Derivatives Notes. 10/23.

$$\mathrm{Call}(K_n, T) = \sum_{j \geq n} \left(\mathrm{Call}(K_j, T) - \mathrm{Call}(K_{j+1}, T) \right)$$

$$= \sum_{j \geq n} CS(K_j, K_{j+1}, T)$$

$$CS(K_j, K_{j+1}, T) = \sum_{i \geq j} B(K_i, K_{i+1}, K_{i+2}, T)$$

$$\text{Call}(K_n, T) = \sum_{j=n}^{\infty} \sum_{i \geq j} B(K_i, K_{i+1}, K_{i+2}, T)$$

$$= \sum_{j=n}^{\infty} (j+1-n) B(K_j, K_{j+1}, K_{j+2}, T)$$

$$= \sum_{j=n}^{\infty} (j+1-n) \delta K \left(\frac{B(K_j, K_{j+1}, K_{j+2}, T)}{\delta K} \right)$$

$$= \sum_{j=n}^{\infty} (K_{j+1} - K_n)\, w(K_{j+1}, T)$$

$$= \sum_{j=0}^{\infty} (K_{j+1} - K_n)^+ w(K_{j+1}, T)$$

$$w(K_{j+1}, T) = \frac{B(K_j, K_{j+1}, K_{j+2}, T)}{\delta K}$$

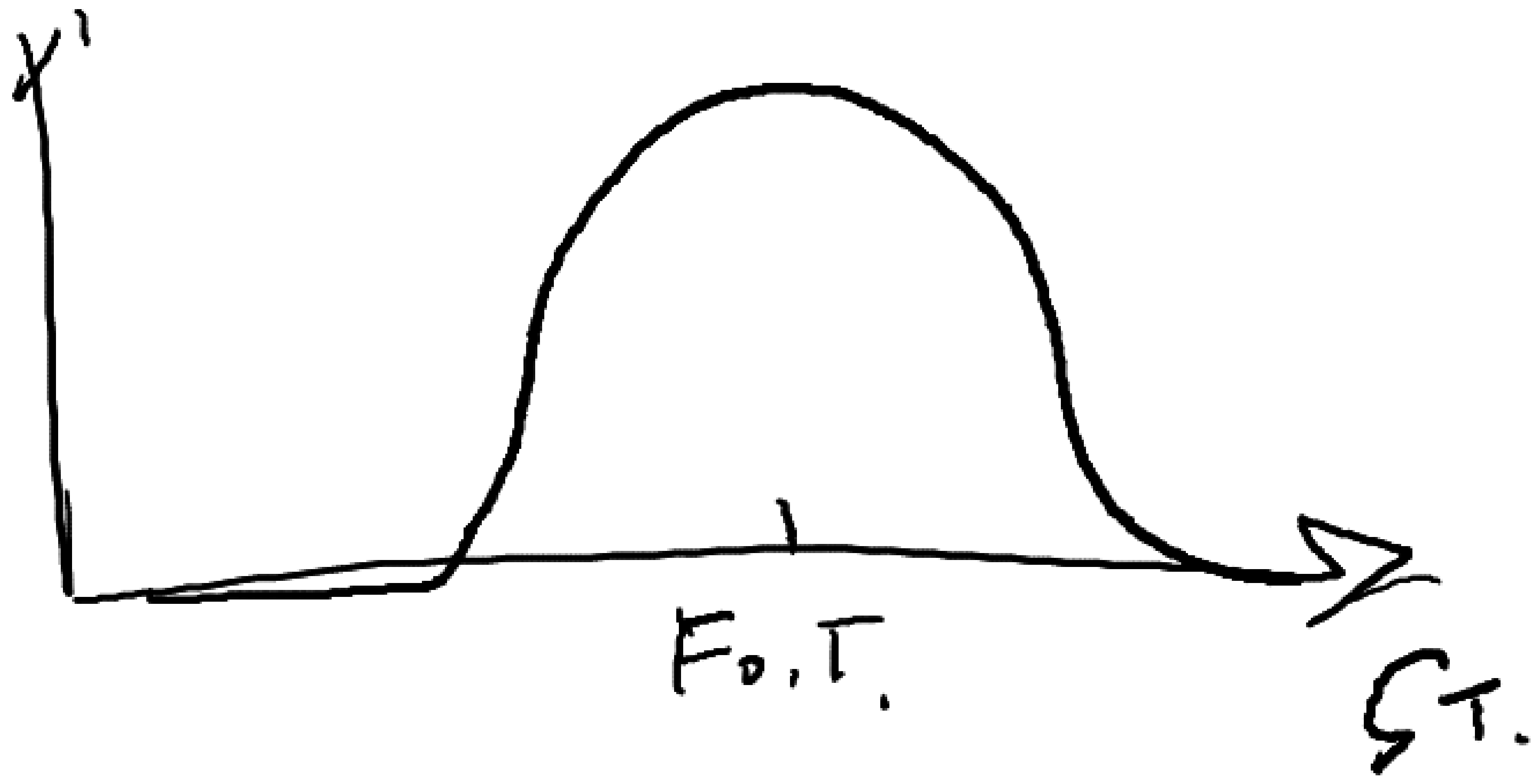
The weights correspond to values of Butterfly spreads centered at each K_j. In particular they are positive.

