Q2.

Suppose a, b, c are zero bond values of a maturity payment of \$1

$$a = \$1 \cdot \frac{1}{1 + T \times TLIBOR(0)}$$

$$b = \$1 \cdot \frac{1}{1 + (T+0.25) \times T3MLIBOR(0)}$$

$$c = \$1 \cdot \frac{1}{1 + (T+0.5) \times T6MLIBOR(0)}$$

Based on no arbitrage at $t=0$ current state.
the implied 3MLIBOR(T) should be.

$$d = \$1 \cdot \frac{1}{1 + 0.25 \times 3MLIBOR(T)}$$

$$1 + 0.25 \times 3MLIBOR(T) = \frac{1}{d}..$$

$$3MLIBOR(T) = \frac{4}{d} - 4.$$

where d is the initial investment at $t=T$ for 3M.

Based on no arbitrage at current state

$$a' = d \cdot \frac{1}{1 + T \times TLIBOR(0)} = b$$

That is, financing 2 separate terms of $0 \sim T$ and $T \sim T+3M$, should give the same return as financing 1 term of $0 \sim T+3M$.

$$\Rightarrow \quad d = \frac{b}{a}. \Rightarrow 3MLIBOR(T) = \frac{4a}{b} - 4.$$

Similarly, let's calculate 6M LIBOR(T).

assume e is the investment at $t=T$ for 6 month to get \$1 maturity payment

$$e = \$1 \cdot \frac{1}{1 + 0.5 \cdot 6M\,LIBOR(T)}$$

$$0.5\; 6MLIBOR(T) = \frac{1}{e} - 1$$

$$6MLIBOR(T) = \frac{2}{e} - 2.$$

Also, $a'' = e \cdot \frac{1}{1 + T \times TLIBOR(0)} = c \Rightarrow e = \frac{c}{a}.$

$$\Rightarrow 6MLIBOR(T) = \frac{2a}{c} - 2$$

Now, we know 3MLIBOR(T) and 6MLIBOR(T) and we have their corresponding bond pricing d and e assuming a maturity payment of \$1.

$$d = \$1 \cdot \frac{1}{1 + 0.25 \times 3MLIBOR(T)}$$

$$e = \$1 \cdot \frac{1}{1 + 0.5 \cdot 6MLIBOR(T)}$$

Let's calculate 3M LIBOR(T+3M)

assume f is the investment at t=T+3M with maturity payment of \$1 at t=T+6M.

$$f = \$1 \cdot \frac{1}{1 + 0.25 \times 3MLIBOR(T+3M)}$$

$$d' = f \cdot \frac{1}{1 + 0.25 \times 3MLIBOR(T)} = e \Rightarrow f = \frac{e}{d}$$

$$3MLIBOR(T+3M) = \left(\frac{1}{f} - 1\right) \times 4 = \frac{4d}{e} - 4$$

Now we have everything we need, lets calculate FRA value based on the following

① Borrowing \$1 at $t=T$ using the specified FRA

② Lending \$1 at $t=T$ for 3M

③ Collect return from 1st lending at $t=T+3M$ and lend them again at $t=T+3M$ for 3M.

	Borrow	Lending.
$t=T$.	$+\$1$.	$-\$1$.
$t=T+3M$		$+\$[1+0.25\times 3MLIBOR(T)]$ $-\$[1+0.25\times 3MLIBOR(T)]$
$t=T+6M$	$-\$[1+0.5\times 3MLIBOR(T)]$	$+\$[1+0.25\times 3MLIBOR(T)]$ $\times[1+0.25\times 3MLIBOR(T+3M)]$

$$\Rightarrow \text{total cashflow} = [1+0.25\times 4\times(\tfrac{a}{b}-1)]\times[1+0.25\times 4\times(\tfrac{d}{e}-1)] - [1+0.5\times 4\times(\tfrac{a}{b}-1)]$$

$$= \frac{a}{b}\times\frac{d}{e} - (1+\frac{2a}{b}-2).$$

$$= \frac{a}{b}\times\frac{b/a}{c/a} + 1 - \frac{2a}{b}$$

$$= \frac{a}{c} + 1 - \frac{2a}{b}$$

⇒ We can calculate the implied FRA value to be $\frac{a}{c} + 1 - \frac{2a}{b}$

This matches our expectation when 3M-LIBOR(T) < 6M-LIBOR(T), the FRA value will be positive.

The possible application of this is that, we can form our view on a complicate FRA based on our views of zero rates. For example, by expressing the FRA value in terms of a, b, c, when we have a market view on a, b, c, we can easily determine our view on this particular FRA, and as a result, form a market strategy.