Derivative Securities: Lecture 3 Options, Revised 2019

Sources:
J. Hull Instructor's notes Yahoo Finance
Bloomberg.com

Options

- A **call option** is a contract that allows the holder (long) to purchase an underlying asset from the writer (short) at a fixed price during a specified period of time.

- A **put option** is a contract that allows the holder (long) to sell an underlying asset from the writer (short) at a fixed price during a specified period of time.

- Options can be traded OTC between two counterparties (e.g. banks, or banks and their clients) or in exchanges (CBOE, ISE, NYSE, PCOST, PHLX, BOX).

- Main underlying assets: equity shares, equity indexes, swaps and bonds, futures on bonds, futures on equity indexes, foreign exchange (OTC mostly).

- Many OTC derivatives such as **swaps and strucutured notes** have ``embedded options in them, which makes their study very relevant.

- **Convertible bonds** also contain embedded options.

Why do we study options?

- Optionality is a major component of derivative contracts (the other being forward transacting).

- We want to understand

  -- **pricing** of options and embedded options

  -- the **sensitivity** of option values to underlying assets and risk factors

  -- the **risk** of option positions

- The main approach for doing this is to study pricing models for options and how they depend on related market variables and terms of the contract.

- Ultimately, we will be interested in the sensitivity of options to both **typical** market moves as well as **extreme** market moves

Specifying an Option Contract

- An option contract is specified by
  -- put or call
  -- underlying asset
  -- notional amount
  -- exercise price
  -- maturity date or expiration date
  -- style (American, European)
  -- settlement (cash or physical)

- An American option can be exercised anytime before the expiration date

- A European option can be exercised only at the maturity date

Example: Exchange Traded Equity Option

SPY December 120 Call

Underlying asset: SPY
Notional Amount: 100 Shares
Exercise Price: $120
Expiration date: Friday, December 16 2011
Style: American
Settlement: Physical

- This option trades in the six US options exchange
- Most US exchange-traded options are standardized to a notional of 100 shares
- Expiration is on the 3rd Friday of the expiration month
- Strikes are standardized as well, in increments of $2.50, $5 or $1, depending on the underlying asset and the strike price.
- Regulated by Securities & Exchange Commission,US laws, etc. Centrally cleared by Options Clearing Corporation.

