Derivatives Homework 3.

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Q1. $S_0 = \$50$ $r_f = 10\%$. $K = \$49$

Consider a portfolio of.

$+\Delta$: shares.

-1 : option.

The riskless value of this portfolio gives.

$48\Delta = 53\Delta - 4$.

$\Rightarrow \Delta = 0.8$.

$\Rightarrow$ riskless portfolio $= \begin{cases} \text{long } 0.8 \text{ share.} \\ \text{short } 1 \text{ option.} \end{cases}$

$48 \times 0.8 = 38.4$

$\Rightarrow 0.8 \times 50 - f = 38.4 \times e^{-0.1 \times 2/12}$

$f = 2.23$

$\Rightarrow$ European call option with $K = \$49$ after 2 months has the value of $\$2.23$

Q2. make riskless portfolio.

$-\Delta$: shares.

$+1$: option

$$-85\Delta = -75\Delta + 5.$$

$\Rightarrow \Delta = -0.5 \Rightarrow$ long 0.5 shares.

$$0.5 \times 85 = 42.5.$$

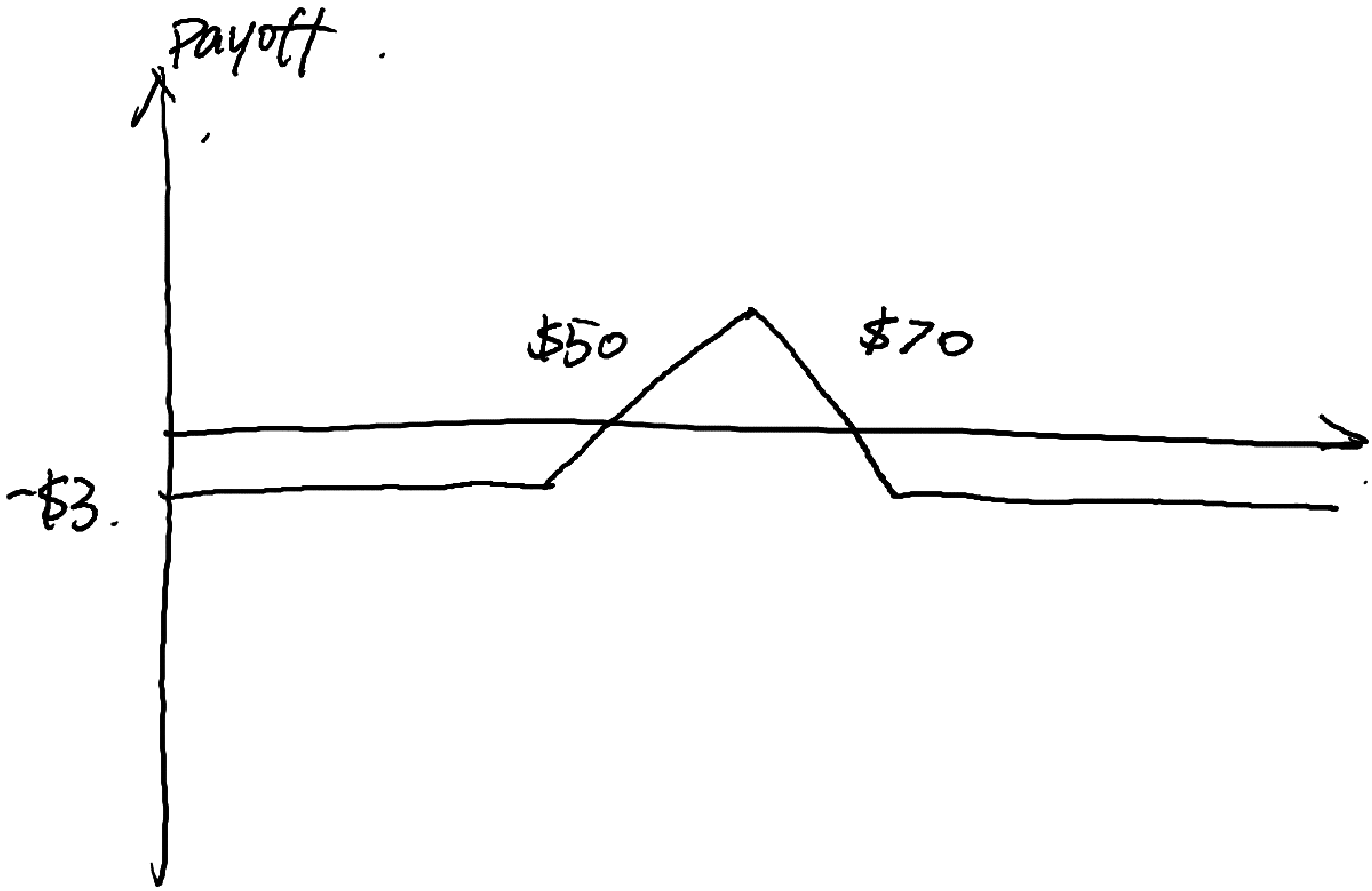
$$\Rightarrow 0.5 \times 80 + f = 42.5 e^{-0.05 \times 4/12}.$$

