Death and revival of chaos

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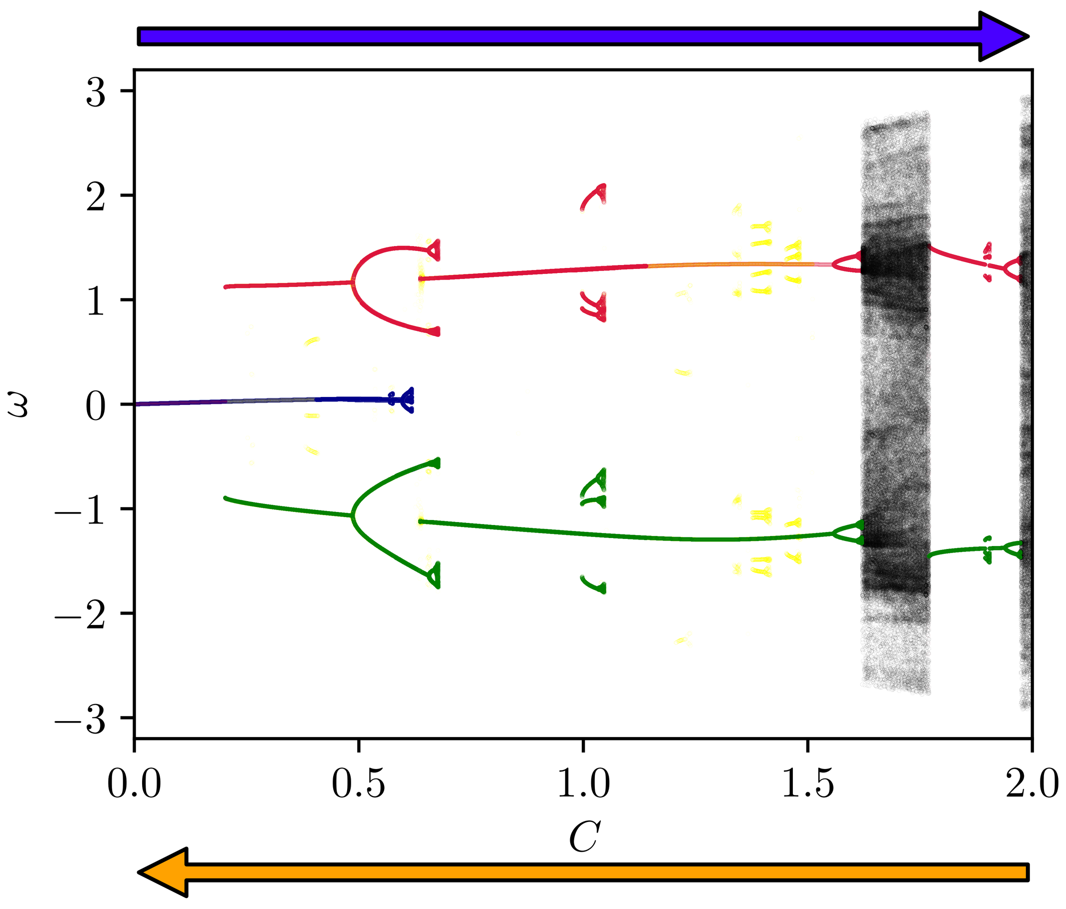
Introduction

