

# Relation Decomposition Algorithms

Source/References:

Database Systems: The Complete Book, and Elmasir/Navathe



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# Decomposition

- Relation Decomposition is done for deriving better normal form relations from a given relation
- Decomposition Algorithms
  - BCNF decomposition algo
  - 3NF synthesis algo



# What is Decomposition?

- Let us say  $R(A1, A2, A3, A4, A5, A6)$  getting decomposed into  $R1(A1, A2, A3, A4)$  and  $R2(A4, A5, A6)$  – schema decomposition.

- Decomposition also mean instance decomposition, for above schema decomposition, we have instance  $r(R)$ , getting decomposed as following –

$$r1 \leftarrow \pi_{a1, a2, a3, a4}(r), \text{ and}$$

$$r2 \leftarrow \pi_{a4, a5, a6}(r)$$

- Decomposition should guarantee  $r = r1 * r2$



# Example

- R(SSN, FNAME, SALARY, DNO, DNAME, MGRSSN)

FD (Minimal) Set:

SSN  $\rightarrow$  FNAME, SALARY, DNO

**DNO  $\rightarrow$  DNAME, MGRSSN** (violates BCNF)

Key: SSN.

ssn	fname	salary	dno	dname	mgrssn
103	Alicia	25000	4	Administration	102
104	Ramesh	38000	5	Research	101
105	Joyce	25000	5	Research	101
106	Ahmad	25000	4	Administration	102
107	John	30000	5	Research	101
101	Franklin	40000	5	Research	101
102	Jennifer	43000	4	Administration	102
108	James	56000	5	Research	101



# $r(R)$ decomposed into $r_1(R_1)$ and $r_2(R_2)$

ssn	fname	salary	dno
103	Alicia	25000	4
104	Ramesh	38000	5
105	Joyce	25000	5
106	Ahmad	25000	4
107	John	30000	5
101	Franklin	40000	5
102	Jennifer	43000	4
108	James	56000	5

Decomposition should guarantee  
 $r_1 * r_2 = r$

dno	dname	mgrssn
4	Administration	102
5	Research	101



# Requirements for relational decomposition

- Should be attribute preserving
- Should be lossless
- Should be FD preserving



# Decomposition should be “lossless”

- As already stated, decomposition should guarantee
$$\mathbf{r} = \mathbf{r1} * \mathbf{r2}$$
- When it does, then it is “loss-less join decomposition” or simply “loss-less decomposition”
- It is called “loss-less” because natural join of decomposed relation results into original relation, that is there is no loss of “information”
- Lossy decomposition normally results into additional tuples (also referred as spurious tuples) on having natural join of decomposed relations.



# Lossless Decomposition Examples

- Given relation  $R(\text{PNO}, \text{PNAME}, \text{ESSN}, \text{HOURS})$ , and FDs-

$\text{PNO} \rightarrow \text{PNAME}$

$\text{PNAME} \rightarrow \text{PNO}$

$\{\text{PNO}, \text{ESSN}\} \rightarrow \text{HOURS}$

$\{\text{PNAME}, \text{ESSN}\} \rightarrow \text{HOURS}$

Keys:  $\{\text{PNO}, \text{ESSN}\}, \{\text{PNAME}, \text{ESSN}\}$

- Relation is in **3NF**
- Loss-lessly, there are two possible decompositions-

$R_1(\text{PNO}, \text{ESSN}, \text{HOURS}), R_2(\text{PNO}, \text{PNAME})$ , or

$R_1(\text{PNAME}, \text{ESSN}, \text{HOURS}), R_2(\text{PNO}, \text{PNAME})$





# Lossless Decomposition Examples

SSN	PNO	PNAME	HOURS
101	1	PATR	38
101	2	XURT	20
102	1	PATR	64
103	2	XURT	58

SSN	PNO	HOURS
101	1	38
101	2	20
102	1	64
103	2	58

SSN	PNAME	HOURS
101	PATR	38
101	XURT	20
102	PATR	64
103	XURT	58

PNO	PNAME
1	PATR
2	XURT

PNO	PNAME
1	PATR
2	XURT



# Decomposition Examples

- In given relation  $R(\text{PNO}, \text{PNAME}, \text{ESSN}, \text{HOURS})$ , let us say PName is no more unique; then our FD set is shortened to-