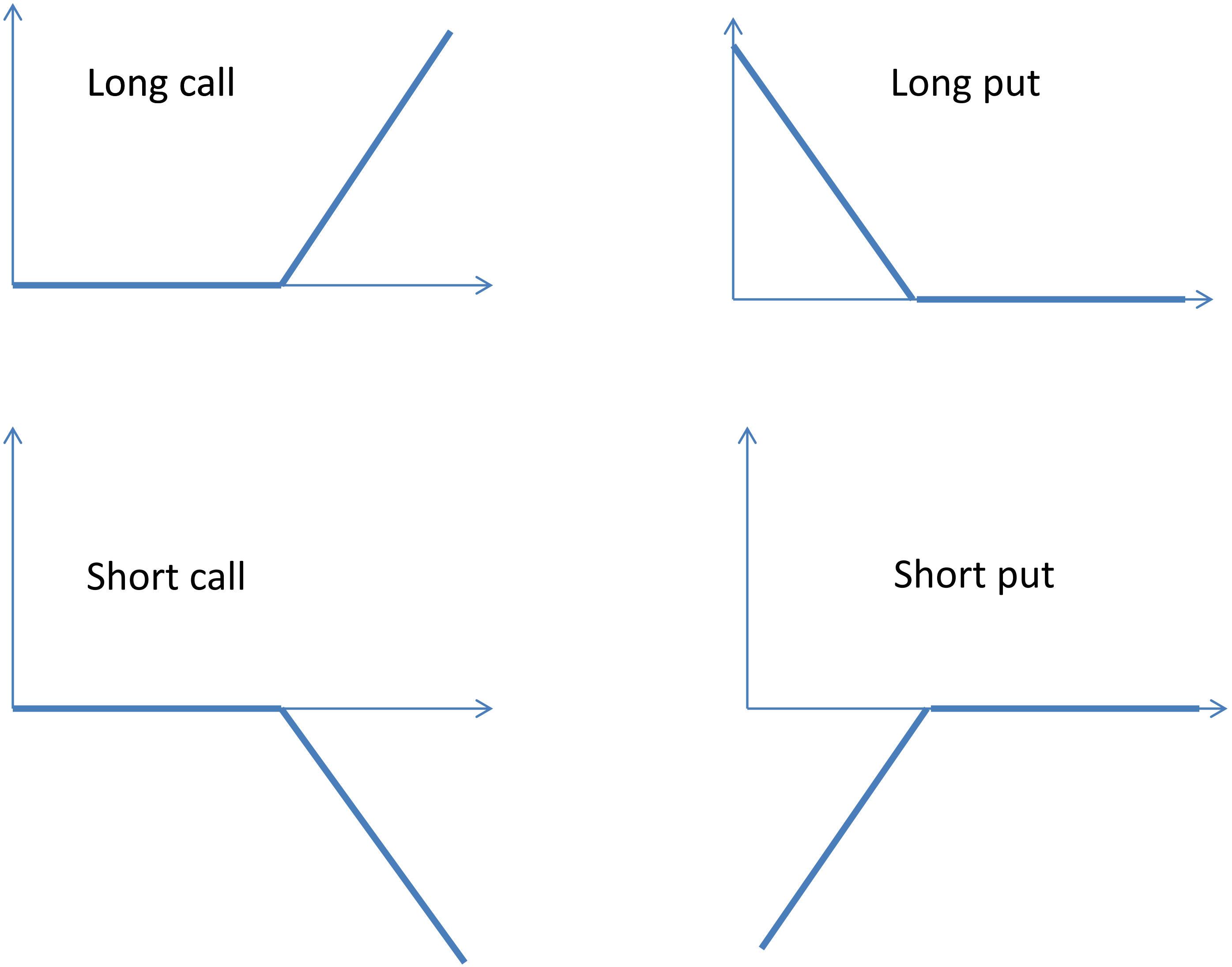
Example: an OTC currency option

120 day USD/JPY 85 Put

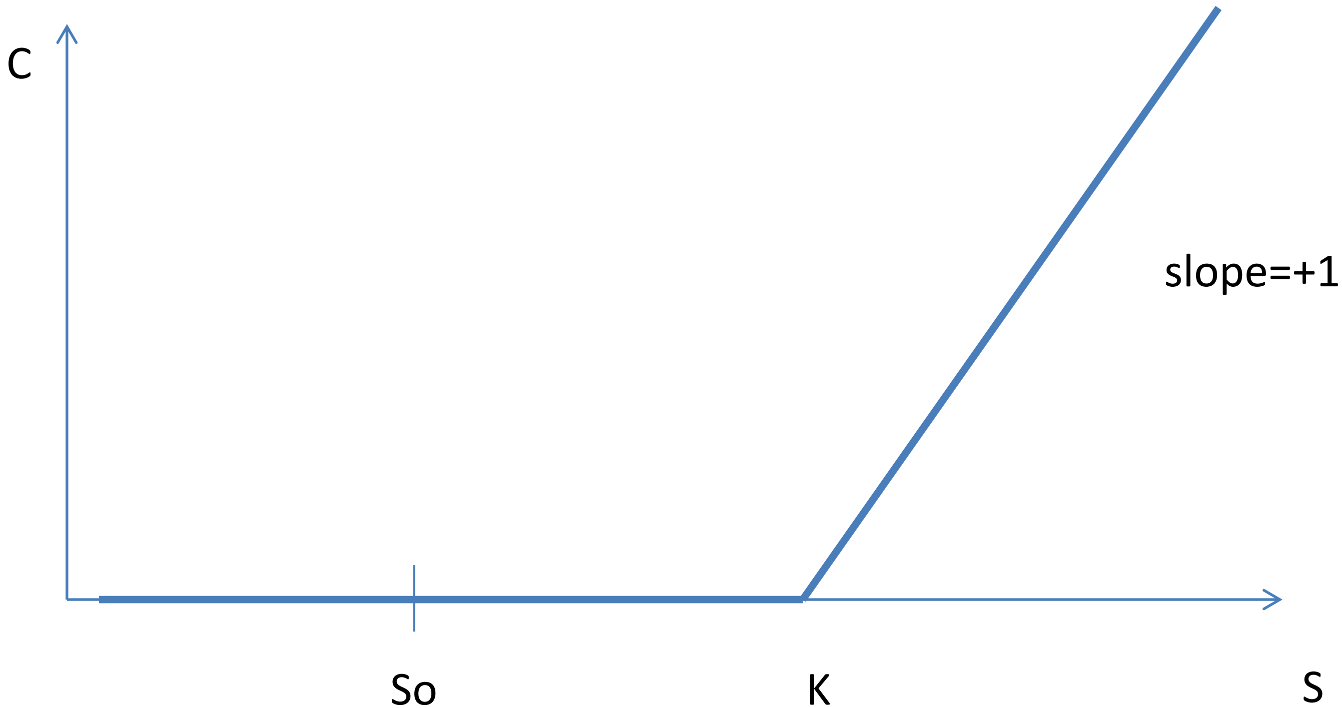
- Underlying asset: USD/JPY
- Notional amount: USD 40,000,000
- Trade date: Sep 19 2011
- Expiration date: Jan 17 2012
- Style: European
- Settlement: Cash

- OTC contract between banks or banks and clients
- Notional not standardized (minimum notional ~ 10 MM USD)
- Strikes are not necessarily standardized
- Governed by interbank agreements.
- Not centrally cleared.

Basic positions & profit diagrams



A closer look at the call payoff



Payoff = max(S-K,0)

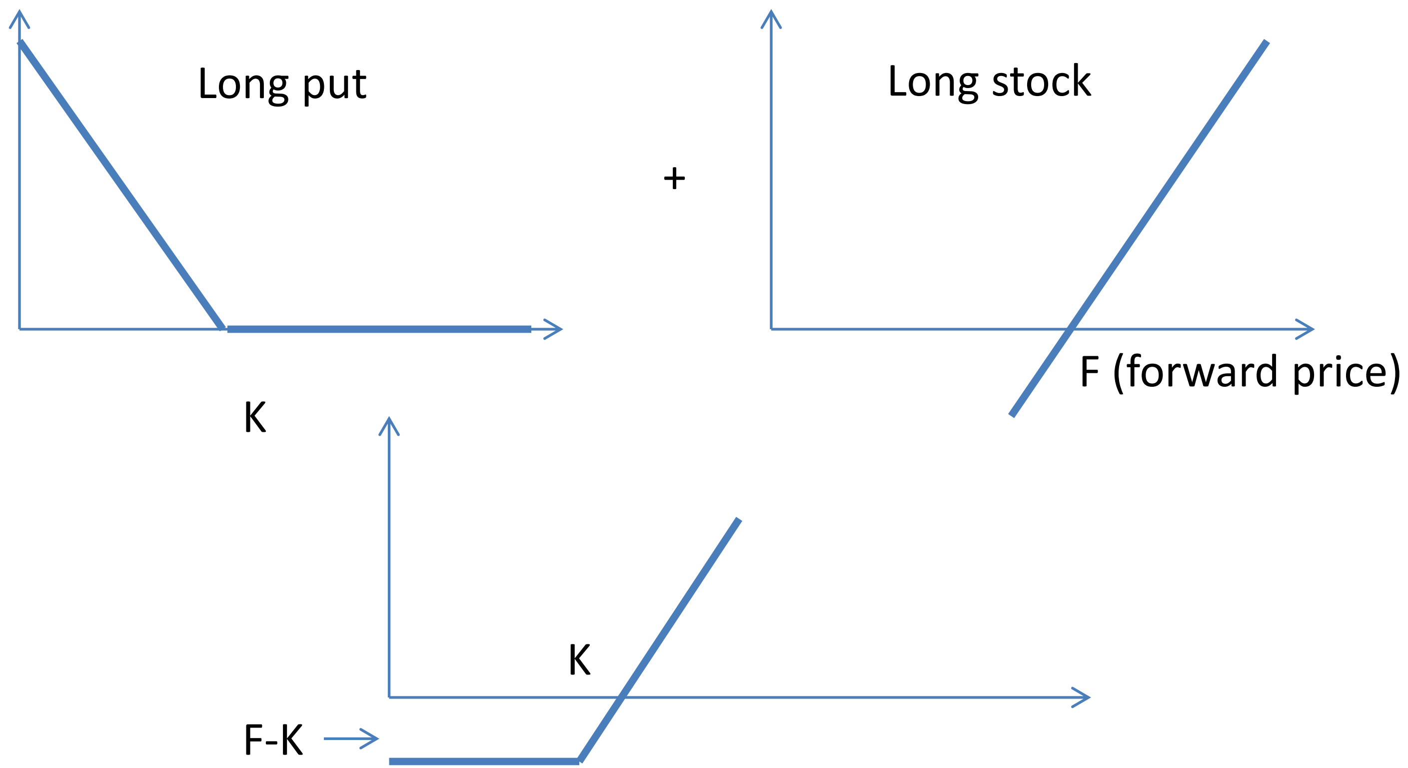
If So<K, the option is **out-of-the-money**

If So>K, the option is **in-the-money**

Put-Call Parity

- This principle applies to European options but is also widely use to analyze American-style options as well.

