Distributed Control Systems

Course Project 2019
Distributed Dual Gradient Tracking for Microgrid Control

The project deals with the design and implementation of a distributed optimization algorithm in order to solve an optimization problem arising in microgrid control.

Task 1

Suppose we have N agents that want to cooperatively solve the concave program

$$\max_{\lambda} \sum_{i=1}^{N} q_i(\lambda) \tag{1}$$

where $\lambda \in \mathbb{R}^S$, and $q_i : \mathbb{R}^S \to \mathbb{R}$ are strictly concave quadratic functions, i.e.,

$$q_i(\lambda) = -(\lambda^\top Q_i \lambda + r_i^\top \lambda), \qquad i \in \{1, \ldots, N\},$$

where Q_i is a positive definite matrix. Design a software, written in MATLAB, implementing the Distributed Gradient Tracking algorithm presented in [1]. Use the software to solve a random instance of problem (1).

Task 2

Consider the microgrid control problem described in [2, Section 5], i.e.,

$$\begin{aligned} \min_{p_1,\ldots,p_N} \quad & \sum_{i=1}^{N} f_i(p_i) \\ \text{subj. to} \quad & \sum_{i \in \texttt{GEN}} p^\tau_{\texttt{gen},i} + \sum_{i \in \texttt{STOR}} p^\tau_{\texttt{stor},i} + \sum_{i \in \texttt{CONL}} p^\tau_{\texttt{conl},i} + p^\tau_{\texttt{tr}} - D^\tau = 0, \\ & \qquad\qquad \forall\, \tau \in [0, T], \\ & p_i \in X_i, \qquad\qquad \forall\, i \in \{1, \ldots, N\}, \end{aligned} \tag{2}$$

with slightly modified cost functions f_i and local sets X_i, summarized in Table 1. In particular, we added a small strictly convex perturbation to each cost (except for $i \in \texttt{GEN}$ which are already strictly convex), with a small positive $\varepsilon > 0$. Moreover, to make all the local sets compact, we also consider box constraints for $i \in \texttt{CONL}$. Derive the dual problem of (2), when dualizing only the coupling constraints $\sum_{i \in \texttt{GEN}} p^\tau_{\texttt{gen},i} + \sum_{i \in \texttt{STOR}} p^\tau_{\texttt{stor},i} + \sum_{i \in \texttt{CONL}} p^\tau_{\texttt{conl},i} + p^\tau_{\texttt{tr}} - D^\tau = 0$. The dual problem should have a structure similar to problem (1).

Index	Cost function $f_i(p_i)$	Local constraints X_i
GEN	$\sum_{\tau=0}^{T} \left[\alpha_1 p^{\tau}_{\texttt{gen},i} + \alpha_2 \left(p^{\tau}_{\texttt{gen},i}\right)^2\right]$	$\underline{p} \leq p^{\tau}_{\texttt{gen},i} \leq \bar{p}, \quad \tau \in [0, T]$ $\underline{r} \leq p^{\tau+1}_{\texttt{gen},i} - p^{\tau}_{\texttt{gen},i} \leq \bar{r}, \quad \tau \in [0, T-1]$
STOR	$\varepsilon \lVert p_{\texttt{stor},i} \rVert^2$	$-d_{\texttt{stor}} \leq p^{\tau}_{\texttt{stor},i} \leq c_{\texttt{stor}}, \quad \tau \in [0, T]$ $q^{\tau+1}_{\texttt{stor},i} = q^{\tau}_{\texttt{stor},i} + p^{\tau}_{\texttt{stor},i}, \quad \tau \in [0, T-1]$ $0 \leq q^{\tau}_{\texttt{stor},i} \leq q_{\max}, \quad \tau \in [0, T]$
CONL	$\sum_{\tau=0}^{T} \beta \max\{0, p^{\tau}_{\texttt{des},i} - p^{\tau}_{\texttt{conl},i}\} + \varepsilon \lVert p_{\texttt{conl},i} \rVert^2$	$-P \leq p^{\tau}_{\texttt{conl},i} \leq P, \quad \tau \in [0, T]$
$\{N\}$	$\sum_{\tau=0}^{T} (-c_1 p^{\tau}_{\texttt{tr}} + c_2 \lvert p^{\tau}_{\texttt{tr}} \rvert) + \varepsilon \lVert p_{\texttt{tr},i} \rVert^2$	$-E \leq p^{\tau}_{\texttt{tr}} \leq E, \quad \tau \in [0, T]$

Table 1: *Cost functions and local constraints for the microgrid control problem.*

Task 3

Make the necessary modifications to the software developed in Task 1 to solve an instance of the derived dual problem. The software should also return the optimal solution of the primal problem (2). The problem data must be generated randomly. Generate a set of simulations and show the results.

Notes

1. Any other information and material necessary for the project development will be given during project "meetings".
2. The project report must be written in Latex and follow the main structure of the attached template.
3. Any email for project support must have the subject DCS2019"NAME OF THE GROUP" "rest of the subject".

References

[1] G. Qu and N. Li, "Harnessing smoothness to accelerate distributed optimization," *IEEE Transactions on Control of Network Systems*, vol. 5, no. 3, pp. 1245–1260, 2018.

[2] I. Notarnicola and G. Notarstefano, "Constraint coupled distributed optimization: Relaxation and duality approach," *preprint arXiv:1711.09221*, 2017.