$\text{PNO} \rightarrow \text{PNAME}$

$\{\text{PNO}, \text{ESSN}\} \rightarrow \text{HOURS}$

Keys:  $\{\text{PNO}, \text{ESSN}\}$

Relation is in **1NF**

- Now, with modified set of FDs, second decomposition is no more lossless, it is lossy?

$R_1(\text{PNO}, \text{ESSN}, \text{HOURS}), R_2(\text{PNO}, \text{PNAME})$

$R_1(\text{PNAME}, \text{ESSN}, \text{HOURS}), R_2(\text{PNO}, \text{PNAME})$



# Decomposition Examples

SSN	PNO	PNAME	HOURS
101	1	PATR	38
101	2	XURT	20
102	1	PATR	64
103	3	XURT	58

SSN	PNO	HOURS
101	1	38
101	2	20
102	1	64
103	3	58

SSN	PNAME	HOURS
101	PATR	38
101	XURT	20
102	PATR	64
103	XURT	58

PNO	PNAME
1	PATR
2	XURT
3	XURT

PNO	PNAME
1	PATR
2	XURT
3	XURT

ON JOIN we do not get back original relation



# Binary Lossless Decomposition

- A test for binary decomposition: Let us say  $R$  is decomposed into  $R_1$  and  $R_2$ , then it is lossless, when
  - We have one of following FD  
 $(R_1 \cap R_2) \rightarrow (R_1 - R_2)$ , or  
 $(R_1 \cap R_2) \rightarrow (R_2 - R_1)$ , or
  - Or in other words, common attribute(s) are key of one of the decomposed relation.



# BCNF Decomposition Algorithm

(source: Ullman et al.)

- Input: A relation **R** with set (minimal) of FD's **F**.
- Output: A decomposition of R into a collection of relations, all of which are in BCNF.
- Method:

$T \leftarrow R$ , and  $G \leftarrow F$

Repeat till T is in BCNF.

- If there is a FD  $X \rightarrow Y$  that violates BCNF condition. Compute  $X^+$ , and choose  $X^+$  as one relation as R1, and another R2 as  $\{(R - X^+) \cup X\}$
- Map FD set G on R1 and R2 (determine FDs on R1 and R2)
  - Recursively repeat the algo on R1 and R2



# BCNF Decomposition Algorithm

(source: Ullman et al.)

- Replace relation  $R$  with two relations  $R_1$  and  $R_2$  such that

$$R_1 = X^+ \text{ (with } X \text{ as key).}$$

$$R_2 = R - (X^+ - X), \text{ or } R - X^+ \cup X.$$

- Project given FD's  $G$  onto the two new relations.



# **BCNF Decomposition Algorithm**

## **Worked Examples**



# BCNF Decomposition– Example #1

Given:

$R(\text{SSN}, \text{FNAME}, \text{PNO}, \text{PNAME}, \text{HOURS})$

$F = \{ \{ \text{SSN}, \text{PNO} \} \rightarrow \text{HOURS}; \text{SSN} \rightarrow \text{FNAME}; \text{PNO} \rightarrow \text{PNAME} \}$

Key:  $\{ \text{SSN}, \text{PNO} \}$

R

SSN	FNAME	PNO	PNAME	HOURS
101	Sumit	1	P-1	38
101	Sumit	2	P-2	20
102	Vipul	1	P-1	64
103	Ajay	2	P-2	58

FD  $\text{SSN} \rightarrow \text{FNAME}$

violates BCNF requirement;

