Calibration of IRS curves

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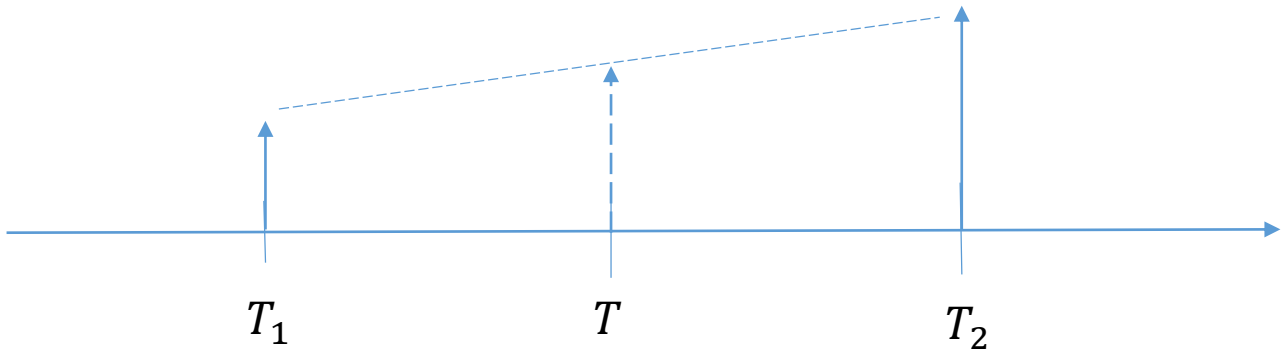
Derivative Securities 2019

Calibration of IRS curves

- Like with futures and forwards, the goal of curve calibration or curve construction is to be able to price derivatives based on current market information
- Market information: current LIBOR and SWAP rates quoted on standard tenors
- Any book of swaps will have coupons and tenors which do not correspond necessarily to today's quotes (swap rates and standard tenors)
- Curve will allow us to price by fitting the market to a smooth curve to obtain discount factors for every date in the future.
- Usually, the dates are interpolated using a monthly grid.

Interpolation of Zeros

Zero rates are interpolated linearly between ``grid dates''



$$R(T) = \frac{T_2 - T}{T_2 - T_1} R(T_1) + \frac{T - T_1}{T_2 - T_1} R(T_1)$$

Forward curve discretization

Grid dates

$T_1 = 1\ month$

$T_2 = 3\ months$

$T_3 = 6\ months$

$T_j = (6 + 3 \times j)\ months$

$1 \leq j \leq 120$

Cash Instruments

1-month LIBOR

3 month LIBOR

6 month LIBOR

Swaps

s/a swaps rates for tenors

1Y , 2Y, 3Y, 4Y. 5Y, 7Y
10Y 30Y

Note: we could also use other information: ED futures, FRAs…

Todays LIBOR rates (spot)

USD	10-01-2019	09-30-2019	09-27-2019	09-26-2019	09-25-2019
USD LIBOR - overnight	1.82613 %	1.82150 %	1.82713 %	1.82988 %	1.84075 %
USD LIBOR - 1 week	1.89275 %	1.91150 %	1.91325 %	1.92975 %	1.94688 %
USD LIBOR - 2 weeks	-	-	-	-	-
USD LIBOR - 1 month	2.01088 %	2.01563 %	2.03150 %	2.04350 %	2.05363 %
USD LIBOR - 2 months	2.05963 %	2.06538 %	2.07088 %	2.08650 %	2.08950 %
USD LIBOR - 3 months	2.08863 %	2.08513 %	2.09863 %	2.10438 %	2.09963 %
USD LIBOR - 4 months	-	-	-	-	-
USD LIBOR - 5 months	-	-	-	-	-
USD LIBOR - 6 months	2.05650 %	2.05563 %	2.06300 %	2.06438 %	2.04413 %
USD LIBOR - 7 months	-	-	-	-	-
USD LIBOR - 8 months	-	-	-	-	-
USD LIBOR - 9 months	-	-	-	-	-
USD LIBOR - 10 months	-	-	-	-	-
USD LIBOR - 11 months	-	-	-	-	-
USD LIBOR - 12 months	2.03550 %	2.03213 %	2.04325 %	2.02013 %	1.98500 %

Today’s swap rates

USD Swaps Rates

Current Interest Rate Swap Rates - USD. **Libor Rates are available** Here

get this free widget

TradeStation TradeStation

Interest Rate Swaps

Week | Month | Year | 3 Yrs

2-Oct-19	Last	BPS
1-Year	***1.750%***	***-3.0***
2-Year	1.550%	-3.0
3-Year	1.480%	-2.0
5-Year	1.440%	-2.0
7-Year	1.470%	-1.0
10-Year	1.530%	-1.0
30-Year	1.690%	-1.0

