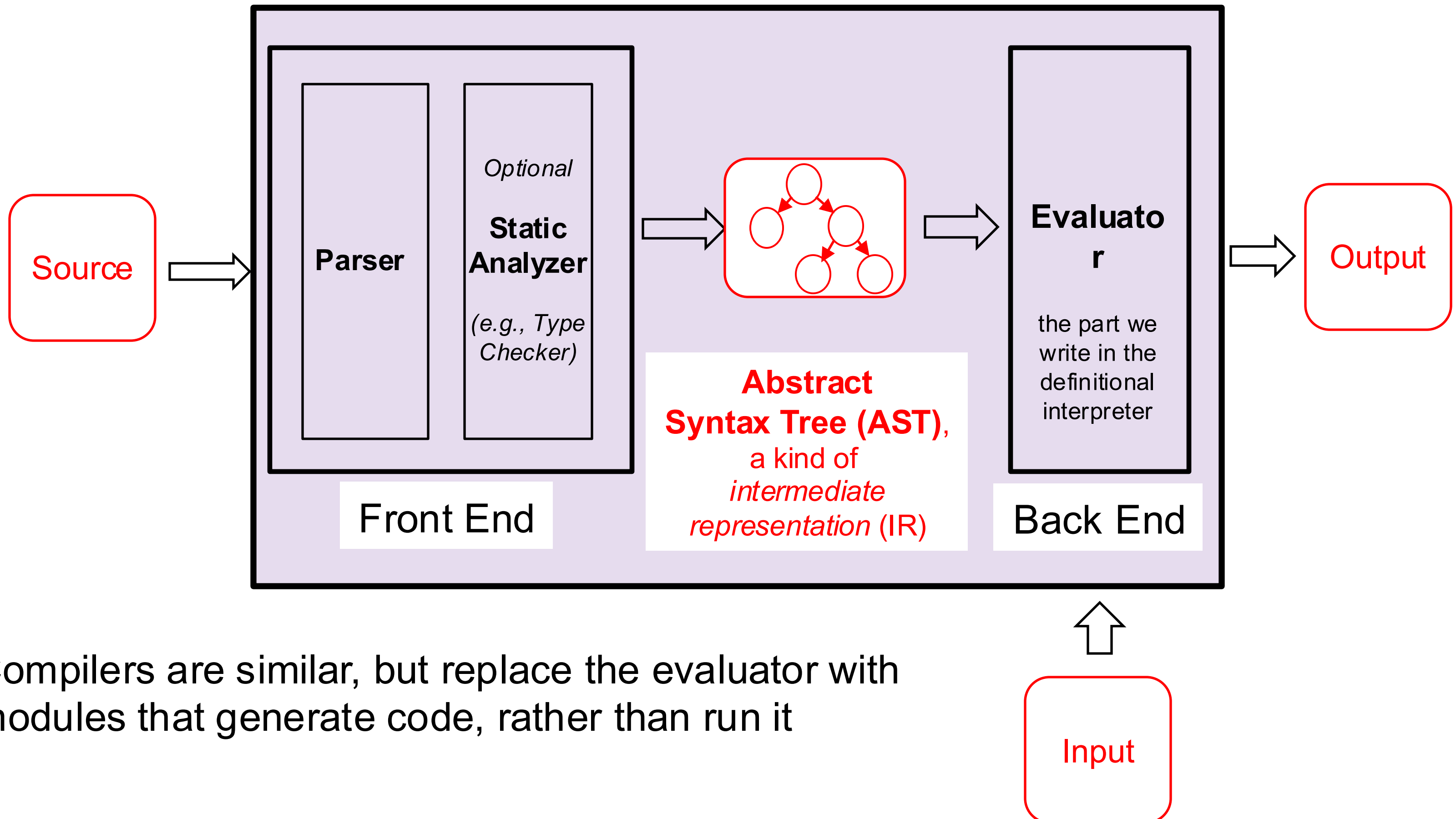
CMSC 330: Organization of Programming Languages

Context Free Grammars

Interpreters

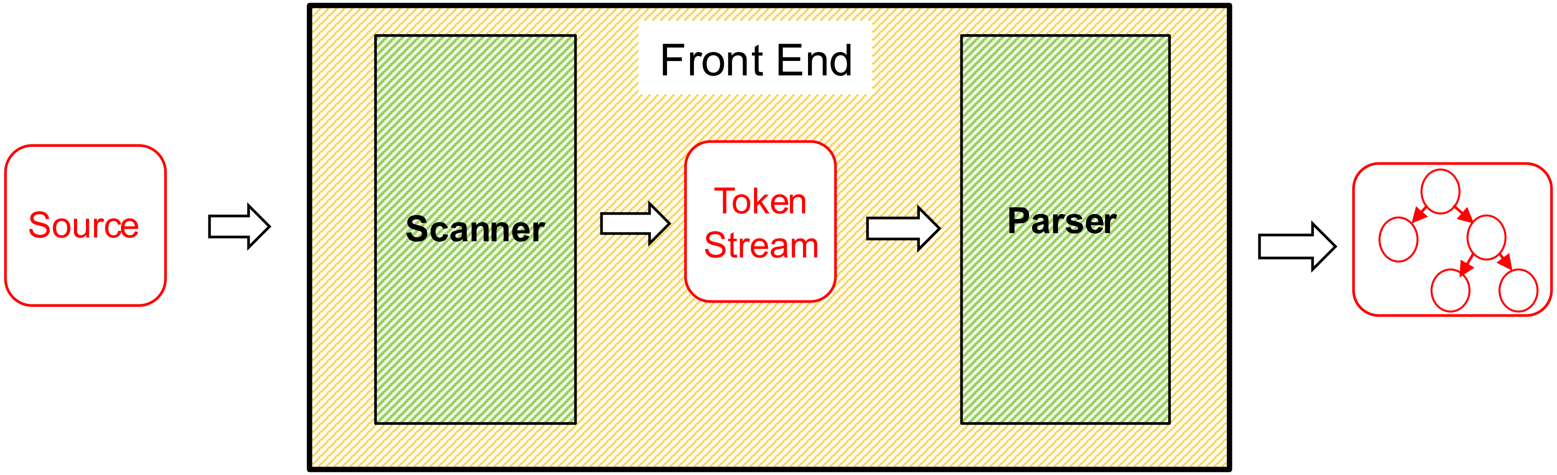


Compilers are similar, but replace the evaluator with modules that generate code, rather than run it

