Derivative Note, 12。

pricing measures.

aka. pricing kernels.

aka pricing models.

= . probability measures of future market scenarios which are used to pricing derivatives by discounting expected cash flows.

eg. derivatives based on SPX

the pricing model must satisfy.

$$\frac{\delta S_t}{S_t} = \sigma_t V_t \sqrt{\delta t} + r_t \delta t - q_t \Delta t .$$

How to compute σ, r, q in practice?

A: use realtime derivatives market price.

find forward rates from term rates

using. $r_{t,t+\delta t} = \dfrac{(t+\Delta t)R_{t+\delta t} - tR_t}{\delta t}$.

Similarly for dividends

$$\bar{q}_T = \frac{1}{T}\int_0^T q_t\,dt = \frac{1}{n\Delta t}\sum_{i=0}^{n-1} q_{t_i}(t_{i+1}-t_i)$$

$$t \in [T_i, T_{i+1}] \Rightarrow q_t = \frac{T_{i+1}\bar{q}_{T_{i+1}} - T_i\bar{q}_{T_i}}{T_{i+1}-T_i}.$$

Similarly for volatilities.

$$\bar{\sigma}_T^2 = \frac{1}{T}\int_0^T \sigma_t^2 \, dT = \frac{1}{n\delta t} \sum_{j=0}^{n-1} \sigma_{ti}^2 (t_{i+1} - t_i)$$

$$t \in [T_i, T_{i+1}] \Rightarrow \sigma_t^2 = \frac{T_{i+1} \bar{\sigma}_{T_{i+1}}^2 - T_i \bar{\sigma}_{T_i}^2}{T_{i+1} - T_i}$$

Trinomial Tree.

$$
(n,j)\;\begin{array}{lll}
\nearrow & U. & +\sigma_{max}\sqrt{\delta t} \\
\rightarrow & M. & 0. \\
\searrow & D. & -\sigma_{max}\sqrt{\delta t}
\end{array}
$$

$$P_{u,n} = \frac{1}{2} f_{n,j} \left(1 - \frac{\sigma_{max}\sqrt{\Delta t}}{2}\right) + \frac{\mu_n \sqrt{\Delta t}}{2\sigma_{max}}.$$

$$P_{M,n} = 1 - f_{n,j}. \quad f_{n,j} = \frac{\sigma_n^2}{\sigma_{max}^2}, \quad \mu_n = r_n - q_n.$$

$$P_{D,n} = \frac{1}{2} f_{n,j} \left(1 + \frac{\sigma_{max}\sqrt{\Delta t}}{2}\right) - \frac{\mu_n \sqrt{\Delta t}}{2\sigma_{max}}.$$

Barrier options.

executed when underlying asset crosses a given level.

Down & Out Calls.

= standard call with the additional provision that the contract is void if the underlying asset price goes below some level.

Reverse knock-outs.

= Up and Out calls

One-touch

delivers a cash payoff if a barrier is hit within a certain time-period.