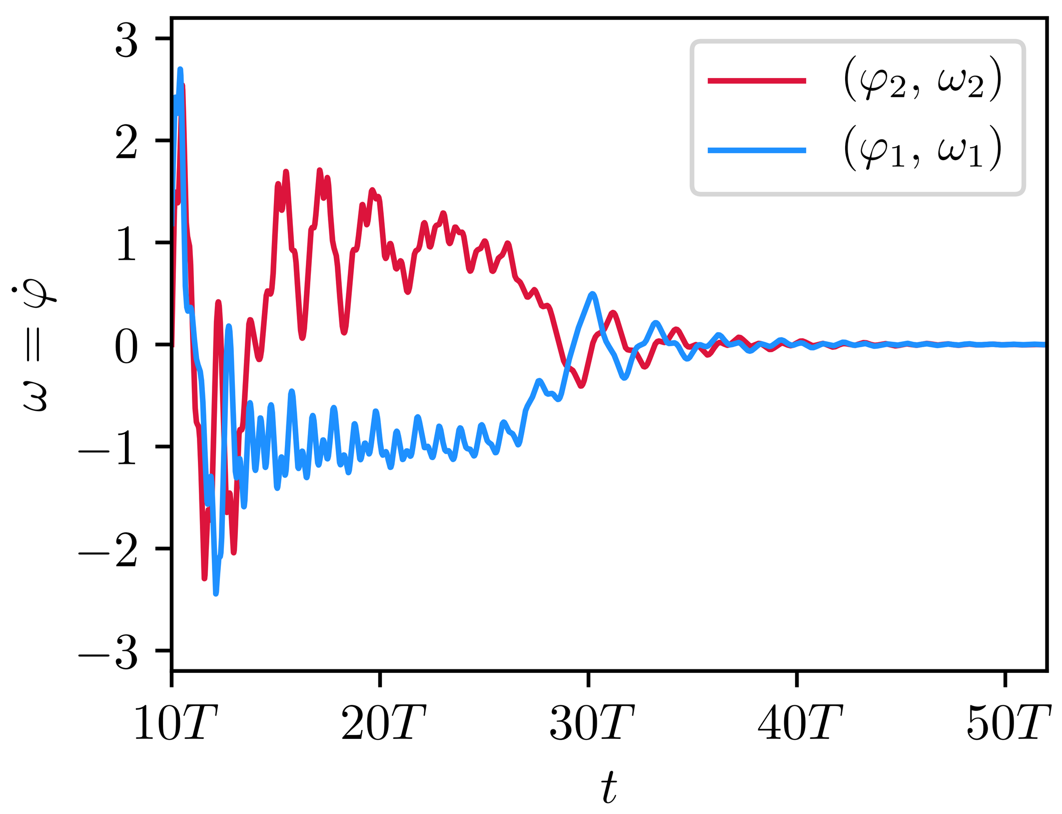
- Study of "climate change scenarios" in dynamical systems
- Simple paradigmatic model with a continuous parameter shift

$$\ddot{\varphi} = -\gamma^2 \sin\varphi - 2\beta\dot{\varphi} + C(t)\cos(\varphi)\cos(t)$$

- Pendulum with moving suspension and decreasing driving, $C(t)$ [1]
- $\omega \equiv \dot{\varphi}$ angular velocity, $T = 2\pi$
- $C(t) = C$ =const. **frozen-in system**
- Frozen in attractors: **Bifurcation diagram**