Implementing the Front End

- Goal: Convert program text into an Abstract Syntax Tree
- ASTs are easier to work with
  - Analyze, optimize, execute the program
- Do this using regular expressions?
  - Won't work!
  - Regular expressions cannot reliably parse paired braces {{ … }}, parentheses (((…))), etc.
- Instead: Regexps for tokens (**scanning**), and Context Free Grammars for **parsing** tokens

Front End – Scanner and Parser



- Scanner / lexer converts program source into tokens (keywords, variable names, operators, numbers, etc.) using regular expressions

- Parser converts tokens into an AST (abstract syntax tree). Parsers recognize strings defined as context free grammars

Context-Free Grammar (CFG)

- A way of describing sets of strings (= languages)
  - The notation $L(G)$ denotes the language of strings defined by grammar G

- Example grammar G is $S \to 0S \mid 1S \mid \varepsilon$

  which says that string $s' \in L(G)$ iff
  - $s' = \varepsilon$, or $\exists s \in L(G)$ such that $s' = 0s$, or $s' = 1s$

- Grammar is same as regular expression $(0|1)^*$
  - Generates / accepts the same set of strings

CFGs Are Expressive

- CFGs subsume REs, DFAs, NFAs
  - There is a CFG that generates any regular language
  - But: REs are often better notation for those languages

- And CFGs can define languages regexps cannot
  - $S \rightarrow (S) \mid \varepsilon$ // represents balanced pairs of ( )'s

- As a result, CFGs often used as the basis of parsers for programming languages

Parsing with CFGs

- CFGs formally define languages, but they do not define an *algorithm* for accepting strings
- Several styles of algorithm; each works only for less expressive forms of CFG
  - LL(k) parsing ← We will discuss this next lecture
  - LR(k) parsing
  - LALR(k) parsing
  - SLR(k) parsing
- Tools exist for building parsers from grammars
  - JavaCC, Yacc, etc.

Formal Definition: Context-Free Grammar

- A CFG G is a 4-tuple (Σ, N, P, S)

$S \to 0S1 \mid \varepsilon$

represents same # of 0's, 1's

  - Σ – alphabet (finite set of symbols, or terminals)
    - Often written in lowercase

01: $S \to 0S1 \to 0\varepsilon 1 \to 01$

  - N – a finite, nonempty set of nonterminal symbols
    - Often written in UPPERCASE
    - It must be that $N \cap \Sigma = \varnothing$

0011 $S \to 0S1 \to 00S11 \to 00\varepsilon 11 \to 0011$

can only generate $0^x 1^x$

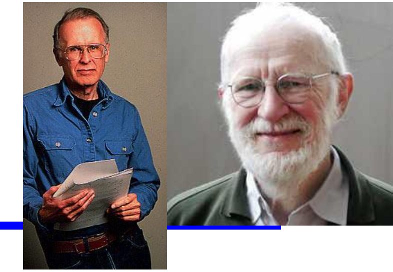
  - P – a set of productions of the form $N \to (\Sigma|N)^*$
    - Informally: the nonterminal can be replaced by the string of zero or more terminals / nonterminals to the right of the →
    - Can think of productions as rewriting rules (more later)
  - $S \in N$ – the start symbol

Notational Shortcuts

$$
\begin{array}{llll}
S \to aBc & & S \to aBc & \text{// S is start symbol} \\
A \to aA & & & \\
\quad | \; b & & \text{// } A \to b & \\
\quad | & & \text{// } A \to \varepsilon &
\end{array}
$$

- A production is of the form
  - left-hand side (LHS) → right hand side (RHS)
- If not specified
  - Assume LHS of first production is the start symbol
- Productions with the same LHS
  - Are usually combined with |
- If a production has an empty RHS
  - It means the RHS is ε

Aside: Backus-Naur Form



- Context-free grammar production rules are also called Backus-Naur Form or BNF
  - Designed by John Backus and Peter Naur
    - Chair and Secretary of the Algol committee in the early 1960s. Used this notation to describe Algol in 1962

- A production $A \rightarrow B\ c\ D$
  is written in BNF as <A> ::= <B> c <D>

abba : $S \rightarrow aSa \rightarrow abSba$
$ab\varepsilon ba \rightarrow abba$

$S \rightarrow aSa \mid bSb \mid a \mid b \mid \varepsilon$

aba ? $aSa \rightarrow aba$

Generating Strings

- Think of a grammar as generating strings by rewriting
  - Beginning with the start symbol, repeatedly rewrite a nonterminal per a production in the grammar (replace LHS with RHS)
- Example grammar G

  $S \rightarrow 0S \mid 1S \mid \varepsilon$

  011: 0S → 01S → 011S → 011

- Generate string 011 from G as follows:

  $S \Rightarrow 0S$ // using $S \rightarrow 0S$

  $\Rightarrow 01S$ // using $S \rightarrow 1S$

  $\Rightarrow 011S$ // using $S \rightarrow 1S$

  $\Rightarrow 011$ // using $S \rightarrow \varepsilon$

Accepting Strings (Informally)

