PROBABILITY

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13.3.4 [1]

In answering a question on a multiple choice test,a student either knows the answer or guesses.Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses.Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{4}$.what is the probability that the student knows the answer given that he answered it correctly?

Solution: : Let $X \in \{0,1\}$ where 0 denotes a guess and 1 denotes that he knows the answer. Let $Y \in \{0,1\}$ where 0 being the case when the answer is incorrect and 1 being the case that the answer is correct.

From the given information,

Random variable	**Description**
X=0	Student guesses the answer
X=1	Student knows the answer
Y=0	Answer is incorrect
Y=1	Answer is correct

Table 13.3.4.2: Random variables X and Y

Pr(Event)	**Value**
Pr(Y=1 | X=0)	0.25
Pr(Y=1|X=1)	1
Pr(X=0)	0.75
Pr(X=1)	0.25

Table 13.3.4.4: Probability of events X and Y

The probability that the student knows the answer and he answered it correctly is

$$\Pr\left(X=1|Y=1\right)=\frac{\Pr\left(Y=1|X=1\right)\Pr\left(X=1\right)}{\Pr\left(Y=1|X=1\right)\Pr\left(X=1\right)+\Pr\left(Y=1|X=0\right)\Pr\left(X=0\right)} \tag{13.3.4.1}$$

Hence,

$$\Pr\left(X=1|Y=1\right)=\frac{0.25}{0.25+0.25\times 0.75} \tag{13.3.4.2}$$

$$\Pr\left(X=1|Y=1\right)=0.571 \tag{13.3.4.3}$$

[1] Read question numbers as (CHAPTER NUMBER).(EXERCISE NUMBER).(QUESTION NUMBER)