- A position long put + long forward is equivalent to long call (up to a cash position) as shown by the diagram below:



Put-Call Parity

Call payoff Put payoff

$$\max(S_T - K, 0) - \max\left(K - S_T\, 0\right) = S_T - K$$

$$= S_T - F_T + F_T - K$$

Forward payoff Cash

- Since, by definition, the ATM forward contract has zero value, we have, in terms of the option premia,

$$\boxed{Call\left(K,T\right) - Put\left(K,T\right) = PV\left(F_T - K\right)}$$

- Arbitrage relation between the fair values of European-style puts and calls

Put-Call Parity in terms of forward & spot prices

- If the options are at-the-money forward,

$$K = F_T \quad \Rightarrow \quad Call(F_T, T) = Put(F_T, T)$$

- In general, we have

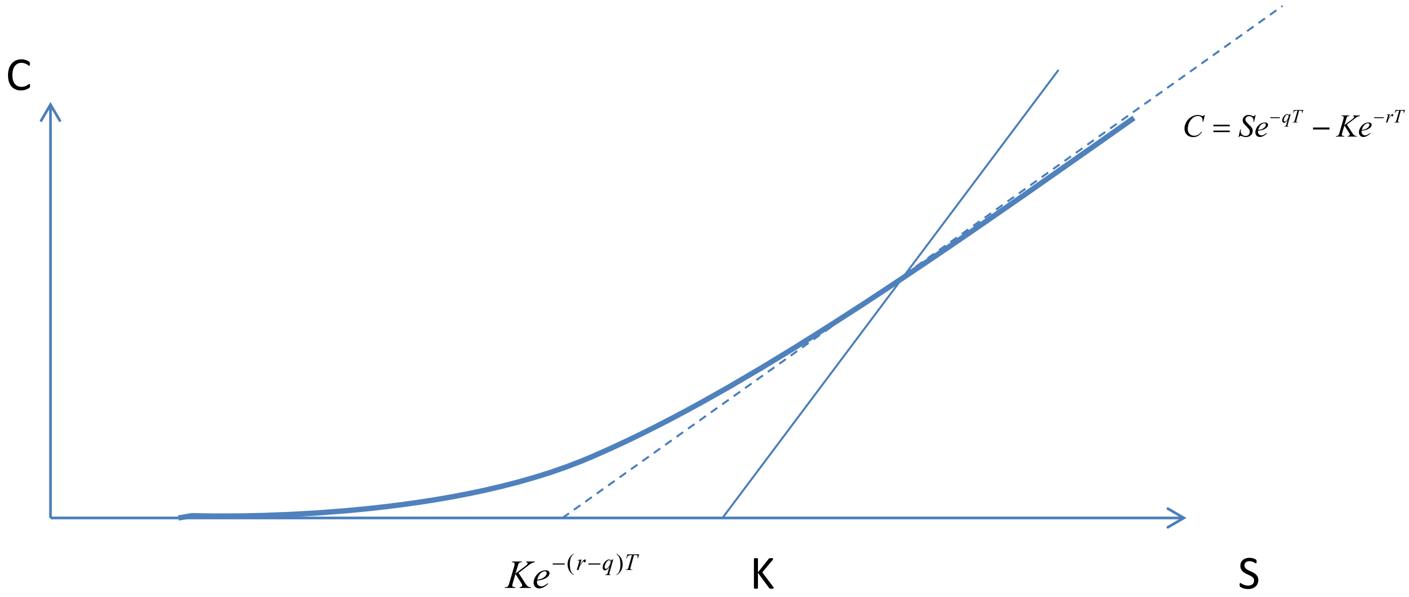
$$\begin{aligned} Call(K,T) - Put(K,T) &= PV(F_T - K) \\ &= e^{-rT}\left(e^{(r-q)T} S_0 - K\right) \\ &= e^{-qT} S_0 - e^{-rT} K \end{aligned}$$

$q =$ dividend yield for the stock over period $(0,T)$

$r =$ funding rate over the period $(0,T)$

Basic properties of options: calls

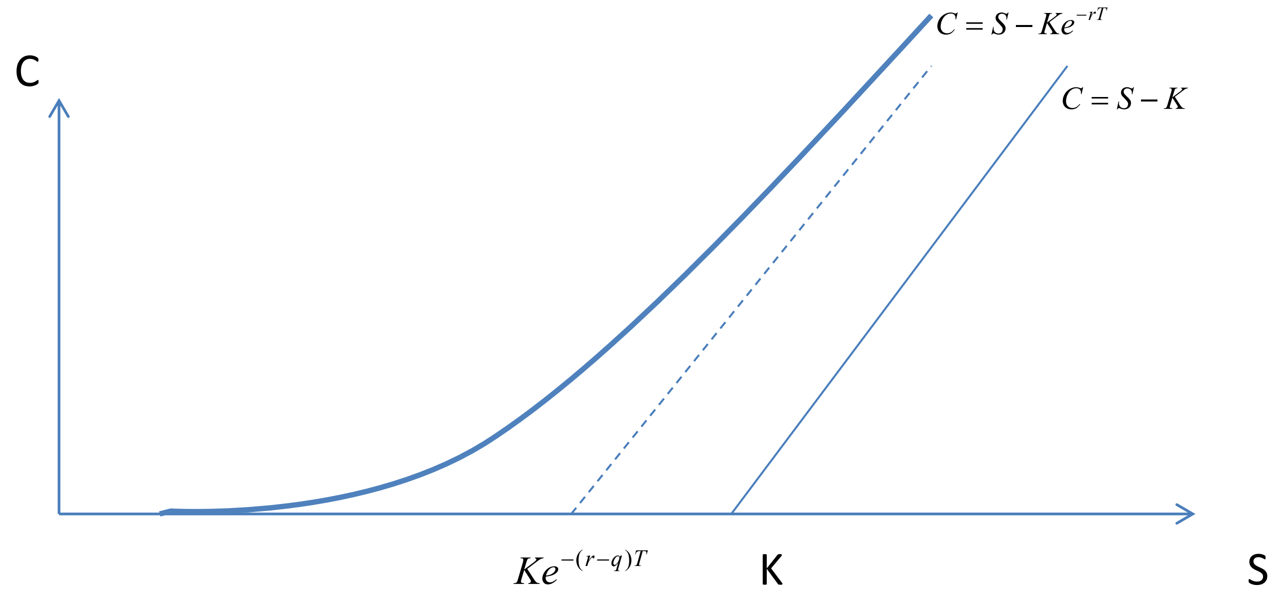
$$Call(S,K,T) > 0, \quad Call(S,K,T) > Se^{-qT} - Ke^{-rT}$$
$$Call(S,K,T) \approx Se^{-qT} - Ke^{-rT}, \; S/K >> 1$$



