Tipping-like phenomena in generic dynamical systems

Bálint Kaszás[1,4], Ulrike Feudel[2], Tamás Tél[1,3]

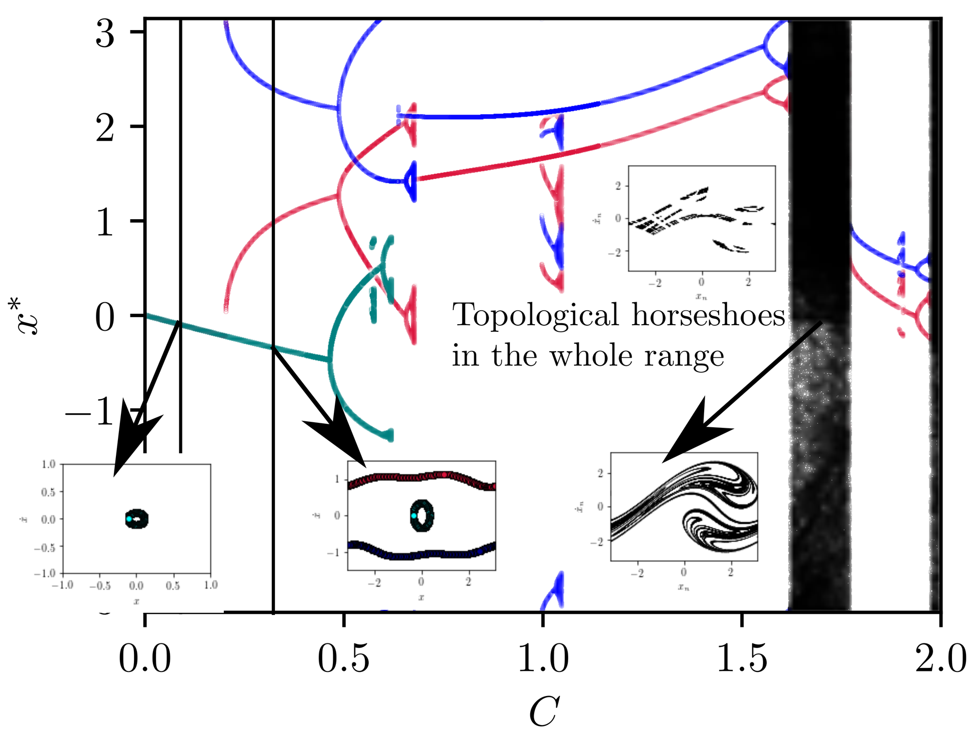
[1]Institute for Theoretical Physics, Eötvös Loránd University, Budapest, Hungary
[2]Theoretical Physics/ Complex Systems, ICBM, Carl von Ossietzky University Oldenburg, Germany
[3]MTA-ELTE Theoretical Physics Research Group
[4]kaszasb@caesar.elte.hu

Motivation

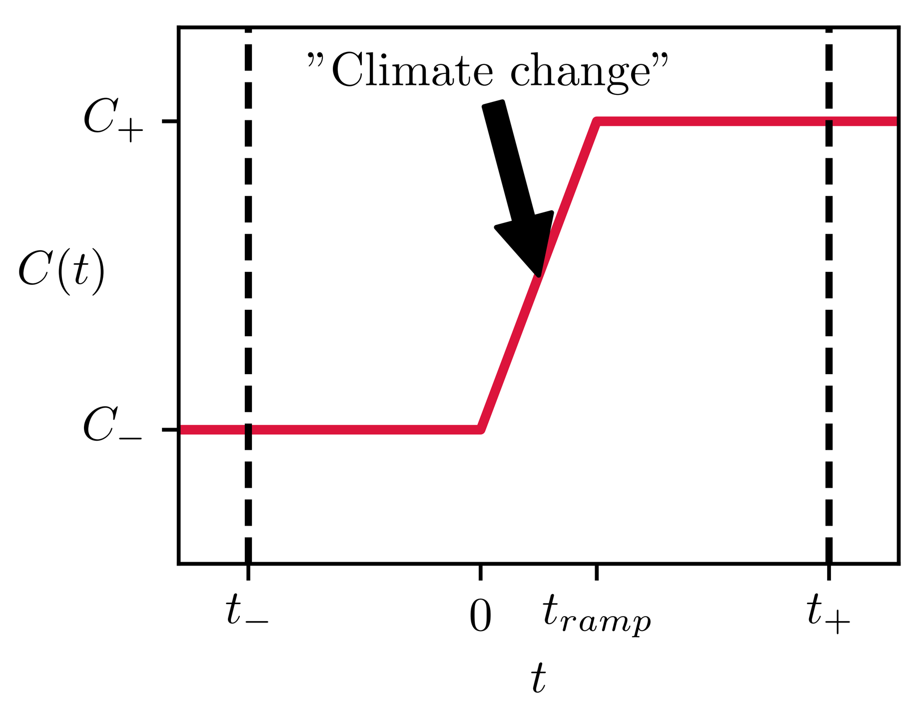
- Tipping transitions are found to be numerous in dynamical systems with parameter drift that exhibit multistability. Examples include
- Climate science
- Ecology (called regime shifts)
  See [1] for a minireview.