$$f = 1.80.$$

$\Rightarrow$ option value \$1.80.

Q3.

a).



b) $\text{Cost} = C_{\$50} - 2 \cdot C_{\$60} + C_{\$70}$

$= \$7.5 - 2 \times \$3 + \$1.5$

$= \$3.0$

c). There will be a positive profit for stock prices at option maturity between \$53 and \$67.

Q4. Use put-call parity.

$$c - p = S_0 - PV(K).$$

$$C_{\$50} = \$94 - PV(\$50) + P_{\$50}.$$

$$= \$94 - \$50 \cdot \frac{91}{100} + \$3$$

$$= \$94 - \$45.5 + \$3.$$

$$= \$51.5.$$

similarly.

$$C_{\$60} = \$94 - PV(\$60) + P_{\$60}.$$

$$= \$94 - \$ \cdot \frac{6 \times 91}{10} + \$5.$$

$$= \$94 - \$54.6 + \$5.$$

$$= \$44.4$$

Q5. a)

a. Buy. 1 call option with $K=a$.
Sell 1 call option with $K=b$.

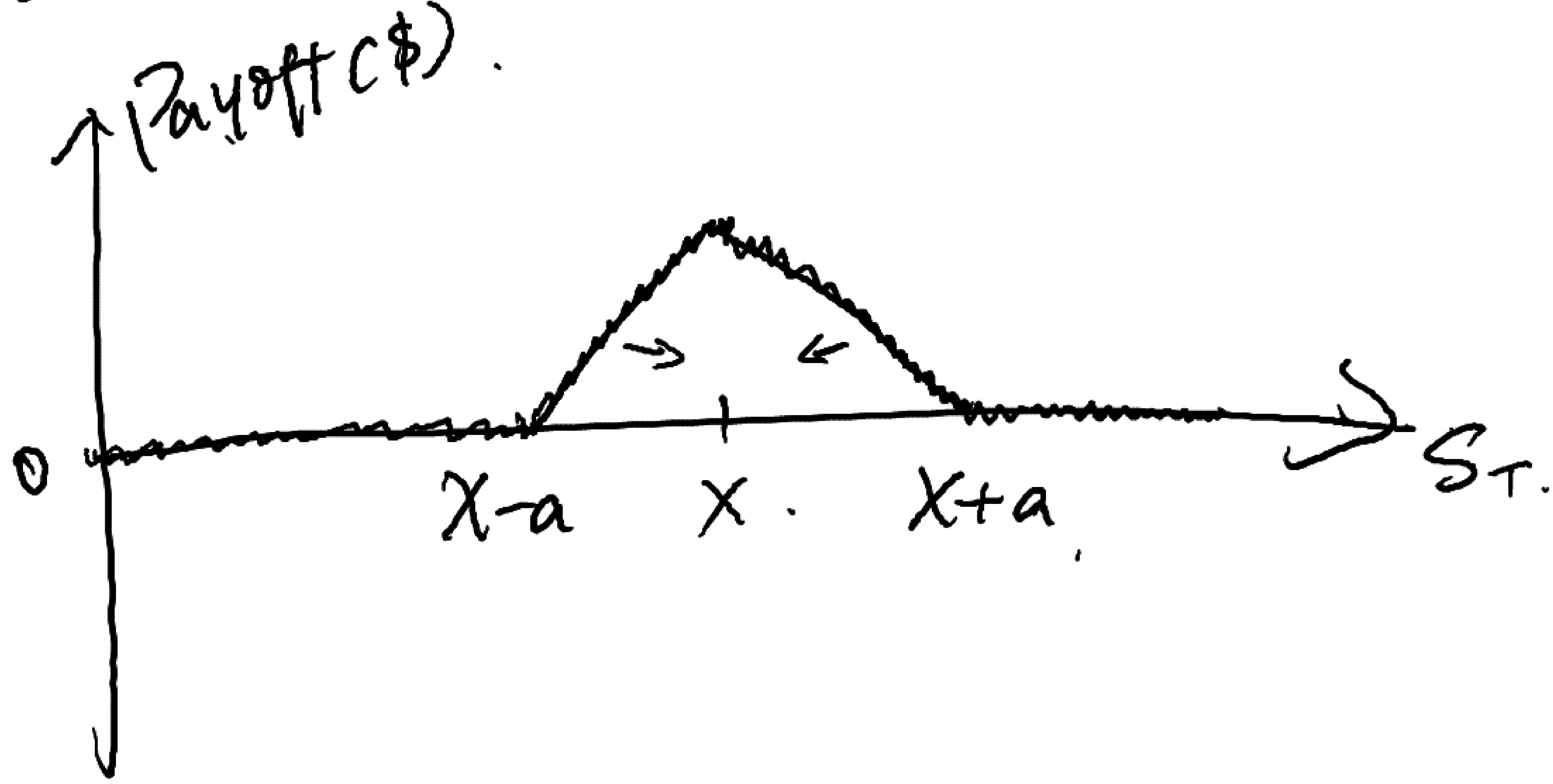
b. Buy 1 call option with $K=a$.
Buy 1 put option with $K=a$

c. Buy 1 call option with $K=a$.
Sell 2 call options with $K=b$.
Buy 1 call option with $K=c$.

b). b will hedge my risk against volatility. because the more volatile the more payoff from b. The cost is the cost of 1 call option plus 1 put option.

Q6.