Unknowns: forward rates and zeros for grid points (T_i)

$$F_i = F(0, T_{i-1}, T_i)$$

$$Z_i = Z(0, T_i)$$

Swap rates can be derived from the standard formula

$$S_i = \frac{1 - Z_i}{A_i}$$

where Ai is the corresponding value of an annuity

Penalty function for smoothing

$$J(F) = a\sum_{i=3}^{n-1}(F_{i+1} + F_{i-1} - 2F_i)^2 + b\sum_{i=3}^{n-1}(F_{i+1} - F_{i-1})^2 + b$$

Corresponds to

$$J(f) = p\int_0^{\infty}(f''(t))^2 dt + q\int_0^{\infty}(f'(t))^2 dt$$

Note: it is known that the functions that minimize the latter functional under ``price constraints'' are piecewise polynomials of degree 3 which are ``glued'' at gridpoints with continuous second derivatives.

Solving for the curve points

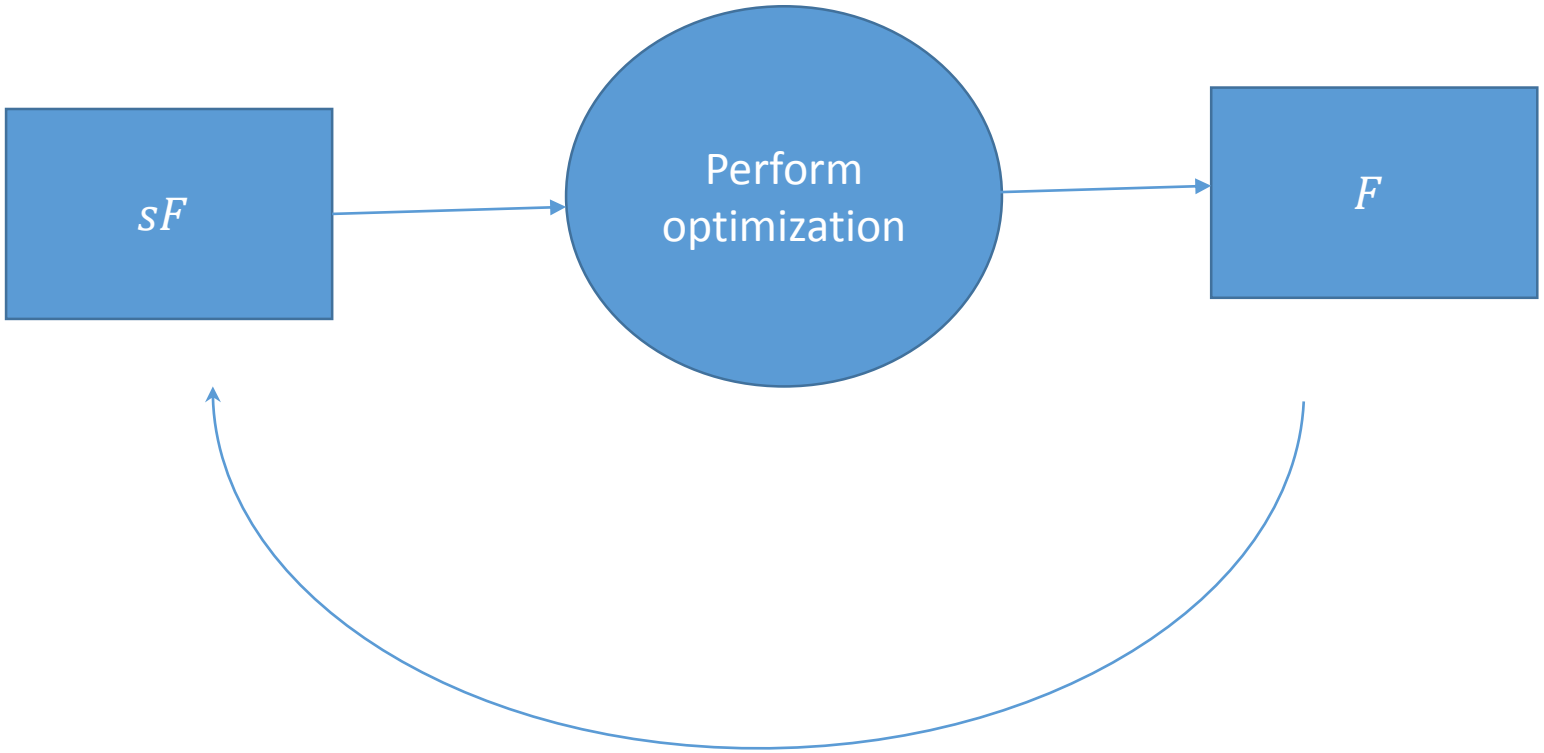
- Hagan and West (Applied Math Finance (2006) recognized that there are many solutions which can match the data.

