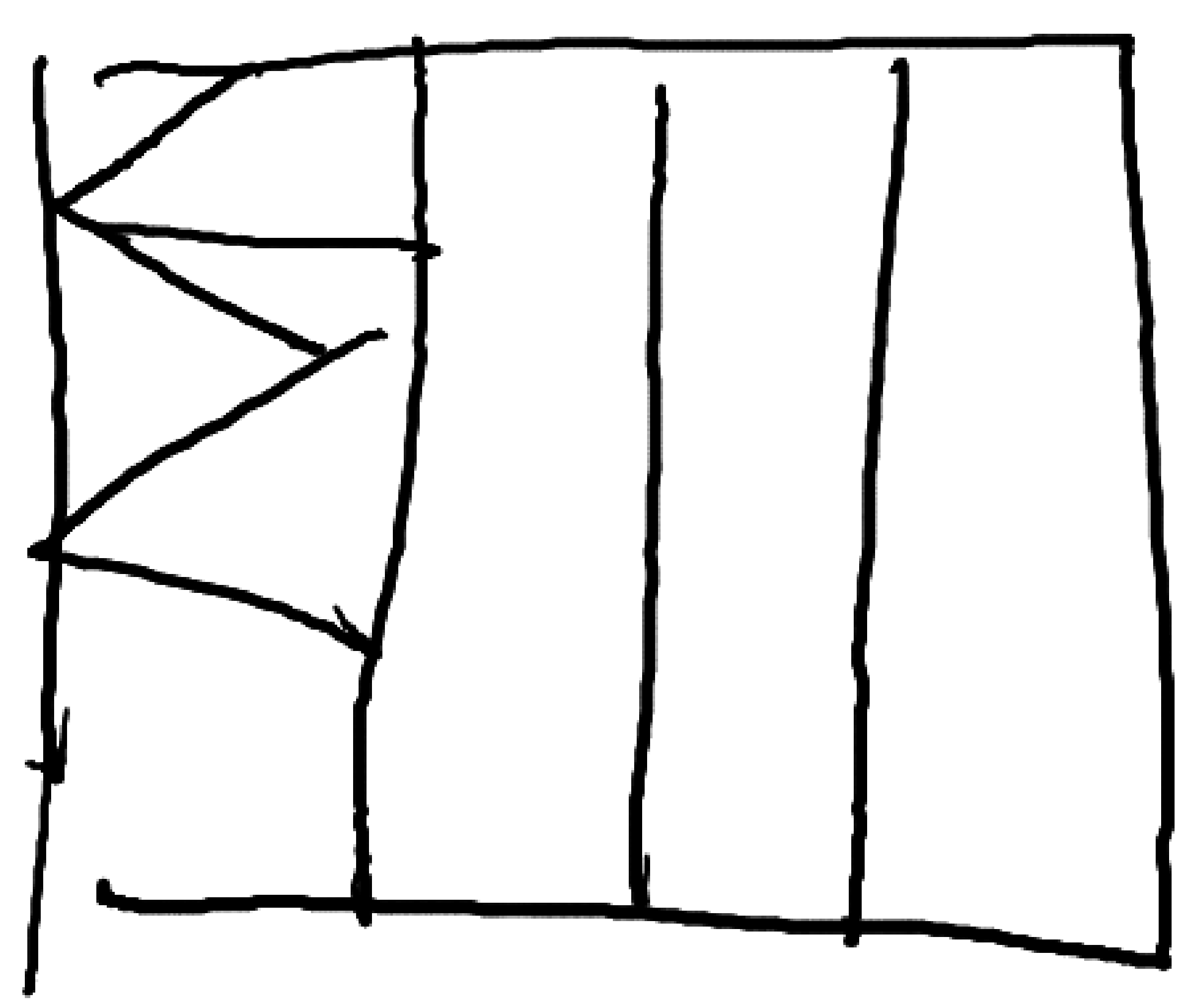
Derivatives Note 10/30.