Call premium is increasing in S/K and asymptotic to PV(F-K).

If there are no dividend payments then C>Max(S-K,0)

$$Call(S,K,T) > 0, \quad Call(S,K,T) > S - Ke^{-rT}$$



Call premium is increasing in S/K and asymptotic to PV(F-K).

American-style vs. European-Style calls

$$Call_{am}(K,T) \geq Call_{eu}(K,T)$$

always

$$\text{If } \; q = 0,\; Call_{eu}(K,T) \geq (S - Ke^{-rT})^{+} > (S-K)^{+}$$

$$\therefore \qquad Call_{am}(K,T) > (S-K)^{+}$$

if S>K

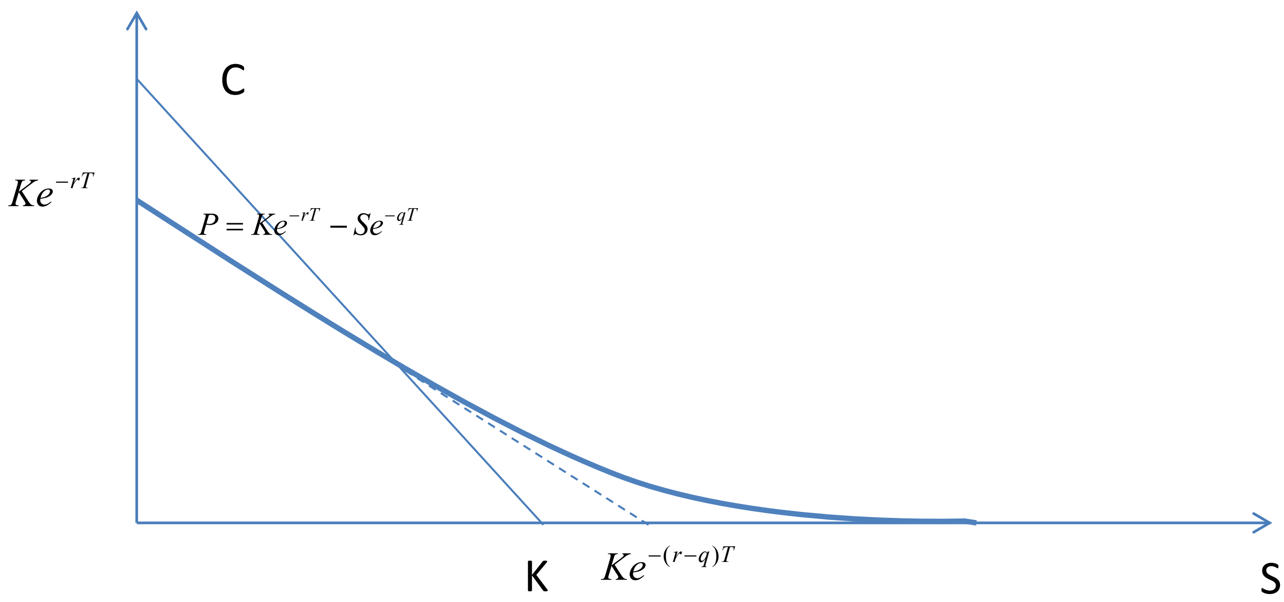
$$\therefore \; \boxed{Call_{am}(K,T) = Call_{eu}(K,T)}$$

- If the stock does not pay dividends over the life of the option, there is no early-exercise premium.

- More generally, if a commodity does not have a positive convenience yield, then American-style and European-style options have the same premium.

Basic properties of puts

$$Put(S,K,T) > 0, \quad Put(S,K,T) > Ke^{-rT} - Se^{-qT}$$
$$Put(S,K,T) \approx Ke^{-rT} - Se^{-qT}, \quad S/K << 1$$



- Put premium is decreasing in S/K and asymptotic to PV(K-F).
- The asymptotic is below intrinsic value if r>0
- American puts have early exercise premium if r>0

Puts vs. Calls, philosophically

- There is complete symmetry between puts and calls in several respects.

- Put = ``Call on Cash'' using stock as the currency.

- Best context for this is FX

N-day USD Call/ JPY put with strike 75 Y , notional 10,000,000 USD, is

-- an option to buy 10 MM USD at 75 JPY per dollar N days from now (Dollar Call)

-- an option to sell 750 MM JPY at 0.013333 USD per JPY N days from now (Yen Put)

- In FX, the foreign interest rate plays the role of dividend. $r = r_d,\ q = r_f$

SPY November 2011 Options (partial view)