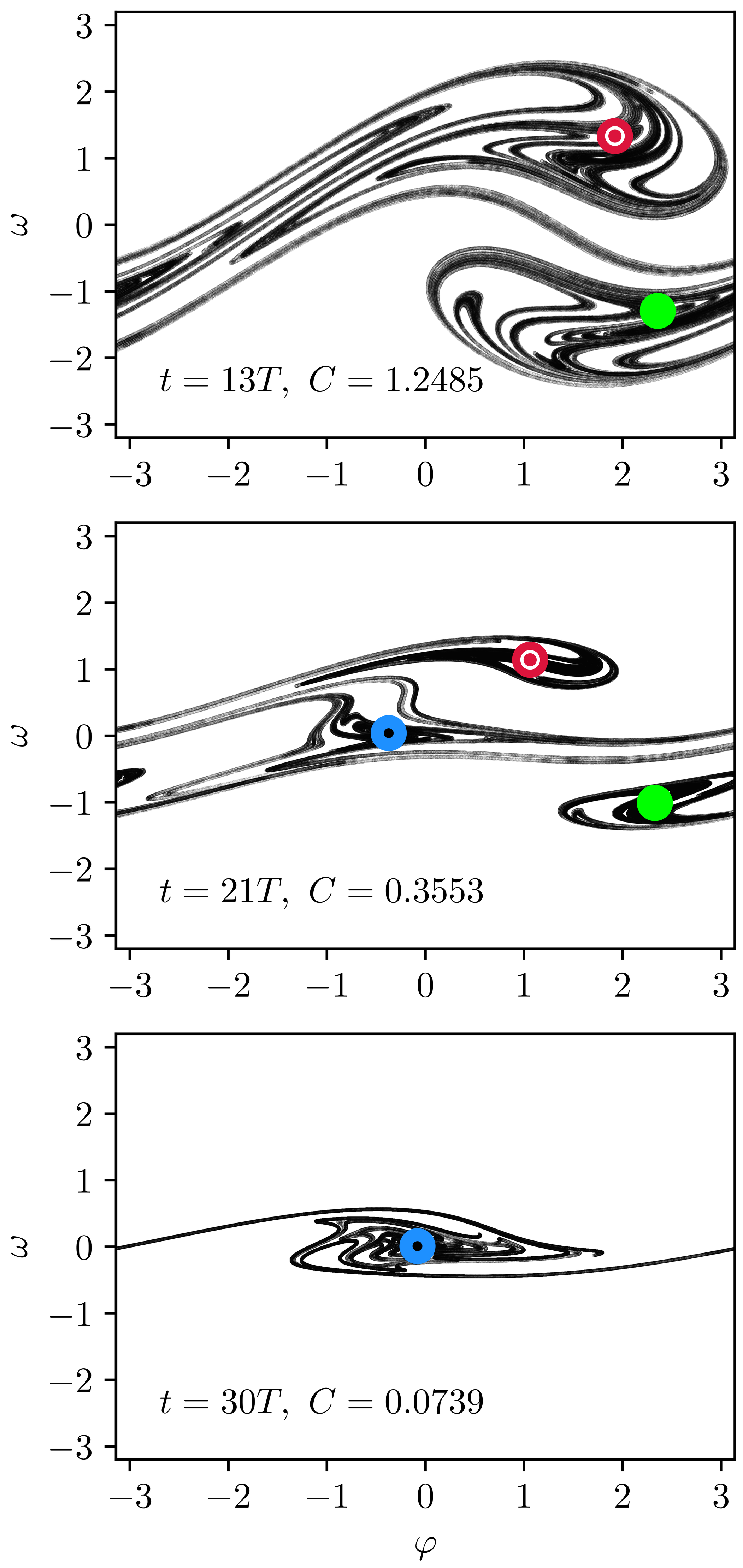
Trajectories



- Small difference in initial conditions ⇒ **internal variability**
- Individual trajectories are **not representative** of the switching off process

Ensemble approach

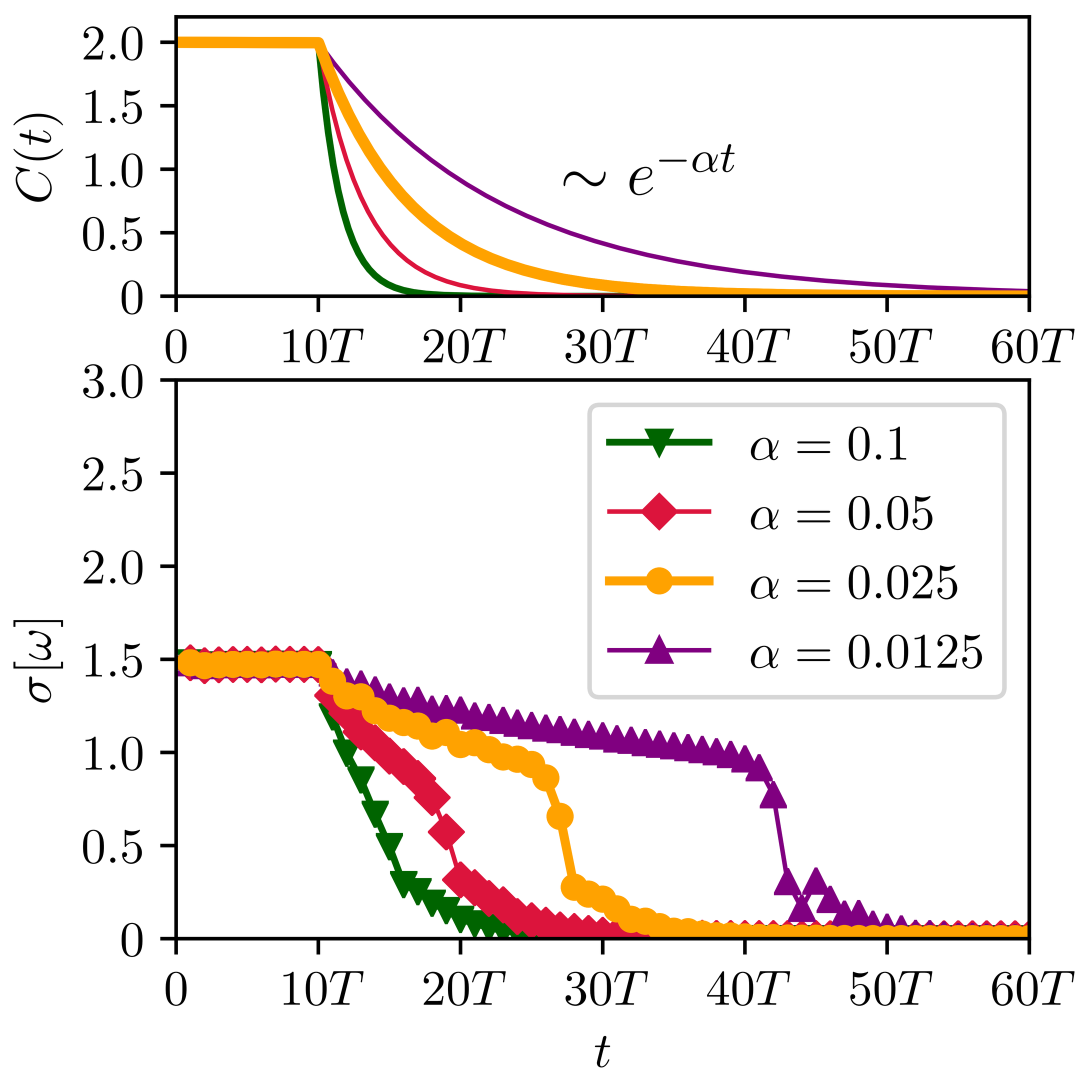
- Following an ensemble of 10^7 trajectories in **phase space**
- Applying a **stroboscopic map**



