PROBABILITY

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13.1.6 [1] **A coin is tossed three times where**
(i) E:head on third toss, F:head on first two tosses
(ii)E:atleast two heads,F:atmost two heads
(iii)E:atmost two tails,F:atleast one tail
determine P($E \mid F$)

Solution: : In an experiment of tossing a coin 3 times, random variable $X \in \{0,1,2,3\}$ follows binomial distribution.
By using the binomial distribution formula :
Pr(X=k)=${}^{n}C_k \times p^k \times (1-p)^{n-k}$

Random Variable	Values	Description
X	{0,1,2,3}	Number of heads or tails in a respective cases

Table 13.1.6.2: Random variable X

Variable	Description
k	total number of success
p	probability of success of individual trial
n	number of trials =3

Table 13.1.6.4: variable and Description

i E:head on third toss, F:head on first two tosses
By using product rule,

$$Pr(F) = \frac{1}{2} \times \frac{1}{2} \tag{13.1.6.1}$$

$$Pr(F) = 0.25 \tag{13.1.6.2}$$

$$Pr(EF) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \tag{13.1.6.3}$$

$$Pr(EF) = 0.125 \tag{13.1.6.4}$$

$$Pr(E \mid F) = \frac{Pr(EF)}{Pr(F)} \tag{13.1.6.5}$$

$$Pr(E \mid F) = 0.5 \tag{13.1.6.6}$$

[1]Read question numbers as (CHAPTER NUMBER).(EXERCISE NUMBER).(QUESTION NUMBER)

ii E:atleast two heads,F:atmost two heads

$$Pr(F) = Pr(X \leq 2) \tag{13.1.6.7}$$
$$Pr(F) = Pr(X = 0) + Pr(X = 1) + Pr(X = 2) \tag{13.1.6.8}$$
$$Pr(F) =^3 C_0(\frac{1}{2})^3 +^3 C_1(\frac{1}{2})^3 +^3 C_2(\frac{1}{2})^3 \tag{13.1.6.9}$$
$$Pr(F) = 0.875 \tag{13.1.6.10}$$
$$Pr(EF) = Pr(X = 2) \tag{13.1.6.11}$$
$$Pr(EF) =^3 C_2(\frac{1}{2})^3 \tag{13.1.6.12}$$
$$Pr(EF) = 0.375 \tag{13.1.6.13}$$
$$Pr(E \mid F) = \frac{Pr(EF)}{Pr(F)} \tag{13.1.6.14}$$
$$Pr(E \mid F) = 0.428 \tag{13.1.6.15}$$

iii E:atmost two tails,F:atleast one tail

$$Pr(F) = Pr(X \geq 1) \tag{13.1.6.16}$$
$$Pr(F) = 1 - Pr(X = 0) \tag{13.1.6.17}$$
$$Pr(F) = 0.875 \tag{13.1.6.18}$$
$$Pr(EF) = Pr(X = 1) + Pr(X = 2) \tag{13.1.6.19}$$
$$Pr(EF) =^3 C_1(\frac{1}{2})^3 +^3 C_2(\frac{1}{2})^3 \tag{13.1.6.20}$$
$$Pr(EF) = 0.75 \tag{13.1.6.21}$$
$$Pr(E \mid F) = \frac{Pr(EF)}{Pr(F)} \tag{13.1.6.22}$$
$$Pr(E \mid F) = 0.857 \tag{13.1.6.23}$$