hw2

October 27, 2019

Homework 2 - Bolun Zhang

1 Question 1 You believe that interest rates will rise. Accordingly, you

1.1 A. Buy a December Eurodollar futures - False

How the Eurodollar futures contract works For example, if on a particular day an investor buys a single three-month contract at 95.00 (implied settlement LIBOR of 5.00%):

if at the close of business on that day, the contract price has risen to 95.01 (implying a LIBOR decrease to 4.99%), US\$25 will be paid into the investor's margin account; or if at the close of business on that day, the contract price has fallen to 94.99 (implying a LIBOR increase to 5.01%), US\$25 will be deducted from the investor's margin account.

Therefore, an invester who long the future would want the LIBOR rate to decrease. Now I expect the LIBOR rate to increase, which mean I would not want to buy the future.

1.2 B. Sell a December Eurodollar futures - True

Similar to above, I would want to sell the future because I bet the rate goes up and the price goes down.

1.3 C. Pay fixed on a 5-year swap - True

If I believe the interest rates will rise, I believe the floating leg rate of a swap will rise. Therefore, I would rather pay the fixed leg and receive the floating leg payment.

1.4 D. Receive fixed on a 5-year swap - False

Similar to C, I believe the floating rate will rise. Then, I would not want to receive fixed payment and pay the floating rate.

2 Question 2 Consider a FRA in which you will borrow money for 6 months on December 20, 2019 (T) based on the 3-month LIBOR rate:

2.1 Please see attached handwritten problem solving.

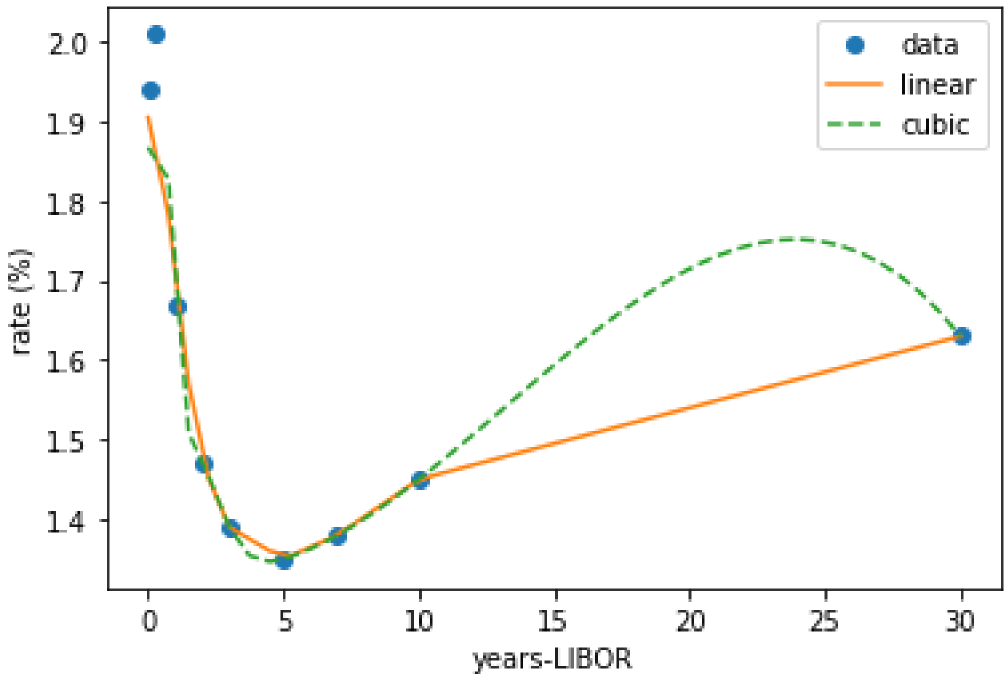
3 Question 3 Build a smooth discount curve (Z-curve) with monthly granularity for LIBOR rates on October 8, 2019, using the following information:

[18]:
```python
from scipy.interpolate import interp1d
import numpy as np
import matplotlib.pyplot as plt

data = []
data += [(30/365, 1.94)]
data += [(90/365, 2.01)]
data += [(1, 1.67)]
data += [(2, 1.47)]
data += [(3, 1.39)]
data += [(5, 1.35)]
data += [(7, 1.38)]
data += [(10, 1.45)]
data += [(30, 1.63)]
x, y = [], []
for (t,r) in data:
    x += [t]
    y += [r]

x = np.asarray(x)
y = np.asarray(y)

f = interp1d(x, y,fill_value="extrapolate")
f2 = interp1d(x, y, kind='cubic',fill_value="extrapolate")
xnew = np.linspace(0, 30, num=41, endpoint=True)
plt.plot(x, y, 'o', xnew, f(xnew), '-', xnew, f2(xnew), '--')
plt.legend(['data', 'linear', 'cubic'], loc='best')
plt.xlabel('years-LIBOR')
plt.ylabel('rate (%)')
plt.show()
```



4 Question 4 Build the curve using the Hagan-West iteration method explained in p. 92 of that paper.

4.1 Plot using Hagan West method

Q2.