Introduction

Our goal is to study processes in dynamical systems that are analogous to climate change. The system under study has a bifurcation diagram that can be considered typical



- Three dynamically different asymptotic states exist. Transition between them (**tipping**) is possible when the parameter C drifts in time.
- Are there any additional types of tipping except the recently discovered **rate-induced [2]** and **partial tipping [3]** due to topological complexity of the dynamics (presence of permanent and transient chaos)?
- Consider the following **parameter drift scenario**



References, Acknowledgements

[1] U. Feudel, A. N. Pisarchik, K. Showalter, Chaos **28**, 033501 (2018)
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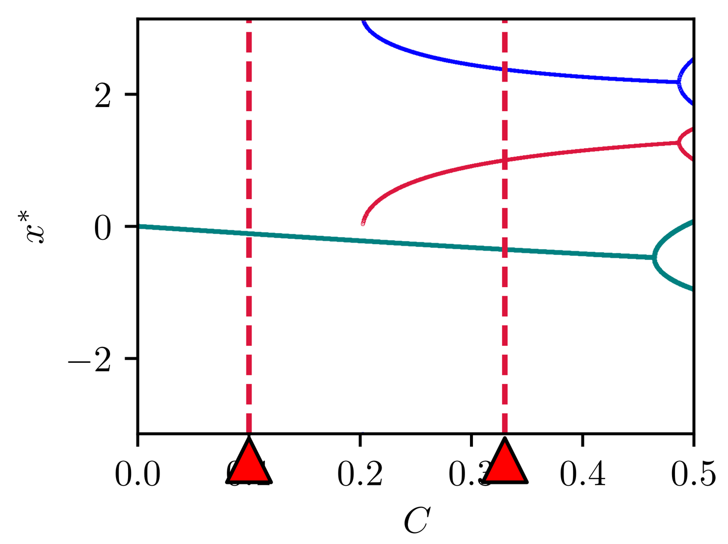
Methods