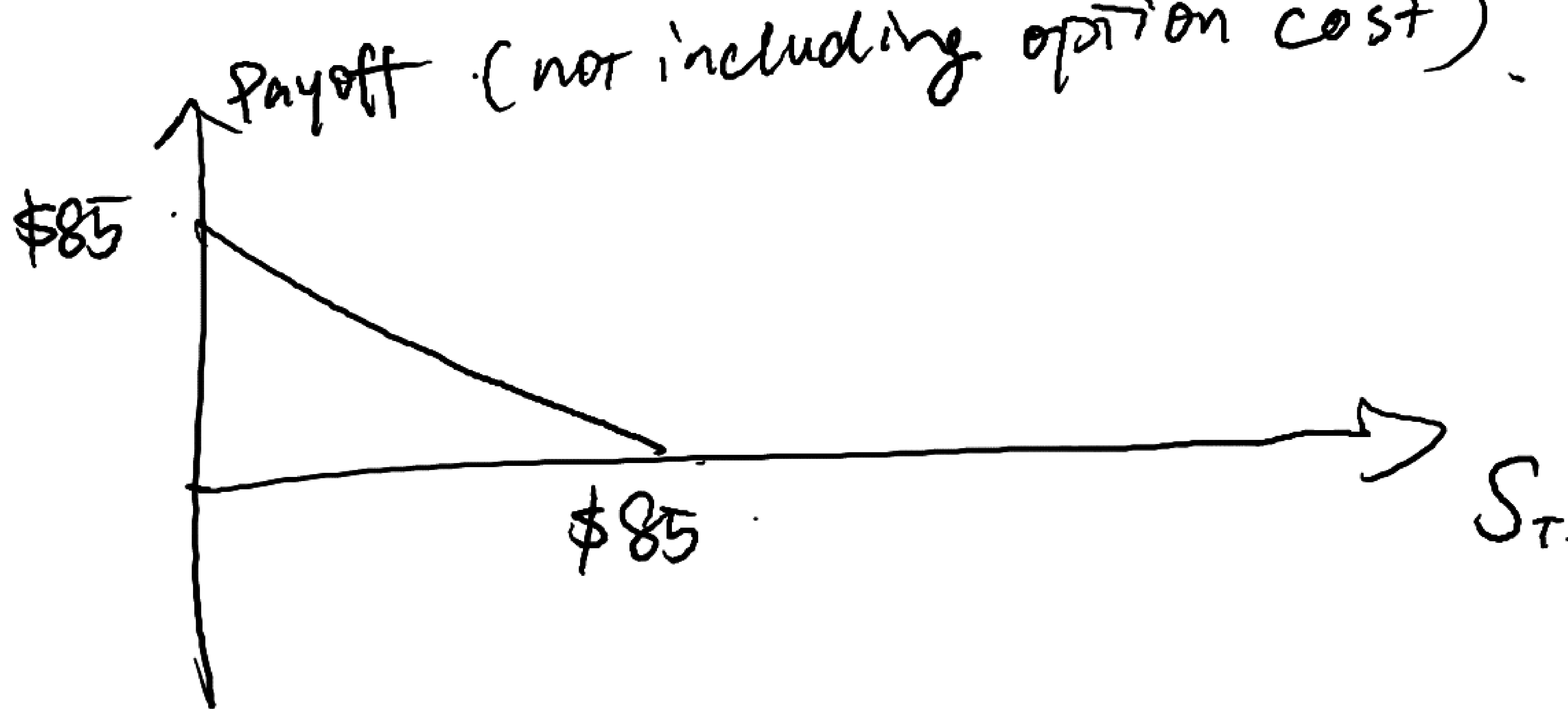
a).



The payoff converges to $0 as the low and high strike prices converge.

b). He believes the stock price will not move far from its current value

Q7. a).



b). As long as,

$S_T \le \$85 - FV(\$4)$

$\Rightarrow S_T \le \$80.95$.

IBM price less than or equal to $80.95.
You will make money.

Q8.

S_0, K, T, σ, r, D. all these 6 factors can affect option prices.

eg. a scheduled dividend payoff within the lifespan of a put option will increase the put option's price.

Increasing volatility σ will also increase option prices.

Q9. by put call parity.

$$c - p = S_0 - PV(K)$$

$$57.5 - 3 = 100 - PV(50)$$

$$\Rightarrow PV(50) = 45.5$$

$$\$100 \times \frac{45.5}{50} = \$91.$$

$$C - 5 = 100 - PV(60)$$

$$\Rightarrow C = 100 - 60 \times \frac{91}{100} + 5$$

$$= 100 - 54.6 + 5$$

$$= 50.4$$

$\Rightarrow$. price of option with $K = \$60$ is $\$50.4$

Q10.

a).

Stock price S_T.	Portfolio value ($).
$S_T \le \$45$	$1000 S_T + 3,000$
$S_T > \$45$	$45000 + 3000 = 48000$

This portfolio will have a min value of $3000 and max value of $48000. It does not protect the down payment under extreme case where $S_T = \$0$.

b).

S_T	Portfolio value.
$S_T \le \$35$	$35000 - 3000 = \$32000$
$S_T > \$35$	$1000 S_T - 3000$

This portfolio insures an minimum of \$32000
It has no cap on upside gain, but may have moderate loss.

