Matrix Chain Multiplication

A = 1 × 3 (P0 P1)
B = 3 × 1 (P1 P2)
C = 1 × 2 (P2 P3)
D = 2 × 3 (P3 P4)

E = ABCD (1 2 3 4) → 11

I) A(BCD) → 24
II) (ABC)D → 18
III) (AB)(CD) → 12

B(CD) → 24
(BC)D → 33

A(BC) → 11
(AB)C → 11

Recurrence Relation

$$c(i,j) = \min_{i \le k < j} \{ c(i,k) + c(k+1,j) + P_{i-1} P_k P_j \}$$

3 × 1 × 3 = 9

3 × 1 | 1 × 3

B(CD) = 1 × 2 × 3 = 6

1 × 3 | 3 × 3

A

1 × 3 × 3 = 9

9 + 6 = 15

Total = 15 + 9 = 24

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3 × 2 | 2 × 3 → 3 × 2 × 3 = 18

(BC)D

(BC) = 3 × 1 × 2 = 6

18 + 6 = 24

1 × 3 × 3 = 9

A((BC)D) = 24 + 9 = 33

$A = 1 \times 3$ $B = 3 \times 1$ $C = 1 \times 2$ $D = 2 \times 3$

$P_0\ P_1$ $P_1\ P_2$ $P_2\ P_3$ $P_3\ P_4$

$i = 1$

$J = 4$

$i\ \ k$ $\quad k+1\ \ J$

$$c(1,4) = \min \begin{cases} c(1,1) + c(2,4) + P_0 P_1 P_4 \quad \text{---} (1) \\ \underline{A(BCD)} \\ c(1,2) + c(3,4) + P_0 P_2 P_4 \quad - (2) \\ \underline{(AB)(CD)} \\ c(1,3) + c(4,4) + P_0 P_3 P_4 \quad - (3) \\ \underline{(ABC)D} \end{cases}$$

1×2 $\quad$ 2×3

$B(CD)$ $\quad 6$

$i = 2,\ J = 4$

$$c(2,4) = \min_{i \le k < J} \begin{cases} c(2,2) + c(3,4) + P_1 P_2 P_4 \\ 0 \\ c(2,3) + c(4,4) + P_1 P_3 P_4 \\ (BC)D \qquad 0 \end{cases}$$

BCD

$i == J$:

result = 0

$$c(3,4) = \min \begin{cases} \overset{0}{c(3,3)} + \overset{0}{c(4,4)} + \\ P_2 P_3 P_4 \\ 1 * 2 * 3 \end{cases}$$

DSA Interview Question

$$\frac{{}^{2*3}C_3}{3+1}$$

$$\frac{{}^{6}C_3}{4}$$

$$\frac{6!}{3!\,3!\,4} = \frac{6\times5\times4}{3\times2\times4}$$

$$= 5$$

4 Matrix → 5 Possibilities

10 Matrix → Catalan Number

↳ Posible number of ways for matrix multiplication

$$\begin{cases} \frac{{}^{2n}C_n}{n+1} \end{cases}$$

$$\frac{{}^{2*9}C_9}{10} = \frac{{}^{18}C_9}{10}$$

$$\frac{18!}{9!\,9!\,10} = \frac{18\times17\times16\times15\times14\times13\times12\times11\times10}{9\times8\times7\times6\times10\times5\times4\times3\times2}$$

$$= 4862$$

≈ 12 →

$$\frac{2^n}{n^n}$$

3 times $a = 3$, target $= 13$

3 times $a = 5$, target $= 11$

3 times $a = 4$, target $= 12$

3 times $a = 2$, target $= 14$

Sum of subset

set = {3, 4, 5, 2}

Sum = 6 {4, 2}

Output = True

$4^4 = 256$

Recursive Solution or Brute force approach

$O(2^n)$ → Exponential

isSS(4, 6) — n, sum

including

isSS(3, 3) — {3}

exclude

isSS(3, 6)

include

isSS(2, -1) — {3, 4}

false

exclude

isSS(2, 3)

include

isSS(1, -2)

false

Include

isSS(2, 2)

Include

isSS(1, -3)

false

Exclude

isSS(1, 2)

exclude

isSS(1, 3)

isSS(0, 2)

isSS(0, 0)

True

isSS(0, 1)

false

isSS(0, 3)

false

{ Sum < 0 or

n == 0

→ return false

{ Sum = 0

→ return True

Dynamic Programming

$\Theta(n)$(sum)

Exponential ⟶ Polynomial

Tabulation

		J → 0	1	2	3	4	5	6
0	0	T	f	f	f	f	f	f
1	3	T	f	f	T	f	f	f
2	4	T	f	f	T	T	f	f
3	5	T	f	f	T	T	T	f
4	2	T	f	T	T	T	T	T

Sum ↓

T → final result

M(i)(0) = T

if J < input(i):

M(i)(J) = M(i-1)(J)

else:

Exclude

M(i)(J) = M(i-1)(J) OR M(i-1)(J - input(i))

↳ Include

f

M(1)(3) = M(0)(3) OR M(0)(0) T

$$M(1)(6) = \underbrace{M(0)(6)}_{f} \text{ OR } \underbrace{M(0)(3)}_{f}$$