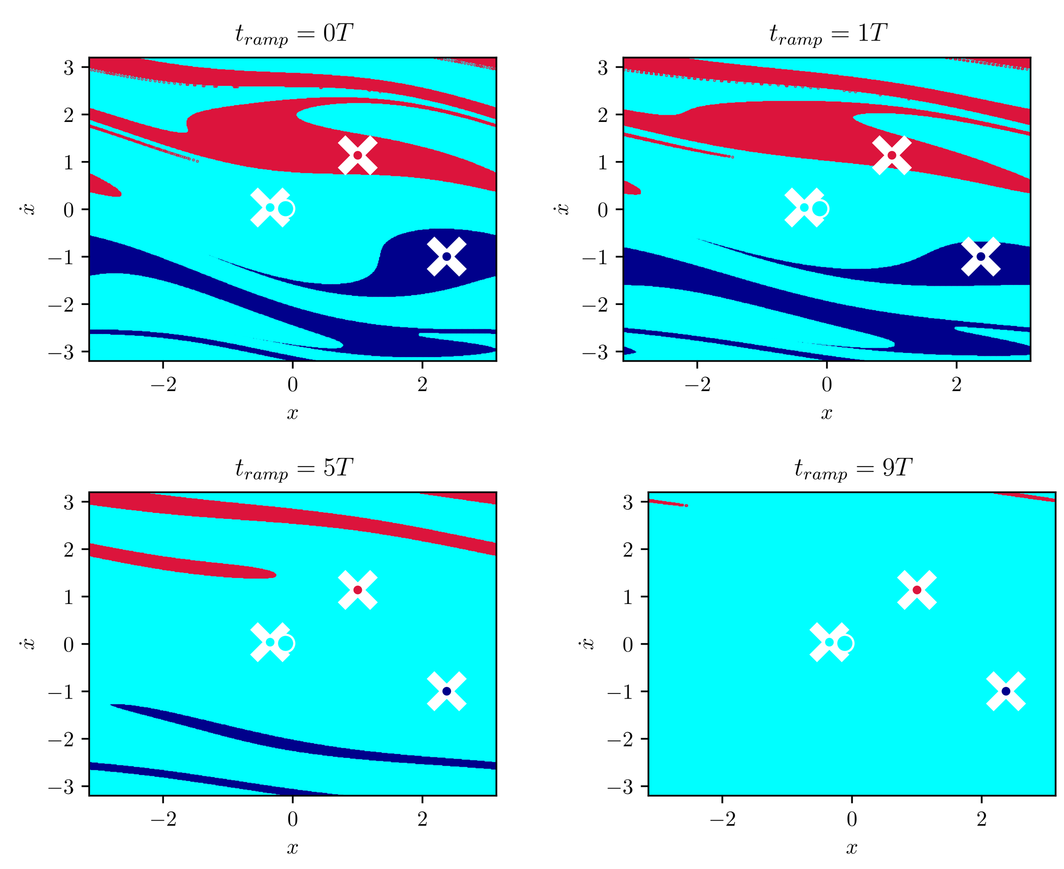
- Ensemble methods are applied to construct the pullback (or snapshot) attractors [4] and basins.
- Basins of attraction belonging to the attractors at t_+ are constructed at the **time instant t_-.**
- If the intersection of the basins of attraction at t_+ and t_- is non-empty, these trajectories are said to **tip**. The measure of this intersection yields the probability of transition between attractors.

$$P_{A_1,A_2} = \frac{|B_2^{sc}(t_-,t_+) \cap B_1(t_-)|}{|B_1(t_-)|} \qquad (1)$$

Scenario dependent basins I.

Parameter drift through a typical bifurcation with the scenario $t_- = 0$, $t_+ = 100T$, $C_- = 0.1$, $C_+ = 0.33$, while changing the time of the parameter drift, t_{ramp}.

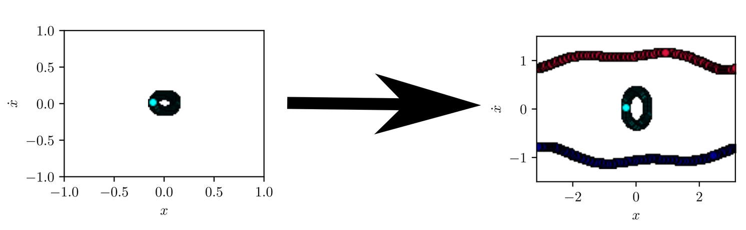


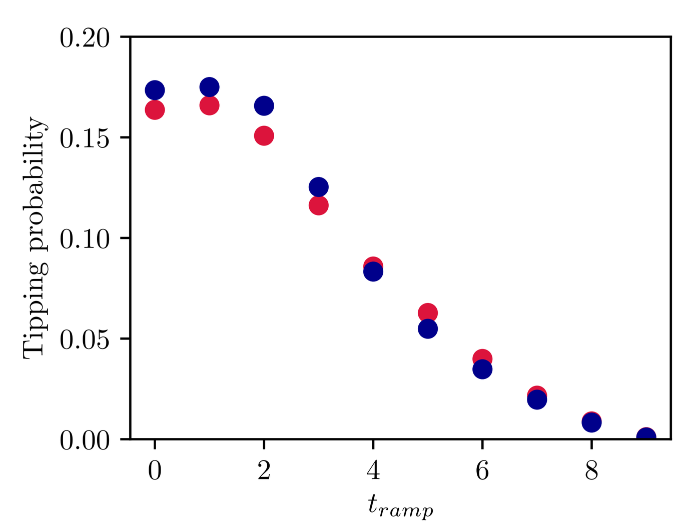


Basin boundaries are smooth. Attractors at t_+ do not necessarily belong to their basins at t_-. For slow drifts there is no tipping. This also holds when t_- is increased.

