappa_{rM})\ \boldsymbol{i}_{RM} + \dot{d}_M\ \boldsymbol{j}_{RM} \approx \boxed{\dot{s}_M}\ \boldsymbol{i}_{RM} + \boxed{\dot{d}_M}\ \boldsymbol{j}_{RM}$$

Control model

- Differential equation of the manipulator

$$\dot{d}_M = v_{xM} \sin \Delta\theta_M + v_{yM} \cos \Delta\theta_M$$
$$= \sin \Delta\theta_M (v(t) + \dot{a} \cos\alpha - a \sin\alpha \ \dot{\alpha} - a \sin\alpha \ \dot{\theta})$$
$$+ \cos \Delta\theta_M ((2L + l_P)\dot{\theta} + a \cos\alpha \ \dot{\theta} + \dot{a} \sin\alpha + a \cos\alpha \ \dot{\alpha})$$

$$\dot{s}_M = v_{xM} \cos \Delta\theta_M - v_{yM} \sin \Delta\theta_M$$
$$= \cos \Delta\theta_M (v(t) + \dot{a} \cos\alpha - a \sin\alpha \ \dot{\alpha} - a \sin\alpha \ \dot{\theta})$$
$$- \sin \Delta\theta_M ((2L + l_P)\dot{\theta} + a \cos\alpha \ \dot{\theta} + \dot{a} \sin\alpha + a \cos\alpha \ \dot{\alpha})$$

$$\Delta\dot{\theta}_M = \dot{\theta} - \dot{\theta}_{rM}$$
$$= \dot{\theta} - \kappa_{rM} \dot{s}_M$$
$$= \dot{\theta} - \kappa_{rM} (\cos \Delta\theta_M (v(t) + \dot{a} \cos\alpha - a \sin\alpha \ \dot{\alpha} - a \sin\alpha \ \dot{\theta})$$
$$- \sin \Delta\theta_M ((2L + l_P)\dot{\theta} + a \cos\alpha \ \dot{\theta} + \dot{a} \sin\alpha + a \cos\alpha \ \dot{\alpha}))$$

Control model

- Nonlinear state equations

$$\boldsymbol{x}^{\mathrm{T}} = [d, \Delta\theta, d_M, \Delta\theta_M, a, \alpha, \kappa_r, \kappa_{rM}] \qquad \boldsymbol{u}^{\mathrm{T}} = [\tilde{\delta}, \dot{a}, \dot{\alpha}] := [u_1, u_2, u_3]$$

$$\boldsymbol{z}^{\mathrm{T}} = [\dot{\kappa}_r, \dot{\kappa}_{rM}] := [z_1, z_2, z_3] \qquad \boldsymbol{y}^{\mathrm{T}} = [d, d_V, d_M, \Delta\alpha]$$

$$\dot{d} = v(t) \sin \Delta\theta \tag{1a}$$

$$\Delta\dot{\theta} = v(t)(u_1 - \kappa_r) \tag{1b}$$

$$\begin{aligned}\dot{d}_M = {} & \sin \Delta\theta_M (v(t) + u_2 \cos\alpha - (v(t)u_1 + u_3)a \sin\alpha) \\ & + \cos \Delta\theta_M ((2L + l_P)v(t)u_1 + u_2 \sin\alpha + (v(t)u_1 + u_3)a\cos\alpha)\end{aligned} \tag{1c}$$

$$\begin{aligned}\Delta\dot{\theta}_M = {} & v(t)u_1 - \kappa_{rM}(\cos \Delta\theta_M (v(t) + u_2 \cos\alpha - (v(t)u_1 + u_3)a \sin\alpha) \\ & - \sin \Delta\theta_M ((2L + l_P)v(t)u_1 + u_2 \sin\alpha + (v(t)u_1 + u_3)a\cos\alpha)\end{aligned} \tag{1d}$$