- Due to the nature of the optimization problem, a single solution may not be guaranteed. Optimizer gets stuck in a local minimum.

- Solution: smooth the F-rate curve which is the local solution and perform optimization in an iterative way until a smooth solution with zero error and minimal penalty is achieved.

$$sF_i = \frac{1}{2m+1} \sum_{j=-m}^{+m} F_{i+j}$$

Iteration procedure



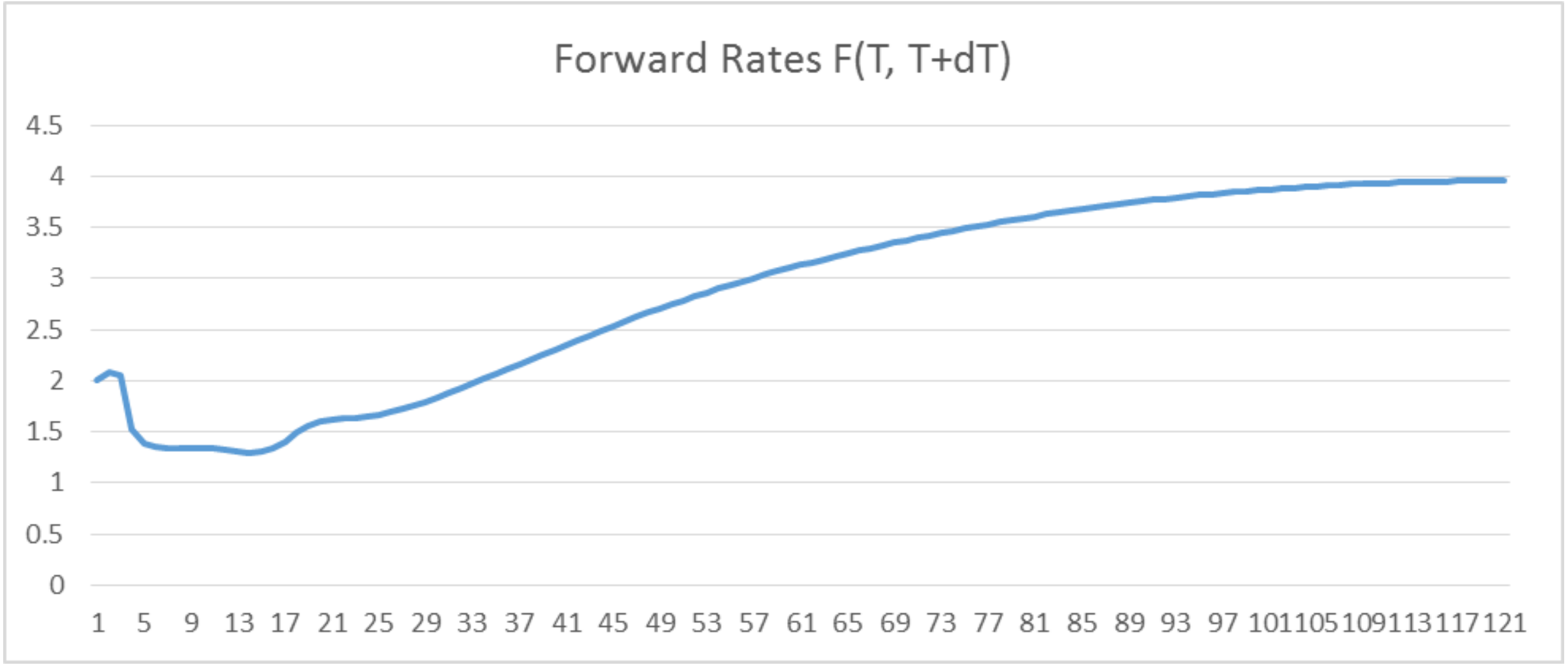
Iterate to get globally smooth forward curve
that fits the data