- Checking if $s \in L(G)$ is called acceptance
  - Algorithm: Find a rewriting from G's start symbol that yields s
    - 011 $\in L(G)$ according to the previous rewriting
- Terminology
  - Such a sequence of rewrites is a derivation or parse
  - Discovering the derivation is called parsing

Derivations

- Notation
  - $\Rightarrow$ indicates a derivation of one step
  - $\Rightarrow^+$ indicates a derivation of one or more steps
  - $\Rightarrow^*$ indicates a derivation of zero or more steps
- Example
  - $S \rightarrow 0S \mid 1S \mid \varepsilon$

010 : S ⇒ 0S ⇒ 01S ⇒ 010S ⇒ 010ε ⇒ 010

- For the string 010
  - $S \Rightarrow 0S \Rightarrow 01S \Rightarrow 010S \Rightarrow 010$
  - $S \Rightarrow^+ 010$
  - $010 \Rightarrow^* 010$

Language Generated by Grammar

- L(G) the language defined by G is

$$L(G) = \{ s \in \Sigma^* \mid S \Rightarrow^+ s \}$$

  - S is the start symbol of the grammar
  - Σ is the alphabet for that grammar

- In other words
  - All strings over Σ that can be derived from the start symbol via one or more productions

Quiz #1

Consider the grammar

$$S \rightarrow bS \mid T$$
$$T \rightarrow aT \mid U$$
$$U \rightarrow cU \mid \varepsilon$$

Which of the following strings is generated by this grammar?

~~A.~~ aba
B. ccc
~~C.~~ bab
~~D.~~ ca

Starts w/ any # of b's
then any num of a's
then c's

Quiz #1

Consider the grammar

$S \to bS \mid T$

$T \to aT \mid U$

$U \to cU \mid \varepsilon$

Which of the following strings is generated by this grammar?

A. aba

$\boxed{\text{B. ccc}}$

C. bab

D. ca

Quiz #2

- Consider the grammar

  $S \rightarrow bS \mid T$

  $T \rightarrow aT \mid U$

  $U \rightarrow cU \mid \varepsilon$

  S => T => aT => aaT => aaU => aacU =>

- Which of the following is a derivation of the string aac?

  A. $S \Rightarrow T \Rightarrow aT \Rightarrow aTaT \Rightarrow aaT \Rightarrow aacU \Rightarrow aac$

  B. $S \Rightarrow T \Rightarrow U \Rightarrow aU \Rightarrow aaU \Rightarrow aacU \Rightarrow aac$

  C. $S \Rightarrow aT \Rightarrow aaT \Rightarrow aaU \Rightarrow aacU \Rightarrow aac$

  D. $S \Rightarrow T \Rightarrow aT \Rightarrow aaT \Rightarrow aaU \Rightarrow aacU \Rightarrow aac$

Quiz #2

- Consider the grammar

  $S \rightarrow bS \mid T$

  $T \rightarrow aT \mid U$

  $U \rightarrow cU \mid \varepsilon$

- Which of the following is a derivation of the string aac?

A. $S \Rightarrow T \Rightarrow aT \Rightarrow aTaT \Rightarrow aaT \Rightarrow aacU \Rightarrow aac$

B. $S \Rightarrow T \Rightarrow U \Rightarrow aU \Rightarrow aaU \Rightarrow aacU \Rightarrow aac$

C. $S \Rightarrow aT \Rightarrow aaT \Rightarrow aaU \Rightarrow aacU \Rightarrow aac$

D. $\boxed{S \Rightarrow T \Rightarrow aT \Rightarrow aaT \Rightarrow aaU \Rightarrow aacU \Rightarrow aac}$

Quiz #3

Consider the grammar

$S \rightarrow bS \mid T$

$T \rightarrow aT \mid U$

$U \rightarrow cU \mid \varepsilon$

Which of the following regular expressions accepts the same language as this grammar?

A. (a|b|c)*

B. b*a*c*

C. (b|ba|bac)*

D. bac*

Quiz #3

Consider the grammar

$S \rightarrow bS \mid T$

$T \rightarrow aT \mid U$

$U \rightarrow cU \mid \varepsilon$

Which of the following regular expressions accepts the same language as this grammar?

A. (a|b|c)*

B. $\boxed{\text{b*a*c*}}$

C. (b|ba|bac)*

D. bac*

Designing Grammars

1. Use recursive productions to generate an arbitrary number of symbols

   $A \rightarrow xA \mid \varepsilon$ // Zero or more x's

   $A \rightarrow yA \mid y$ // One or more y's

2. Use separate non-terminals to generate disjoint parts of a language, and then combine in a production

   a*b* // a's followed by b's

   $S \rightarrow AB$

   $A \rightarrow aA \mid \varepsilon$ // Zero or more a's

   $B \rightarrow bB \mid \varepsilon$ // Zero or more b's

Designing Grammars

3. To generate languages with matching, balanced, or related numbers of symbols, write productions which generate strings from the middle

   $\{a^n b^n \mid n \geq 0\}$ // N a's followed by N b's

   $S \rightarrow aSb \mid \varepsilon$

   Example derivation: $S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aabb$

   $\{a^n b^{2n} \mid n \geq 0\}$ // N a's followed by 2N b's

   $S \rightarrow aSbb \mid \varepsilon$

   Example derivation: $S \Rightarrow aSbb \Rightarrow aaSbbbb \Rightarrow aabbbb$

Designing Grammars

4. For a language that is the union of other languages, use separate nonterminals for each part of the union and then combine

$\{ a^n(b^m|c^m) \mid m > n \geq 0\}$

Can be rewritten as

$\{ a^nb^m \mid m > n \geq 0\} \cup \{ a^nc^m \mid m > n \geq 0\}$

$S \rightarrow T \mid V$

$T \rightarrow aTb \mid U$

$U \rightarrow Ub \mid b$

$V \rightarrow aVc \mid W$

$W \rightarrow Wc \mid c$

Practice

- Try to make a grammar which accepts
  - $0^*|1^*$
  - 0^n1^n where $n \geq 0$

$S \rightarrow A \mid B$
$A \rightarrow 0A \mid \varepsilon$
$B \rightarrow 1B \mid \varepsilon$

$S \rightarrow 0S1 \mid \varepsilon$

- Give some example strings from this language
  - $S \rightarrow 0 \mid 1S$
    - 0, 10, 110, 1110, 11110, …
  - What language is it, as a regexp?
    - 1*0

Quiz #4

Which of the following grammars describes the same language as 0^n1^m where $m \leq n$?