$$\dot{\alpha} = u_3 \tag{1e}$$

$$\dot{\kappa}_r = z_1 \tag{1f}$$

$$\dot{\kappa}_{rM} = z_2 \tag{1g}$$

Control model

- Equilibrium and linearization
  for $\boldsymbol{u} = \mathbf{0},\ \boldsymbol{z} = \mathbf{0}$

$$\longrightarrow \boldsymbol{x}_e^{\mathrm{T}} = [d_e, \Delta\theta_e, d_{Me}, \Delta\theta_{Me}, \mathrm{a}_e, \alpha_e, \kappa_{re}, \kappa_{rMe}]$$
$$= [0, 0, 0, 0, \mathrm{a}_e, \alpha_e, 0, 0] \qquad v(t) > 0 \text{ und } a_e > 0$$

$$\longrightarrow \Delta\dot{d} = v(t)\Delta(\Delta\theta) \tag{1a}$$
$$\Delta(\Delta\dot{\theta}) = v(t)(\Delta u_1 - \Delta\kappa_r) \tag{1b}$$
$$\Delta\dot{d}_M = v(t)\Delta(\Delta\theta_M) + (2L + l_P + a_e\cos\alpha_e)v(t)\Delta u_1 + \sin\alpha_e\Delta u_2 + a_e\cos\alpha_e\Delta u_3 \tag{1c}$$
$$\Delta(\Delta\dot{\theta}_M) = v(t)(\Delta u_1 - \Delta\kappa_{rM}) \tag{1d}$$
$$\Delta\dot{\alpha} = \Delta u_3 \tag{1e}$$
$$\Delta\dot{\kappa}_r = \Delta z_1 \tag{1f}$$
$$\Delta\dot{\kappa}_{rM} = \Delta z_2 \tag{1g}$$

Control model

- Linear state space model

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_c(t)\boldsymbol{x}(t) + \boldsymbol{B}_c(t)\boldsymbol{u}(t) + \boldsymbol{E}_c\boldsymbol{z}(t)$$
$$\boldsymbol{y}(t) = \boldsymbol{C}_c\boldsymbol{x}(t)$$

$$\boldsymbol{A}_c(t) = \begin{bmatrix} 0 & v(t) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -v(t) & 0 \\ 0 & 0 & 0 & v(t) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -v(t) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \boldsymbol{E}_c = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\boldsymbol{B}_c(t) = \begin{bmatrix} 0 & 0 & 0 \\ v(t) & 0 & 0 \\ (2L + l_P + a_e \cos\alpha_e)v(t) & \sin\alpha_e & a_e\cos\alpha_e \\ v(t) & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \boldsymbol{C}_c = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & L & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

CONTROL DESIGN

MPC design

MPC design

Model predictive control - fundamentals

- Prediction model
- Optimizer:
  - Cost function
  - Constraints

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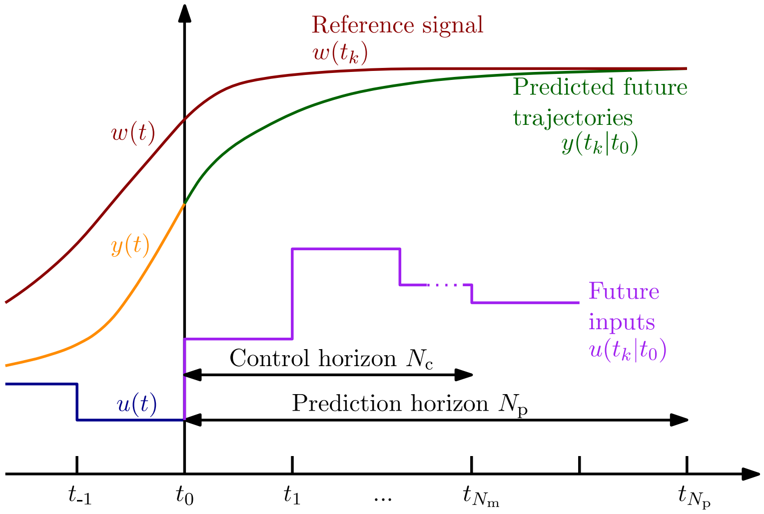


