

AI1110 Assignment 23 in L^AT_EX

Chakka Surya Saketh
AI22BTECH11005

10.15.1.9: Question In a hurdle race, a player has to cross 10 hurdles. The probability that he will clear each hurdle is $\frac{5}{6}$. What is the probability that he will knock down fewer than 2 hurdles?

Solution:

$$X \sim \text{Bin}(n, p), \quad (1)$$

$$p = \frac{5}{6}, \quad (2)$$

$$n = 10 \quad (3)$$

where p is the probability of a person knocking down a hurdle and n is the number of hurdles

The desired probability is

Let i be the number of hurdles he knocks down.

$$\therefore \Pr(X = i) = {}^nC_i p^i (1 - p)^{n-i} \quad (4)$$

$$(5)$$

Let Cumulative Distribution function be:

$$F_X(i) = \Pr(X \leq i) \quad (6)$$

$$\Pr(X = i) = {}^{10}C_i p^i (1 - p)^{10-i} \quad (7)$$

$$\therefore F_X(i) = \sum_{r=0}^i {}^{10}C_r p^r (1 - p)^{10-r} \quad (8)$$

$$\therefore F_X(1) = \Pr(X \leq 1) \quad (9)$$

$$= \sum_{i=0}^1 \Pr(X = i) \quad (10)$$

$$= \sum_{i=0}^1 {}^{10}C_i \left(\frac{1}{6}\right)^i \left(\frac{5}{6}\right)^{10-i} \quad (11)$$

$$= \left(\frac{5}{6}\right)^{10} + (10) \left(\frac{1}{6}\right) \left(\frac{5}{6}\right)^9 \quad (12)$$

$$= \frac{(5)^{10}}{2 * (6)^9} \quad (13)$$

\therefore The required probability is

$$\frac{(5)^{10}}{2 * (6)^9}$$