Iteration stops when there is no improvement on penalty function

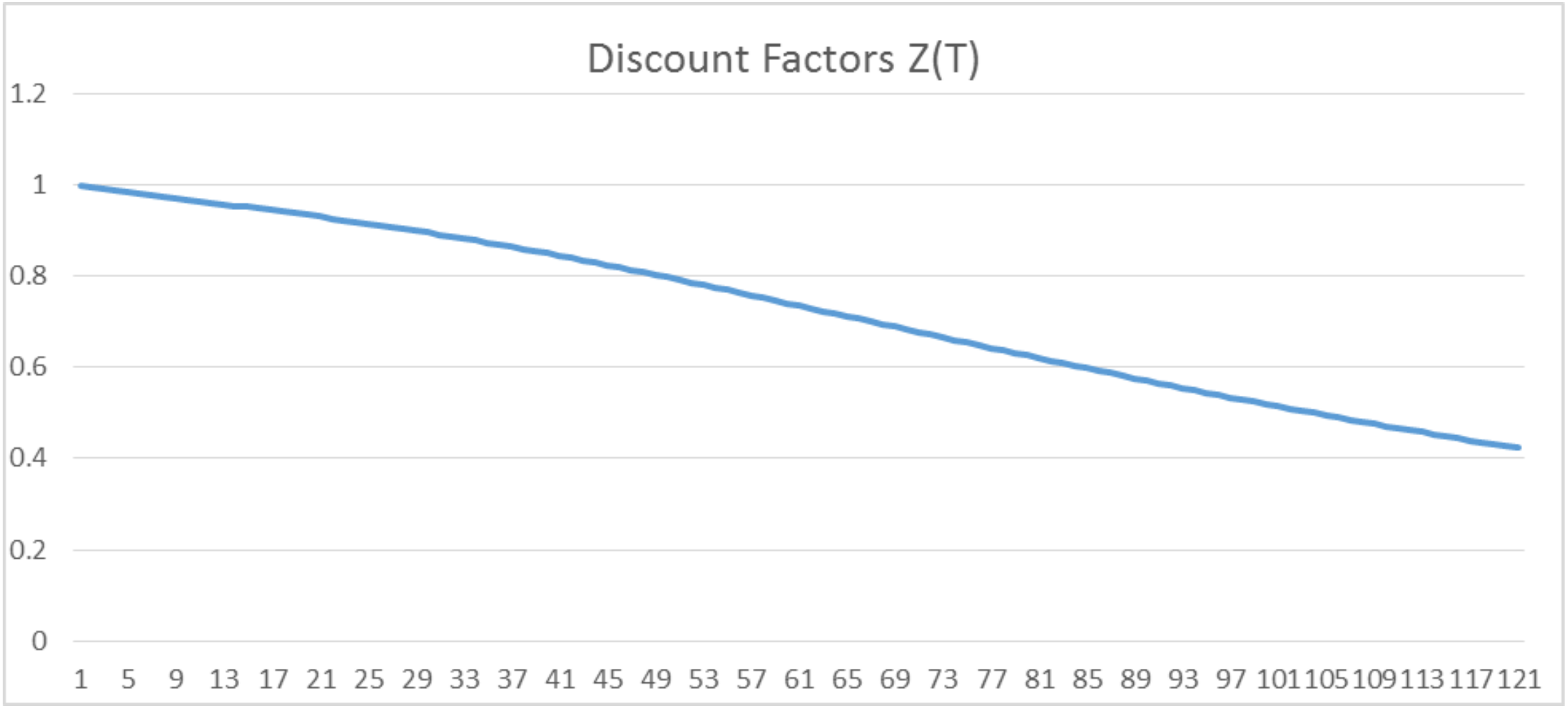
Results using 10/02/2019 data

COMUPUTED AFTER FITTING			CUBIC SPLINE SMOOTHING				DATA	
Fitted price	ERROR	ERROR^2					LIBOR	Rate
2.01	0.00	0.00		CURVATURE	SLOPE		1-month	2.01
2.08	0.00	0.00	Smoothness Penalty	0.170632291	0.8675093	1.038142	3-month	2.08
2.05	0.00	0.00	Error penalty			0.00	6-month	2.05
	0.00	0.00	Objective Function			1.059995	SWAPS	
1.75	0.00	0.00					1-year	1.75
1.55	0.00	0.00					2-year	1.55
1.48	0.00	0.00					3-year	1.48
1.44	0.00	0.00					4-year	1.44
1.47	0.00	0.00					5-year	1.47
1.53	0.00	0.00					7-year	1.53
1.69	0.00	0.00					10-year	1.69
2.67	0.00	0.00					30-year	2.67

Output for Forward Rates



Output for Zeros



Output for 3m LIBOR/ 3M interval swaps