Tipping probabilities

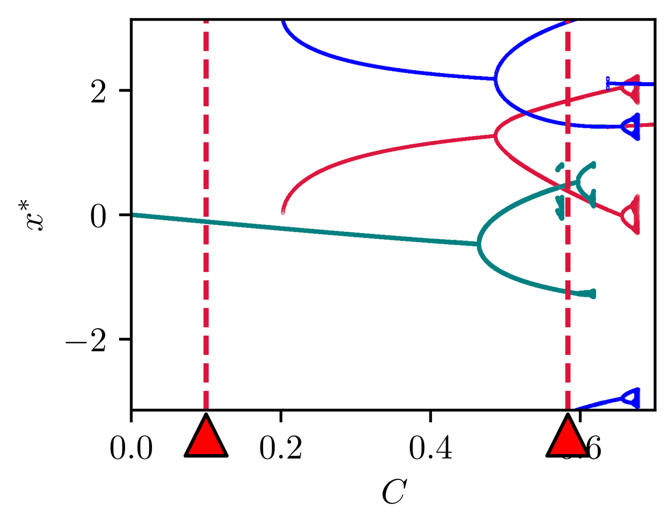
Probability is computed according to (1). This quantity depends on the parameters of the scenario, t_-, t_{ramp}, t_+ (due to **transient chaos**).