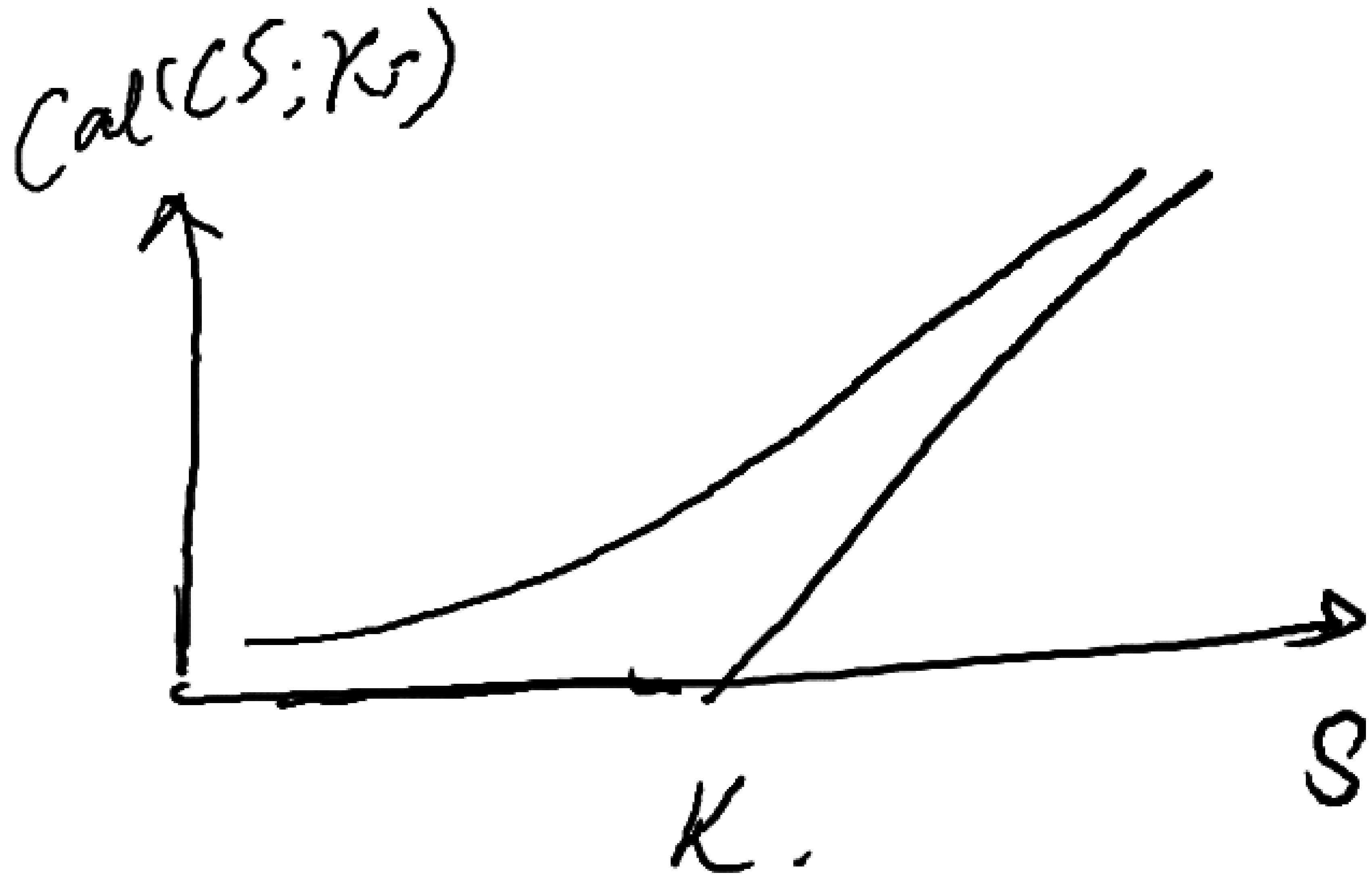
$$\mathrm{Call}(K,T) = e^{-rT} \sum_j \max(K_j - K, 0)\, p(K_j, T)$$

$$= e^{-rT} E^{p}\{\max(S_T - K, 0)\}$$

$$P_f = e^{-rT} E^Q\{f(S_T)\}.$$

A pricing measure = a probability of future prices of the underlying asset with the above property.





Option fair value is a smoothed out version of the payoff.

the Black-Scholes Model

Log return are normal.

$$X = Z\sigma\sqrt{T} - \frac{\sigma^2}{2}T + (r-q)T.$$

$$Z \sim N(0, 1)$$