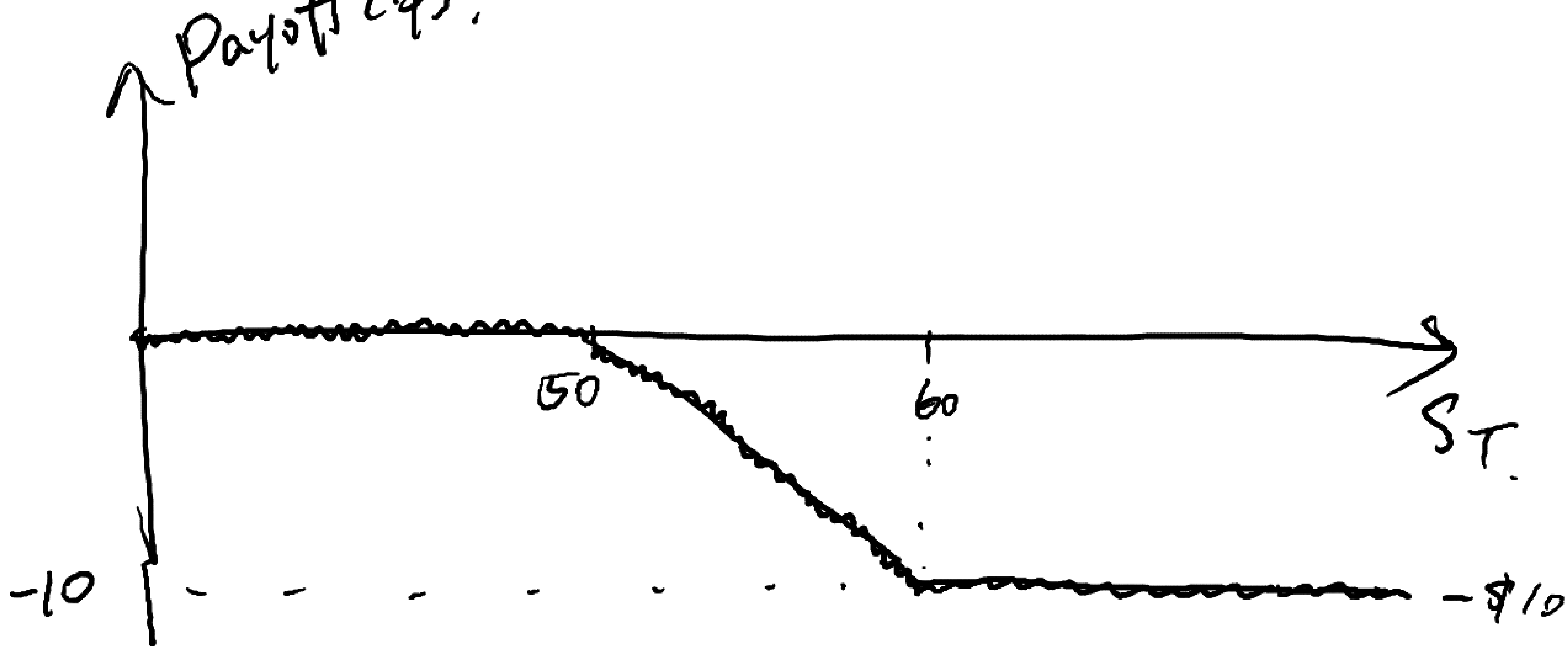
c).