Inspired by [Borrelli 2011]

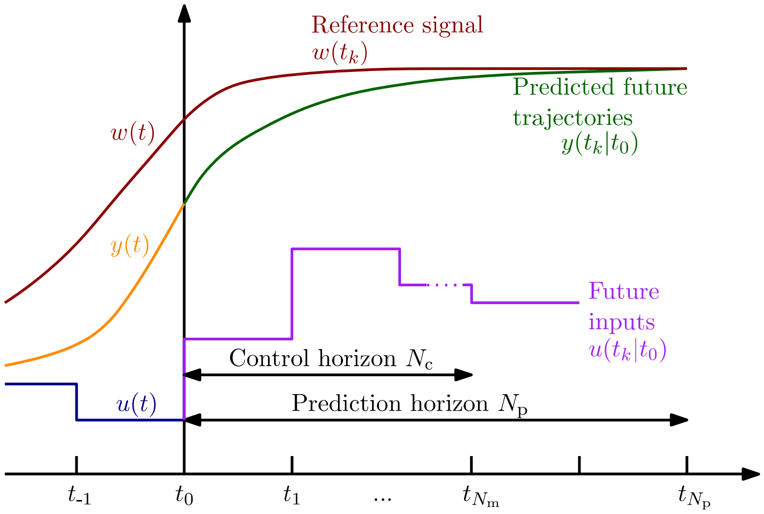
State of the Art | Simulation environment | **Control concept** | Results | Conclusion & further research

MPC design

Model predictive control - fundamentals

- Prediction model
- Optimizer:
  - Cost function
  - Constraints

$$\min J(x,u)$$
$$\text{s.t. } \dot{x} = f(x,u)$$
$$c(x,u) \leq 0$$



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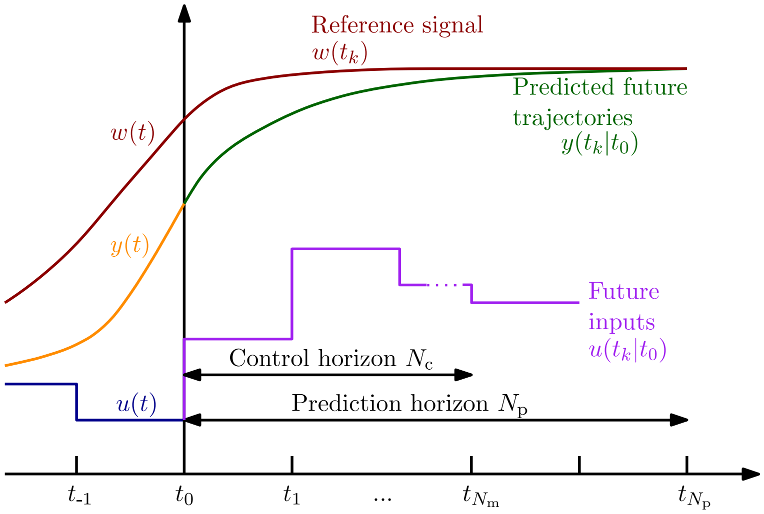