SPY=$119.50, Expiration Date, Nov 18 2011, 43 trading days left

CALLS							PUTS					
Last	Change	Bid	Ask	Volume	Open Int	Strike	Last	Change	Bid	Ask	Volume	Open Int
12.5	0.83	12.29	12.35	39	5,185	110	2.88	0.24	2.85	2.87	1,452	79,332
11.74	1.54	11.47	11.59	46	4,000	111	3.15	0.29	3.06	3.11	236	21,256
10.97	0.73	10.69	10.81	43	4,229	112	3.33	0.23	3.31	3.35	224	49,388
10.1	0.4	10.01	10.04	617	7,244	113	3.62	0.29	3.58	3.62	1,460	34,591
9.3	0.5	9.24	9.26	800	5,480	114	3.86	0.24	3.83	3.87	2,903	45,415
8.6	0.46	8.55	8.57	1,480	12,266	115	4.13	0.22	4.09	4.11	1,546	95,122
7.87	0.54	7.86	7.87	1,091	12,448	116	4.44	0.26	4.45	4.47	2,117	36,139
7.22	0.5	7.19	7.21	1,505	6,763	117	4.76	0.26	4.78	4.8	1,953	48,040
6.63	0.4	6.54	6.56	2,143	22,235	118	5.15	0.35	5.11	5.14	3,500	42,238
5.95	0.39	5.95	5.97	3,929	12,458	119	5.54	0.35	5.51	5.55	4,684	13,423
5.3	0.47	5.34	5.36	4,200	27,501	120	5.87	0.24	5.91	5.93	4,669	63,099
4.74	0.45	4.77	4.78	3,034	26,806	121	6.36	0.36	6.33	6.34	2,835	24,034
4.3	0.31	4.26	4.27	2,791	26,410	122	6.82	0.35	6.8	6.81	1,558	17,818
3.74	0.34	3.71	3.73	2,269	16,416	123	7.31	0.45	7.26	7.28	1,970	5,564
3.27	0.29	3.23	3.24	1,703	40,432	124	7.88	0.44	7.86	7.88	1,433	33,682
2.79	0.37	2.81	2.82	1,525	57,405	125	8.38	0.43	8.4	8.42	635	30,826
2.46	0.25	2.42	2.43	1,626	26,484	126	9.01	0.48	8.97	8.99	865	8,792
2.09	0.22	2.03	2.05	1,797	41,838	127	9.56	0.39	9.56	9.59	419	14,686
1.76	0.16	1.71	1.72	1,625	11,681	128	10.23	0.44	10.3	10.3	503	6,495
1.41	0.2	1.42	1.45	1,530	7,092	129	10.97	0.6	10.9	11.1	109	1,668

European options: implying forward price from option prices

Put call parity

$$C(K,T) - P(K,T) = e^{-RT}(F - K)$$

$$F = K + e^{RT}\big(C(K,T) - P(K,T)\big)$$

In practice, one uses the strike for which the difference between the put and the call is smallest.

$$K^* = \arg\max_K |C(K^*,T) - P(K^*,T)|$$

$$F = K^* + e^{RT}\big(C(K^*,T) - P(K^*,T)\big)$$

Implied Dividend Yield

- The implied dividend yield is the yield that makes Put-Call parity true.

$$C_{eur}(K,T) - P_{eur}(K,T) = Se^{-qT} - Ke^{-rT}$$

$$q = q(K,T) = -\frac{1}{T}\ln\left(\frac{C_{eur}(K,T) - P_{eur}(K,T) + Ke^{-rT}}{S}\right)$$

- Option markets contain information about funding rates and dividends. If the options are European-style, q should be roughly independent of K.

- If the options are American-style, we can still use the market to estimate the dividend yield.

CALLS			Strike	PUTS			
Bid	Ask	Mid		Bid	Ask	Mid	IDIV
12.29	12.35	12.32	110	2.85	2.87	2.86	0.33%
11.47	11.59	11.53	111	3.06	3.11	3.09	0.41%
10.69	10.81	10.75	112	3.31	3.35	3.33	0.53%
10.01	10.04	10.03	113	3.58	3.62	3.6	0.51%
9.24	9.26	9.25	114	3.83	3.87	3.85	0.63%
8.55	8.57	8.56	115	4.09	4.11	4.1	0.34%
7.86	7.87	7.865	116	4.45	4.47	4.46	0.61%
7.19	7.21	7.2	117	4.78	4.8	4.79	0.59%
6.54	6.56	6.55	118	5.11	5.14	5.13	0.52%
5.95	5.97	5.96	119	5.51	5.55	5.53	0.49%
5.34	5.36	5.35	120	5.91	5.93	5.92	0.49%
4.77	4.78	4.775	121	6.33	6.34	6.34	0.45%
4.26	4.27	4.265	122	6.8	6.81	6.81	0.35%
3.71	3.73	3.72	123	7.26	7.28	7.27	0.40%
3.23	3.24	3.235	124	7.86	7.88	7.87	0.82%
2.81	2.82	2.815	125	8.4	8.42	8.41	0.62%
2.42	2.43	2.425	126	8.97	8.99	8.98	0.43%
2.03	2.05	2.04	127	9.56	9.59	9.58	0.33%
1.71	1.72	1.715	128	10.3	10.3	10.3	0.53%
1.42	1.45	1.435	129	10.9	11.1	11	0.43%

SPY=119.50
FF=0.10%

Average IDIV around
The money=0.49%

Implied Dividend Yields from Option prices (American)

The effect of implying dividends from American-style options

- American in-the-money puts are higher than the European counterparts
- **IDIV is less than q for low strikes, IDIV is greater than q for high strikes**

$$S >> K \quad \Rightarrow \quad C_{am}(K,T) > C_{eur}(K,T) \ \ \& \ \ P_{am}(K,T) \approx P_{eur}(K,T)$$

$$\therefore IDIV = -\frac{1}{T}\ln\left(\frac{C_{am} - P_{am} + Ke^{-rT}}{S}\right) < -\frac{1}{T}\ln\left(\frac{C_{eur} - P_{eur} + Ke^{-rT}}{S}\right) \approx q$$

$$K \ >> \ S \quad \Rightarrow \quad P_{am}(K,T) > P_{eur}(K,T) \ \ \& \ \ C_{am}(K,T) \approx C_{eur}(K,T)$$

$$\therefore IDIV = -\frac{1}{T}\ln\left(\frac{C_{am} - P_{am} + Ke^{-rT}}{S}\right) > -\frac{1}{T}\ln\left(\frac{C_{eur} - P_{eur} + Ke^{-rT}}{S}\right) \approx q$$

XOM January 2013 options (near the money)

Calls			Strike	Puts				
Symbol	Bid	Ask		Symbol	Bid	Ask	IDIV	C-P
XOM1301	15.75	16.5	60	XOM1301	5.7	5.8	1.84%	10.4
XOM1301	12.45	12.7	65	XOM1301	7.4	7.6	2.16%	5.08
XOM1301	9.55	9.7	70	XOM1301	9.5	9.75	2.26%	0
XOM1301	8.3	8.45	72.5	XOM1301	10.9	11	2.32%	-2.55
XOM1301	7.1	7.3	75	XOM1301	12.1	12.4	2.30%	-5.03
XOM1301	6.05	6.25	77.5	XOM1301	13.6	13.8	2.31%	-7.53
XOM1301	5.15	5.3	80	XOM1301	15.1	15.4	2.27%	-9.98