~~A.~~ $S \rightarrow 0S1 \mid \varepsilon$

~~B.~~ $S \rightarrow 0S1 \mid S1 \mid \varepsilon$

C. $S \rightarrow 0S1 \mid 0S \mid \varepsilon$

D. $S \rightarrow SS \mid 0 \mid 1 \mid \varepsilon$

Quiz #4

Which of the following grammars describes the same language as 0^n1^m where $m \le n$?

A. $S \rightarrow 0S1 \mid \varepsilon$ *same number of 0 and 1*

B. $S \rightarrow 0S1 \mid S1 \mid \varepsilon$ *more 1's*

C. $\boxed{S \rightarrow 0S1 \mid 0S \mid \varepsilon}$ *more 0's*

D. $S \rightarrow SS \mid 0 \mid 1 \mid \varepsilon$ *no control of the number*

Parse Trees

- Parse tree shows how a string is produced by a grammar
  - *Will be useful for spotting ambiguity; discussed later*
  - Root node of parse tree is the start symbol
  - Every internal node is a nonterminal
  - Children of an internal node
    - Are symbols on RHS of production applied to nonterminal
  - Every leaf node is a terminal or ε

- Reading the leaves left to right
  - Shows the string corresponding to the tree

Parse Tree Example

S

S

$S \rightarrow aS \mid T$

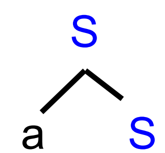
$T \rightarrow bT \mid U$

$U \rightarrow cU \mid \varepsilon$

Parse Tree Example

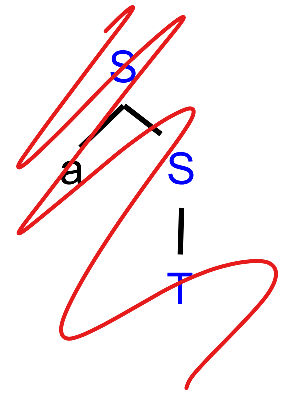
$S \Rightarrow aS$

$S \rightarrow aS \mid T$
$T \rightarrow bT \mid U$
$U \rightarrow cU \mid \varepsilon$

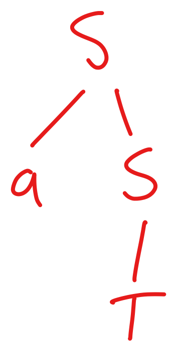


Parse Tree Example

$$S \Rightarrow aS \Rightarrow aT$$



$S \rightarrow aS \mid T$
$T \rightarrow bT \mid U$
$U \rightarrow cU \mid \varepsilon$



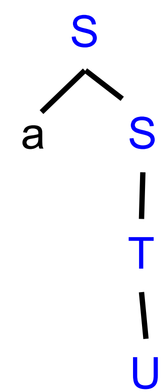
Parse Tree Example

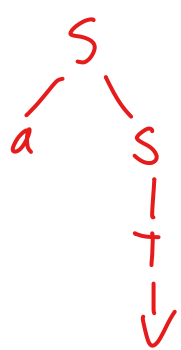
$S \Rightarrow aS \Rightarrow aT \Rightarrow aU$

$S \rightarrow aS \mid T$

$T \rightarrow bT \mid U$

$U \rightarrow cU \mid \varepsilon$

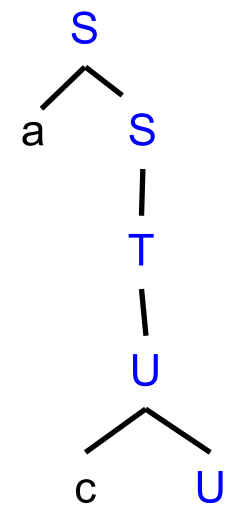




Parse Tree Example

$S \Rightarrow aS \Rightarrow aT \Rightarrow aU \Rightarrow acU$

$S \rightarrow aS \mid T$
$T \rightarrow bT \mid U$
$U \rightarrow cU \mid \varepsilon$



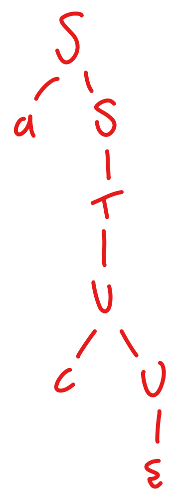
Parse Tree Example

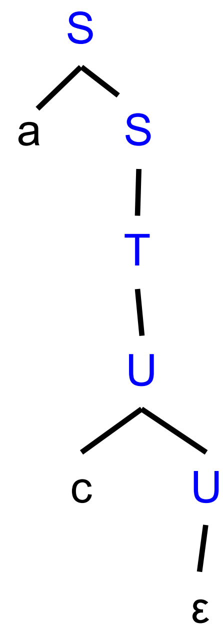
$S \Rightarrow aS \Rightarrow aT \Rightarrow aU \Rightarrow acU \Rightarrow ac$

