

**MIT Sloan Finance Problems  
and Solutions Collection  
Finance Theory I**

Part 2

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# 1 Questions

## 1.4 Forward and Futures

1. During the summer you had to spend some time with your uncle, who is a wheat farmer. Your uncle, knowing you are studying for an MBA at Sloan, asked your help. He is afraid that the price of wheat will fall, which will have a severe impact on his profits. Thus he asks you to compute the 1yr forward price of wheat. He tells you that its current price is \$3.4 per bushel and interest rates are at 4%. However, he also says that it is relatively expensive to store wheat for one year. Assume that this cost, which must be paid upfront, runs at about \$0.1 per bushel. What is the 1yr forward price of wheat?

2. The Wall Street Journal gives the following futures prices for gold on September 6, 2006:

Maturity	Oct	Dec	Jun 07	Dec 07
Futures price (\$/oz)	635.60	641.80	660.60	678.70

and the spot price of gold is \$633.50/oz. Compute the (effective annualize) interest rate implied by the futures prices for the corresponding maturities.

3. Suppose that in 3 months the cost of a pound of Colombian coffee will be either \$1.25 or \$2.25. The current price is \$1.75 per pound.
  - (a) What are the risks faced by a hotel chain who is a large purchaser of coffee?
  - (b) What are the risks faced by a Colombian coffee farmer?
  - (c) If the delivery price of coffee turns out to be \$2.25, should the farmer have forgone entering into a futures contract? Why or why not?
4. Consider a 6-month forward contract (delivers one unit of the security) on a security that is expected to pay a \$1 dividend in three months. The annual risk-free rate of interest is 5%. The security price is \$20. What forward price should the contract stipulate, so that the current value of entering into the contract is zero?
5. Spot and futures prices for Gold and the S&P in September 2007 are given below.

	07-September	07-December	08-June
COMEX Gold (\$/oz)	\$693	\$706.42	\$726.7
CME S&P 500	\$1453.55	\$1472.4	\$1493.7

Table 1: Gold and S&amp;P 500 Prices on September 7, 2007

- (a) Use prices for Gold to calculate the effective annualized interest rate for Dec 2007 and June 2008. Assume that the convenience yield for Gold is zero.
  - (b) Suppose you are the owner of a small gold mine and would like to fix the revenue generated by your future production. Explain how the futures market enables such hedges.
6. Use the same set of information given in the problem above.
- (a) Use S&P 500 future prices to calculate the implied dividend yield on S&P 500. For simplicity, assume you can borrow or deposit money at the rates implied by Gold's futures prices.
  - (b) Now suppose you believe that we are headed for a slow-down in economic activity and that the dividend yield will be lower than the value implied in part (a). What June-2008 contracts you would buy or sell to make money, assuming your view is correct? Again, assume you can borrow or deposit money at the rates implied by Gold's futures prices.
7. The Wall Street Journal gives the following futures prices for crude oil on September 6, 2006:

Maturity	Oct	Dec	Jun 07	Dec 07
Futures price (\$/barrel)	67.50	69.60	72.66	73.49

and the spot price of oil is \$67.50/barrel. Use the interest rates you found in the previous problem.

- (a) Compute the net convenience yield (in effective annual rate) for these maturities. (You can use the market information provided in the above problem.)
  - (b) Briefly discuss the convenience yield you obtained.
8. The data is the same as in the two problems above. You are running a refinery and need 10 million barrels of oil in three months.
- (a) How do you use oil futures to hedge the oil price risk? The contract size is 1,000 barrels for futures.