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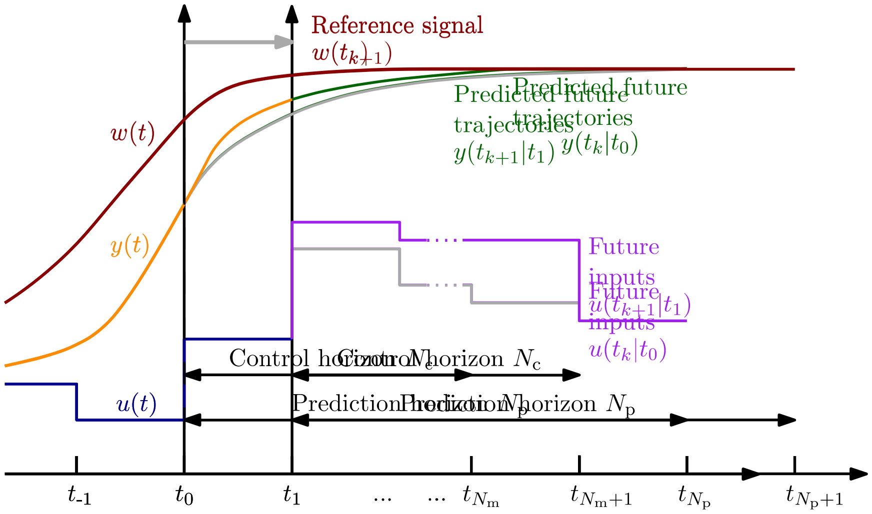

Inspired by [Borrelli 2011]

Control design

- Time continuous system model

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_c \boldsymbol{x}(t) + \boldsymbol{B}_c \boldsymbol{u}(t) + \boldsymbol{E}_c \boldsymbol{z}(t)$$
$$\boldsymbol{y}(t) = \boldsymbol{C}_c \boldsymbol{x}(t)$$

- d model with *T*

$$\boldsymbol{A}(k) = e^{\boldsymbol{A}_c T} \approx \boldsymbol{I} + \boldsymbol{A}_c T + \frac{1}{2} \boldsymbol{A}_c^2 T^2$$
$$\boldsymbol{B}(k) = \int_0^T e^{\boldsymbol{A}_c (T-\tau)} \boldsymbol{B}_c \mathrm{d}\tau$$
$$\boldsymbol{E}(k) = \int_0^T e^{\boldsymbol{A}_c (T-\tau)} \boldsymbol{E}_c \mathrm{d}\tau$$
$$\boldsymbol{C}(k) = \boldsymbol{C}_c$$

- Time discrete prediction model

$$\boldsymbol{x}(k+1) = \boldsymbol{A}(k)\boldsymbol{x}(k) + \boldsymbol{B}(k)\boldsymbol{u}(k) + \boldsymbol{E}(k)\boldsymbol{z}(k)$$
$$\boldsymbol{y}(k) = \boldsymbol{C}(k)\boldsymbol{x}(k)$$

Control design

- Time discrete prediction model:

$$\boldsymbol{x}(k+1) = \boldsymbol{A}(k)\boldsymbol{x}(k) + \boldsymbol{B}(k)\boldsymbol{u}(k) + \boldsymbol{E}(k)\boldsymbol{z}(k)$$
$$\boldsymbol{y}(k) = \boldsymbol{C}(k)\boldsymbol{x}(k)$$

- Cost function

$$J(\boldsymbol{x}, \boldsymbol{u}) = \sum_{k=1}^{N} \boldsymbol{x}_k^{\mathrm{T}} \boldsymbol{Q} \boldsymbol{x}_k + \sum_{k=0}^{N-1} \boldsymbol{u}_k^{\mathrm{T}} \boldsymbol{R} \boldsymbol{u}_k$$

$$\text{mit} \quad \boldsymbol{Q} = \mathrm{diag}(G_d, G_{\Delta\theta}, G_{dM}, 0, G_{\Delta\alpha}, 0, 0) \qquad \boldsymbol{Q} \geq 0$$
$$\text{und} \quad \boldsymbol{R} = \mathrm{diag}(G_{\tilde{\delta}}, G_{\dot{a}}, G_{\dot{\alpha}}) \qquad \boldsymbol{R} > 0$$