$S \rightarrow aS \mid T$

$T \rightarrow bT \mid U$

$U \rightarrow cU \mid \varepsilon$





CFGs and ASTs

- An abstract syntax tree is a data structure that represents a parsed input, e.g., a program expression
  - An AST can be expressed with an OCaml datatype that is very close to the CFG that describes the language syntax

CFG for arithmetic expressions:

- $E \rightarrow$ a | b | c | d
  - | E+E
  - | E-E
  - | E*E
  - | (E)

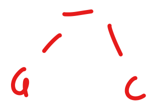
AST:

```
type expr = A | B | C | D
   | Plus of expr * expr
   | Minus of expr * expr
   | Mult of expr * expr
```

Eventual Goal: Parse a CFG to get an AST

CFG (string):

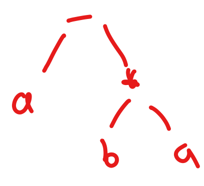
- E → a | b | c | d
  | E+E
  | E-E
  | E*E
  | (E)

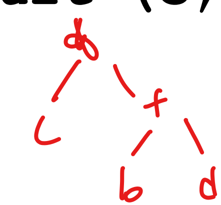


AST definition (OCaml):

```
type expr = A | B | C | D
   | Plus of expr * expr
   | Minus of expr * expr
   | Mult of expr * expr
```

a-c	parses to	`Minus (A, C)`
a-(b*a)	parses to	`Minus (A, Mult (B,A))`
c*(b+d)	parses to	`Mult (C, Plus (B,D))`



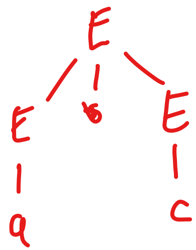


Parse Trees for Expressions

- A parse tree shows the structure of an expression as it corresponds to a grammar

  E → a | b | c | d | E+E | E-E | E*E | (E)

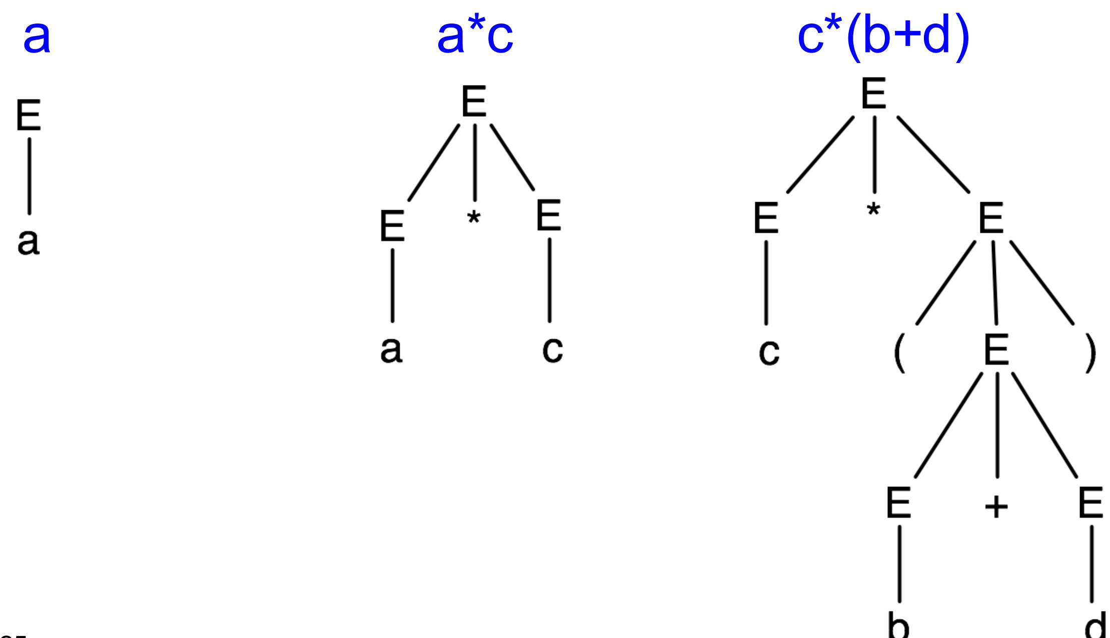
a a*c c*(b+d)



Parse Trees for Expressions

- A parse tree shows the structure of an expression as it corresponds to a grammar

  $E \rightarrow a \mid b \mid c \mid d \mid E+E \mid E\text{-}E \mid E*E \mid (E)$

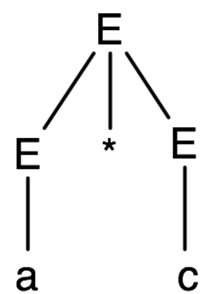


Abstract Syntax Trees

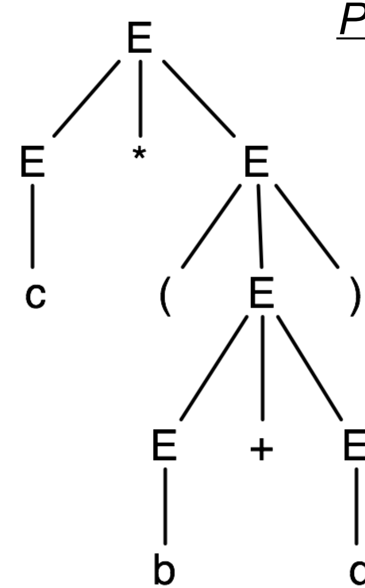
We will show how to parse to an AST in the next lecture

- A parse tree and an AST are not the same thing
  - The latter is a data structure produced by parsing

a*c

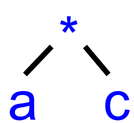


c*(b+d)

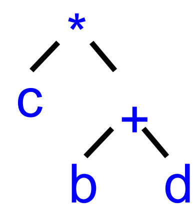


Parse trees

ASTs



Mult(A,C)



