Derivative Securities Fall 2011 Lecture 2: Forward & Futures Markets evised all 201

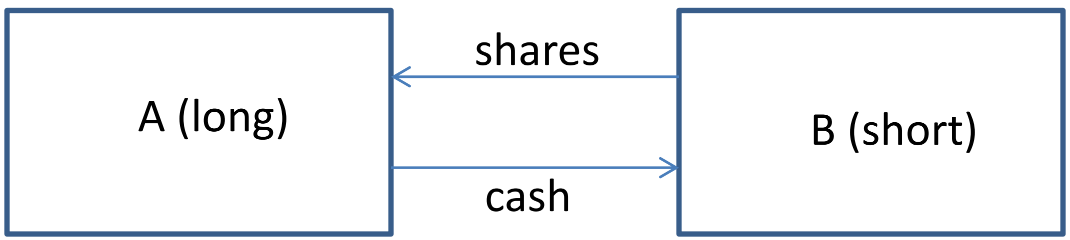
Sources:

Instructor's notes,
J.C. Hull , Chapters 2, 3
CME Group (www.cme.com)
Energy Information Administration (www.eia.gov)
Yahoo!Finance
Bloomberg (www.bloomberg.com)

Contract for Forward Delivery

(commodity, stocks, currencies, OTC)

- Example: Forward contract for exchanging 1,000000 shares of XYX at K dollars per share



(at time T)

- Short must hedge by buying the stock at the starting date

1. borrow to buy the stock (t=0)
2. collect dividends, if any (0<t<T)
3. exchange stock for stipulated price (t=T)
4. return loan + interest (t=T)

Forward Price

- On trade date: short borrows $N\ S_0$ dollars and buys N shares
- On delivery date, short: - delivers shares
  - pays back loan, $-\ N\ S_0 e^{rT}$
  - receives + *N K* dollars
  - has accumulated dividends + *N* $\sum_i d_i e^{r(T-T_i)}$

Profit Loss $= -\ NS_0 e^{rT} + N\ K + N \sum_i d_i e^{r(T-T_i)}$

The price at which the short breaks even solves

$$-\ NS_0 e^{rT} + N\ K + N \sum_i d_i e^{r(T-T_i)} = 0 \qquad \therefore \qquad \boxed{K = F_T = e^{rT}\left[S_0 - \sum_i d_i e^{-rT_i}\right]}$$

Evolution of Forward Price

- If the forward contract is negotiated at the forward price, its value at time 0 is zero. In principle, no cash-flows are required initially.

- If K is different from the forward price, then there should be an initial cash-flow $= e^{-rT}\left(F_{0T} - K\right)$ by the long to the short because the contract is **off-market**.

- At time *t*, the new forward price is

$$F_{tT} = e^{r(T-t)}\left[S_t - \sum_{T_i > t} e^{-r(T_i - t)} d_i\right]$$

and the value of the forward contract for the long is

$$V_t = Ne^{-r(T-t)}\left[F_{t,T} - F_{0,T}\right]$$

- This is the amount that the short would be willing to pay to ``get out of the contract''. It is known as the Mark-to-Market value of the forward contract.

Futures contracts

- Forward contracts are generally negotiated bilaterally on the basis on spot price and cost of carry (interest rate, dividends) : credit risk

- Futures contracts are **exchange-traded versions of forwards,** where the settlement price is negotiated directly on an exchange and there is daily MTM.

Example: **E-mini S&P futures from Chicago Mercantile Exchange**

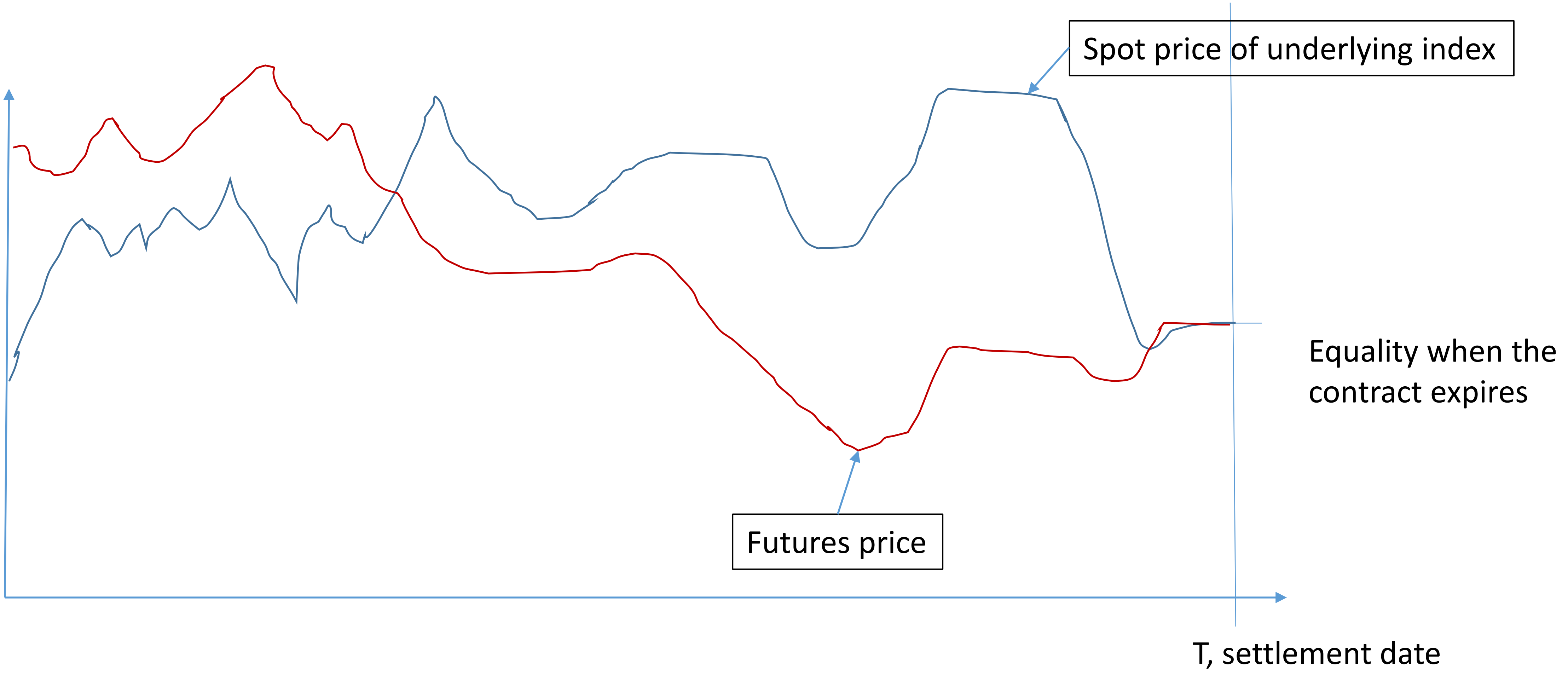
-- Contract Size: $50 times the S&P futures price

-- Tick size (minimum variation): 0.25 pts = $12.50

-- Contract Months: Mar, Jun, Sep, Dec

-- Settlement: Third Friday of the contract month (a.m.), in cash.

Mechanics of Futures Prices

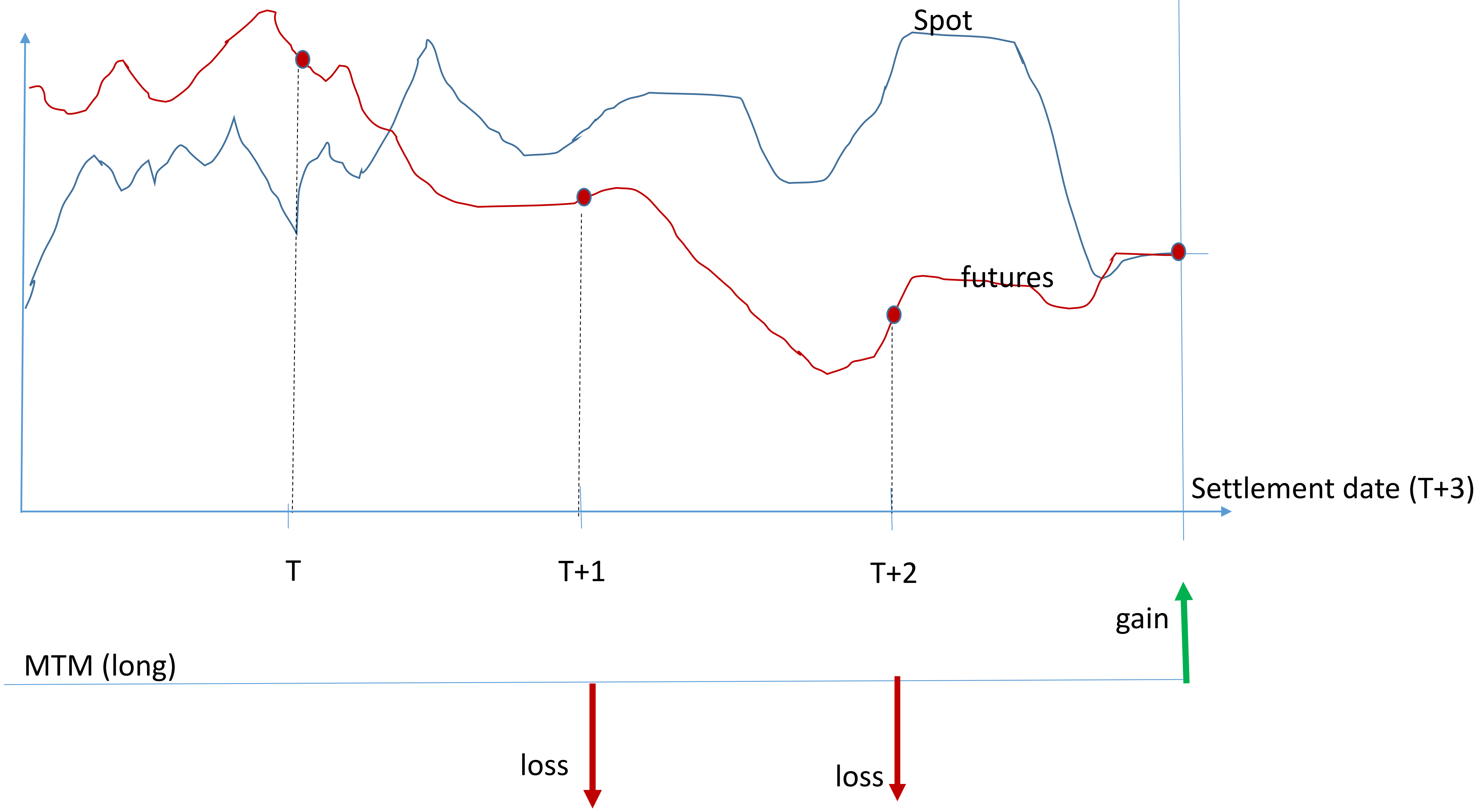


The futures price settles on the spot price on the settlement date.