- (b) Suppose that you can also rent a storing facility for 10 million barrels of oil for three months at an annualized cost of 5% (in terms of the value of oil stored). Describe how you can utilize it to lock into a fixed oil price for your future demand.
  - (c) Which of these two strategies is better? Explain why.
9. The data is the same above. Now suppose instead that you are not in the oil business but can also rent the storage facility at the same cost. Can you take advantage of the current market conditions and the rental opportunity? If yes, please explain how (i.e., describe the actions you need to take). If not, briefly explain why.
10. The current price of silver is \$13.50 per ounce. The storage costs are \$0.10 per ounce per year payable quarterly at the beginning of each quarter and the interest rate is 5% APR compounded quarterly (1.25% per quarter).
- (a) Calculate the future price of silver for delivery in nine months. Assume that silver is held for investment only and that the convenience yield of holding silver is zero.
  - (b) Suppose the actual price of the futures contract traded in the market is below the price you calculated in part (a). How would you construct a risk-free trading strategy to make money? What if the actual price is higher? To get full credit, say precisely what you will buy or sell, and how much money you will borrow or deposit into a bank account and for how long.
11. A pension plan currently has \$50M in S&P 500 index and \$50M in one-year zero-coupon bonds. Assume that the one-year interest rate is 6%. Assume that the current quote on the S&P 500 index is 1,350, each futures contract is written on 250 units of the index and the dividend yield on the index is approximately 3% per year, i.e., \$1,000 invested in the index yields \$30 in dividends at the end of the year.
- (a) Suppose you invest  $\$1,350 \times 250$  in one-year zero-coupon bonds and at the same time enter into a single futures contract on S&P 500 index with one year to maturity. Assume that in one year the index finishes at 1,200. What is the total value of your position? How does this compare with buying 250 units of the index and holding them for a year? Assume that in one year the index finishes at 1,400. Repeat the analysis.
  - (b) If this plan decides to switch to a 70/30 stock/bond mix for a period of one year, how would you implement this strategy using S&P 500 futures? How many contracts with one year to maturity

would you need? Assume that the index finishes the year at 1,400, describe the plan's portfolio in one year and one day from now (right after the futures expire). What is the stock/bond mix?

- 12.** Spot price for soybean meal is \$152.70 per ton and the 12-month soybean-meal futures is traded at \$148.00. The 1-year interest rate is 3%.
- (a) What is the net convenience yield on soybean meal for the 12 month period?
  - (b) You need 1,000 tons of soybean meal in 12 months. How would you lock into a price today using the futures contracts? (The size for each soybean-meal futures contract is 100 tons.)
- 13.** The spot price for smoked salmon is \$5,000 per ton and its six-month futures price is \$4,800. The monthly interest rate is .0025 (.25%).
- (a) What is the average monthly net convenience yield on smoked salmon for the next six months?
  - (b) If you are a manager of Bread&Circus and need 10 tons of smoked salmon in six months. How can you avoid the risk in the price of smoked salmon over the next six months using futures?
  - (c) Suppose that your net convenience yield for smoked salmon is 1.2%. How does this change your hedging strategy?
- 14.** A wine wholesaler needs 100,000 gallons of Cheap Chardonnay for delivery in Boston in June 2007. A producer offers to deliver the wine at that time for \$500,000 paid now, in December 2006.
- The wholesaler can also buy Cheap Chardonnay futures contracts for June 2007. The current futures price is \$51,000 for each 10,000 gallon futures contract.
- The wholesaler is determined to lock in the cost of the 100,000 gallons needed in June.
- (a) The wholesaler considers the futures contract, but worries that the contract will not lock in her cost, because futures prices may fluctuate widely between now and June. Is her concern justified? Why or why not?
  - (b) Do you recommend that the wholesaler pay the producer now or take a long position in Chardonnay futures? (Additional assumptions may be needed to answer. Make sure they are reasonable.) Explain briefly.

15. You are a distributor of canola seed and need to make deliveries of 10,000 bushels one month from now. You currently have no canola seed in inventory. The current spot price of canola seed is \$7.45 per bushel and the futures price for delivery in one month is \$7.65. You would like to hedge the uncertainty about the spot price one month from now.
- (a) If your storage cost is \$.15 per bushel (paid at the end of month), what would you do?
  - (b) Suppose that in the short run, your storage cost increases to \$.25 per bushel. What would you do?
16. Assume perfect markets: no transaction costs and no constraints. In addition assume that the one-month risk-free interest rate will remain constant over a three-month period. Two futures contracts with two and three months maturity are traded on a financial asset without any intermediate payout. The price for these contracts are  $F_2 = \$100$  and  $F_3 = \$101$ , respectively.
- (a) What is the spot price of the underlying asset today?
  - (b) Suppose that a one-month futures contract is trading at price  $F_1 = \$98$ . Does this imply an arbitrage opportunity? How would you take advantage of this opportunity? To get full credit, be precise on what you would buy or sell, and how much money you would deposit into a bank account and/or borrow.
17. Assume perfect markets: no transaction costs and no constraints. The one-month risk-free interest rate will remain constant over a six-month period. Two futures contracts are traded on a financial asset without payouts: a three-month (futures price  $F(t, t + 3)$ ) and a six-month (futures price  $F(t, t + 6)$ ) contract. You can observe that  $F(t, t + 3) = \$120$  and  $F(t, t + 6) = \$122$ .
- (a) What is the spot price of the underlying asset at time  $t$ ?
  - (b) Suppose that a three-month futures contract is trading at price  $F(t, t + 3) = \$119.5$ . Does this imply an arbitrage opportunity? How would you take advantage of this opportunity?