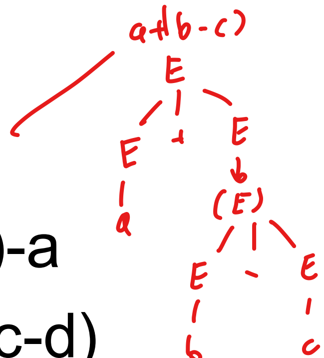
Mult(C,Plus(B,D))

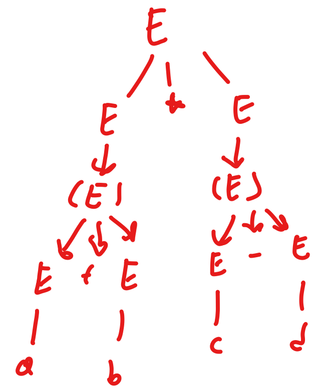
Practice

E → a | b | c | d | E+E | E-E | E*E | (E)

Make a parse tree for…

- a*b
- a+(b-c)
- d*(d+b)-a
- (a+b)*(c-d)
- a+(b-c)*d





Leftmost and Rightmost Derivation

- Leftmost derivation
  - Leftmost nonterminal is replaced in each step
- Rightmost derivation
  - Rightmost nonterminal is replaced in each step
- Example
  - Grammar
    - $S \to AB$, $A \to a$, $B \to b$
  - Leftmost derivation for "ab"
    - $S \Rightarrow AB \Rightarrow aB \Rightarrow ab$
  - Rightmost derivation for "ab"
    - $S \Rightarrow AB \Rightarrow Ab \Rightarrow ab$

Parse Tree For Derivations

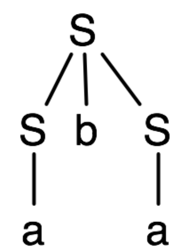
- Parse tree may be same for both leftmost & rightmost derivations
  - Example Grammar: S → a | SbS   String: aba

    Leftmost Derivation

    S ⇒ SbS ⇒ abS ⇒ aba

    Rightmost Derivation

    S ⇒ SbS ⇒ Sba ⇒ aba



  - Parse trees don't show order productions are applied
- Every parse tree has a unique leftmost and a unique rightmost derivation

Parse Tree For Derivations (cont.)

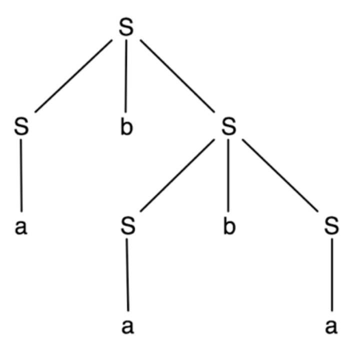
- Not every string has a unique parse tree
  - Example Grammar: S → a | SbS   String: ababa

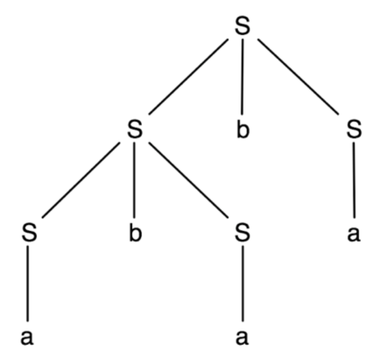
    Leftmost Derivation

    S ⇒ SbS ⇒ abS ⇒ abSbS ⇒ ababS ⇒ ababa

    Another Leftmost Derivation

    S ⇒ SbS ⇒ SbSbS ⇒ abSbS ⇒ ababS ⇒ ababa





Ambiguity

- A grammar is ambiguous if a string may have multiple leftmost derivations

I saw a girl with a telescope.

Ambiguity

- A grammar is ambiguous if a string may have multiple leftmost derivations
  - Equivalent to multiple parse trees
  - Can be hard to determine

1. $S \rightarrow aS \mid T$
   $T \rightarrow bT \mid U$
   $U \rightarrow cU \mid \varepsilon$

   No

2. $S \rightarrow T \mid T$
   $T \rightarrow Tx \mid Tx \mid x \mid x$

   Yes

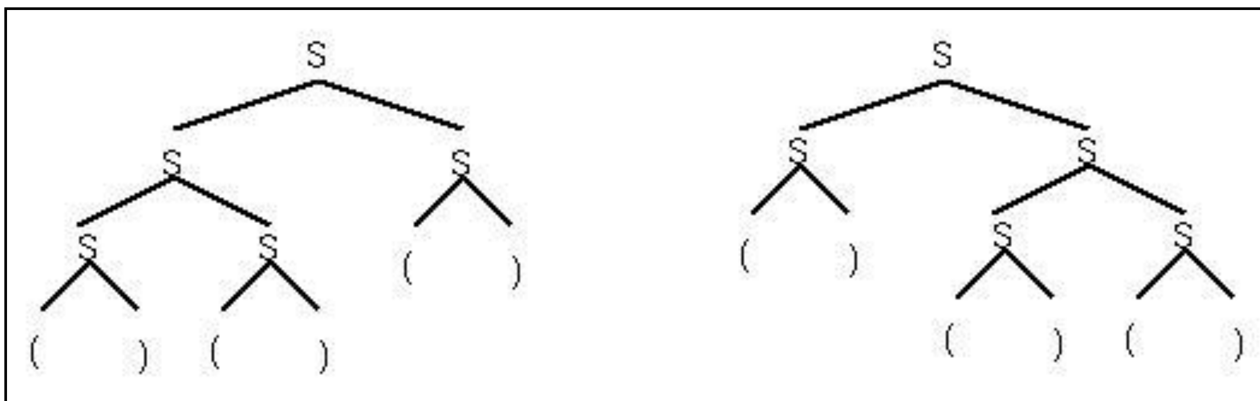
3. $S \rightarrow SS \mid () \mid (S)$

   ?