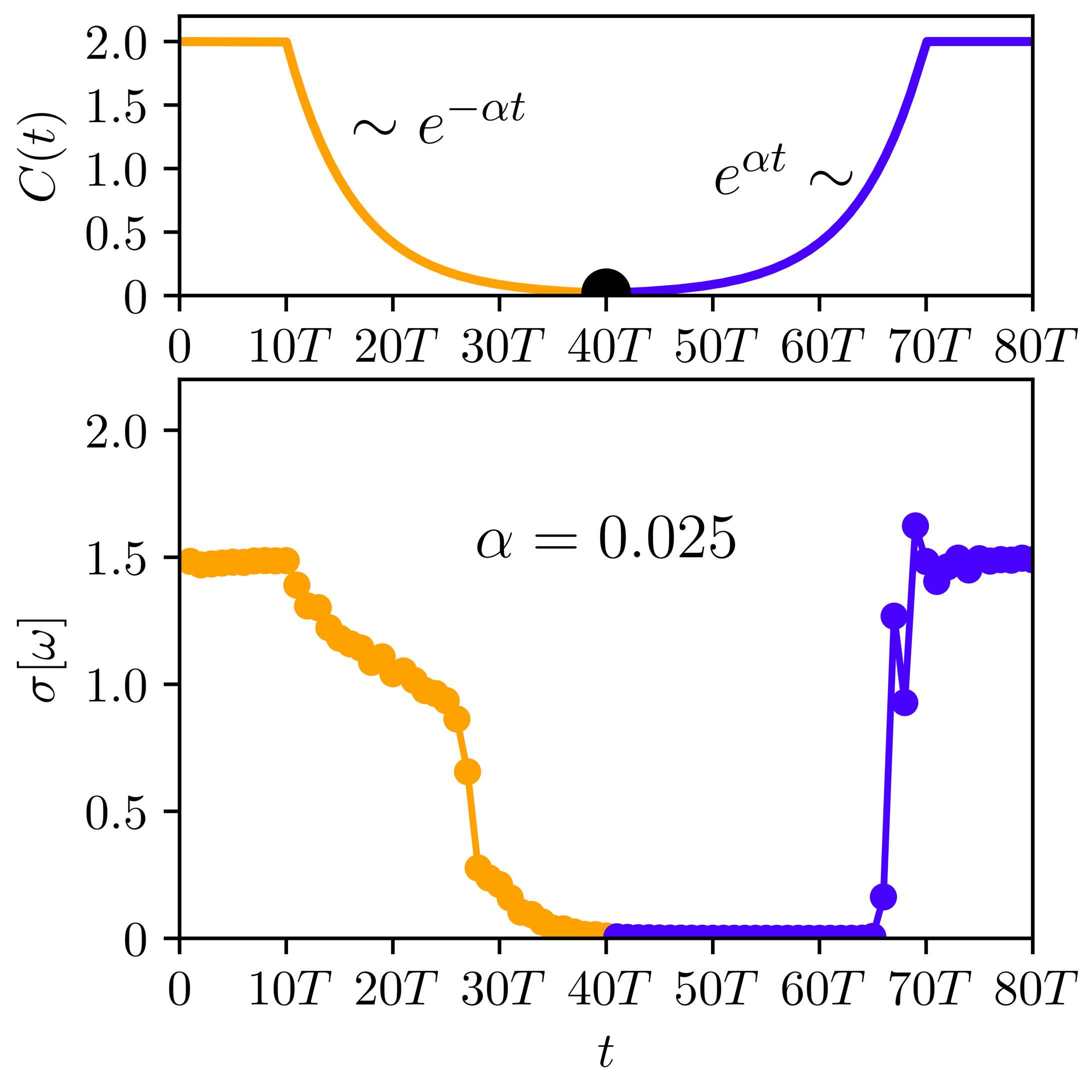
- The ensemble is never similar to the frozen-in attractors (●●●)
- Extension of the ensemble ∼ variability