## 1.5 Options

1. A stock price is currently \$50. It is known that at the end of two months it will be either \$53 or \$48. The risk-free interest rate is 10% per annum with continuous compounding. What is the value of a two-month European call option with a strike price of \$49?
2. A stock price is currently \$80. It is known that at the end of four months it will be either \$75 or \$85. The risk-free interest rate is 5% per annum with continuous compounding. What is the value of a four-month European put option with a strike price of \$80?
3. Today's price of three traded call options on BackBay.com, all expiring in one month, are as follows:

Strike Price	Option Price
\$50	$\$7\frac{1}{2}$
60	\$3
70	$\$1\frac{1}{2}$

You are considering buying a “butterfly spread” consisting of the following positions:

- Buy 1 call at strike price of \$50
  - Sell (write) 2 calls at strike price of \$60
  - Buy 1 call at strike price of \$70.
- (a) Plot the payoff of your total position for different values of the stock price on the maturity date.
  - (b) What is the dollar investment required to establish the spread?
  - (c) For what stock prices on the maturity date will you be making an overall profit?
4. You are given the following prices:

Security	Maturity (years)	Strike	Price (\$)
JEK stock	-	-	94
Put on JEK stock	1	50	3
Put on JEK stock	1	60	5
Call on JEK	1	50	?
Call on JEK	1	60	?
Tbill (FV=100)	1	-	91

What is the price of the two call options?

5. (a) Here are the payoff diagrams of some popular trading strategies using just put and call options with same maturities. How would you replicate them? Identify the number and strikes of call or put options that have to be bought or sold in order to generate these payoffs. (All angles are 45 degrees!)

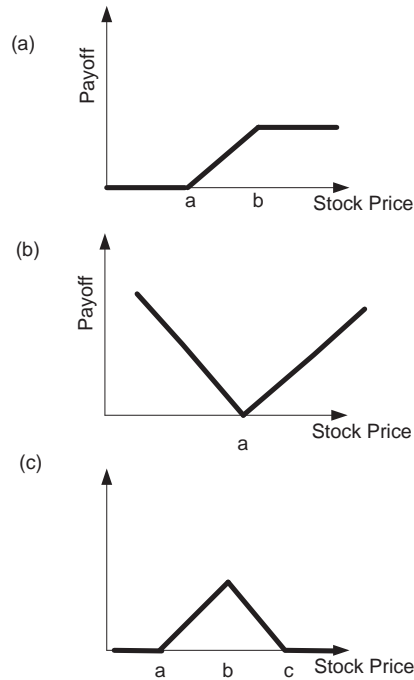


Figure 1: Payoff Diagrams

- (b) Now suppose for institutional reasons, you are short on volatility in this market, i.e. you will lose money if the market becomes volatile. For example you can imagine that you are an investment bank working on a few major M&A deals which may fall apart if the market goes down too much or goes up too much. In either case, you will lose money if the market becomes volatile. Explain which if any of the above three payoffs would work well to hedge your exposure. What is the cost of this hedge to you?
6. A *butterfly spread* is a combination of option positions that involve three strike prices. To create a butterfly spread, a trader purchases an option with a low strike price and an option with a high strike price and sells two options with an intermediate strike price. For this problem,



assume that the intermediate strike price is halfway between the low and the high strike prices and that the options are European. Denote the intermediate strike price by  $X$ , the low strike price by  $X - a$ , and the high strike price by  $X + a$ ,  $a > 0$ .