Fix by forcing leftmost derivi

Ambiguity (cont.)

- Example
  - Grammar: S → SS | () | (S)   String: ()()()
  - 2 distinct (leftmost) derivations (and parse trees)
    - $S \Rightarrow \underline{S}S \Rightarrow \underline{S}SS \Rightarrow ()\underline{S}S \Rightarrow ()()\underline{S} \Rightarrow ()()()$
    - $S \Rightarrow \underline{S}S \Rightarrow ()\underline{S} \Rightarrow ()\underline{S}S \Rightarrow ()()\underline{S} \Rightarrow ()()()$



CFGs for Programming Languages

- Recall that our goal is to describe programming languages with CFGs

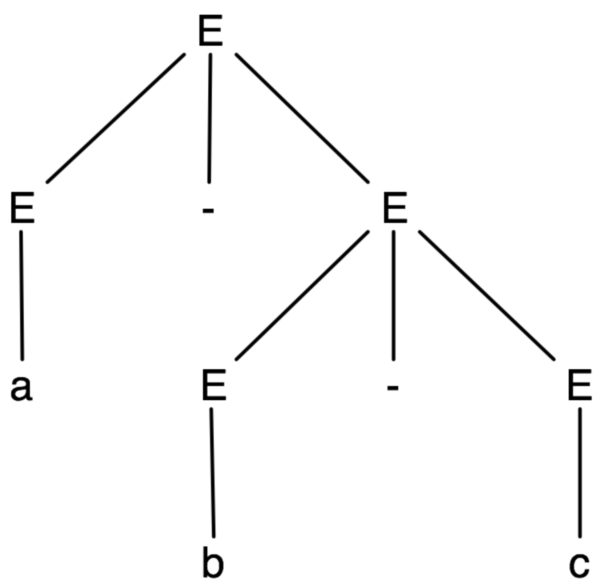
- We had the following example which describes limited arithmetic expressions

  $E \rightarrow a \mid b \mid c \mid E+E \mid E-E \mid E*E \mid (E)$

- What's wrong with using this grammar?
  - It's ambiguous!

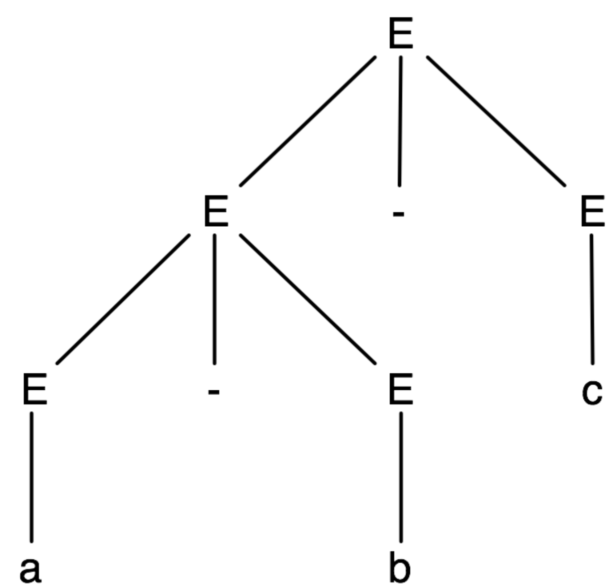
Example: a-b-c

$E \Rightarrow E\text{-}E \Rightarrow a\text{-}E \Rightarrow a\text{-}E\text{-}E \Rightarrow a\text{-}b\text{-}E \Rightarrow a\text{-}b\text{-}c$



Corresponds to a-(b-c)

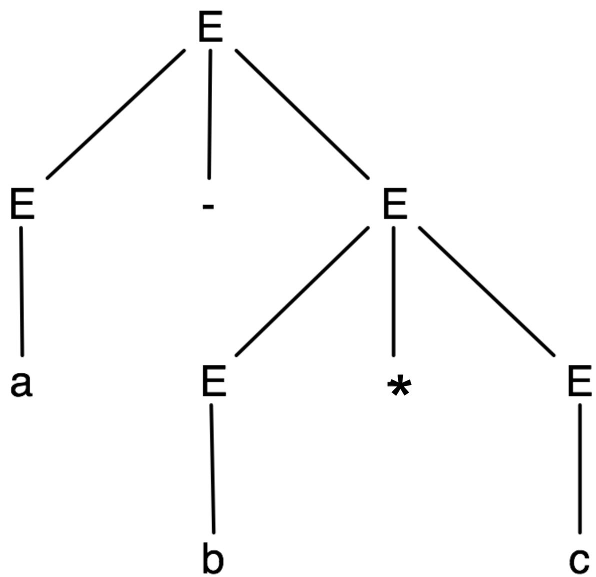
$E \Rightarrow E\text{-}E \Rightarrow E\text{-}E\text{-}E \Rightarrow a\text{-}E\text{-}E \Rightarrow a\text{-}b\text{-}E \Rightarrow a\text{-}b\text{-}c$



Corresponds to (a-b)-c

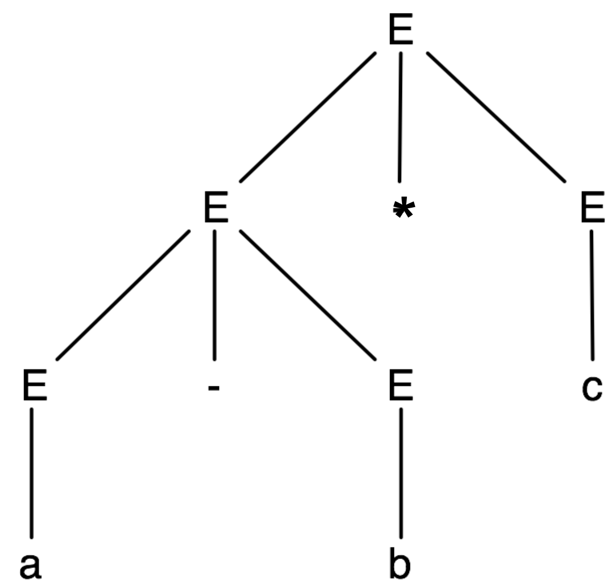
Example: a-b*c

E ⇒ E-E ⇒ a-E ⇒ a-E*E ⇒
a-b*E ⇒ a-b*c



Corresponds to a-(b*c)

E ⇒ E-E ⇒ E-E*E ⇒
a-E*E ⇒ a-b*E ⇒ a-b*c



Corresponds to (a-b)*c

Another Example: If-Then-Else

Aka the dangling else problem

<stmt> → <assignment> | <if-stmt> | ...

