

PROBABILITY

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13.1.6 ¹ A coin is tossed three times where

(i) E:head on third toss, F:head on first two tosses

(ii)E:atleast two heads,F:atmost two heads

(iii)E:atmost two tails,F:atleast one tail

determine $P(E | F)$

Solution: : In an experiment of tossing a coin 3 times, random variable $X \in \{0, 1, 2, 3\}$ follows binomial distribution.

By using the binomial distribution formula :

$$\Pr(X=k)={}^nC_k \times p^k \times (1-p)^{n-k}$$

| Random Variable | Values | Description |
|-----------------|-----------|--|
| X | {0,1,2,3} | Number of heads or tails in a respective cases |

Table 13.1.6.2: Random variable X

| Variable | Description |
|----------|--|
| k | total number of success |
| p | probability of success of individual trial |
| n | number of trials =3 |

Table 13.1.6.4: variable and Description

i E:head on third toss, F:head on first two tosses

| PARAMETER | DESCRIPTION |
|-----------|----------------------------------|
| E | It is the head on the third toss |
| F | Head on the first two tosses |

Table 13.1.6.6: parameter and Description

By using product rule,

¹Read question numbers as (CHAPTER NUMBER).(EXERCISE NUMBER).(QUESTION NUMBER)

$$Pr(F) = \frac{1}{2} \times \frac{1}{2} \quad (13.1.6.1)$$

$$Pr(F) = 0.25 \quad (13.1.6.2)$$

$$Pr(EF) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \quad (13.1.6.3)$$

$$Pr(EF) = 0.125 \quad (13.1.6.4)$$

$$Pr(E | F) = \frac{Pr(EF)}{Pr(F)} \quad (13.1.6.5)$$

$$Pr(E | F) = 0.5 \quad (13.1.6.6)$$

ii E:atleast two heads,F:atmost two heads

| PARAMETER | DESCRIPTION | Distribution parameter |
|-----------|------------------------------------|------------------------|
| E | Atleast 2 heads when tossed thrice | $X \geq 2$ |
| F | Atmost 2 heads when tossed thrice | $X \leq 2$ |

Table 13.1.6.8: parameter and Description

$$Pr(F) = Pr(X \leq 2) \quad (13.1.6.7)$$

$$Pr(F) = Pr(X = 0) + Pr(X = 1) + Pr(X = 2) \quad (13.1.6.8)$$

$$Pr(F) = {}^3C_0\left(\frac{1}{2}\right)^3 + {}^3C_1\left(\frac{1}{2}\right)^3 + {}^3C_2\left(\frac{1}{2}\right)^3 \quad (13.1.6.9)$$

$$Pr(F) = 0.875 \quad (13.1.6.10)$$

$$Pr(EF) = Pr(X = 2) \quad (13.1.6.11)$$

$$Pr(EF) = {}^3C_2\left(\frac{1}{2}\right)^3 \quad (13.1.6.12)$$

$$Pr(EF) = 0.375 \quad (13.1.6.13)$$

$$Pr(E | F) = \frac{Pr(EF)}{Pr(F)} \quad (13.1.6.14)$$

$$Pr(E | F) = 0.428 \quad (13.1.6.15)$$

iii E:atmost two tails,F:atleast one tail

| PARAMETER | DESCRIPTION | DISTRIBUTION PARAMETER |
|-----------|-------------------------------------|------------------------|
| E | Atmost two tails when tossed thrice | $X \leq 2$ |
| F | Atleast one tail when tossed thrice | $X \geq 1$ |

Table 13.1.6.10: parameter and Description

$$Pr(F) = Pr(X \geq 1) \quad (13.1.6.16)$$

$$Pr(F) = 1 - Pr(X = 0) \quad (13.1.6.17)$$

$$Pr(F) = 0.875 \quad (13.1.6.18)$$

$$Pr(EF) = Pr(X = 1) + Pr(X = 2) \quad (13.1.6.19)$$

$$Pr(EF) = {}^3C_1\left(\frac{1}{2}\right)^3 + {}^3C_2\left(\frac{1}{2}\right)^3 \quad (13.1.6.20)$$

$$Pr(EF) = 0.75 \quad (13.1.6.21)$$

$$Pr(E \mid F) = \frac{Pr(EF)}{Pr(F)} \quad (13.1.6.22)$$

$$Pr(E \mid F) = 0.857 \quad (13.1.6.23)$$