Homework 3

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Qualitative Accuracy

Unfortunately I budgeted my time poorly this week and ran out of time to code up a numeric convergence test, however in lieu of that a qualitative one will at least be a little helpful. As show in Fig. 1 and 2 using the Verlet method at a Jupiter mass of $1\text{-}10\mathrm{M}_J$ we can obtain a (visually) circular orbit. However, above that the orbit starts oscillating and at around $100\mathrm{M}_J$ both the Earth and Sun have moved from where the center of mass had been and Earth's orbit is now oscillating wildly. Finally at $1000\mathrm{M}_J$ we have too much mass and the system breaks apart.

Numeric Accuracy

Checking my values against the provided values (only for Earth unfortunately) I obtained values that were, on average, within 0.001 of the provided values in both the x and y dimensions. However, while the average was fairly low, there were a significant amount of values that were ~ 2 above what was provided. Ultimately checking numeric accuracy here was not going to work out well as I used a modified version of the Verlet method (called the Velocity Verlet) as opposed to the one suggested in the book (called the Basic Verlet).

Physical Stability

As seen in Fig. 5, 6, 13, and 14 both the Earth and the Sun were relatively stable for Jupiter having 1-10 times its mass. Above that both their orbits became much more unstable and by the time Jupiter gets 1000 times its original mass the system is completely unstable and breaks apart almost immediately.

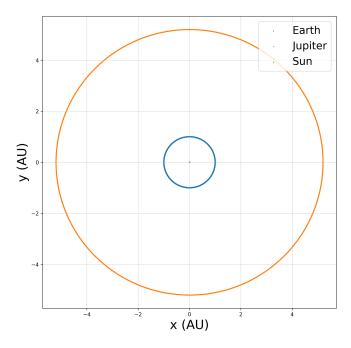


Fig. 1. Orbits of the Sun-Earth-Jupiter system over 10 periods of Jupiter (\sim 118 years) with the mass of Jupiter at $1{\rm M}_J$.

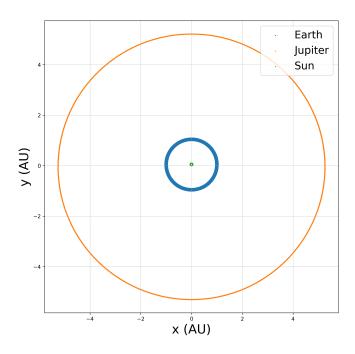


Fig. 2. Orbits of the Sun-Earth-Jupiter system over 10 periods of Jupiter (\sim 118 years) with the mass of Jupiter at $10{\rm M}_J$.

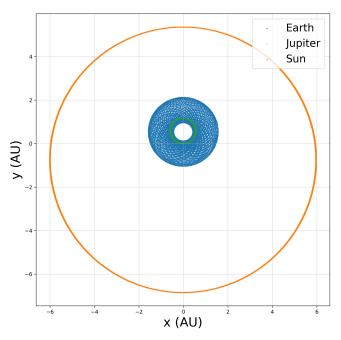


Fig. 3. Orbits of the Sun-Earth-Jupiter system over 10 periods of Jupiter (\sim 118 years) with the mass of Jupiter at $100 {\rm M}_J$.

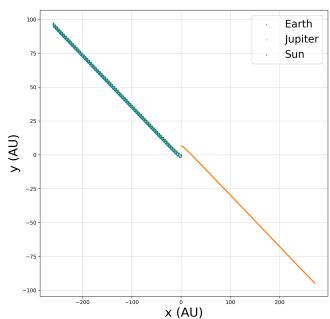


Fig. 4. Orbits of the Sun-Earth-Jupiter system over 10 periods of Jupiter (\sim 118 years) with the mass of Jupiter at $1000 {\rm M}_J$.

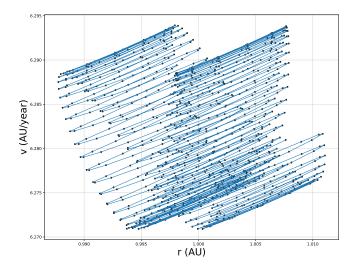


Fig. 5. Poincare section of Earth sampled over 1000 (\sim 1000 years) Earth orbital periods scale with Jupiter at $1{\rm M}_J$.

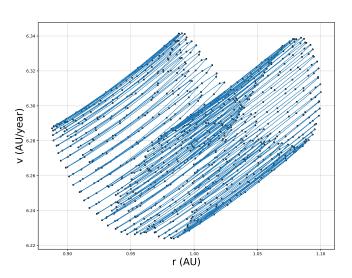


Fig. 6. Poincare section of Earth sampled over 1000 (\sim 1000 years) Earth orbital periods scale with Jupiter at $10{\rm M}_J$.