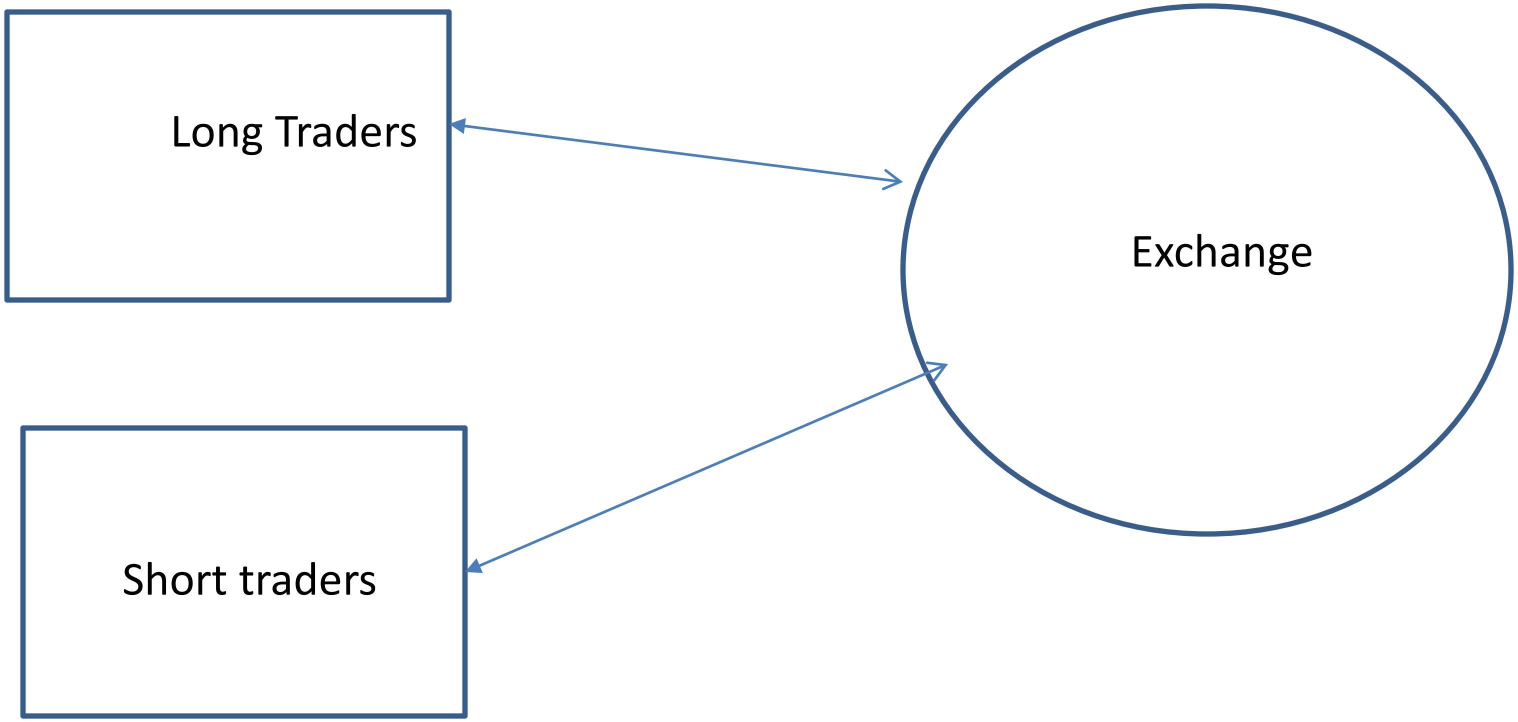
Futures Rules

- Futures contracts are standardized and traded in exchanges
- **Central trading** : exchange matches buyers and sellers
- **Central clearing**: exchange is counterparty in all trades (no credit risk between buyer/seller)
- **Mark-to-market**: trading accounts are debited/credited for the difference between the trade price and the daily settlement price OR the difference between successive daily settlement prices.
- **Margin requirements** imposed by CCP on traders.

Mark-to-Market



Schematic structure of a futures exchange



- In futures exchanges, the traders face the credit risk of the clearinghouse instead of the risk of individual counterparties. The CH collects margin and marks-to-market daily its member accounts.

CME E-mini S&P 500 contract (daily quotes)

E-mini S&P intraday chart

How futures work (ex. E-Mini S&P)

- Traders post bids and offers and quantities on Futures contracts during the trading session

- Traders can buy (go long) contracts or sell (go short contracts). For every long there is a short (trades cleared by the exchange).

- On the settlement date, the contract is worth exactly the **S&P 500 index times the contract size**

- During the lifetime of the contract, if the futures price changes the PNL for a trader long N contracts is $\Delta E = N \times 50 \times \Delta f$ where *f* is the futures price (mark-to-market).

- Example: a trader is long 20 Sep 11 contracts. At noon, F=1140, and at 3:30 PM its value is 1163.50. The traders ``paper'' profit is

  20 * 50 * (1163.50-1140)= 1000*23.50=$23,500.

``Locking in'' profit / loss intraday

12:00 noon: Go long 20 E-mini S&P Sep at 1140

3:30 PM : Go short 20 E-mini S&P at 1163

5:00 PM: E-mini settles at 1130

MTM from long = 1130-1140 = - 10
MTM from short = -1130+1163=+33
MTM = 1163-1140= + 23
Total open position = 0 contracts !

For 20 contracts = 20 * 50 * 23 = $23,000.

Futures versus forwards with the same settlement date

- Suppose that a trader is long one contract at the start of day 1 and carries the position to expiration, n days later. Assume contract size 1 for simplicity. The profit/loss is

$$\Delta E = \sum_{1}^{n} \left(f_i - f_{i-1}\right) e^{r(n-i)\Delta t}$$

- If the position is $e^{-r(n-i)\Delta t}$ contracts on day *i,* then, accordingly

$$\Delta E = \sum_{i=1}^{n} e^{-r(n-i)\Delta t} e^{r(n-i)\Delta t} \left(f_i - f_{i-1}\right) = \sum_{i=1}^{n} \left(f_i - f_{i-1}\right)$$

$$= f_n - f_0$$

$$= I_n - f_0$$

Since futures have zero cost, the PNL from this strategy matches exactly that of a Forward contract on the index. Conclusion: $f_{0,T} = F_{0,T}$

Equivalence of Futures and Forwards

- The argument of the previous slide shows that if the funding rates are constant, then index futures satisfy

$$\boxed{f_{0,T} = F_{0,T}}$$

- The same argument applies if interest rates can be ``locked in'' in the future, i.e. if the interest rates for date *i* is r_i
- In particular, the futures markets provide an interesting relation between the spot price of the underlying index, the funding rate and the dividend flows pro vided by holding the index (cash & carry arbitrage).
- They can be used to **gain exposure to the underlying index** or to **arbitrage the carry costs.**
- **Futures can be used for price discovery in forward transactions.**

Continuous Dividends

- Equity indexes have a very frequent flow of small dividend payments

$$I_t = \sum_{i=1}^{m} w_i S_i$$

- Currencies have a ``dividend'' corresponding to the foreign interest rate

- In these cases, it is convenient to model the dividends as a continuous **dividend yield *q*:** the dividend paid over one day is modeled as
  $I \cdot q\Delta t$

- A one-day investment in the basket underlying an equity index has total PnL (including financing for one day)

$$\Delta I + qI\Delta t - rI\Delta t$$

Capital gain/loss   Dividend income   Financing cost

Forward price and continuous dividends

- Invest in $e^{-q(T-t)}$ units of the underlying basket of stocks at time *t, financing daily*, and assuming zero initial equity. The PnL for any given day is

$$\begin{aligned}\Delta E_t &= rE_t\Delta t + e^{-q(T-t)}\left(\Delta I_t + qI_t\Delta t - rI_t\Delta t\right)\\ &= rE_t\Delta t + e^{-q(T-t)}e^{+(q-r)(T-t)}\Delta\left(e^{-(q-r)(T-t)}I_t\right)\\ &= rE_t\Delta t + e^{-r(T-t)}\Delta\left(e^{-(q-r)(T-t)}I_t\right)\end{aligned}$$

Hence

$$\Delta E_t - rE_t\Delta t = e^{-r(T-t)}\Delta\left(e^{-(q-r)(T-t)}I_t\right)$$

$$e^{-rt}\left(\Delta E_t - rE_t\Delta t\right) = e^{-rT}\Delta\left(e^{-(q-r)(T-t)}I_t\right)$$

$$\Delta\left(e^{-rt}E_t\right) = e^{-rT}\Delta\left(e^{-(q-r)(T-t)}I_t\right)$$

$$\boxed{\therefore\quad E_T - e^{-rT}E_0 = E_T = I_T - e^{-(q-r)T}I_0}$$

Final equity

Cash & Carry Argument

- The trader that does this strategy can deliver the basket, which has market price I_T , against a payment of $I_0 e^{(r-q)T}$ or settle for the difference in cash.

- Conclusion: for continuous dividends, we have

$$\boxed{F_{0,T} = I_0 e^{(r-q)T}}$$

- One can view the forward or futures price for a basket that trades liquidly in the spot market as the market's estimate of the financing costs and dividend stream for the basket of stocks.

- This leads to an arbitrage strategy between index futures and cash equities or exchange-traded funds (ETFs). Traders can estimate whether the basket is ``rich'' or ``cheap'' relative to the futures on a given maturity.

Example

- On Sep 13 2011, at 11:40 am, the December E Mini contract is trading at 1161.00. The index value is 1165.88. There are 68 trading days before settlement. Therefore

$$r - q = \frac{252}{68} \ln\left(\frac{1161.00}{1165.88}\right) = -0.0155442 = -1.554\%$$

$$r = 0.12\% \text{ (Fed Funds)}$$

$$q = 1.554 + 0.12 = 1.674\% \quad \text{(implied dividend yield)}$$

- The dividend yield for the **SPY ETF** listed in *Yahoo!Finance* is 1.99%. If we take this as the reference yield, the E-mini futures is **expensive** and an arbitrage trade could be possible. (Must take into acct also bid/ask and other transaction costs. The ``profit'' would be 1.99-1.67=0.32% (annualized).

