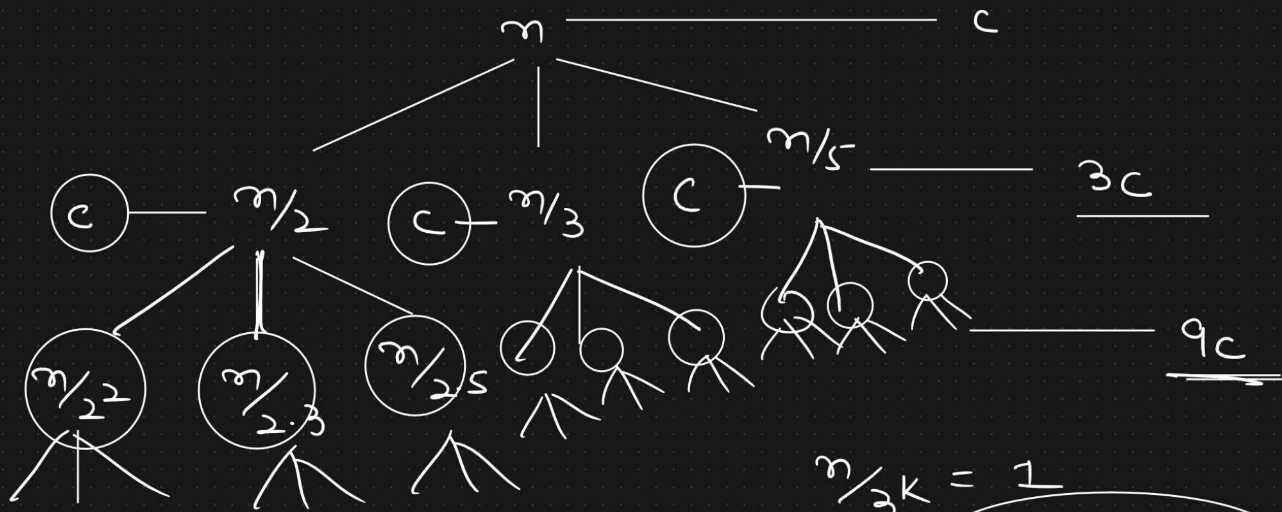
Assignment Problem

2) $T(n) = T(n/2) + T(n/3) + T(n/5) + c$



$n/3^k = 1$

$k = \log_3 n$

$\frac{n}{2^k} = 1 \qquad n = 2^k \qquad k = \log_2 n$ — * Higher value

$n/5^k = 1$

$k = \log_5 n$

$(3)^0 c + (3)^1 c + (3)^2 c + \ldots + (3)^k c$

$c\left((3)^0 + (3)^1 + (3)^2 + \ldots + (3)^{\log_2 n}\right)$

↳ GP series

$r = 3 \qquad r > 1$

$a = 1$

Property Number 7

$S = \frac{a(r^n - 1)}{r - 1} = c\left(\frac{1(3^{\log_2 n} - 1)}{3 - 1}\right)$

$= c\left(\frac{n^{\log_2 3} - 1}{2}\right)$

$$\Rightarrow \underline{\underline{O(n^{1.5})}}$$

1)

$$T(n) = \begin{cases} 1 & n = 1 \\ 2T(n/2) + n & n > 1 \end{cases}$$

<u>Substitution Method</u>;

$$\boxed{T(n) = 2T\left(n/2\right) + n}$$

$$= 2\left(2T\left(n/2^2\right) + n/2\right) + n$$

$$\Rightarrow 2^2\, T\left(n/2^2\right) + 2n$$

$$\Rightarrow 2^2\left(2T\left(n/2^3\right) + \frac{n}{2^2}\right) + 2n$$

$$\Rightarrow 2^3\, T\left(\frac{n}{2^3}\right) + 3n$$

k times

$$\frac{n}{2^k} = 1$$

$$n = 2^k$$

$$\log_2 n = k$$

$$2^k\, T\left(\frac{n}{2^k}\right) + k \cdot n$$

$$2^{\log_2 n}\, T\left(\frac{n}{2^{\log_2 n}}\right) + \log_2 n \cdot n$$

$$n^{\cancel{\log_2^2}\,1}\; T\left(\frac{\cancel{n}\,1}{\cancel{n^{\log_2^2}}\,1}\right) + \log_2 n \cdot n$$