- (a) Graph the payoff diagrams at maturity of the butterfly spread in which the underlying options are call options. Holding the intermediate strike price fixed, what happens to the payoffs as the low and high strike price converge to the intermediate price?
  - (b) Suppose that a trader purchases a butterfly spread (using call options) for which the intermediate strike price equals to today's stock price. Based only on this trade, what is the trader's view of the future direction of the market?
7. IBM shares are now traded at \$80.00. The term structure of interest rates is flat at 5%.
- (a) Plot the terminal payoff from a European put option on 1 share of IBM with a exercise price of \$85 and a maturity of 3 months (not including the price of the option).
  - (b) Suppose that you purchase the put option at \$4.00 from the market. Specify the ranges of IBM share price at the options maturity date for which you will be making a net profit.
8. Suppose that after a recent news about the economy, IBM share price remains the same, but the prices of its options shot up. How is this possible?
9. You are given the following information. Use this information to determine the unknown prices.

Table 2: Stock, Options, and T-bill prices

Secutiry	Maturity (years)	Strike	Price(\$)
401 Stock	-	-	\$100
Put on 401 Stock	1	\$50	\$3
Put on 401 Stock	1	\$60	\$5
Calle on 401 Stock	1	\$50	\$57.50
Calle on 401 Stock	1	\$60	?
Tbill(FV=100)	1	-	?

10. Joseph Jones, a manager at Computer Science, Inc. (CSI), received 1,000 shares of company stock as part of his compensation package. The stock currently sells at \$40 at share. Joseph would like to defer selling the stock until the next tax year. In January, however, he will

need to sell all his holdings to provide for a down payment on his new house. Joseph is worried about the price risk involved in keeping his shares. At current prices, he would receive \$40,000 for the stock. If the value of his stock holdings falls below \$35,000, his ability to come up with the necessary down payment would be jeopardized. On the other hand, if the stock value rises to \$45,000, he would be able to maintain a small cash reserve even after making the down payment. Joseph considers three investment strategies:

- (a) Strategy A is to write January call options on the CSI shares with strike price \$45. These calls are currently selling for \$3 each.
- (b) Strategy B is to buy January put options on CSI with strike price \$35. These options also sell for \$3 each.
- (c) Strategy C is to establish a zero-cost collar by writing the January calls and buying the January puts.

Evaluation each of these strategies with respect to Joseph's investment goals. What are the advantages and disadvantages of each? Which would you recommend?

11. You write a call option with strike \$50 and buy a call with strike \$60. The options are on the same stock and have the same maturity date. One of the calls sells for \$3; the other sells for \$9. (Assume zero interest rate.)
- (a) Draw the payoff graph for this strategy at the option maturity date.
  - (b) Draw the profit graph for this strategy.
  - (c) What is the break-even point for this strategy? Are you bullish or bearish on the stock?
12. Consider an increasingly popular deposit contract with payoffs linked to the performance on the S&P 500 Index on the U.S. stock market. For every dollar invested in the contract, the rate of return in one year is equal to 60% of the realized rate of return of the S&P 500 Index during this year if this rate of return is positive; otherwise, you get your money back. In essence, you are protected from the downside risk of the S&P 500 Index, while you are still able to participate in the upside potential of the stock market. The one year riskless interest rate is 10%. For simplicity, assume that the stocks in the index do not pay dividends.
- (a) Draw a graph for the payoff one year from now for a one dollar investment in the contract with the horizontal axis being the re-

- alized rate of return on the S&P 500 Index. Also write down the payoff symbolically.
- (b) Show that the payoff one year from now for a one dollar investment in this contract is the payoff to a portfolio of a default-free bond and a European call option on the S&P 500 Index.
  - (c) Suppose that the rates of return on the S&P 500 Index can take two possible values one year from now, 20% and -20% with probabilities 60% and 40%, respectively. Do you make money or lose money investing in this contract? If so, how much?
13. What is a lower bound for the price of 3-month call option on a non-dividend-paying stock when the stock price is \$50, the strike price is \$45, and the 3-month risk-free interest rate is 8%? Explain briefly.
14. Draw position (payoff) diagrams for each of the following trades. Each put or call option is written on 100 shares of the same stock and has the same 6-month maturity. The current stock price is \$50 per share.
- (a) Buy 100 shares, buy a put with an exercise price of \$40, sell a call with an exercise price of \$60.
  - (b) Same as (a), except that you borrow \$4902. The semi-annual interest rate is 2%, so you will have to repay  $4902 \times 1.02 = 5000$  after six months.
  - (c) Buy a put and a call with exercise price of \$50, sell a put with exercise price of \$40, sell a call with an exercise price of \$60.
15. Explain how you could generate the same payoffs as in part a of last question without purchasing any shares.
16. Ineffable Corporations stock price is currently \$100. At the end of 3 months it will be either \$110 or \$90.91. The risk-free interest rate is 2% per annum. What is the value of a 3-month European call option with a strike price of \$100? Calculate your answer to this problem using
- (a) replication.
  - (b) the risk-neutral method.
17. State whether the following statements are true or false. In each case, provide a brief explanation.
- (a) In a risk averse world, the binomial model states that, other things being equal, the greater the probability of an up movement in the stock price, the lower the value of a European put option.