$\text{SSN}^+ = \{ \text{SSN}, \text{FNAME} \}$

$R_1(\text{SSN}, \text{FNAME});$

$R_2(\text{SSN}, \text{PNO}, \text{PNAME}, \text{HOURS})$





# Decomposition of R based on FD

## SSN $\rightarrow$ FNAME

R2

SSN	PNO	PNAME	HOURS
101	1	P-1	38
101	2	P-2	20
102	1	P-1	64
103	2	P-2	58

FD set F2 :

{SSN, PNO}  $\rightarrow$  HOURS

PNO  $\rightarrow$  PNAME

R1

SSN	FNAME
101	Sumit
102	Vipul
103	Ajay

FD set F1:

SSN  $\rightarrow$  FNAME



# Decomposition of R2 based on FD

## $PNO \rightarrow PNAME$

R2(SSN, PNO, PNAME, HOURS)

$\{SSN, PNO\} \rightarrow HOURS$

$PNO \rightarrow PNAME$

R22

SSN	PNO	HOURS
101	1	38
101	2	20
102	1	64
103	2	58

FDs:  $\{SSN, PNO\} \rightarrow HOURS$

FD  $PNO \rightarrow PNAME$

violates BCNF requirement;

$PNO^+ = \{PNO, PNAME\}$

R21(PNO, PNAME);

R22(SSN, PNO, HOURS)

R21

PNO	PNAME
1	P-1
2	P-2

FDs:  $PNO \rightarrow PNAME$



# Final BCNF decomposition

Given:

$R(\text{SSN}, \text{FNAME}, \text{PNO}, \text{PNAME}, \text{HOURS})$

$F = \{ \{ \text{SSN}, \text{PNO} \} \rightarrow \text{HOURS},$

$\text{SSN} \rightarrow \text{FNAME}, \text{PNO} \rightarrow \text{PNAME} \}$

R22

SSN	PNO	HOURS
101	1	38
101	2	20
102	1	64
103	2	58

FDs:  $\{ \text{SSN}, \text{PNO} \} \rightarrow \text{HOURS}$

R1

SSN	FNAME
101	Sumit
102	Vipul
103	Ajay

FDs:  $\text{SSN} \rightarrow \text{FNAME}$

R21

PNO	PNAME
1	P-1
2	P-2

FDs:  $\text{PNO} \rightarrow \text{PNAME}$



# BCNF Decomposition– Example #2

- Consider relation **Medicine**(TradeName, GenericName, BatchNo, Stock, MRP, TaxRate, Manufacturer), and FDs
  - TradeName  $\rightarrow$  GenericName
  - TradeName  $\rightarrow$  Manufacturer
  - BatchNo  $\rightarrow$  TradeName
  - BatchNo  $\rightarrow$  Stock
  - BatchNo  $\rightarrow$  MRP
  - GenericName  $\rightarrow$  TaxRate
- Key: BatchNo
- FD TradeName  $\rightarrow$  GenericName violates BCNF requirement, and TradeName<sup>+</sup> = {TradeName, GenericName, Manufacturer, TaxRate};
- Therefore, the relation **Medicine**(TradeName, GenericName, BatchNo, Stock, MRP, TaxRate, Manufacturer) decomposed to:
  - Medicine1**( TradeName<sup>+</sup> ), and
  - Medicine2**( {Medicine – TradeName<sup>+</sup>} U TradeName )



# BCNF Decomposition– Example #2

- New relations and their projected FDs you have as following.
- **Medicine1**(TradeName, GenericName, Manufacturer, TaxRate)
  - FDs
    - TradeName  $\rightarrow$  GenericName
    - TradeName  $\rightarrow$  Manufacturer
    - GenericName  $\rightarrow$  TaxRate
- **Medicine2**(BatchNo, TradeName, Stock, MRP)
  - FDs:
    - BatchNo  $\rightarrow$  TradeName
    - BatchNo  $\rightarrow$  Stock
    - BatchNo  $\rightarrow$  MRP
- Are both of these in BCNF ?