XOM=71.97

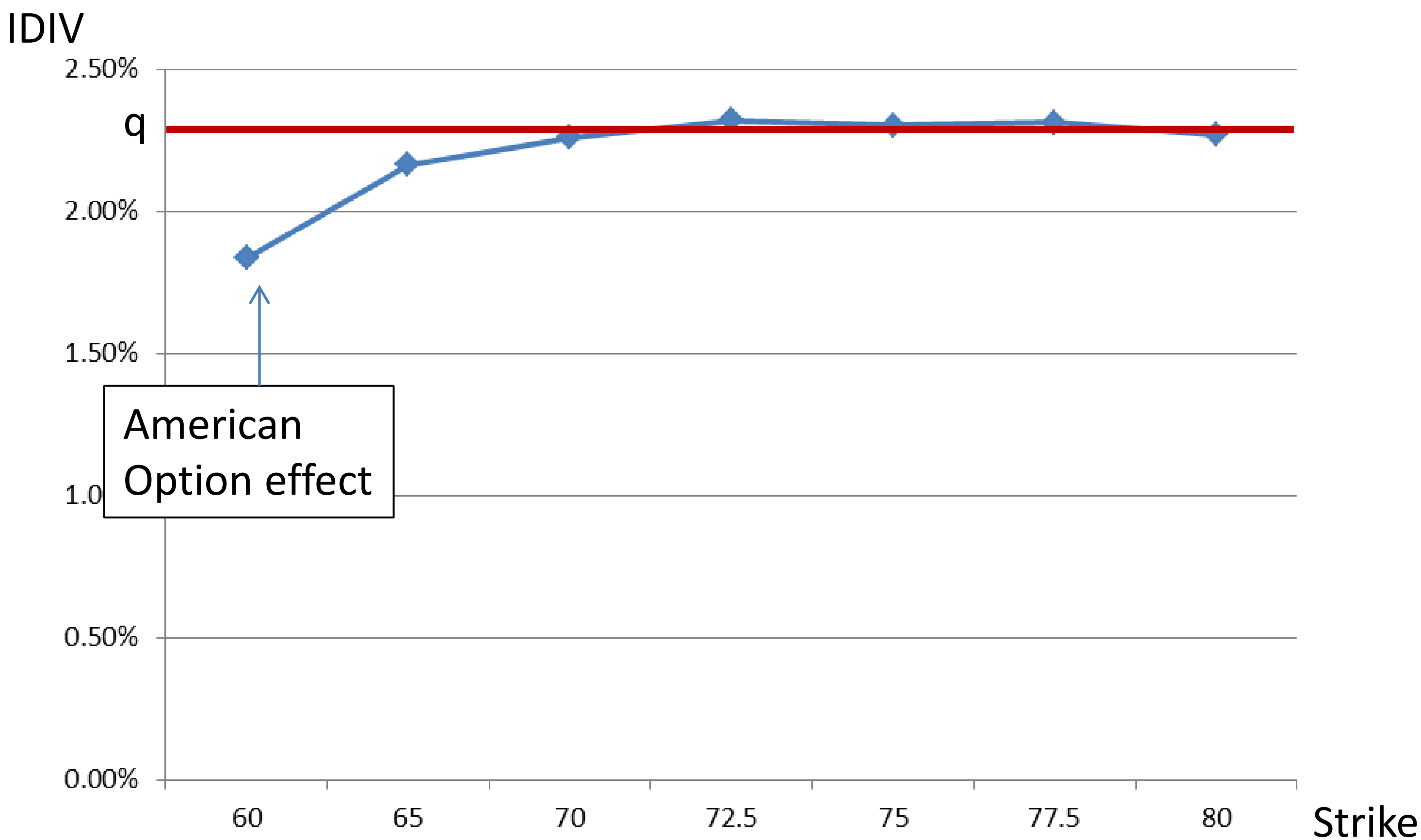
Implied Dividend

Call-Put

XOM January 2013 options, IDIV

Actual distributions

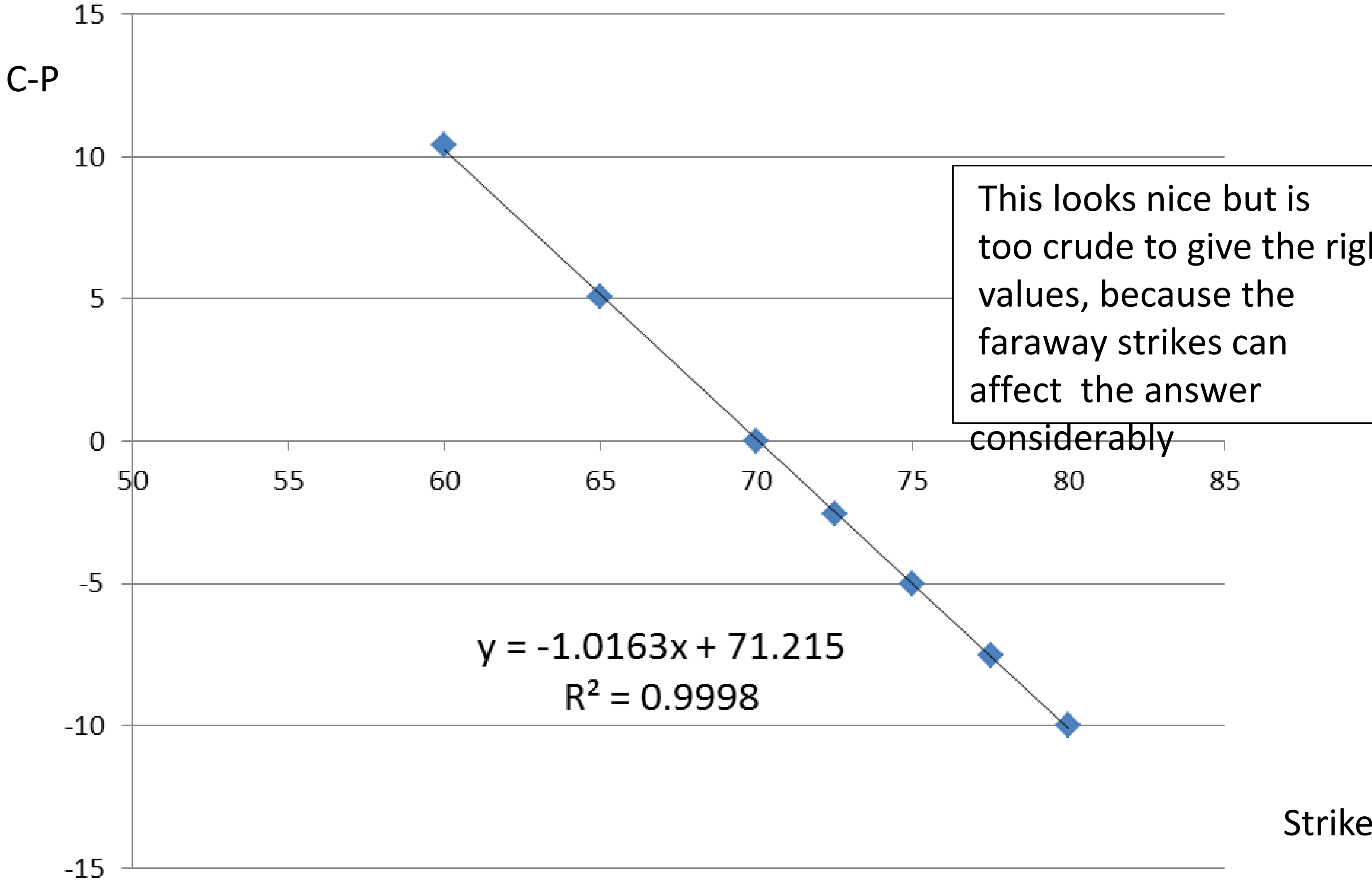
Date	Dividends
8/10/2011	0.47
5/11/2011	0.47
2/8/2011	0.44
11/9/2010	0.44
8/11/2010	0.44
5/11/2010	0.44
2/8/2010	0.42
11/9/2009	0.42
8/11/2009	0.42
5/11/2009	0.42
2/6/2009	0.4



