

PROBABILITY

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13.3.4 ¹

In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{4}$, what is the probability that the student knows the answer given that he answered it correctly?

Solution: Let $X \in \{0, 1\}$ where 0 denotes a guess and 1 denotes that he knows the answer. Let $Y \in \{0, 1\}$ where 0 being the case when the answer is incorrect and 1 being the case that the answer is correct. From the given information,

Random variable	Description
X=0	Student guesses the answer
X=1	Student knows the answer
Y=0	Answer is incorrect
Y=1	Answer is correct

Table 2: Random variables X and Y

Pr(Event)	Value
$\Pr(Y=1 X=0)$	0.25
$\Pr(Y=1 X=1)$	1
$\Pr(X=0)$	0.75
$\Pr(X=1)$	0.25

Table 4: Probability of events X and Y

The probability that the student knows the answer and he answered it correctly is

$$\Pr(X = 1|Y = 1) = \frac{\Pr(Y = 1|X = 1) \Pr(X = 1)}{\Pr(Y = 1|X = 1) \Pr(X = 1) + \Pr(Y = 1|X = 0) \Pr(X = 0)}$$

Hence,

$$\Pr(X = 1|Y = 1) = \frac{0.25}{0.25 + 0.25 \times 0.75}$$

$$\Pr(X = 1|Y = 1) = 0.571$$

¹Read question numbers as (CHAPTER NUMBER).(EXERCISE NUMBER).(QUESTION NUMBER)