Statistical properties

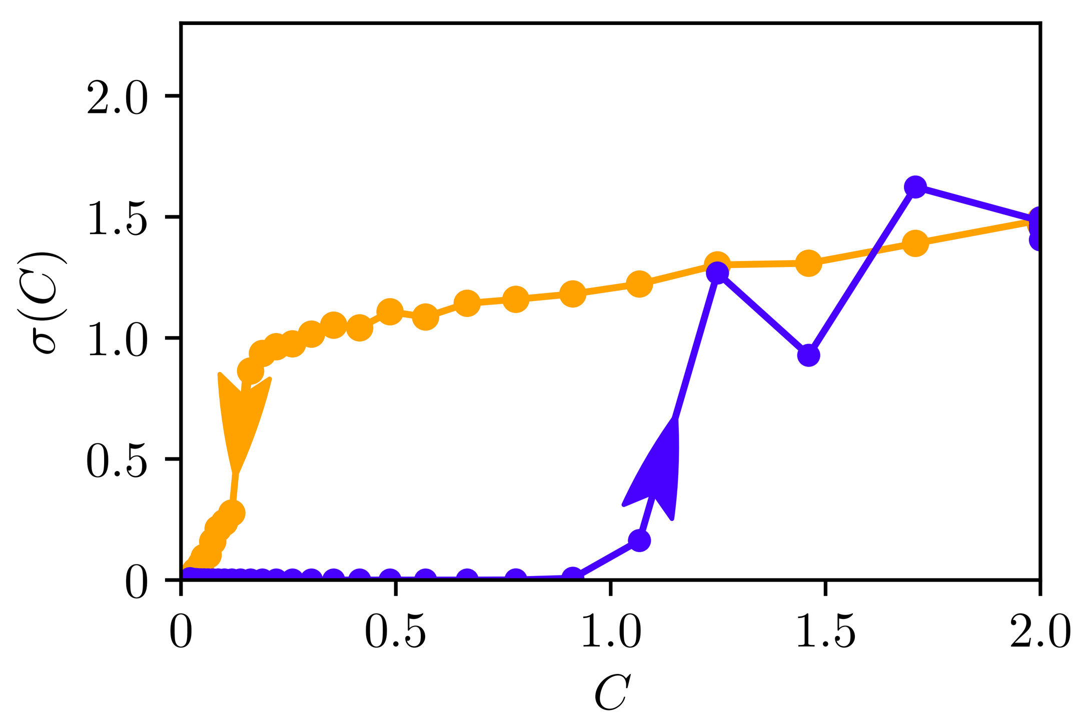
- Standard deviation
- $\sigma[\omega] = \left(\langle\omega^2\rangle - \langle\omega\rangle^2\right)^{1/2}$
- Measures internal variability