- Future vector sequences

$$\mathbf{x} = [\boldsymbol{x}_1^{\mathrm{T}}, \dots, \boldsymbol{x}_N^{\mathrm{T}}]^{\mathrm{T}}$$
$$\mathbf{u} = [\boldsymbol{u}_0^{\mathrm{T}}, \boldsymbol{u}_1^{\mathrm{T}}, \dots, \boldsymbol{u}_{N-1}^{\mathrm{T}}]^{\mathrm{T}}$$
$$\mathbf{z} = [\boldsymbol{z}_0^{\mathrm{T}}, \boldsymbol{z}_1^{\mathrm{T}}, \dots, \boldsymbol{z}_{N-1}^{\mathrm{T}}]^{\mathrm{T}}$$
$$\mathbf{y} = [\boldsymbol{y}_1^{\mathrm{T}}, \dots, \boldsymbol{y}_N^{\mathrm{T}}]^{\mathrm{T}}$$

Control design

- Prädiktion

$$\mathbf{x} = \boldsymbol{\mathcal{A}} \boldsymbol{x}_0 + \boldsymbol{\mathcal{B}} \mathbf{u} + \boldsymbol{\mathcal{E}} \mathbf{z}$$

$$\boldsymbol{\mathcal{A}} = \begin{bmatrix} \boldsymbol{A} \\ \boldsymbol{A}^2 \\ \vdots \\ \boldsymbol{A}^N \end{bmatrix}, \quad \boldsymbol{\mathcal{B}} = \begin{bmatrix} \boldsymbol{B} & \boldsymbol{0} & \dots & \boldsymbol{0} \\ \boldsymbol{A}\boldsymbol{B} & \boldsymbol{B} & \dots & \boldsymbol{0} \\ \vdots & \ddots & \ddots & \vdots \\ \boldsymbol{A}^{N-1}\boldsymbol{B} & \dots & \boldsymbol{A}\boldsymbol{B} & \boldsymbol{B} \end{bmatrix}, \quad \boldsymbol{\mathcal{E}} = \begin{bmatrix} \boldsymbol{E} & \boldsymbol{0} & \dots & \boldsymbol{0} \\ \boldsymbol{A}\boldsymbol{E} & \boldsymbol{E} & \dots & \boldsymbol{0} \\ \vdots & \ddots & \ddots & \vdots \\ \boldsymbol{A}^{N-1}\boldsymbol{E} & \dots & \boldsymbol{A}\boldsymbol{E} & \boldsymbol{E} \end{bmatrix}$$

$$\mathbf{y} = \boldsymbol{\mathcal{C}}(\boldsymbol{\mathcal{A}} \boldsymbol{x}_0 + \boldsymbol{\mathcal{B}} \mathbf{u} + \boldsymbol{\mathcal{E}} \mathbf{z}) \quad \text{mit} \quad \boldsymbol{\mathcal{C}} = \text{blkdiag}(\underbrace{\boldsymbol{C}, \dots, \boldsymbol{C}}_{N-\text{mal}})$$

- Kostenfunktion

$$J(\mathbf{x}, \mathbf{u}) = \mathbf{x}^{\mathrm{T}} \boldsymbol{\mathcal{Q}} \mathbf{x} + \mathbf{u}^{\mathrm{T}} \boldsymbol{\mathcal{R}} \mathbf{u} \quad \text{mit} \quad \boldsymbol{\mathcal{Q}} = \text{blkdiag}\,(\underbrace{\boldsymbol{Q}, \dots, \boldsymbol{Q}}_{N-\text{mal}}) \quad \text{und} \quad \boldsymbol{\mathcal{R}} = \text{blkdiag}\,(\underbrace{\boldsymbol{R}, \dots, \boldsymbol{R}}_{N-\text{mal}})$$