S_T	Portfolio value.
$S_T \le \$35$	$\$35000$.
$\$35 < S_T < \$45$	$1000\, S_T$.
$S_T \ge \$45$	$\$45000$

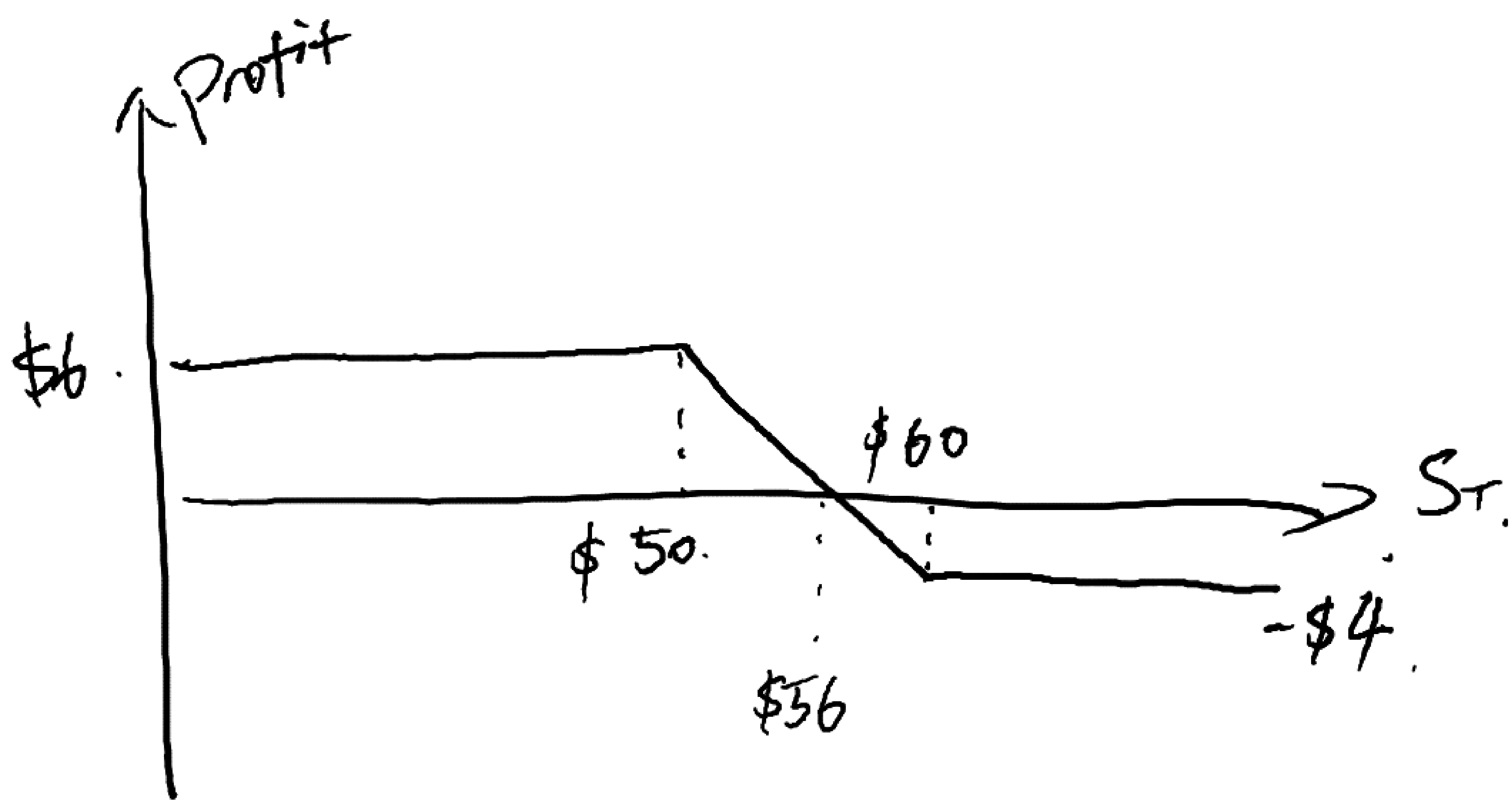
This has cap on both downside loss and upside gain.

This strategy mostly fits Jones because it insures the downpayment to some extend and also leaves the opportunity to get $45000.

Q11 a).



b).



