PROBABILITY

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13.1.6 [1] **A coin is tossed three times where**
(i) E:head on third toss, F:head on first two tosses
(ii)E:atleast two heads,F:atmost two heads
(iii)E:atmost two tails,F:atleast one tail
determine P($E \mid F$)

Solution: : In an experiment of tossing a coin 3 times, random variable $X \in \{0, 1, 2, 3\}$ follows binomial distribution.
By using the binomial distribution formula :
$\Pr(X=k) = {}^nC_k \times p^k \times (1-p)^{n-k}$

Random Variable	Values	Description
X	{0,1,2,3}	Number of heads or tails in a respective cases

Table 13.1.6.2: Random variable X

Variable	Description
k	total number of success
p	probability of success of individual trial
n	number of trials =3

Table 13.1.6.4: variable and Description

   i E:head on third toss, F:head on first two tosses
By using product rule,
$\Pr(F) = \frac{1}{2} \times \frac{1}{2}$
$\Pr(F) = 0.25$
$\Pr(EF) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$
$\Pr(EF) = 0.125$
$\Pr(E \mid F) = \frac{Pr(EF)}{Pr(F)}$
$\Pr(E \mid F) = 0.5$

   ii E:atleast two heads,F:atmost two heads
$Pr(F) = Pr(X \leq 2)$
$Pr(F) = Pr(X = 0) + Pr(X = 1) + Pr(X = 2)$

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[1]Read question numbers as (CHAPTER NUMBER).(EXERCISE NUMBER).(QUESTION NUMBER)

$Pr(F)= {}^3C_0(\frac{1}{2})^3 + {}^3C_1(\frac{1}{2})^3 + {}^3C_2(\frac{1}{2})^3$
$Pr(F){=}0.875$
$Pr(EF){=}Pr(X=2)$
$Pr(EF){=}{}^3C_2(\frac{1}{2})^3$
$Pr(EF){=}0.375$
Pr(E | F)$=\frac{Pr(EF)}{Pr(F)}$
Pr(E | F)$=0.428$

iii E:atmost two tails,F:atleast one tail
$Pr(F){=}Pr(X \geq 1)$
$Pr(F){=}1{-}Pr(X=0)$
$Pr(F){=}0.875$
$Pr(EF){=}Pr(X=1){+}Pr(X=2)$
$Pr(EF){=}{}^3C_1(\frac{1}{2})^3{+}{}^3C_2(\frac{1}{2})^3$
$Pr(EF){=}0.75$
Pr(E | F)$=\frac{Pr(EF)}{Pr(F)}$
Pr(E | F)$=0.857$