  PNL for 68 days = 0.086 % or 8.6 basis points. (Not much…. ☺)

- Futures are in line with the basket.

Currency Forwards & Futures

- Currency trading (FX) can be viewed as investing in foreign overnight deposits and hence earn interest (which is like a continuous dividend) as well as currency appreciation depreciation

- Usually, term rates are quoted in simply compounded terms.

$$F_{0,T} = Se^{(r_d - r_f)T} = S\left(\frac{1+R_d T}{1+R_f T}\right); \qquad S = \text{spot rate}$$

- Example: Spot USD/BRL (Brazilian Real)=1.7135 (Bloomberg) Brazil 1 year rate= 10.94%, US 1 year rate =0.09%

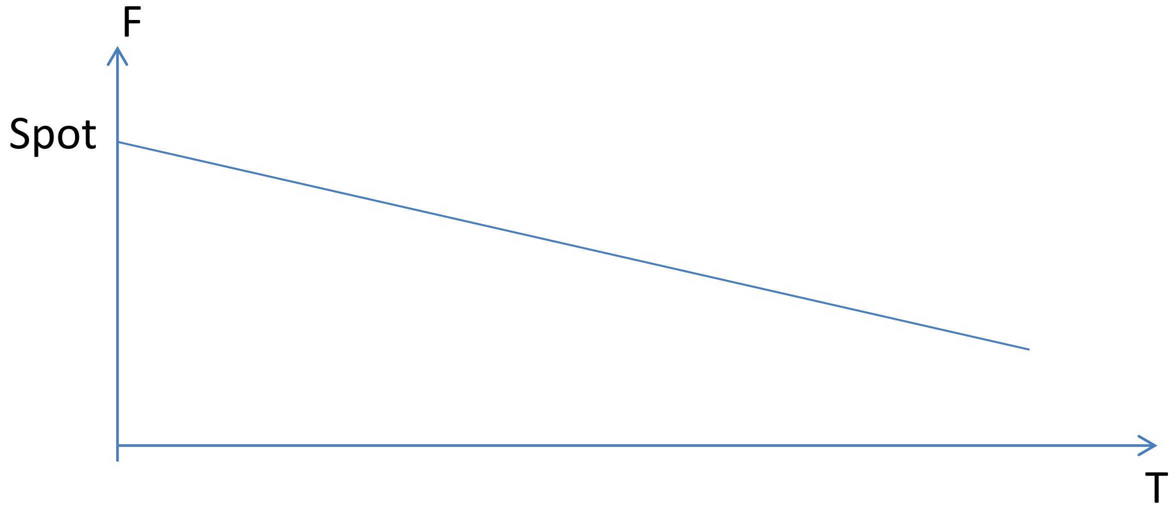
$$F_{0,1}^{USD/BRL} = \frac{1}{1.7135} \times \left(\frac{1+0.0009}{1+0.1094}\right) = 0.583601 \times 0.902199$$
$$= 0.526524$$

$$F_{0,1}^{BRL/USD} = \frac{1}{0.526524} = 1.8999$$

Rates are **interbank rates** (LIBOR, etc)

Currency Forward Curves

- Quoting vs. the dollar, for simplicity (USD=domestic currency)
- If Rf>Rd then the forward is lower than the spot (*downward sloping*)
  Associated with positive carry
- If Rf<Rd then the forward is lower than the spot (*upward sloping*)
  Associated with negative carry



Foreign interest rate > Domestic Interest rate: Forward is cheaper than spot

CME FX Futures (Example EUR/USD)

- Contract size: USD 125,000
- Month listings: Six months in the March quarterly cycle (Mar, Jun, Sep, Dec)
- Minimum price increment = $ 0.0001 per contract ($12.50/contract)
- Last trade: 9:16 AM (CT), second business day before the 3rd Wednesday of the contract month (usually Monday)

Dec 2011 Euro-Globex Futures

December 2011 Euro contract (intraday– 5 min intervals)

Example: Hedging receivables

- European car exporter wants to hedge his dollar revenues until December. He expects to collect $30,000,000 USD for the sale of cars by Dec 16 and believes that **EUR will be volatile** with respect to the dollar. How can he hedge?

- Using the Dec 11 contract as a reference, EUR/USD=1.3712 (futures-implied exchange rate)
  Notional in Euros: $30,000,000/(1.3712)= EUR 21,878,646
  Number of contracts: 21,878,646/125,000=**175 contracts** (rounded)

- Scenario 1: in December EUR=1.45 USD
  PNL futures= 175*(14500-13712)*12.5= 1,723,750 = 1,188,793 EUR
  Forex = 30,000,000/1.45 = 20,689,655 EUR
  Total = 21,878,448 EUR

- Scenario 2: in December EUR=1.25 USD
  PNL futures= 175*(12500-13712)*12.5= (2,651,250) = (2,121,000) EUR
  Forex = 30,000,000/1.25 = 24,000,000 EUR
  Total = 21,879,000 EUR

This works as if he exchanged his receivables at the rate EUR/USD=1.3712.

Eurodollar Futures (Interest Rates)

- Underlying instrument: **Eurodollar time deposit with 3 months maturity**
  Notional Amount $ 1,000,000

- Eurodollar deposits are bank deposits which are outside the jurisdiction of the Fed (not FDIC insured) offered by major international banks.

- Price quote = 100-3mLIBOR on a 360 day year.
  1bp move in 3mLIBOR corresponds to $25 move in the contract

- Tick size: 0.0025%= $6.25 per contract for nearest month; 0.0050% per contract for all other months ($12.50/contract)

- Contract months: Mar, Jun, Sep, Dec extending for 10 years, plus first 4 months

- Settlement price = 100 – 3m LIBOR

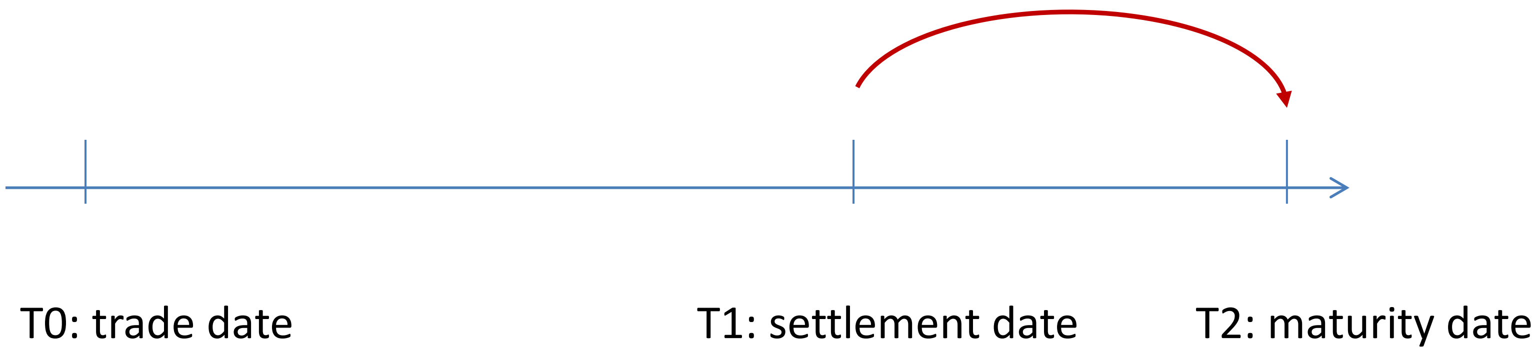
- Last trading = 11AM London time, of the second trading day before the third Wednesday of the month

Example

- The Dec 2011 contract settled on 9/12/2011 at 99.42, implying 3m LIBOR for Dec=100-99.42=0.58%

Forward rate agreements

- A Forward Rate Agreement is an OTC contract to enter into a term loan in the future at a pre-established interest rate

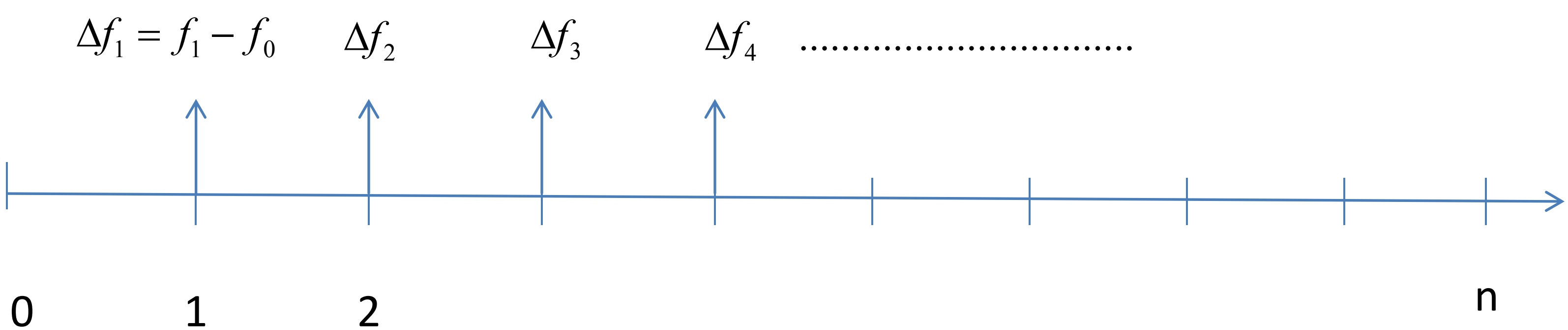


- ED Futures can be used to hedge positions in FRAs and to determine the ``fair value’’ of a FRA