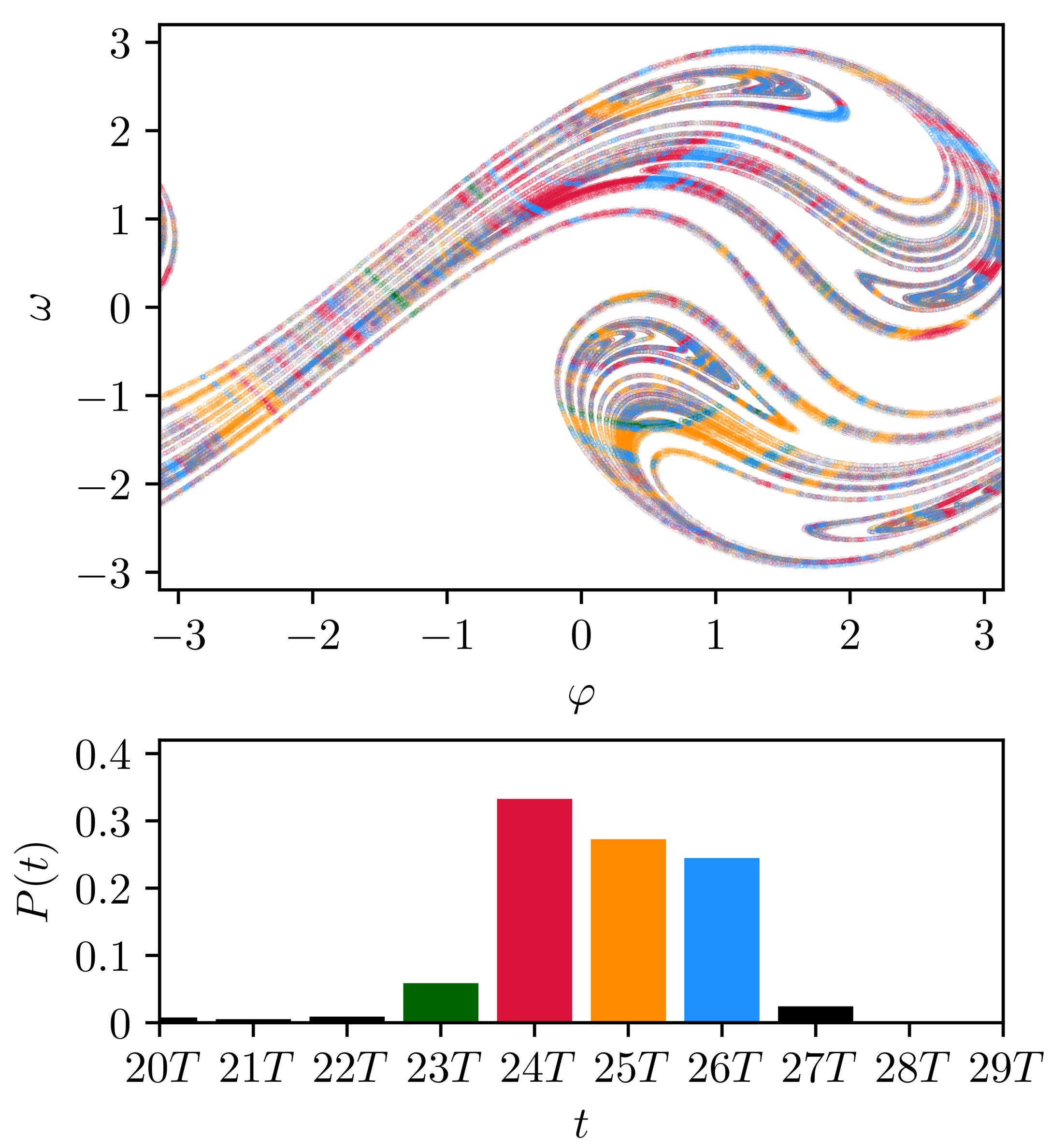
Switching the driving back on



- Disappearance/appearance of chaos
- **Drop/jump** in $\sigma[\omega]$



Distribution of lifetimes



Summary

- Ensemble approach is needed [2, 3]
- Measure of internal variability ∼ $\sigma[\omega]$
- Hysteresis between switching-off and on
- Distribution of lifetimes is fractal-like [4]

References, Acknowledgements

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