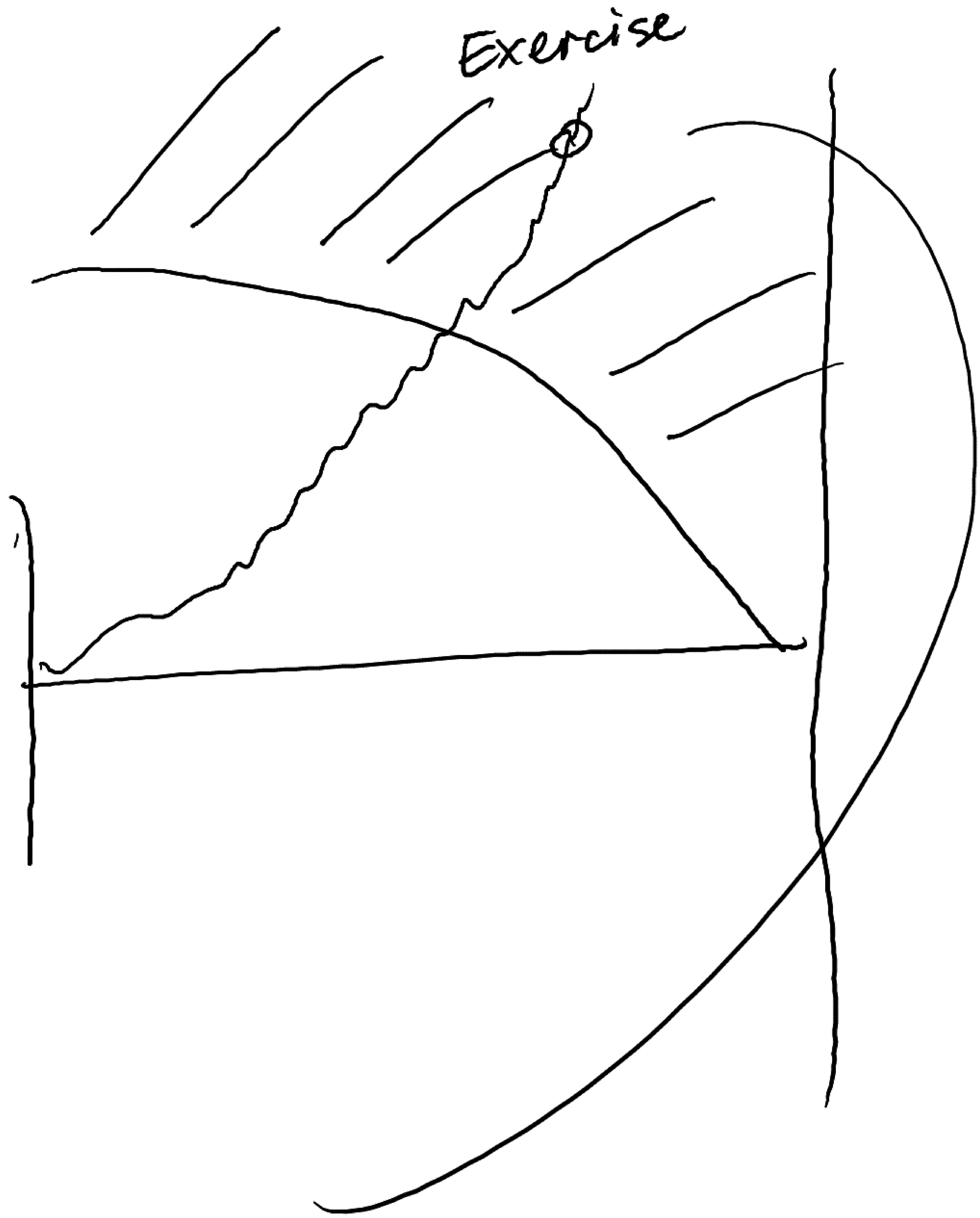
Option Pricing.