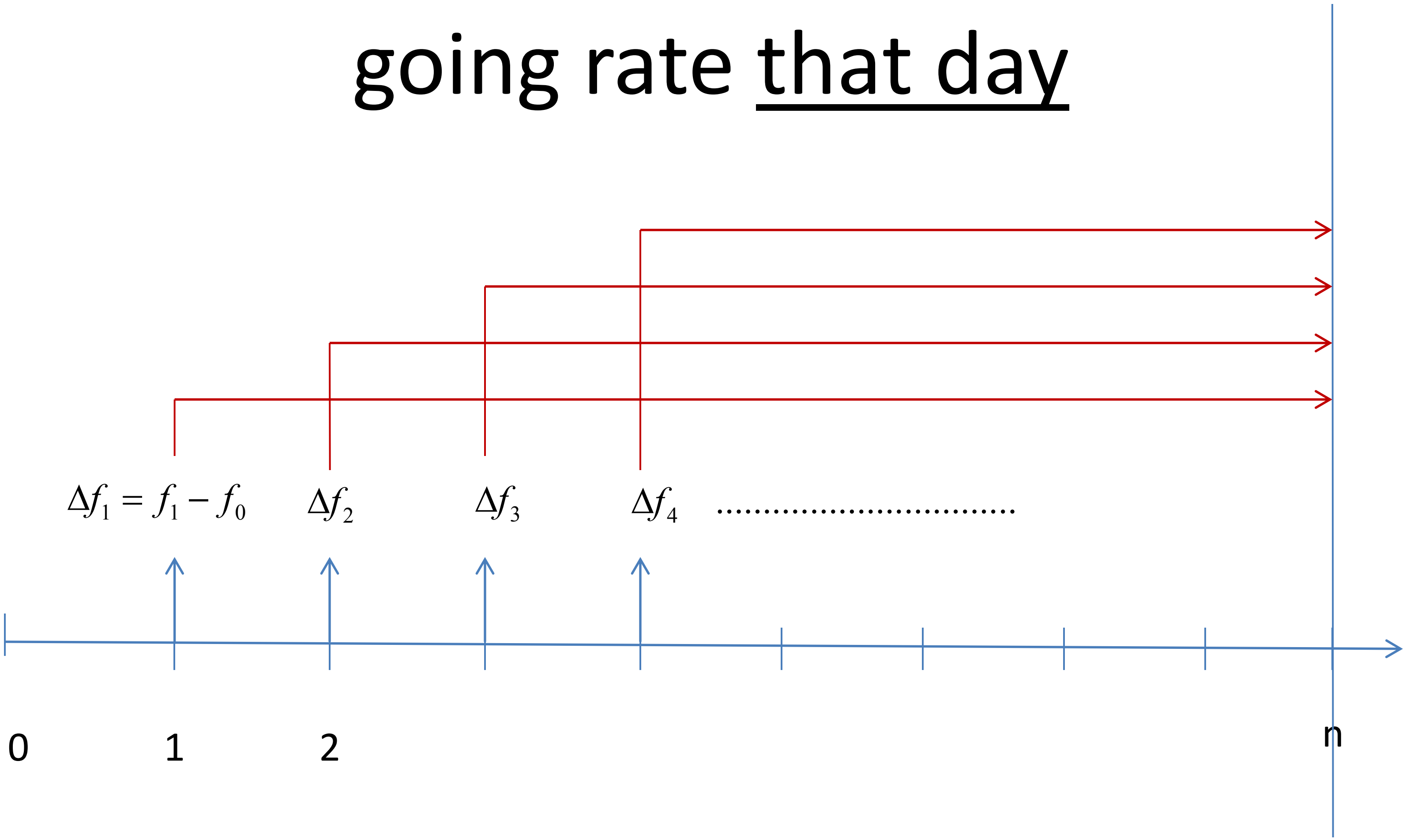
Cash & carry with interest rate futures

- The cost calculation of the cash & carry trade that we described for indices and currencies is modified for ED futures because the financing is **correlated** with the settlement LIBOR rate, so we must take into account the impact on the cash and carry strategy as rates change

- PNL is the forward value of all intermediate cash-flows from futures position



Reinvesting the cash-flows at the going rate <u>that day</u>



- Each cash-flow is invested at the LIBOR term rate at the end of the trading period
- The question is how this change in the reinvestment rate affect the final outcome.

Tailing strategy to hedge FRA

Date t_{i-1} : Position in $e^{-R_{i-1}(T-T_i)}$ contracts

Date t_i : Invest the proceeds at rate R_i

Assume proceeds can be invested at 3m LIBOR rate implied by ED futures -- otherwise we need to consider correlations

$$PNL = \sum_1^n e^{R_i(n-i)\Delta t} e^{-R_{i-1}(n-i)\Delta t}(f_i - f_{i-1})$$

$$= \sum_1^n e^{(R_i - R_{i-1})(n-i)\Delta t}(f_i - f_{i-1})$$

$$\cong \sum_1^n (1 + (R_i - R_{i-1})(n-i)\Delta t)(f_i - f_{i-1})$$

$$= \sum_1^n (f_i - f_{i-1}) + \sum_1^n (f_i - f_{i-1})(R_i - R_{i-1})(n-i)\Delta t$$

$$= f_n - f_0 + \sum_1^n (R_{i-1} - R_i)(R_i - R_{i-1})(n-i)\Delta t$$

$$= f_n - f_0 - \sum_1^n (R_i - R_{i-1})^2 (n-i)\Delta t = \boxed{R_0 - R_n - \sum_1^n (R_i - R_{i-1})^2 (n-i)\Delta t}$$

Convexity in ED Futures

- The tailing strategy with long ED futures produces a PNL equal to

  --- the difference between the Futures Rate and the Settlement Rate

  --- a negative quantity which is quadratic in the rate increments

- A long will ``lock in'' the futures rate for buying a 3m deposit starting at settlement BUT will have a **negative cash-flow from hedging rate moves**

- A short will ``lock in'' the futures rate for selling a 3m deposit starting at settlement BUT will have a **positive cash-flow from hedging rate moves**

- This is known as convexity.
  --- Long Futures/Short Forward = short convexity
  --- Short Futures/Long Forward = long convexity

FRAs and Convexity Adjustment to Futures

- The previous argument shows that the rate the long ``locks in'' should be **lower (cheaper)** than the futures implied rate to compensate for negative convexity:

$$\boxed{R^{Forward} < R^{Futures}}$$

- How much cheaper? We approximate the error term by its ``expected value'', as follows:

$$(R_i - R_{i-1})^2 \approx E(R_i - R_{i-1})^2 = \sigma^2 \Delta t$$

$$\sum_{i=1}^{n} (R_i - R_{i-1})^2 (n-i)\Delta t \approx \sum_{i=1}^{n} \sigma^2 \Delta t (n-i)\Delta t$$

$$= \sigma^2 \sum_{i=1}^{n} (T - T_i)\Delta t = \sigma^2 \int_0^T (T-t)dt$$

$$= \frac{1}{2}\sigma^2 T^2$$

Forward Rates Implied by ED Futures

$$R^{forward} = R^{futures} - \frac{1}{2}\sigma^2 T^2$$

- In practice, the constant in front of the variance may be modified to account for **imperfect correlations** between the funding rate and the futures-implied LIBOR rate.
- The dependence is quadratic in time, which means that the adjustment is negligible for short maturities and increases with maturity
- In the US, ED futures are often used to estimate Forward rates for periods < 5 years, taking into account convexity adjustments.
- This is useful for building interest swap curves, forward rate curves, for OTC trading