# BCNF Decomposition– Example #2

- **Medicine1** is not in BCNF (highlighted FD violates BCNF requirement)
- **Medicine1**(TradeName, GenericName, Manufacturer, TaxRate)
  - FDs
    - TradeName  $\rightarrow$  GenericName
    - TradeName  $\rightarrow$  Manufacturer
    - GenericName  $\rightarrow$  TaxRate
- Repeat the process.



# BCNF Decomposition– Example #2

- Compute GenericName<sup>+</sup>, and do decomposition
- **Medicine11**(GenericName, TaxRate)
  - FDs  
GenericName  $\rightarrow$  TaxRate
- **Medicine12**(TradeName, GenericName, Manufacturer)
  - FDs  
TradeName  $\rightarrow$  GenericName  
TradeName  $\rightarrow$  Manufacturer
- Both are in BCNF ?



# BCNF Decomposition– Example #2

- Here is final BCNF decomposition

**Medicine11**(GenericName, TaxRate)

**Medicine12**(TradeName, GenericName, Manufacturer)

**Medicine2**(BatchNo, TradeName, Stock, MRP)

- Based on semantics of attributes captured in each relation, we can name the final relations as following-

**MedicineBatch**(BatchNo, TradeName, Stock, MRP)

**Medicine**(TradeName, GenericName, Manufacturer)

**GenericMedicine**(GenericName, TaxRate)





# BCNF Decomposition– Example #3

- Determine which Normal Form following relation is?
- Try bringing it in to BCNF. Decomposition should be lossless and should preserve all FDs

Book(ISBN, Title, Author, Publisher, Price, AccessonNo);

FDs:

AccessonNo  $\rightarrow$  ISBN

ISBN  $\rightarrow$  Title, Publisher, Price

Key: ?



# BCNF Decomposition– Example #3

Book(ISBN,Title,Author,Publisher,Price, AccessonNo);

FDs:

AccessonNo  $\rightarrow$  ISBN

ISBN  $\rightarrow$  Title, Publisher, Price

- Key: **AccessonNo, Author**
- Relation is not in BCNF. Let us decompose using BCNF decomposition algo



# BCNF Decomposition– Example #3

Book(ISBN,Title,Author,Publisher,Price, AccessonNo);

FDs:

**AccessonNo  $\rightarrow$  ISBN**

**ISBN  $\rightarrow$  Title, Publisher, Price**

**Key: AccessonNo, Author**

FD ISBN  $\rightarrow$  Title violates BCNF requirement, and we decompose as following -

**R1(ISBN,Title,Publisher, Price)**

**R2(AccessonNo,Author,ISBN)**



# BCNF Decomposition– Example #3

Book(ISBN,Title,Author,Publisher,Price,AccessonNo);

Decomposed to:

R1(ISBN,Title,Publisher, Price)

R2(AccessonNo,Author,ISBN)

FDs on R1

ISBN  $\rightarrow$  Title, Publisher, Price

ISBN is the key and the relation is in BCNF.

FDs for R2:

AccessonNo  $\rightarrow$  ISBN

Key for R2: AccessonNo, Author and therefore R2 is not in BCNF.



# BCNF Decomposition– Example #3

Book(ISBN,Title,Author,Publisher,Price, AccessonNo);

FDs:

AccessonNo  $\rightarrow$  ISBN

ISBN  $\rightarrow$  Title, Publisher, Price

Key: AccessonNo, Author

- Final BCNF decomposition-

R1(ISBN,Title,Publisher, Price)

R21(AccessonNo, ISBN)

R22(Author, AccessonNo)



# BCNF Decomposition Algorithm

- BCNF decomposition algorithm that we have seen here-
  - Guarantees loss-less decomposition.
  - However this does not guarantee Dependency Preservations
- Here are few such example (following slides)-



# BCNF Decomposition Algorithm

- Consider our example #1 again. We have given relation  $R(\text{PNO}, \text{PNAME}, \text{ESSN}, \text{HOURS})$ , and FDs