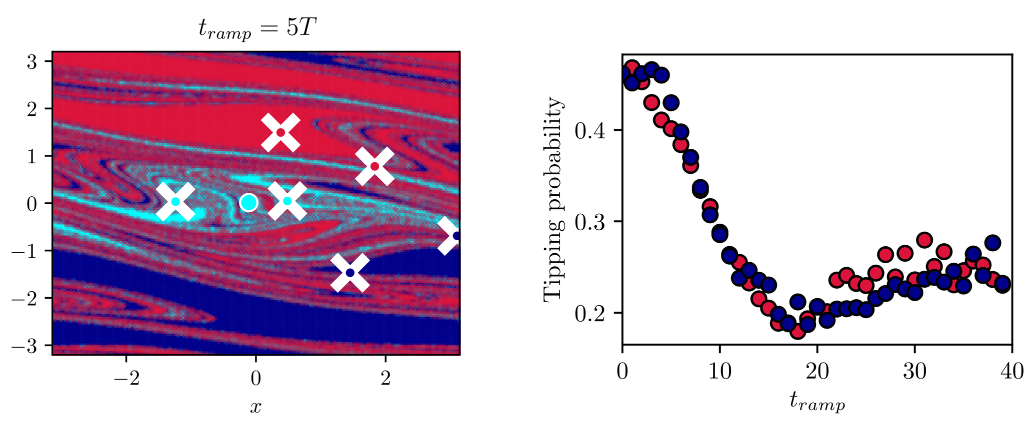




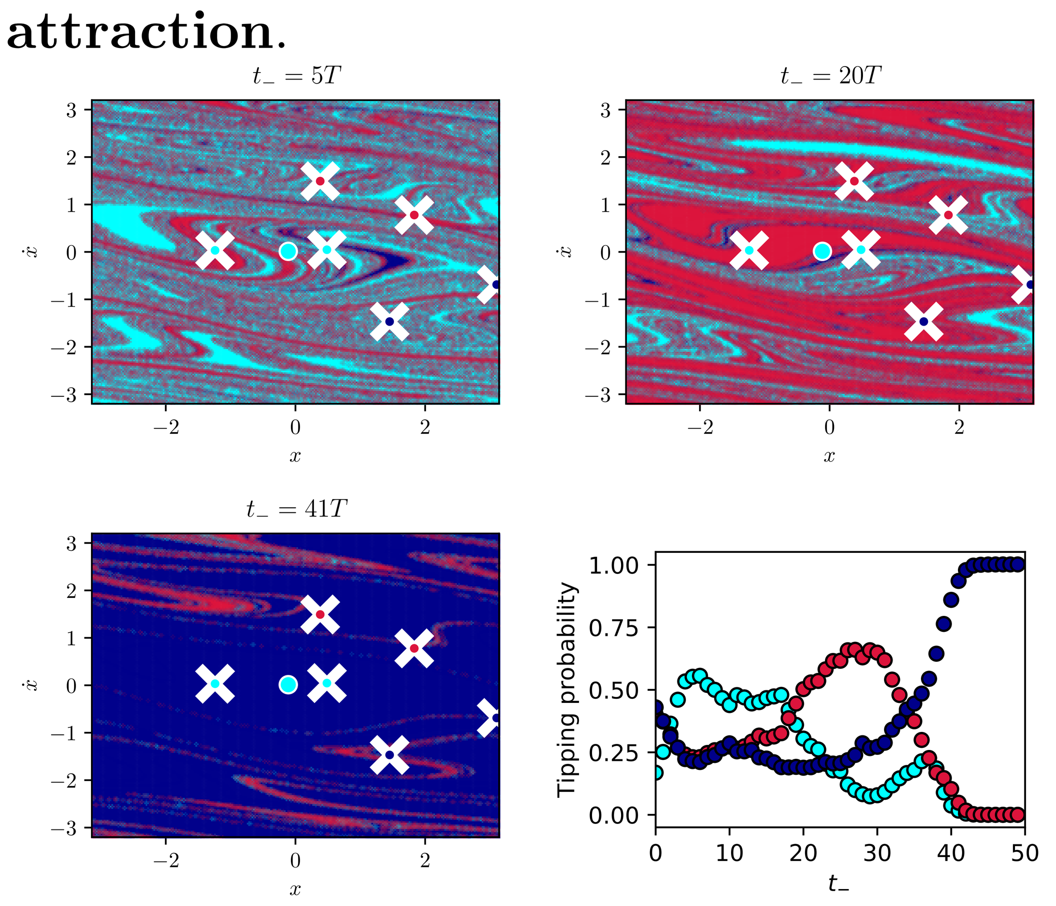
Scenario dependent basins II.

Parameter drift through multiple bifurcations with $t_- = 0$, $t_+ = 100T$, $C_- = 0.1$, $C_+ = 0.5835$.

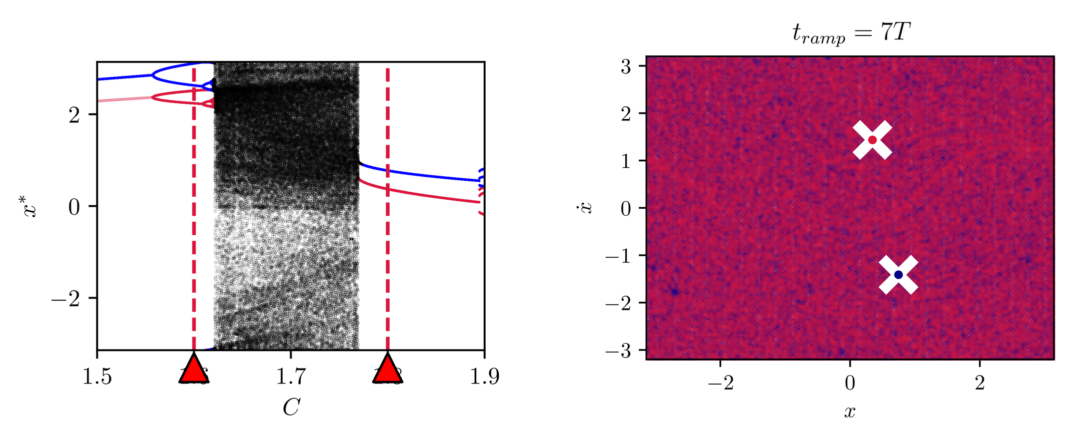




In the following, $t_{ramp} = 5T$, $C_- = 0.1$, $C_+ = 0.5835$, $t_+ = 100T$. The probability of tipping depends heavily on t_-, as a result of the **fractal structure of the basins of attraction**.



Drifting through chaotic attractors



Drift through a chaotic regime with $t_- = 0$, $t_+ = 100T$, $C_- = 1.6$, $C_+ = 1.8$. The basin becomes riddled, the probability of tipping to either of the attractors is 1/2, signaling complete unpredictability.

Summary

In typical dynamical systems with parameter drift that exhibit topological complexity, we have found the following mechanisms for tipping

- Fractality-induced tipping
- Transient chaos induced tipping
- Chaotic attractor induced tipping

All of these are partial tippings, the probability can be computed by the use of scenario-dependent basins of attraction.