Commodity Futures
Example: Light Sweet Crude Oil (WTI) Futures

- Contract unit: 1,000 barrels
- Quote: dollars & cents per barrel

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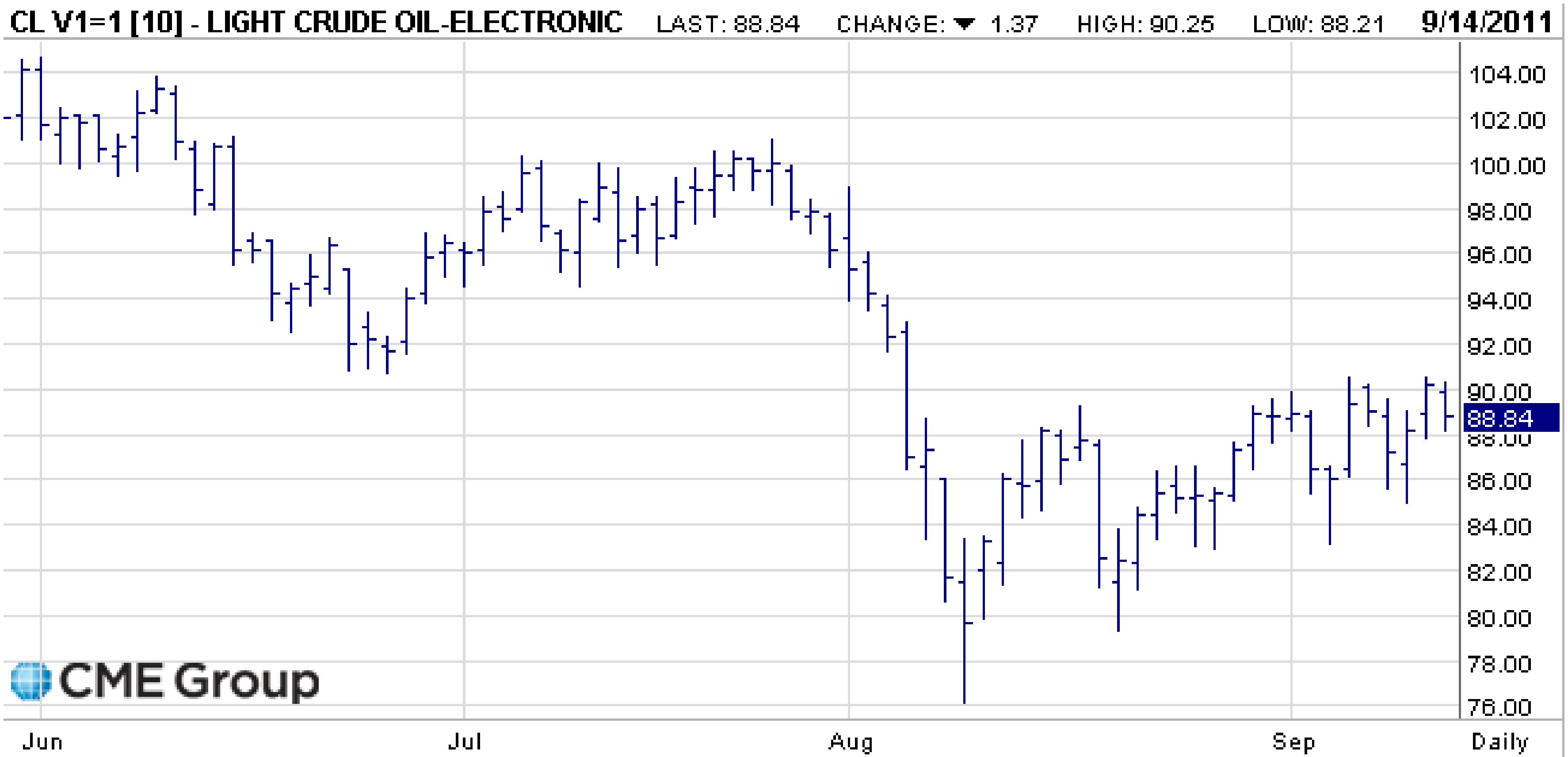
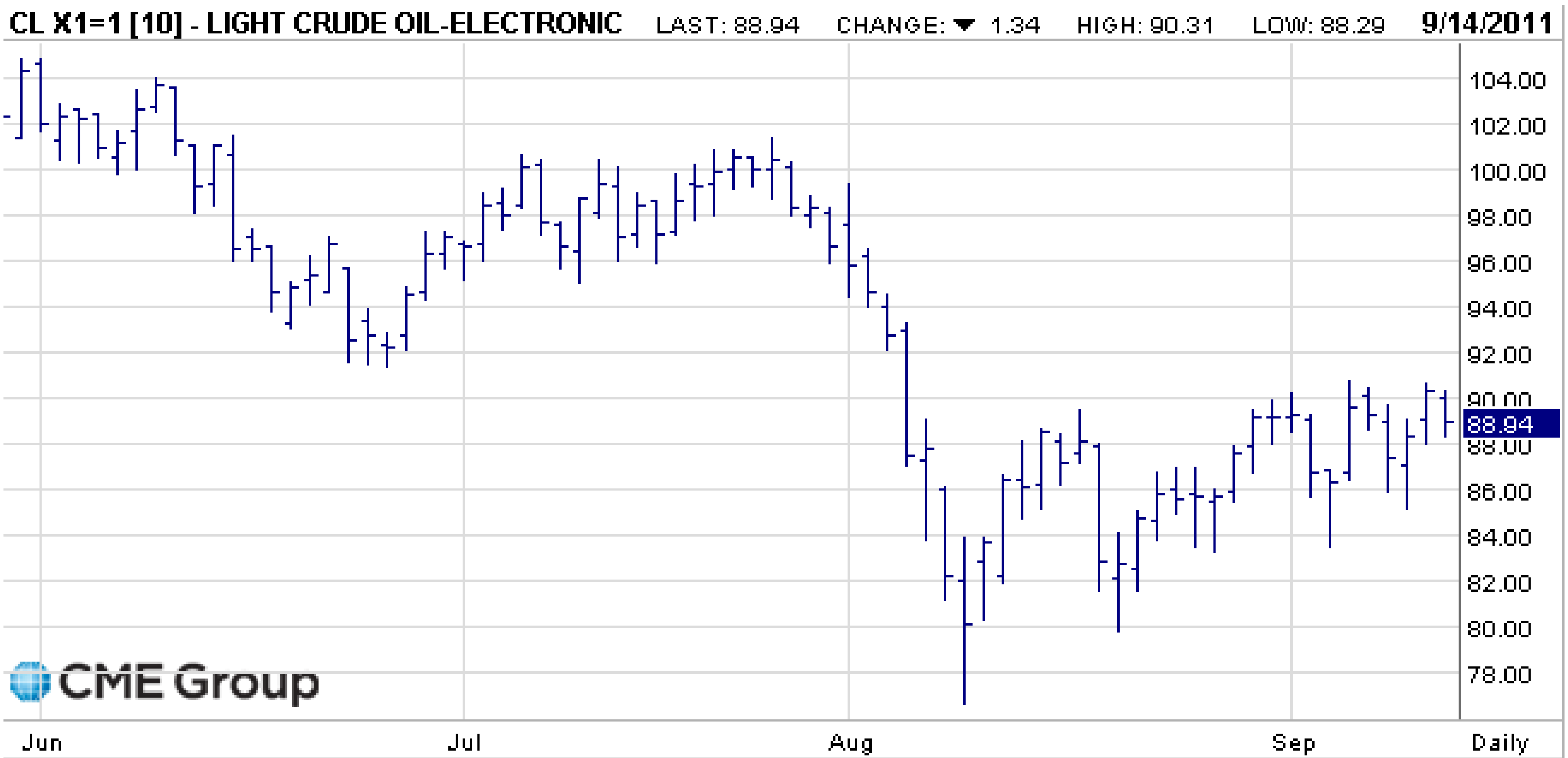
- Minimum size: $0.01 per barrel ($10)
- Listed contracts: consecutive months for the current year and next 5 years
- Delivery: Physical FOB (expenses for the seller) at any storage facility in Cushing OK with pipeline access to select facilities (TEPPCO), Cushing Storage, Equilon Pipeline Co.). Grade and quality are specified in the contract.
- Delivery on any day of the delivery month.
- Last trading date: last day of the month before the delivery month.

CL V1 (Oct 2011)



CL X1 (Nov 11)



CL X1=1 [10] - LIGHT CRUDE OIL-ELECTRONIC
LAST: 88.94
CHANGE: ▼ 1.34
HIGH: 90.31
LOW: 88.29
9/14/2011
104.00
102.00
100.00
98.00
96.00
94.00
92.00
90.00
88.94
88.00
86.00
84.00
82.00
80.00
78.00
CME Group
Jun
Jul
Aug
Sep
Daily

CL M2 (June 2012)

Term-structure of Futures

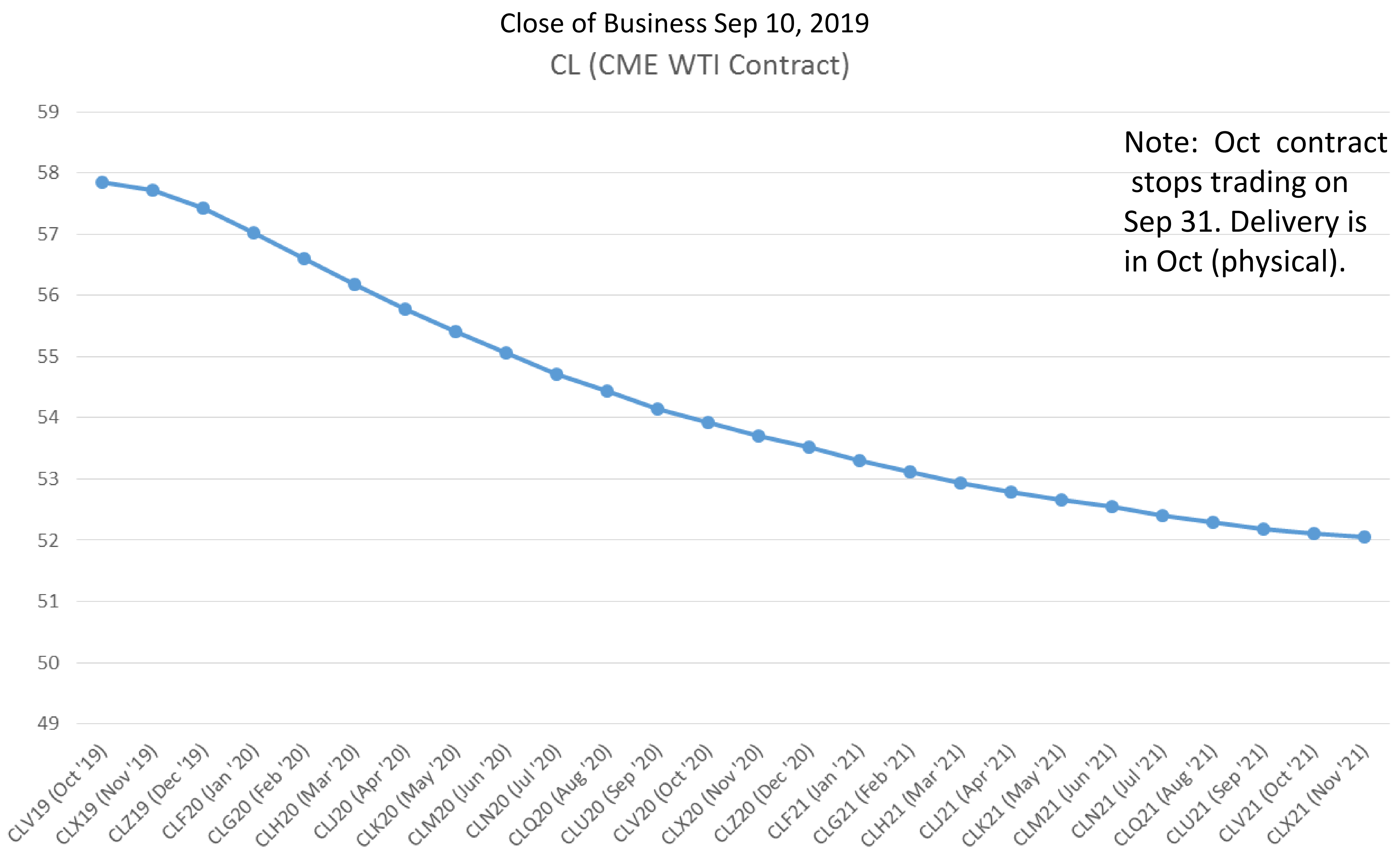
- The Term Structure of Futures (on a given contract) is the function which matches time-to-maturity with futures price

- The TS informs about several important characteristics of the product, such as

  -- convenience yield
  -- storage costs
  -- risk (volatility) in the spot market
  -- how events in the future are expected to affect the underlying spot price.