$\underline{T(1) = 1}$

$$n \cdot \underline{T(1)} + \log_2 n * n$$

$$\underline{n} + \underline{n * \log_2 n}$$

$$\underline{\underline{O(n \log_2 n)}}$$

3)

$$T(n) = \begin{cases} 1 & n = 1 \\ 8T(n/2) + n^2 & n > 1 \end{cases}$$

$$T(n) = 8T(n/2) + n^2$$

$$= 8\left(8T\left(n/2^2\right) + \left(\frac{n}{2}\right)^2\right) + n^2$$

$$\overset{2}{\cancel{8}} * \frac{n^2}{\cancel{4}} + n^2$$

$$= 8^2 T\left(\frac{n}{2^2}\right) + \underline{3n^2}$$

$$\overset{4}{\cancel{8^2}} * \frac{n^2}{\cancel{16}} = 4n^2$$

$$= 8^2\left(8\,T\left(\frac{n}{2^3}\right) + \left(\frac{n}{2^2}\right)^2\right) + 3n^2$$

$$= 8^3 T\left(\frac{n}{2^3}\right) + 7n^2$$

$$\frac{n}{2^k} = 1$$

$\Big\downarrow$ k times

$$K = \log_2 n$$

$$8^k \, T\left(n/2^k\right) + (2^k - 1)\, n^2$$

$$\log_2 8 = \log_2 2^3 = 3$$

$$8^{\log_2 n} \, T\left(\frac{n}{2^{\log_2 n}}\right) + \left(2^{\log_2 n} - 1\right) n^2$$

$$\Downarrow$$

$$n^{\log_2 \cancel{8}^{\,3}} \, T\left(\frac{n}{n^{\cancel{\log_2 2}}}\right) + \left(n^{\cancel{\log_2 2}} - 1\right) n^2$$

$$T(1) = 1$$

$$n^3 \, T\left(\frac{\cancel{n}\ 1}{\cancel{n}}\right) + n^3 - n^2$$

$$n^3 + n^3 - n^2$$

$$O(n^3)$$

1)

$$f(n) = n - 10$$
$$g(n) = n + 10$$

$$f(n) = \Theta(g(n))$$
$$\Downarrow$$
Big O &

Omega

Large values of n

Big O

$$f(n) \le c \cdot g(n)$$
$$n - 10 \le c \cdot (n + 10)$$
$$c = 1$$
$$f(n) = O(g(n)) \checkmark$$

Omega

$$f(n) \geq c \cdot g(n)$$
$$n - 10 \geq c \cdot (n + 10)$$
$$c = \frac{1}{2}$$
$$n = 1000$$
$$990 \geq \frac{\cancel{1010}^{505}}{\cancel{2}}$$
$$f(n) = \Omega(g(n))$$

$$f(n) = \Theta(g(n))$$

2) $f(n) = n \qquad g(n) = n$

$$f(n) = \Theta(g(n)) \longrightarrow \text{True}$$

$$f(n) <= c \cdot g(n)$$
$$f(n) >= c \cdot g(n)$$

Big O: $n <= c \cdot n$, $c = 1$

omega: $n >= c \cdot n$, $c = 1$

3) $64^{\log_2 n} \cdot 32^{\log_2 n} = O(n^5)$ $\hookrightarrow$ false

$$\Downarrow$$
$$n^{\log_2 64} \cdot n^{\log_2 32}$$
$$n^6 \cdot n^5 = n^{11}$$

$$\log_2 64 = \log_2 2^6 = 6$$
$$\log_2 32 = \log_2 2^5 = 5$$

$f(n) <= c \cdot g(n)$

$\boxed{n^{11} <= c \cdot n^5} \rightarrow$ Not a Valid Big O

$\rightarrow c = n^6$

4) $\frac{4^n}{2^n} = O(2^n) \longrightarrow$ True

$\frac{2^n * \cancel{2^n}}{\cancel{2^n}} = O(2^n)$

$2^n <= c \cdot 2^n$

$c = 1$

5) $\underbrace{128^{\log_2^n} \cdot n^2}_{f(n)} = \Theta(\underbrace{n^9}_{g(n)})$

$n^{\log_2^{128}} \cdot n^2$

$n^{\log_2^{2^7}} \cdot n^2$

$n^7 \cdot n^2 = n^9$

$f(n) = \Theta(g(n))$

$n^9 = \Theta(n^9)$

Big O	omega
$n^9 <= c \cdot n^9$	$n^9 >= c \cdot n^9$
$c = 1$	$c = 1$