Last calendar year's distributions = 1.82/71.97=2.53%

Options markets imply a slightly lower dividend yield (2.29%), but close.

Regressing C-P on Strike Price



This looks nice but is too crude to give the right values, because the faraway strikes can affect the answer considerably

Arbitrage strategy based on Put-Call Parity

- Based on cash-and-carry
- If C-P > PV(F-K), sell call, buy put and buy stock (conversion)
- If C-P < PV(F-K), buy call, sell put and short stock (reversal)
- More precisely: if C-P > PV(F-K) then

-- sell 1 call

-- buy 1 put

This is a synthetic short forward with strike K.

Note: the proceeds are greater than the upfront fee for entering into a long forward with price K. [So it's a profitable trade ☺].

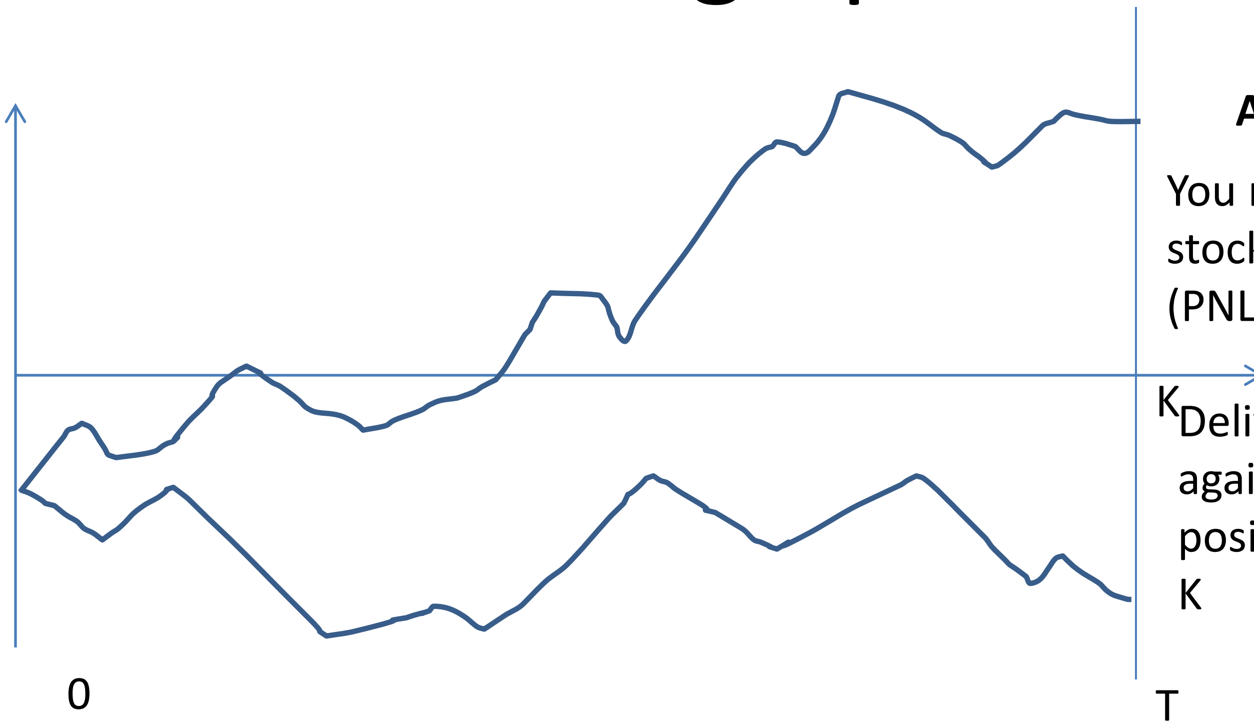
-- **Cash & carry:** borrow $$, buy 100 shares of stock, invest the proceeds of the option trade. Collect stock dividends if any and deliver the stock against the short forward. This pays K, which gives a total PNL = (F-K)+K=F, enough to pay back the loan with the dividends collected.

With a graph

At time 0

1. Sell the Conversion

2. Go long stock (financing)



At time T

You now have the stock by carrying it (PNL = -F).

Deliver the stock against the option position and collect K

Hedging PNL= -(loan + interest) + (dividends) = - F

Option PNL = K   (deliver stock and get K)

Net PNL = K-F + FV(C-P) = FV[C-P – PV(F-K)]>0

Conversion and reversals

- Conversions and reversals are simplest examples of **option spreads**

  Reversal: sell put, buy call, short stock, or (synthetic long + physical short)

  Conversion: buy put, short call, buy stock, or (synthetic short + physical long)

- One nice way to thinking about when to do a conversion or a reversal is to compare the implied dividend from the options market and the implied from the forward (or, equivalently, the cost of carry).

$$q_{options} > q_{carry} + \varepsilon \quad \Rightarrow \quad \text{do a reversal}$$
$$q_{options} < q_{carry} - \varepsilon \quad \Rightarrow \quad \text{do a conversion}$$

- In other words, ``collect the most dividends''!