Suppose a, b, c are zero bond values of a maturity payment of \$1

$$a = \$1 \cdot \frac{1}{1 + T \times TLIBOR(0)}$$

$$b = \$1 \cdot \frac{1}{1 + (T+0.25) \times T3MLIBOR(0)}$$

$$c = \$1 \cdot \frac{1}{1 + (T+0.5) \times T6MLIBOR(0)}$$

Based on no arbitrage at $t = 0$ current state. the implied 3MLIBOR(T) should be.

$$d = \$1 \cdot \frac{1}{1 + 0.25 \times 3MLIBOR(T)}$$

$$1 + 0.25 \times 3MLIBOR(T) = \frac{1}{d}..$$

$$3MLIBOR(T) = \frac{4}{d} - 4.$$

where d is the initial investment at $t = T$ for 3M.

Based on no arbitrage at current state

$$a' = d \cdot \frac{1}{1 + T \times TLIBOR(0)} = b$$

That is, financing 2 separate terms of $0 \sim T$ and $T \sim T+3M$, should give the same return as financing 1 term of $0 \sim T+3M$.

$$\Rightarrow \quad d = \frac{b}{a}. \Rightarrow 3MLIBOR(T) = \frac{4a}{b} - 4.$$

Similarly, let's calculate 6M LIBOR(T).

assume e is the investment at $t=T$ for 6 month to get \$1 maturity payment

$$e = \$1 \cdot \frac{1}{1 + 0.5 \cdot 6MLIBOR(T)}$$

$$0.5\, 6MLIBOR(T) = \frac{1}{e} - 1$$

$$6MLIBOR(T) = \frac{2}{e} - 2.$$

Also, $a'' = e \cdot \frac{1}{1 + T \times TLIBOR(0)} = c \Rightarrow e = \frac{c}{a}.$

$$\Rightarrow 6MLIBOR(T) = \frac{2a}{c} - 2$$

Now, we know 3MLIBOR(T) and 6MLIBOR(T) and we have their corresponding bond pricing d and e assuming a maturity payment of \$1.

$$d = \$1 \cdot \frac{1}{1+0.25 \times 3MLIBOR(T)}$$

$$e = \$1 \cdot \frac{1}{1+0.5 \cdot 6MLIBOR(T)}$$

Let's calculate 3M LIBOR(T+3M)

assume f is the investment at t = T+3M with maturity payment of \$1 at t = T+6M.

$$f = \$1 \cdot \frac{1}{1+0.25 \times 3MLIBOR(T+3M)}$$

$$d' = f \cdot \frac{1}{1+0.25 \times 3MLIBOR(T)} = e \Rightarrow f = \frac{e}{d}$$

$$3MLIBOR(T+3M) = \left(\frac{1}{f} - 1\right) \times 4 = \frac{4d}{e} - 4$$

Now we have everything we need, lets calculate FRA value based on the following

① Borrowing \$1 at $t=T$ using the specified FRA

② Lending \$1 at $t=T$ for 3M

③ Collect return from 1st lending at $t=T+3M$ and lend them again at $t=T+3M$ for 3M.

	Borrow	Lending
$t=T$.	$+\$1$.	$-\$1$.
$t=T+3M$		$+\$[1+0.25\times 3M\,LIBOR(T)]$ $-\$[1+0.25\times 3M\,LIBOR(T)]$
$t=T+6M$	$-\$[1+0.5\times 3M\,LIBOR(T)]$	$+\$[1+0.25\times 3M\,LIBOR(T)]$ $\times[1+0.25\times 3M\,LIBOR(T+3M)]$

$$\Rightarrow \text{total cashflow} = [1+0.25\times 4\times(\tfrac{a}{b}-1)]\times[1+0.25\times 4\times(\tfrac{d}{e}-1)] - [1+0.5\times 4\times(\tfrac{a}{b}-1)]$$

$$= \frac{a}{b}\times\frac{d}{e}-(1+\frac{2a}{b}-2).$$

$$= \frac{a}{b}\times\frac{b/a}{c/a}+1-\frac{2a}{b}$$

$$= \frac{a}{c}+1-\frac{2a}{b}$$

⇒ We can calculate the implied FRA value.

to be $\frac{a}{c} + 1 - \frac{2a}{b}$

This matches our expectation when 3M-LIBOR(T) < 6M-LIBOR(T), the FRA value will be positive.

The possible application of this is that, we can form our view on a complicate FRA based on our views of zero rates. For example, by expressing the FRA value in terms of a, b, c, when we have a market view on a, b, c, we can easily determine our view on this particular FRA, and as a result, form a market strategy.