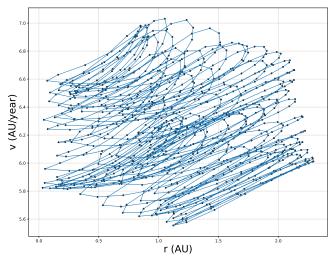


Fig. 7. Poincare section of Earth sampled over 1000 (\sim 1000 years) Earth orbital periods scale with Jupiter at $100 \mathrm{M}_J$.

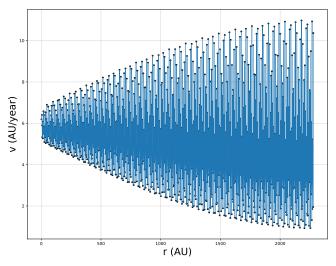


Fig. 8. Poincare section of Earth sampled over 1000 (\sim 1000 years) Earth orbital periods scale with Jupiter at 1000M_J.

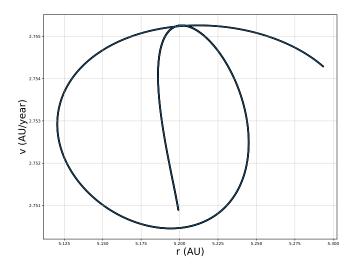


Fig. 9. Poincare section of Jupiter sampled over 1000 (\sim 11,860 years) Jupiter orbital periods scale with Jupiter at $1{\rm M}_J$.

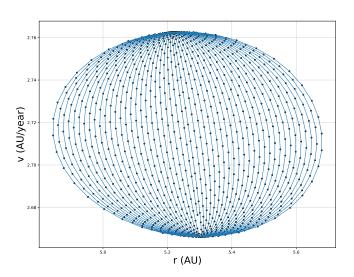


Fig. 10. Poincare section of Jupiter sampled over 1000 (\sim 11,860 years) Jupiter orbital periods scale with Jupiter at $10{\rm M}_J$.

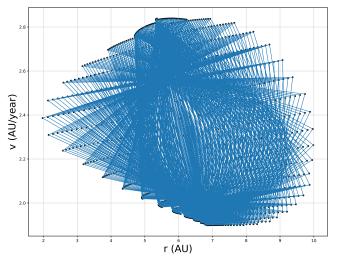


Fig. 11. Poincare section of Jupiter sampled over 1000 (\sim 11,860 years) Jupiter orbital periods scale with Jupiter at $100{\rm M}_J$.

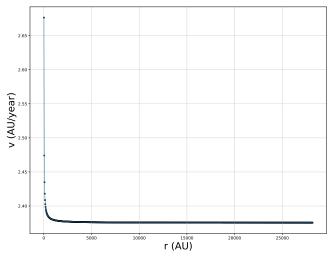


Fig. 12. Poincare section of Jupiter sampled over 1000 (\sim 11,860 years) Jupiter orbital periods scale with Jupiter at $1000 {\rm M}_J$.

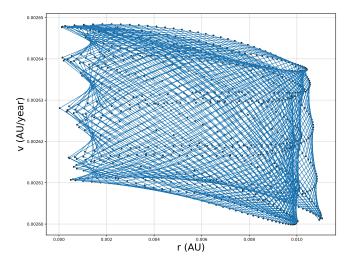


Fig. 13. Poincare section of the Sun sampled over 1000 Earth orbital periods (\sim 1000 years) scale with Jupiter at $1{\rm M}_J$.

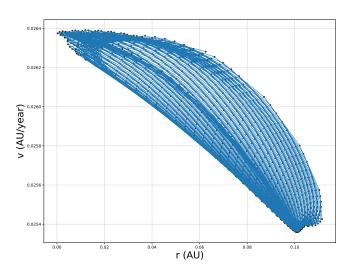


Fig. 14. Poincare section of the Sun sampled over 1000 Earth orbital periods (\sim 1000 years) scale with Jupiter at $10{\rm M}_J$.

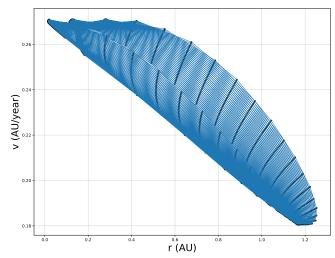


Fig. 15. Poincare section of the Sun sampled over 1000 Earth orbital periods (\sim 1000 years) scale with Jupiter at $100 {\rm M}_J$.

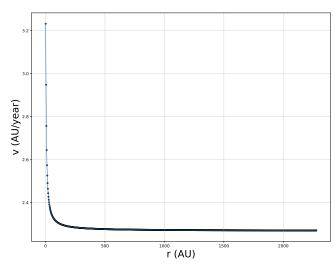


Fig. 16. Poincare section of the Sun sampled over 1000 Earth orbital periods (\sim 1000 years) scale with Jupiter at 1000M_J.