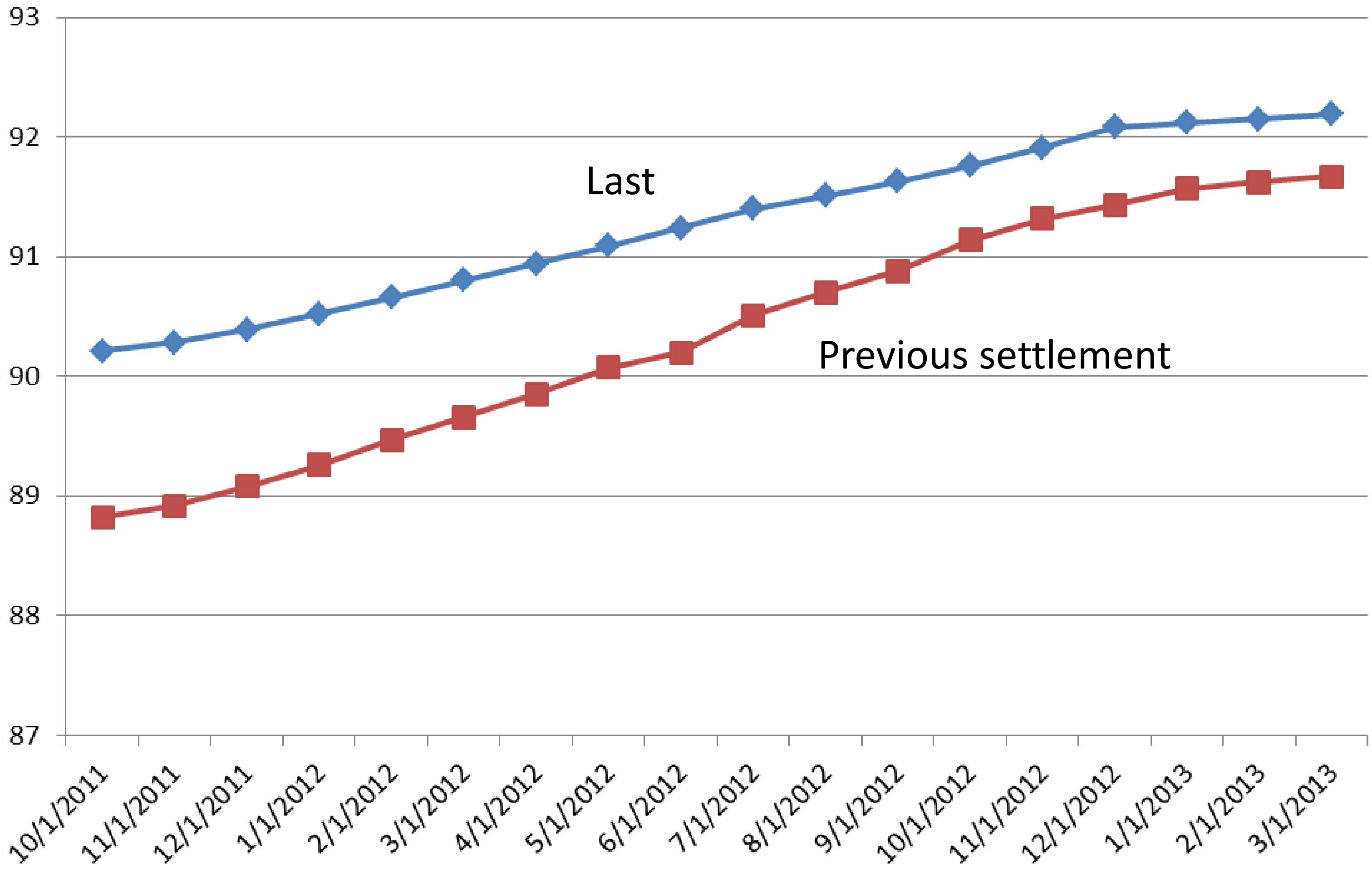
Latest futures price quotes as of Tue, Sep 10th, 2019.

Contract	Last	Change	Open	High	Low	Previous	Volume	Open Int	Time
+ CLY00 (Cash)	57.87s	+1.42	0.00	57.87	57.87	56.45	0	0	09/09/19
+ CLV19 (Oct '19)	57.40s	-0.45	58.03	58.76	57.20	57.85	718,610	299,951	09/10/19
+ CLX19 (Nov '19)	57.29s	-0.44	57.91	58.64	57.09	57.73	137,566	236,968	09/10/19
+ CLZ19 (Dec '19)	57.00s	-0.43	57.60	58.32	56.81	57.43	122,062	281,248	09/10/19
+ CLF20 (Jan '20)	56.60s	-0.43	57.13	57.90	56.43	57.03	42,175	152,164	09/10/19
+ CLG20 (Feb '20)	56.18s	-0.42	56.75	57.43	56.06	56.60	27,215	80,526	09/10/19
+ CLH20 (Mar '20)	55.78s	-0.40	56.27	56.99	55.70	56.18	28,126	111,633	09/10/19
+ CLJ20 (Apr '20)	55.38s	-0.40	56.02	56.54	55.26	55.78	12,744	52,647	09/10/19
+ CLK20 (May '20)	55.02s	-0.39	55.62	56.13	54.97	55.41	9,269	46,257	09/10/19
+ CLM20 (Jun '20)	54.68s	-0.38	55.19	55.81	54.54	55.06	40,781	171,474	09/10/19
+ CLN20 (Jul '20)	54.36s	-0.36	55.20	55.44	54.26	54.72	5,183	44,808	09/10/19
+ CLQ20 (Aug '20)	54.08s	-0.35	54.50	55.08	53.98	54.43	3,360	29,203	09/10/19
+ CLU20 (Sep '20)	53.82s	-0.33	54.28	54.83	53.82	54.15	7,130	54,424	09/10/19
+ CLV20 (Oct '20)	53.60s	-0.32	53.90	54.89	53.12	53.92	2,590	35,517	09/10/19
+ CLX20 (Nov '20)	53.42s	-0.29	54.23	54.23	53.42	53.71	2,899	25,295	09/10/19
+ CLZ20 (Dec '20)	53.26s	-0.27	53.60	54.22	53.14	53.53	52,593	175,119	09/10/19
+ CLF21 (Jan '21)	53.07s	-0.24	53.75	53.75	53.07	53.31	1,931	25,057	09/10/19
		-0.22	0.00	53.25	52.89	53.11	890	12,864	09/10/19



Close of Business Sep 10, 2019
CL (CME WTI Contract)
Note: Oct contract stops trading on Sep 31. Delivery is in Oct (physical).
59
58
57
56
55
54
53
52
51
50
49
CLV19 (Oct '19)
CLX19 (Nov '19)
CLZ19 (Dec '19)
CLF20 (Jan '20)
CLG20 (Feb '20)
CLH20 (Mar '20)
CLJ20 (Apr '20)
CLK20 (May '20)
CLM20 (Jun '20)
CLN20 (Jul '20)
CLQ20 (Aug '20)
CLU20 (Sep '20)
CLV20 (Oct '20)
CLX20 (Nov '20)
CLZ20 (Dec '20)
CLF21 (Jan '21)
CLG21 (Feb '21)
CLH21 (Mar '21)
CLJ21 (Apr '21)
CLK21 (May '21)
CLM21 (Jun '21)
CLN21 (Jul '21)
CLQ21 (Aug '21)
CLU21 (Sep '21)
CLV21 (Oct '21)
CLX21 (Nov '21)

CL Term Structure: Sep 14, 2011



Theoretical Pricing

- No basis between futures and forwards, since commodity is not correlated to funding rate

- Cash & carry costs include transportation and storage and ``convenience'' of having oil to be able to deliver it and replace it later

$$q = \text{(convenience yield) - (transportation) - (storage costs)}$$

$$r = \text{term rate}$$

$$S_0 = \text{``spot''}$$

$$\boxed{F_{0,T} = S_0 e^{(r-q)T}}$$

- The shape of the forward curve depends on the supply/demand of oil on the ground and forecasts thereof.

Contango and Backwardation

- Contango: slope of the futures curve is increasing
- Backwardation: slope of the futures curve is decreasing

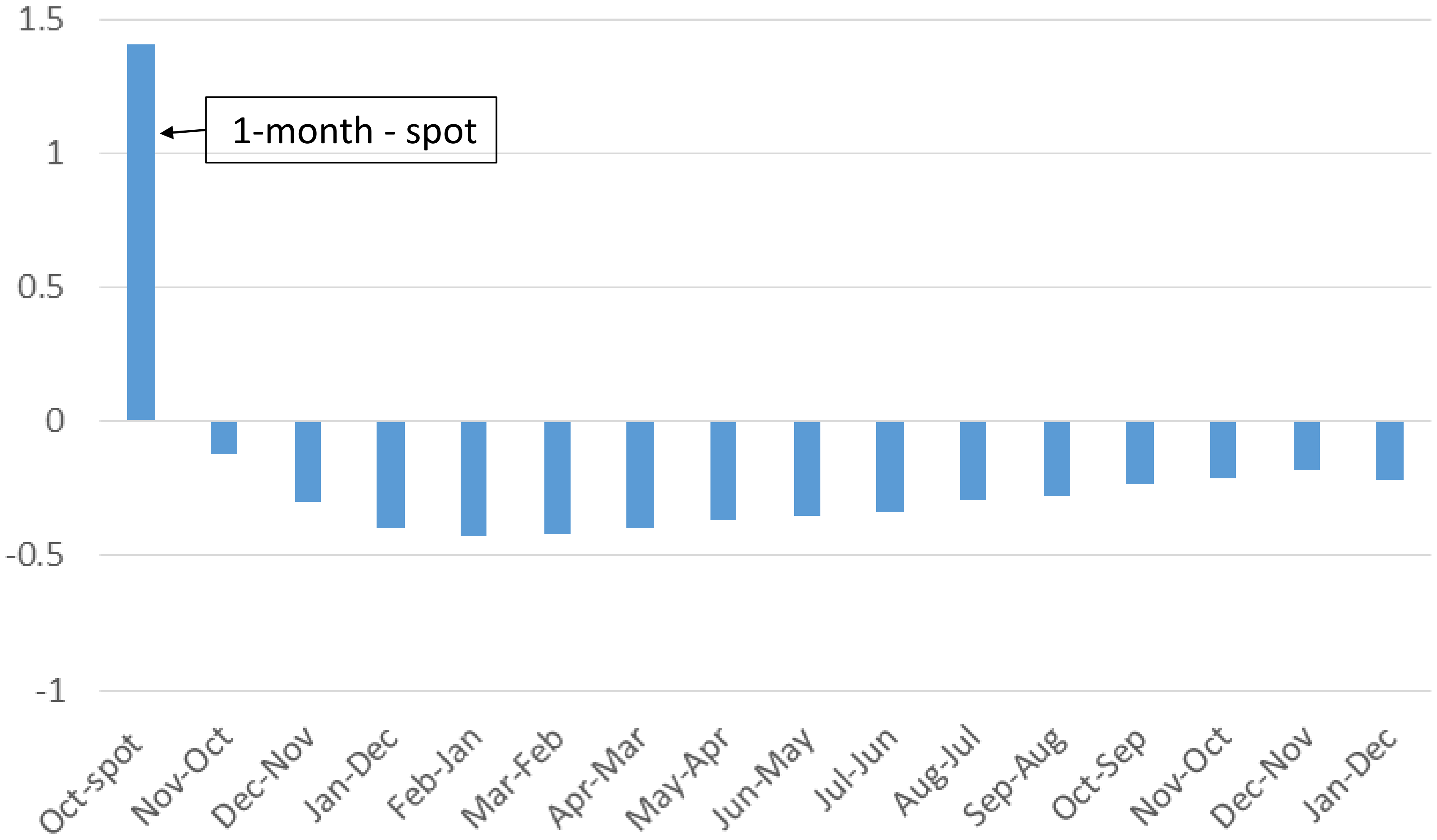
Intuition: Contango holds when storage costs dominate the term structure (i.e. there is no shortage of crude in the spot market)

Backwardation is associated with high demand in the spot market, so that having oil now is more desirable than receiving it later. Low demand for storage.