- (b) By observing the prices of call and put options on a stock, one can recover an estimate of the expected stock return.
- (c) An investor would like to purchase a European call option on an underlying stock index with a strike price of 210 and a time to maturity of 3 months, but this option is not actively traded. However, two otherwise identical call options are traded with strike prices of 200 and 220 respectively, hence the investor can replicate a call with a strike price of 210 by holding a static position in the two traded calls.
- (d) In a binomial world, if a stock is more likely to go up in price than to go down, an increase in volatility would increase the price of a call option and reduce the price of a put option. Note that a *static position* is a position that is chosen initially and not rebalanced through time.

Draw a diagram showing an investor's profit and loss with the terminal stock price for a portfolio consisting of:

18. (a) One share of stock and a short position in one call option  
 (b) Two shares of stock and a short position in one call option  
 (c) One share of stock and a short position in two call options  
 (d) One share of stock and a short position in four call options

You should take into account the cost from purchasing the stock and revenue from selling the calls. For simplicity ignore discounting when combining these costs and revenues with the terminal payoff of the portfolio. For simplicity also assume that the current stock price is equal to the strike price,  $K$ , of the call. Denote the current call price by  $c$ , and the terminal stock price by  $S_T$ .

19. Stock XYZ is worth  $S = \$80$  today. Every 6 months the stock price goes either up by  $u = 1.3$  or down by  $d = 0.8$ . The riskless rate is 6% APR with semiannual compounding. The stock pays no dividends.
- (a) Compute the price of a European call with a maturity of 1 year and a strike price of  $X = \$95$ .
  - (b) Compute the price of an American call with a maturity of 1 year and a strike price of  $X = \$95$ .
  - (c) Compute the price of a European put with a maturity of 1 year and a strike price of  $X = \$95$ .
20. In August 1998 the Bank of Thailand was reported as offering to foreign investors in troubled banks the opportunity to resell their shares back to the central bank within a period of five years for the original purchase

price. “This is to guarantee that at least they will not lose any of the money they plan to invest,” said the Deputy Governor. (*The Wall Street Journal Europe*, August 6, 1998, p.20.) Suppose that (a) the standard deviation of Thai bank shares was about 50 percent a year, (b) the interest rate on the Thai baht was 15%, and (c) the banks were not expected to pay a dividend in this five-year period. How much was this option worth? Assume an investment of 100 million baht.

21. Shares of ePet.com are traded at \$60. In six months, share price could either be \$66 or \$54 with probability 0.6 and 0.4, respectively. The current 6-month risk-free rate is 6%. What is the price of a European put on 100 ePet shares with a strike price of \$64 per share? Would your answer be different if the option is American?
22. Consider again ePet. You want to use ePet shares and the risk-free bond to replicate a payoff in six months that equals the square of ePet's share price. That is, when ePet price goes up to \$66, you have a payoff of  $66^2 = \$4,356$  and when the price goes down to \$54, you have a payoff of  $54^2 = \$2,916$ . Describe the strategy that gives these payoffs. What is the present value of these payoffs?
23. The price of the stock of NewWorld Chemicals Company is \$80. The standard deviation of NewWorld's stock returns is 50%. The 1-year interest rate is 6%.
  - (a) What should be the price of a call on one share of NewWorld with a maturity of 1 year and strike price of \$85? Use the Black-Scholes formula.
  - (b) What should be the price of a put on one share of NewWorld with the same maturity and strike price?
24. You are asked to price some options on ABC stock. ABC's stock price can go up by 15 percent every year, or down by 5 percent. Both outcomes are equally likely. The risk free interest rate is 5 percent per year for the next two years, and the current stock price of ABC is \$100.
  - (a) Find the risk neutral probabilities
  - (b) What is the price of a European Call option on ABC, with strike 100 and maturity 2 years?
  - (c) Describe the strategy to replicate the payoff of the call using the stock and the risk-free bond.