Hard to borrow stocks

- When you finance a long stock, you usually pay interest: FF (plus fee). This is a debit to the cash account.
- When you finance a short stock, you usually receive interest: FF (minus fee). This is usually a credit to the cash account.
- A stock is said to be hard-to-borrow, or special, if it is not easily available for stock-loan and therefore costs more to short.
- Lenders of special stock require an increased rate of interest (like a ``rent''). This extra interest can be viewed as a dividend that is collected by traders who are long and loan the stock at more than FF.
- In this case, since conversions are substitutes for short stock, conversions are expensive or equivalently reversals are attractive.

LNKD December 2011

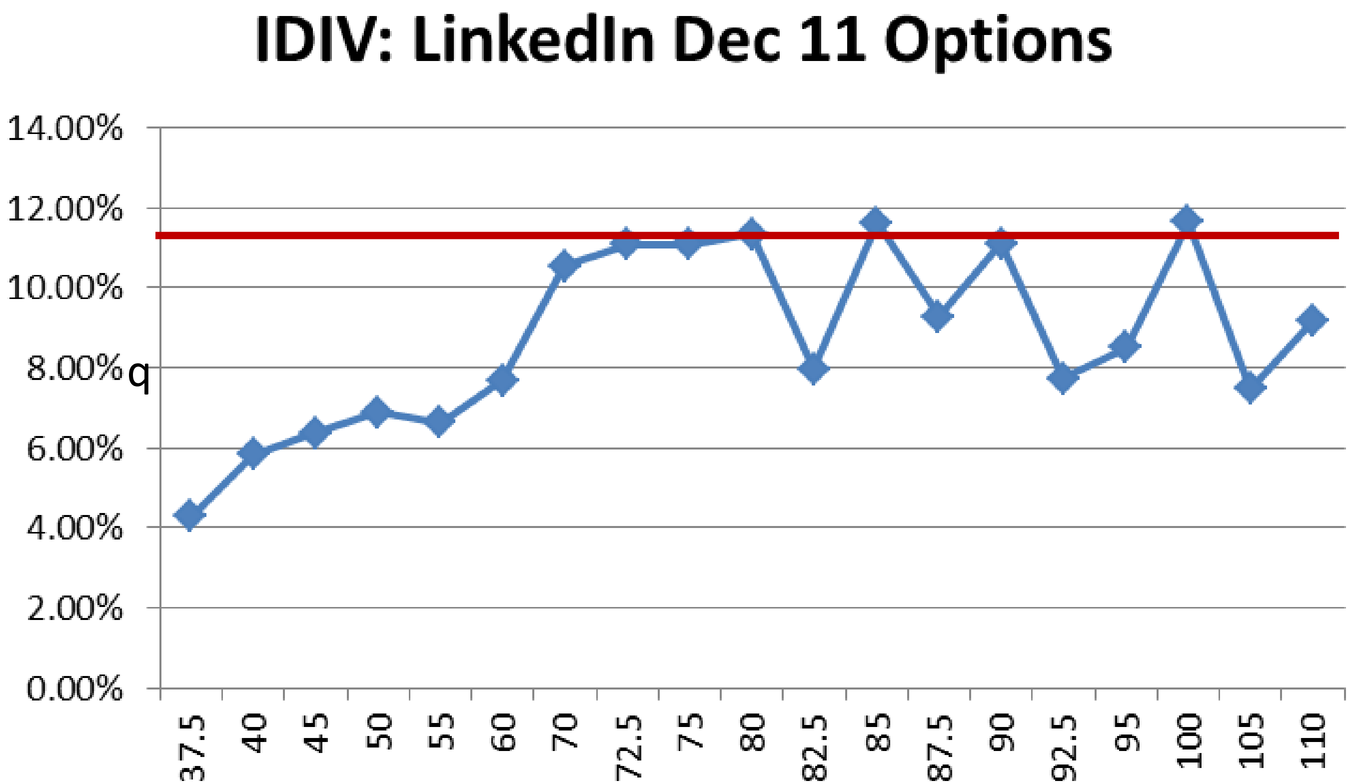
CALLS					PUTS				
Bid	Ask	Volume	Open Int	**STRIKE**	Bid	Ask	Volume	Open Int	IDIV
40	43.5	0	0	37.5	1.3	1.7	10	10	4.30%
37.4	41.2	0	0	40	1.55	2.15	1	4	5.85%
32.9	36.6	0	0	45	2.15	2.65	3	20	6.37%
28.8	32.3	1	1	50	3	3.6	20	77	6.89%
25.1	28.3	0	0	55	4.1	4.7	2	30	6.64%
21.2	24.5	0	0	60	5.5	6	10	88	7.68%
15.2	16.3	5	12	70	8.9	9.5	3	8	10.56%
13.8	14.7	5	52	72.5	9.9	10.7	8	8	11.09%
12.5	13.4	2	15	75	11.1	11.9	10	32	11.09%
10	10.9	10	43	80	13.5	14.6	10	65	11.36%
8.5	11.7	0	0	82.5	15	16.1	8	70	7.97%
8.1	8.9	11	22	85	16.7	17.6	1	96	11.63%
6.9	9.3	0	0	87.5	18.4	19.2	15	27	9.28%
6.3	7.7	32	337	90	20.1	21	3	9	11.11%
5.7	6.5	1	72	92.5	20	23	0	0	7.73%
5	5.5	6	38	95	21.8	24.8	0	0	8.51%
3.8	4.6	2	19	100	26.9	28.8	16	564	11.65%
2.9	4.1	12	110	105	29.9	32.8	0	0	7.48%
2.3	2.65	4	28	110	34.2	37.1	0	0	9.18%

IDIV is not zero although LinkedIn has never paid and has not announced a dividend.

This is due to the cost of shorting the stock:

P-C > PV (F-K) !

Plotting IDIV for LNKD



- Averaging the 75 and 80 strikes leads to $q_{\_option}$=11%, reflecting the difficulty of borrowing LNKD for short-selling.
- LNKD has traded with option-implied q's above 80% since for several months after IPO (August 2011)