Derivatives Note #11

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Greeks & Basic Hedging

☆ general approach to hedging:

stochastic linear model for the returns of A & returns of B.

$$r_A = \alpha + \beta \cdot r_B + \varepsilon$$

return of A.

return of B.

residual
uncorrelated with r_B.
normalized to have mean 0.

$$E(r_A) = \alpha + \beta E(r_B) \Rightarrow \alpha = E(r_A) - \beta E(r_B)$$

$$\begin{aligned} E(r_A r_B) &= \alpha E(r_B) + \beta E(r_B^2) \\ &= E(r_A)E(r_B) - \beta E(r_B)^2 + \beta E(r_B^2). \end{aligned}$$

$$\beta = \frac{E(r_A r_B) - E(r_A)E(r_B)}{E(r_B^2) - E(r_B)^2} = \frac{\mathrm{Corr}(r_A, r_B)}{\mathrm{Var}(r_b)}$$

$\beta =$ regression coefficient of r_A on r_B.

aka. the offset position to take if hedging.

$\frac{\partial C}{\partial S}$ is the exposure.

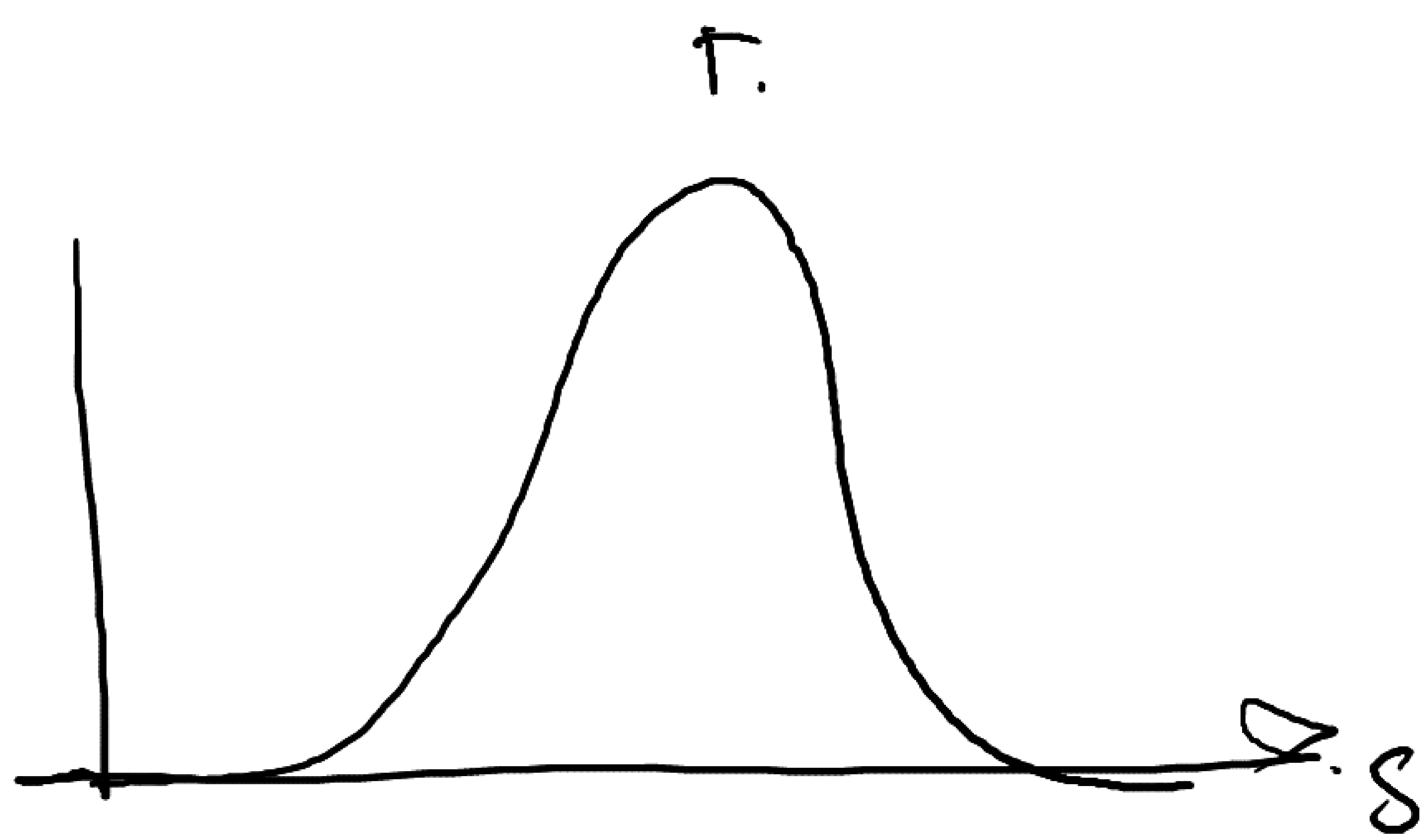
options are non linear functions of the underlying asset

Gamma.

$$\Gamma(S,T,K,r,q,\sigma) = e^{-qT} \frac{e^{-\frac{d_1^2}{2}}}{S\sigma\sqrt{2\pi T}}$$

the change in delta as the stock price moves

Γ is most concentrated at the money.



Vega.

$$Vega = \frac{\partial}{\partial \sigma}\left(e^{-qT} S N(d_1) - e^{-rT} K N(d_2)\right).$$

$$= e^{-qT} S N'(d_1)\sqrt{T}.$$

$$= e^{-qT} S \frac{e^{-\frac{d_1^2}{2}}}{\sqrt{2\pi}}\sqrt{T}.$$

Theta (time decay rate)

$$\frac{\partial \text{ Option value}}{\partial \text{ time to maturity}}$$

$$\Theta = \frac{\sigma S e^{-\frac{d_1^2}{2}}}{\sqrt{2\pi T}}$$