- (d) What is the price of an American option with the same characteristics?
- 25.** You are asked to price some options on KYC stock. KYC's stock price can go up by 15 percent every year, or down by 10 percent. Both outcomes are equally likely. The risk free rate is 5 percent, and the current stock price of KYC is 100.
- (a) Price a European Put option on KYC with maturity of 2 years and a strike price of 100.
- (b) Price an American Put option on KYC with the same characteristics. Is the price different? Why or why not?
- 26.** IBM is currently trading at \$90.29 per share. You believe that IBM will have an expected return of 7% with volatility of 26.1% per year, while annual interest rates are at 0.95%. What is the price of an European put on IBM with a strike price of \$90 and maturity of 1 year?
- 27.** Shares of Ontel will sell for either \$150 or \$80 three months later, with probabilities 0.60 and 0.40, respectively. A European call with an exercise price of \$100 sells for \$25 today, and an identical put sells for \$8. Both options mature in three months. What is a price of a three-month zero-coupon bond with a face of \$100?
- 28.** 401.com's stock is trading at \$100 per share. The stock price will either go up or go down by 25% in each of the next two years. The annual interest rate is 5%.
- (a) Determine the price of a two-year European call option with the strike price  $X = \$110$ .
- (b) Determine the price of a two-year European put option with the strike price  $X = \$110$ .
- (c) Verify that the put-call parity holds.
- (d) Determine the price of a two-year American put option with the strike price  $X = \$110$ .
- (e) What is the replicating portfolio (at every node of the tree) for the American put option with the strike price  $X = \$110$ ?
- 29.** For this problem assume that the risk-free rate of interest for one year loans is 5%. Google stock is selling today for \$500 a share. Assume that in one year Google will either be worth \$600 a share or \$475 a

share and that Google will pay no dividends for at least two years. A call option with an exercise price of \$550 and one year to go until expiration is available for Google stock. What is the value of this call option?

30. A particular stock follows the price movement below.

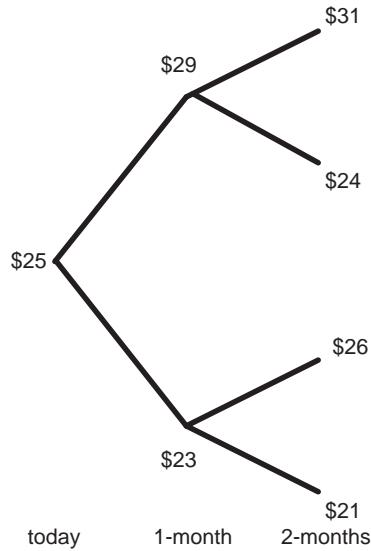


Figure 2: Stock Price Movement

- (a) For this part, suppose the interest rate is fixed at 1% per month. What is the price of a put option with maturity two months, and strike of \$26 ?
- (b) Again, suppose the interest rate is fixed at 1% per month. What is the price of an exotic derivative that in 2-months has a pay off that is a function of the maximum price of the stock during the two month period given by

$$\max(\hat{S} - \$25, 0),$$

where

$$\hat{S} = \max_{0 \leq t \leq 2} S_t.$$

and  $t$  is measured in months.

31. Intel stock is trading at \$120 per share, and the company will not pay any dividends over the next year. Consider an Intel European call option and a European put option, both having an exercise price of

\$124 and both maturing in exactly one year. The simple (annualized) interest rate for borrowing and lending between now and one year from now is 3% for each 6 month period (6.09% per year).

Assume that there are no arbitrage opportunities. Is there enough information to determine which option has the higher market value? If so, which option, the call or the put, has higher market value?

- 32.** Calculate the price of a three-month European put option a non-dividend paying stock with a strike price of \$50 when the current stock price is \$50, the risk free rate is 10% per annum, and the volatility is 30% per annum. What difference does it make to your calculations if a dividend of \$1.50 is expected in two months? Assume that the assumptions made to derive the Black-Scholes formula are valid.
- 33.** It is possible to buy three-month call options and three-month put options on stock X. Both have an exercise price of \$60 and both are worth \$10. Is a six-month call with an exercise price of \$60 more or less valuable than a similar six-month put?