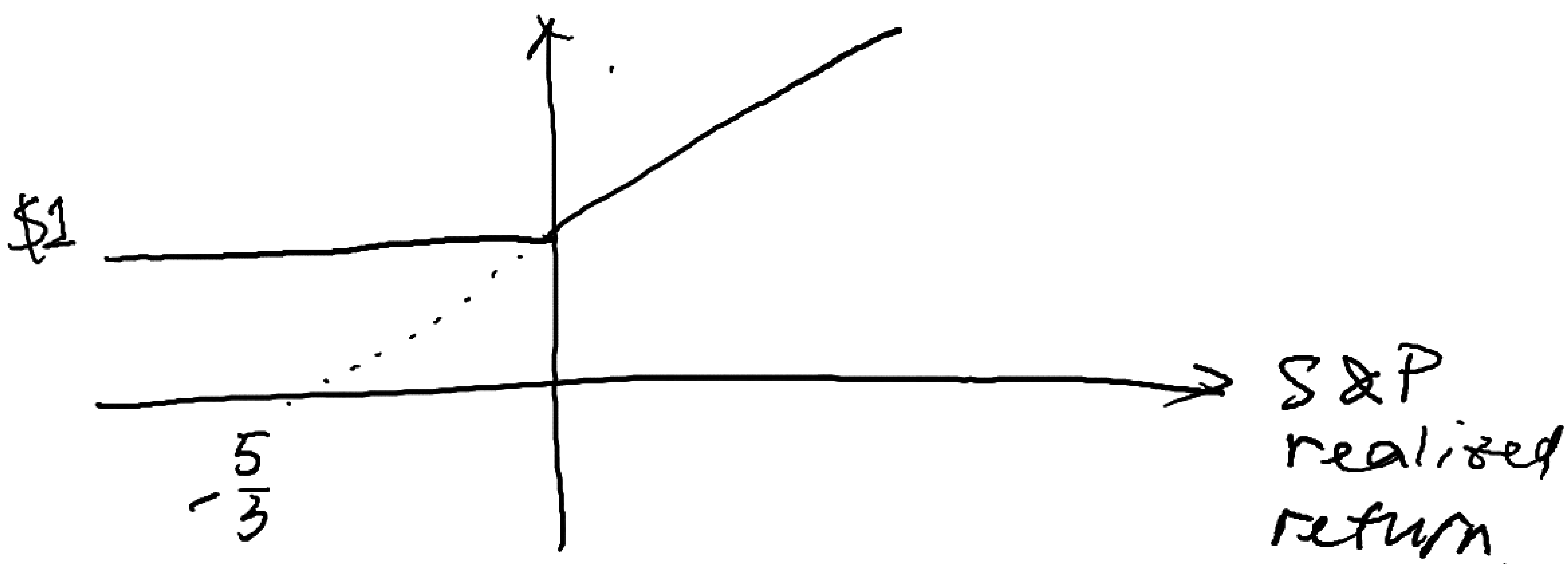
c). Bearish spread.

Breakeven point is \$56.

Q12

a) payoff $= \max\left[0.6 \cdot \left(\frac{S^* - S_0}{S_0}\right) + 1,\ 1\right]$

S^* is S&P index price one year from now. S_0 is current price.



b) The investment can be reproduce with.

$\begin{cases} \text{a risk free discount bond with } FV = \$1 \\ 0.6 \text{ call options that pay } \left(\frac{S^* - S_0}{S_0}, 0\right) \end{cases}$

c). Using binomial tree

$R = 1.1$

$u = 1.2, \ d = 0.8$

$q_u = \frac{1.1 - 0.8}{1.2 - 0.8} = 0.75$

$q_d = \frac{1.2 - 1.1}{1.2 - 0.8} = 0.25$

Use risk-neutral pricing.

$C = \frac{1}{1.1}[0.75 \times 1.12 + 0.25 \times 1] = 0.9909$

Since the offered price \$1 is greater than implied price \$0.9909. You lose money investing it.