In a nutshell: backwardation arises when production cannot meet demand and contango when production can meet demand.

In oil crises, futures term structures are backwarded.

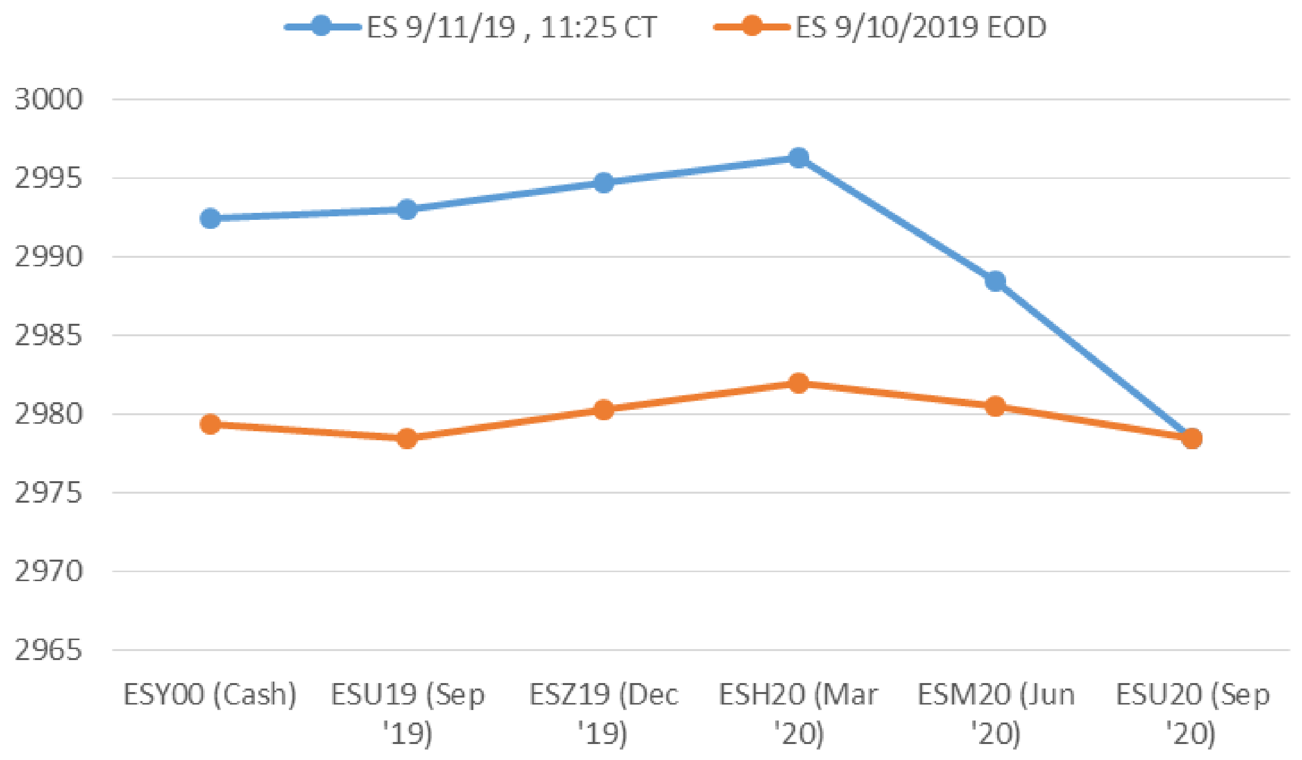
Differences (in $/ barrel) between successive month contracts (Spot= 56.45)



WTI CL Term-structure

CLY00 (Cash)	57.87				Implied r-q	Implied Term r-q	
CLV19 (Oct '19)	57.4	1.4		1.4	(annualized)	(annualized)	Term
CLX19 (Nov '19)	57.29	-0.12	Nov-Oct	-0.12	-1.08%	-1.08%	Nov-Oct
CLZ19 (Dec '19)	57	-0.3	Dec-Nov	-0.3	-2.72%	-1.90%	Dec-Oct
CLF20 (Jan '20)	56.6	-0.4	Jan-Dec	-0.4	-3.64%	-2.48%	Jan-Oct
CLG20 (Feb '20)	56.18	-0.43	Feb-Jan	-0.43	-3.94%	-2.85%	Feb-Oct
CLH20 (Mar '20)	55.78	-0.42	Mar-Feb	-0.42	-3.88%	-3.05%	Mar-Oct
CLJ20 (Apr '20)	55.38	-0.4	Apr-Mar	-0.4	-3.72%	-3.16%	Apr-Oct
CLK20 (May '20)	55.02	-0.37	May-Apr	-0.37	-3.47%	-3.21%	May-Oct
CLM20 (Jun '20)	54.68	-0.35	Jun-May	-0.35	-3.30%	-3.22%	Jun--Oct
CLN20 (Jul '20)	54.36	-0.34	Jul-Jun	-0.34	-3.23%	-3.22%	Jul--Oct
CLQ20 (Aug '20)	54.08	-0.29	Aug-Jul	-0.29	-2.77%	-3.18%	Aug--Oct
CLU20 (Sep '20)	53.82	-0.28	Sep-Aug	-0.28	-2.69%	-3.13%	Sep-Oct
CLV20 (Oct '20)	53.6	-0.23	Oct-Sep	-0.23	-2.22%	-3.06%	Oct--Oct
CLX20 (Nov '20)	53.42	-0.21	Nov-Oct	-0.21	-2.03%	-2.98%	Nov -Oct
CLZ20 (Dec '20)	53.26	-0.18	Dec-Nov	-0.18	-1.75%	-2.89%	Dec-Oct
CLF21 (Jan '21)	53.07	-0.22	Jan-Dec	-0.22	-2.15%	-2.84%	Jan-Oct

E-mini S&P term structure



Dividend yield (SPY, Yahoo finance) = 1.85%

1-year Treasury rate = 1.81%

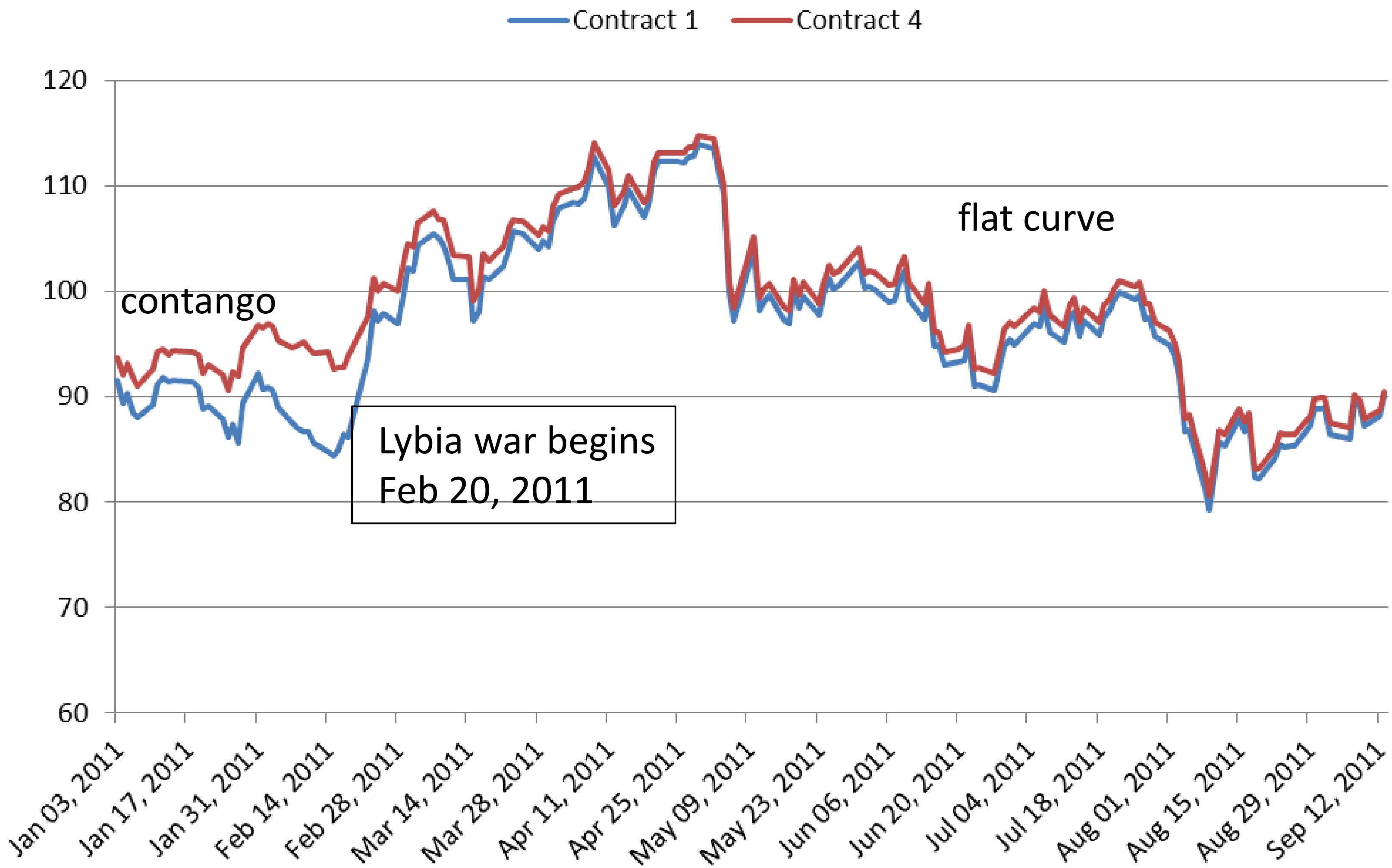
Nominal r-d = =1.81-1.85= -0.04%

eMINI s&p	9/11/2019	11:25 cst		
Contract	Last	Previous	% Change	implied r-q (ANNUALIZED)
ESY00 (Cash)	2992.46	2979.39	0.44%	
ESU19 (Sep '19)	2993	2978.5	0.49%	0.00%
ESZ19 (Dec '19)	2994.75	2980.25	0.49%	0.02%
ESH20 (Mar '20)	2996.25	2982	0.48%	0.07%

E-Mini S&P Futures Term Structure (9/11/2019, downloaded 11:50 AM CT)