$\text{PNO} \rightarrow \text{PNAME} // \text{FD1}$

$\text{PNAME} \rightarrow \text{PNO} // \text{FD2}$

$\{\text{PNO}, \text{ESSN}\} \rightarrow \text{HOURS} // \text{FD3}$

$\{\text{PNAME}, \text{ESSN}\} \rightarrow \text{HOURS} // \text{FD4}$

- Both of following are loss-less BCNF decompositions but are not FD preserving-

$R1(\text{PNO}, \text{ESSN}, \text{HOURS}), R2(\text{PNO}, \text{PNAME}) // \text{Loose FD4}$

$R1(\text{PNAME}, \text{ESSN}, \text{HOURS}), R2(\text{PNO}, \text{PNAME}) // \text{Loose FD3}$



# BCNF Decomposition– Example #4

Given:

$R(ABC)$

$AB \rightarrow C$

$C \rightarrow B$

Key: AB, AC

- FD  $C \rightarrow B$  violates BCNF requirement  
 $C^+ = BC$ ;  $R_1(BC)$ ;  $R_2(AC)$ ;  $F_1\{C \rightarrow B\}$ ;  $F_2\{\}$ .
- Though both relations are in BCNF, we lose FD  $AB \rightarrow C$





# **3NF Synthesis Algorithm and worked Examples**



# 3NF Synthesis Algorithm

- “3-NF Synthesis” algorithm that guarantees lossless and preserves FDs.
- however its resultant relations may not be in BCNF, but surely in 3NF.



# Algorithm – 3NF synthesis

- Find a minimal FD\* set on R.
- For each set of FDs where X is determinant; say  $X \rightarrow A_1$ ,  $X \rightarrow A_2$ , ...  $X \rightarrow A_n$ ; create a relation schema  $R'(X \cup A_1 \cup A_2 \cup \dots \cup A_n)$ .
- If none of the relation created in above step that contains key of R; then create one more relation schema for key of R.
- \*Note: also referred as “minimal base”, “minimal cover”, “canonical cover”



# 3NF Synthesis – Example#1

- Given relation R(ABCDEF), and FDs

$A \rightarrow B$

$A \rightarrow C$

$C \rightarrow D$

$C \rightarrow E$

- Key: AF
- 3NF Decomposition

R1(ABC)

R2(CDE)

R(AF)



# 3NF Synthesis – Example#2

Book(ISBN,Title,Author,Publisher,Price, AccessonNo)

FDs:

AccessonNo  $\rightarrow$  ISBN

ISBN  $\rightarrow$  Title, Publisher, Price

Key: AccessonNo, Author

Decomposition:

R1(AccessonNo,ISBN)

R2(ISBN,Title,Publisher,Price)

R3(AccessonNo,Author)



# 3NF Synthesis – Example#3

- Given relation  $R(A,B,C,D,E)$ , and FDs

$$\{A,B\} \rightarrow C$$

$$C \rightarrow B$$

$$A \rightarrow D$$

Key: ABE

- Using 3NF synthesis algo, we have following decomposition-

$R_1(ABC), R_2(CB); R_3(AD)$

- Since,  $R_2$  we do not need because it is subset of  $R_1$ , and none of determined relation contains the key, we add a relation  $R_4(ABE)$ . Finally we have following 3NF relations –

$R_1(ABC), R_3(AD), R_4(ABE)$



# Requirements for relational decomposition (conclude)

- Should be attribute preserving
- Should be lossless
- Should be FD preserving



# Notion of De-normalization

- We accept less normal form than BCNF for efficiency reasons.





# Exercises



# Exercises

- For following relations, find out FDs (use your understanding of the attribute semantics); find out NF, decompose to best possible normal form without any information and FD loss.
- LibMember(ID, Name, Type, NoOfBooksCanBeIssued, IssueDuration)
- R (StudentID, SPI, CPI\_UptoDate, CPI\_UptoASem, AcadYr, Sem, ProgCode, CourseNo, Grade)
- IssueLog( IssueDate, MemberID, AccessionNo, DueDate, ReturnDate)
- R(A,B,C,D,E); FDs  $\{AB \rightarrow C, C \rightarrow B, A \rightarrow D\}$



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