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Assignment-7

Roll No. : FWC22008

Problem Statement:

Find the maximum area of a triangle which can be inscribed in a given circle

0.1 Solution

Given function is,

$$f(x) = 2x^3r - x^4 \tag{1}$$

Objective function:

$$f(x) = \max_x 2x^3r - x^4 \tag{2}$$

constraints:

$$x > 0 \tag{3}$$

0.1.1 Calculation using normal differentiation

Differentiating (1) yields,

$$\nabla f(x) = 6x^2r - 4x^3 \tag{4}$$

0.1.2 Calculation of Maxima using gradient ascent algorithm

Maxima of the above equation (1), can be calculated from the following expression,

$$x_{n+1} = x_n + \alpha \nabla f(x_n) \tag{5}$$

0.1.3 Calculation of Maxima using gradient ascent algorithm

$$f(x) = 2x^3r - x^4 \tag{6}$$

$$f'(x) = 6x^2r - 4x^3 \tag{7}$$

we have to attain the maximum value of area of triangle. This can be seen in Figure.Using gradient ascent method we can find its maxima.

$$\implies x_{n+1} = x_n + \alpha(6x^2r - 4x^3) \tag{8}$$

Taking $x_0 = 1, \alpha = 0.001$ and precision = 0.00000001, values obtained using python are:

$$\boxed{\text{Maxima} = 0.923176} \tag{9}$$

$$\boxed{\text{Maxima Point} = 1.2900} \tag{10}$$

0.2 Theoritical proof

$$area\ \ of\ \ the\ \ triangle = \frac{1}{2} * b * h \tag{11}$$

where b,base of a triangle is 2*R
h is the height

$$so\ \ area\ \ of\ \ triangle\ \ is\ \ R * h \tag{12}$$

$$where R = \sqrt{(r^2 - (h-r)^2)} \tag{13}$$

r=radius of the circle
area being the positive quantity ,A will be maximum if A^2 is maximum

$$A^2 = R^2h^2 \tag{14}$$

$$Z = R^2h^2 \tag{15}$$

$$where R^2 = 2hr - h^2 \tag{16}$$

$$Z' = 6h^2r - h^4 \tag{17}$$

for maximum value $Z' = 0$
by solving the above equation we get

$$h = \frac{3}{2} * r \tag{18}$$

z'' at h=$\frac{3}{2}$*r is negative
so area is maximum when h=$\frac{3}{2}$*r
by substituting the h value we get

$$R = \sqrt{3} * \frac{r}{2} \tag{19}$$

the maximum area of a triangle is obtained at

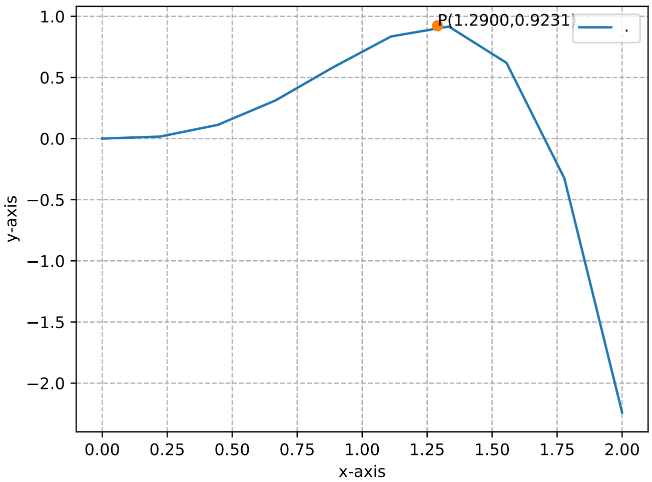
$$A = \frac{3\sqrt{3}}{4}r^2 \tag{20}$$

0.3 Conclusion

1. At first, the given function has been differentiated and it is solved by setting f'(x) equal to zero. By using x values, f(x) values are calculated.

2. Later, the given function f(x) is solved by gradient ascent algorithm to find maxima and the point at which f(x) is maximum.

3. Then, the given function f(x) is solved by gradient descent algorithm to find minima and the point at which f(x) is is minimum.



Download the code
`https://github.com/ballepu1994`