Control design

- Cost function

$$J(\boldsymbol{x}_0, \mathbf{z}, \mathbf{u}) = \mathbf{u}^{\mathrm{T}} \boldsymbol{H} \mathbf{u} + 2(\boldsymbol{x}_0^{\mathrm{T}} \boldsymbol{F} + \mathbf{z}^{\mathrm{T}} \boldsymbol{G}) \mathbf{u}$$

$$\text{with} \quad \boldsymbol{H} = \boldsymbol{\mathcal{B}}^{\mathrm{T}} \boldsymbol{\mathcal{Q}} \boldsymbol{\mathcal{B}} + \boldsymbol{\mathcal{R}} \,, \quad \boldsymbol{F} = \boldsymbol{\mathcal{A}}^{\mathrm{T}} \boldsymbol{\mathcal{Q}} \boldsymbol{\mathcal{B}} \text{ and } \quad \boldsymbol{G} = \boldsymbol{\mathcal{E}}^{\mathrm{T}} \boldsymbol{\mathcal{Q}} \boldsymbol{\mathcal{B}}$$

→ Convex optimization problem

$$\boldsymbol{H} > \mathbf{0} \quad \text{da} \quad \boldsymbol{\mathcal{R}} > \mathbf{0} \quad \text{und} \quad \boldsymbol{\mathcal{B}}^{\mathrm{T}} \boldsymbol{\mathcal{Q}} \boldsymbol{\mathcal{B}} \geq \mathbf{0}$$

→ $\mathbf{u}^*$ is unique

$$\nabla_{\mathbf{u}} J(\boldsymbol{x}_0, \mathbf{u}) = 2\boldsymbol{H}\mathbf{u} + 2\boldsymbol{F}^{\mathrm{T}} \boldsymbol{x}_0 = \mathbf{0}$$

$$\mathbf{u}^*(\boldsymbol{x}_0) = -\boldsymbol{H}^{-1} \boldsymbol{F}^{\mathrm{T}} \boldsymbol{x}_0$$

Control design

- Constraints of the outputs

$$\mathbf{y} = [\boldsymbol{y}_1^{\mathrm{T}}, \dots, \boldsymbol{y}_N^{\mathrm{T}}]^{\mathrm{T}}$$

$$\mathbf{y}_{\min} \leq \mathbf{y} \leq \mathbf{y}_{\max}$$

$$\mathbf{y} = \boldsymbol{\mathcal{C}}(\boldsymbol{\mathcal{A}}\boldsymbol{x}_0 + \boldsymbol{\mathcal{B}}\mathbf{u}) \quad \text{mit} \quad \boldsymbol{\mathcal{C}} = \operatorname{blkdiag}(\underbrace{\boldsymbol{C}, \dots, \boldsymbol{C}}_{N-\text{mal}})$$

$$\underbrace{\begin{bmatrix} \boldsymbol{\mathcal{C}}\boldsymbol{\mathcal{B}} \\ -\boldsymbol{\mathcal{C}}\boldsymbol{\mathcal{B}} \end{bmatrix}}_{\boldsymbol{A}_c} \mathbf{u} \leq \underbrace{\begin{bmatrix} \mathbf{y}_{\max} - \boldsymbol{\mathcal{C}}\boldsymbol{\mathcal{A}}\boldsymbol{x}_0 \\ -\mathbf{y}_{\min} + \boldsymbol{\mathcal{C}}\boldsymbol{\mathcal{A}}\boldsymbol{x}_0 \end{bmatrix}}_{\boldsymbol{b}_c}$$

$\rightarrow$ Quadratic program

$$\boxed{\begin{aligned} \min_{\mathbf{u}} \quad & J(\boldsymbol{x}_0, \mathbf{u}) = \mathbf{u}^{\mathrm{T}}\boldsymbol{H}\mathbf{u} + 2\boldsymbol{x}_0^{\mathrm{T}}\boldsymbol{F}\mathbf{u} \\ \text{s.t.} \quad & \boldsymbol{A}_c\mathbf{u} \leq \boldsymbol{b}_c \\ & \mathbf{u}_{\min} \leq \mathbf{u} \leq \mathbf{u}_{\max} \end{aligned}}$$