→ Black Scholes PDE

Trinomial Tree

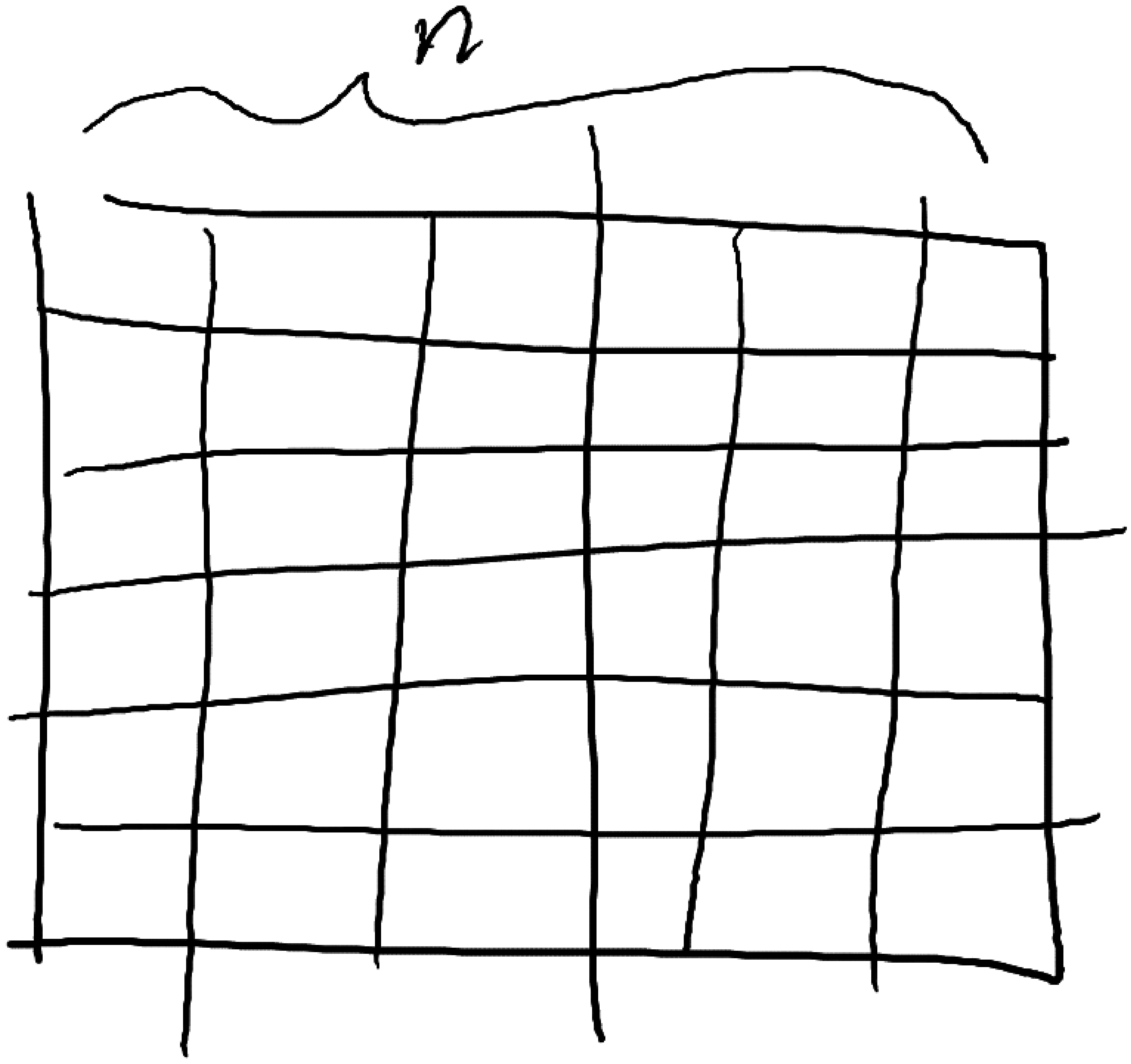


$$C_n^J = \frac{\quad}{1+}$$



Exercise Region

n throwing opportunity



Optimal Stoping Game

Throwing a dice

$$\frac{\partial C(S,t)}{\partial t} + \frac{\sigma^2 S^2}{2}\frac{\partial^2 C(S,t)}{\partial S^2} + (r-q)S\frac{\partial C(S,t)}{\partial S} - rC(S,t) = 0$$

r = interest rate.

q = dividend yield

σ = volatility

$$\frac{\partial C}{\partial t} + \left(r - q - \frac{1}{2}\sigma^2\right)\frac{\partial C}{\partial x} + \frac{1}{2}\sigma^2\frac{\partial^2 C}{\partial x^2} - rC = 0$$

Stability conditions and probabilities

$$p < \frac{1}{2}$$

$$\Rightarrow P_u > 0,\ P_M > 0,\ P_D > 0$$

and $$\frac{|M|\sqrt{\delta t}}{\sigma_{max}} < 1$$

$$C_n^j = \frac{1}{1 + r\Delta t}\left(P_u C_{n+1}^{j+1} + P_M C_{n+1}^{j} + P_D C_{n+1}^{j-1}\right)$$

for American options.

$$C_n^j \geq \max(S_n^j - K, 0) \quad \text{for calls.}$$

$$C_n^j \geq \max(K - S_n^j, 0) \quad \text{for puts}$$

$$C_n^j = \max\left[F(S_n^j), \frac{1}{1+r\delta t}\left(P_u C_{n+1}^{j+1} + P_m C_{n+1}^{j} + P_d C_{n+1}^{j-1}\right) \right]$$

where $F(S)$ is the intrinsic value.

$$ER_{call} = \{(S,t): S > S^*(t)\}.$$

$$ER_{put} = \{(S,t): S < S^{**}(t)\}.$$