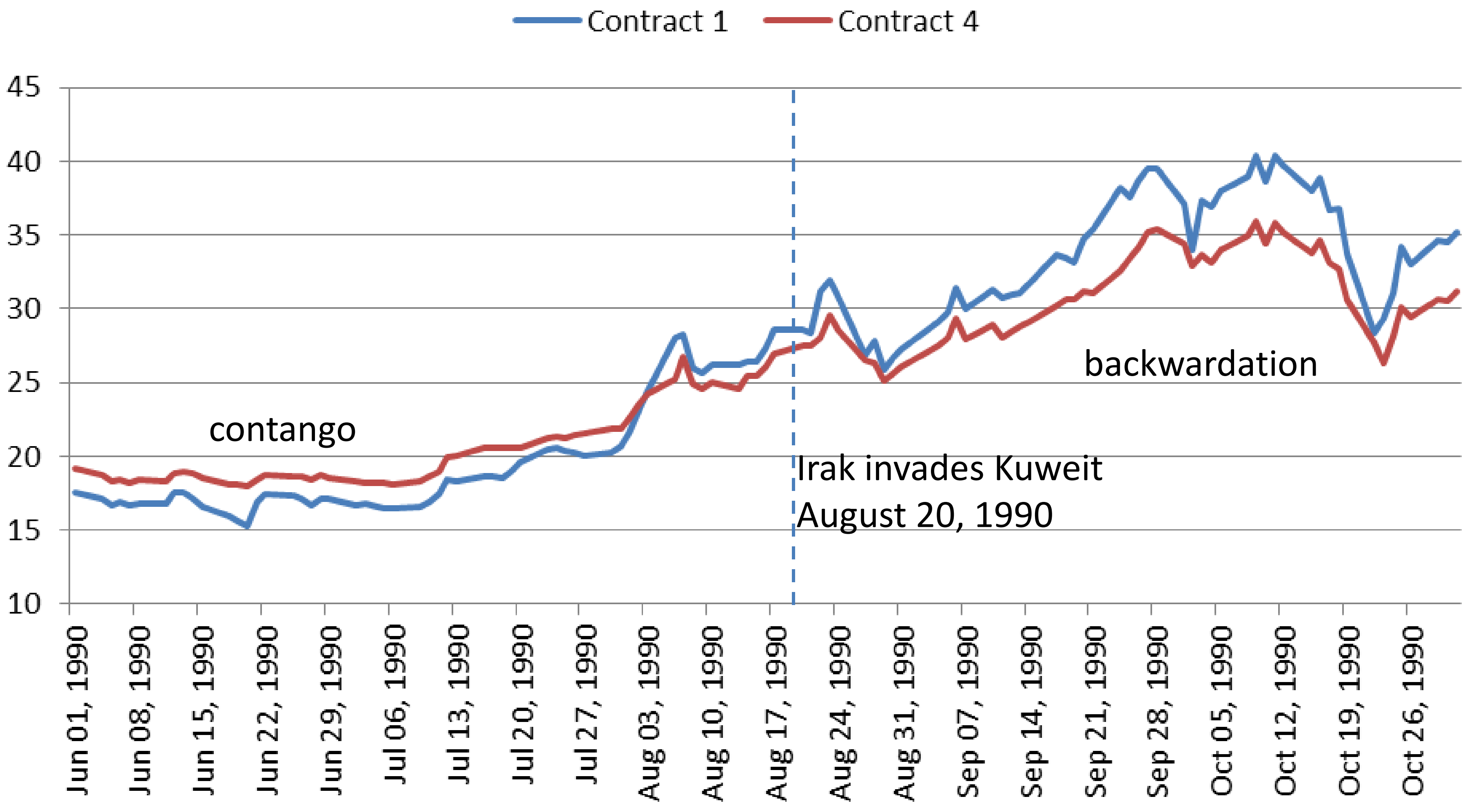
Contract	Last	Change	Previous	Volume	Open Int	Time
ESY00 (Cash)	2992	13.07	2979.39	-	-	11:37 CT
ESU19 (Sep '19)	2993	14.5	2978.5	876,331	2513360	11:37 CT
ESZ19 (Dec '19)	2995	14.5	2980.25	71,641	209453	11:36 CT
ESH20 (Mar '20)	2996	14.25	2982	180	8724	11:25 CT
ESM20 (Jun '20)	2989	8	2980.5	2	1911	10:25 CT
ESU20 (Sep '20)	2979	0	2978.5	1	6	9/10/2019

1st and 4th Month CL contracts in 2011



(source: Energy Information Administration)

CL1 and CL4 around the Kuweit war (1990)



Rolling futures

- Rolling futures means moving from one contract to another as time passes to generate a constant-maturity position across time.
- Application: this allows traders to maintain exposure to the underlying commodity beyond the first expiration.
- Example: the USO (United States Oil Trust) is an ETF (exchange-traded fund) which invests in a rolled futures strategy in CL1 / CL2.

$$I_t = USO,\ F_t^1 = CL1,\ F_t^2 = CL2$$
$$a_t = \text{fraction of the funds invested notionally in } CL1$$

$$\frac{\Delta I_t}{I_t} = \frac{I_{t+1} - I_t}{I_t} = a_t \frac{\Delta F_t^1}{F_t^1} + (1 - a_t) \frac{\Delta F_t^2}{F_t^2} + r\Delta t$$

Rolling futures gives rise to a ``drift'' relative to spot

$$\frac{\Delta I_t}{I_t} = a_t \frac{\Delta F_t^1}{F_t^1} + (1-a_t)\frac{\Delta F_t^2}{F_t^2} + r\Delta t$$

$$F_t^i = S_t e^{(r-q_i)(T-t_i)} \qquad \therefore \qquad \frac{\Delta F_t^i}{F_t^i} = \frac{\Delta S_t}{S_t} - \left(r - q_i\right)\Delta t$$

$$\begin{aligned}
\frac{\Delta I_t}{I_t} &= a_t\left(\frac{\Delta S_t}{S_t} - \left(r - q_1\right)\Delta t\right) + (1-a_t)\left(\frac{\Delta S_t}{S_t} - \left(r - q_2\right)\Delta t\right) + r\Delta t \\
&= \frac{\Delta S_t}{S_t} - r\Delta t + \left(a_t q_1 + (1-a_t)q_2\right)\Delta t + r\Delta t \\
&= \frac{\Delta S_t}{S_t} + \left(a_t q_1 + (1-a_t)q_2\right)\Delta t
\end{aligned}$$

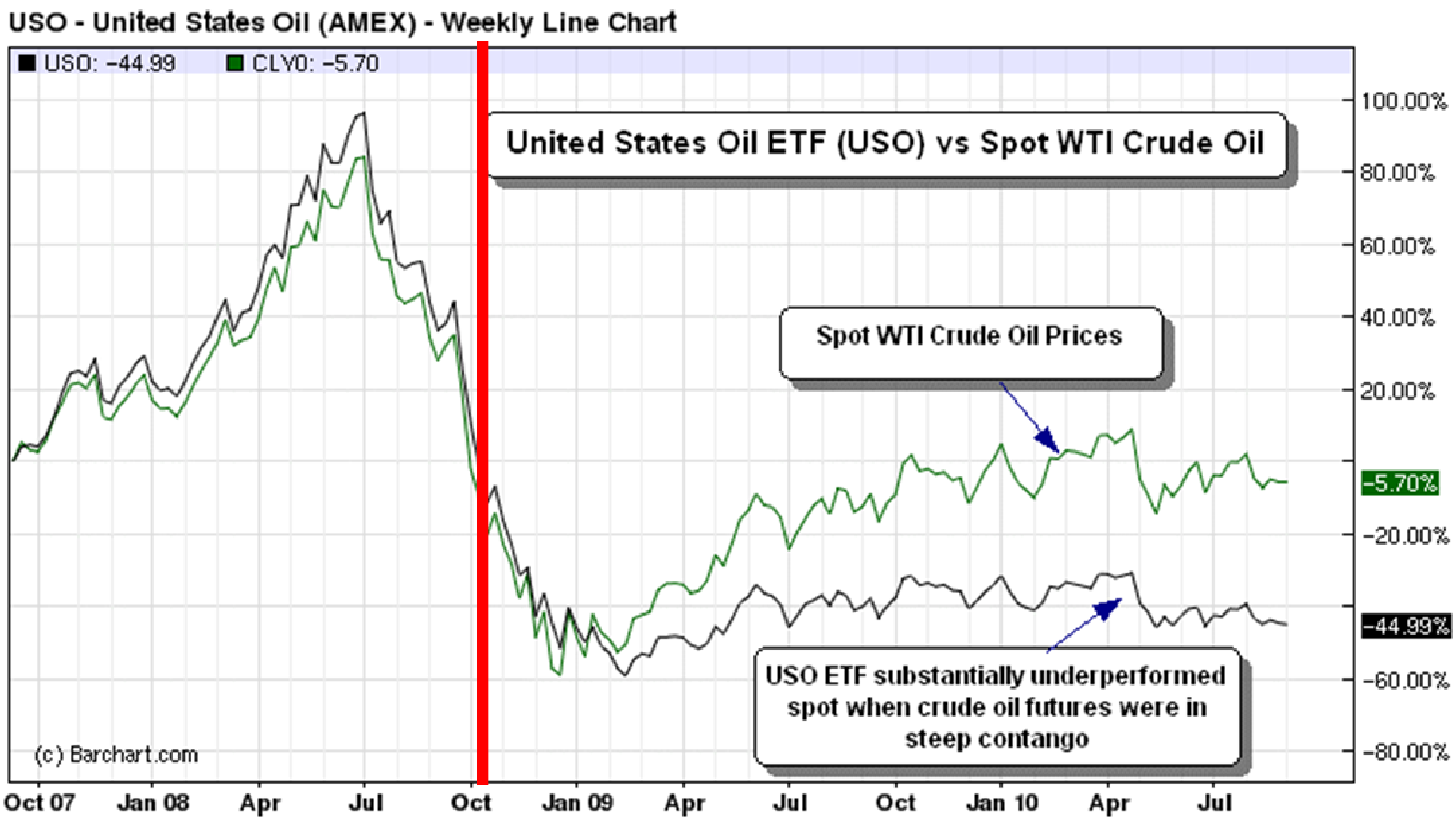
Slope, drift and the performance of rolled strategies

- If $q < 0$ – i.e., low convenience yield, high storage costs, contango situation the rolled future strategy under-performs the commodity

- If $q>0$ -- i.e. high convenience yield, storage costs are low compared to the demand for crude, the rolled strategy outperforms crude

- Conclusion: rolled futures funds like USO should underperform the commodity in times when there is contango (upward sloping futures curve)

USO Fund vs Spot WTI



Implied r-q from ratio between CL4 and CL1