Assignment 2

1) $f(n) = n-10$ $\quad g(n) = n+10$

Find $f(n) = \Theta(g(n))$

Here Big O & Omega

Soln

Big O

$f(n) \leq c \cdot g(n)$

$n-10 \leq c(n+10)$

$c = 2$ this is a Constant

$f(n) = O(g(n))$

Omega

$f(n) \geq c \cdot g(n)$

$n-10 \geq c \cdot (n+10)$

$c = \frac{1}{2}$

$f(n) = \Omega(g(n))$

Hence both Condition is true the it is $f(n) = \Theta(g(n))$ is true

2) $f(n) = n$ $\quad g(n) = n$ Then $f(n) = \Theta(g(n))$

Soln.

Big O

$f(n) \leq c \cdot g(n)$

$n \leq c \cdot n$

R.H.S is multiple of n and Constant so its true

$c = 1$

Omega

$f(n) \geq c \cdot g(n)$

$n \geq c \cdot n$

$c = 1$

Hence the function is true

3) $64^{\log_2 n} \cdot n^{\log_2 n} = O(n^5)$

Soln ⇒ first simplify the equation

$n^{\log_2 64} \cdot n^{\log_2 32}$

then simplify log

$\log_2 64 = \log_2 2^6 \Rightarrow 6$

$\log_2 32 = \log_2 2^5 \Rightarrow 5$

$n^6 \cdot n^5 \Rightarrow n^{11}$

$f(n) <= c \cdot g(n)$

$n^{11} <= c \cdot n^5$

$c = n^6$

Here c is not a constant value so its not a valid Big o function

4) $\frac{4^n}{2^n} = O(2^n)$

Sol: simplify $\frac{2^n * 2^n}{2^n} = O(2^n)$

$f(n) <= c \cdot g(n)$

$2^n <= c \cdot 2^n$

$c = 1$

C is constant so its a valid Big o function

5.) $128^{\log_2 n} \cdot n^2 = \Theta(n^9)$

Soln:- first simplify the f(n)

$n^{\log_2 128} \cdot n^2$

$n^{\log_2 2^7} \cdot n^2$

$n^7 \cdot n^2 \Rightarrow n^9$

Big O

$n^9 <= c \cdot n^9$

$c = 1$

Omega

$n^9 >= c \cdot n^9$

$c = 1$

In both conditions c is constant and both condition are true then its a valid Θ function.