<if-stmt> → if (<expr>) <stmt> |

if (<expr>) <stmt> else <stmt>

(Recall < >' s are used to denote nonterminals)

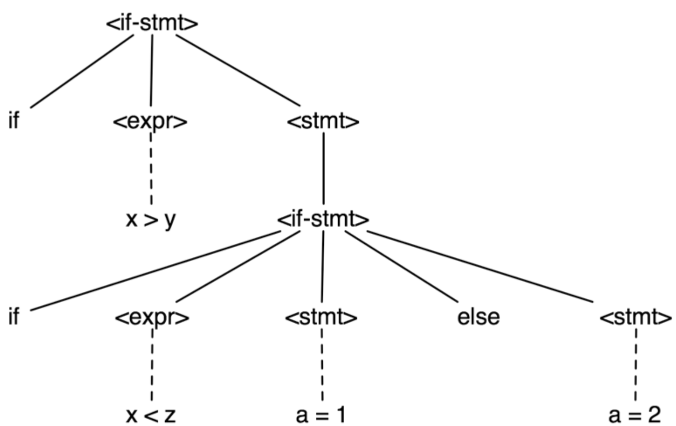
- Consider the following program fragment

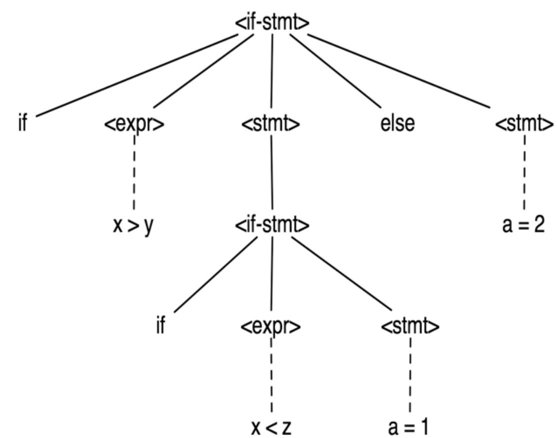
```
if (x > y)
 if (x < z)
  a = 1;
 else a = 2;
```

(Note: Ignore newlines)

Two Parse Trees

```
if (x > y)
   if (x < z)
      a = 1;
   else a = 2;
```





Quiz #5

Which of the following grammars is ambiguous?

A. $S \rightarrow 0SS1 \mid 0S1 \mid \varepsilon$

B. $S \rightarrow A1S1A \mid \varepsilon$

$\quad A \rightarrow 0$

C. $S \rightarrow (S, S, S) \mid 1$

D. None of the above.

Quiz #5

Which of the following grammars is ambiguous?

A. $\boxed{S \rightarrow 0SS1 \mid 0S1 \mid \varepsilon}$

B. $S \rightarrow A1S1A \mid \varepsilon$

   $A \rightarrow 0$

C. $S \rightarrow (S, S, S) \mid 1$

D. None of the above.

$S \rightarrow 0SS1 \rightarrow 0S1 \rightarrow 01$

$S \rightarrow 0S1 \rightarrow 01$

Dealing With Ambiguous Grammars

- Ambiguity is bad
  - Syntax is correct
  - But semantics differ depending on choice
    - Different associativity (a-b)-c vs. a-(b-c)
    - Different precedence (a-b)*c vs. a-(b*c)
    - Different control flow if (if else) vs. if (if) else
- Two approaches
  - Rewrite grammar
    - **Grammars are not unique** – can have multiple grammars for the same language. But result in different parses.
  - Use special parsing rules
    - Depending on parsing tool

Fixing the Expression Grammar

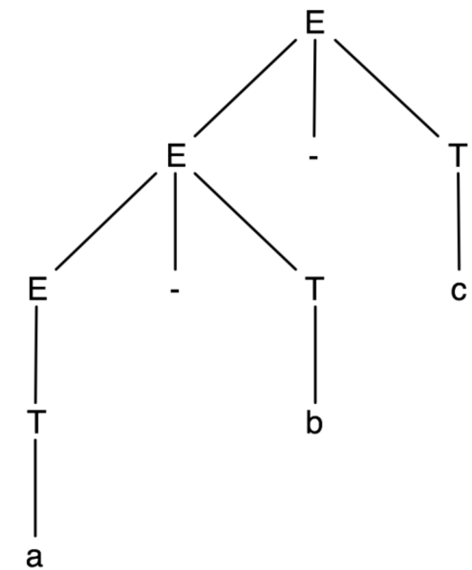
- Require right operand to not be bare expression

  E → E+T | E-T | E*T | T

  T → a | b | c | (E)

- Corresponds to left associativity

- Now only one parse tree for a-b-c
  - Find derivation



What if we want Right Associativity?

- Left-recursive productions
  - Used for left-associative operators
  - Example

    E → E+T | E-T | E*T | T

    T → a | b | c | (E)
- Right-recursive productions
  - Used for right-associative operators
  - Example

    E → T+E | T-E | T*E | T

    T → a | b | c | (E)

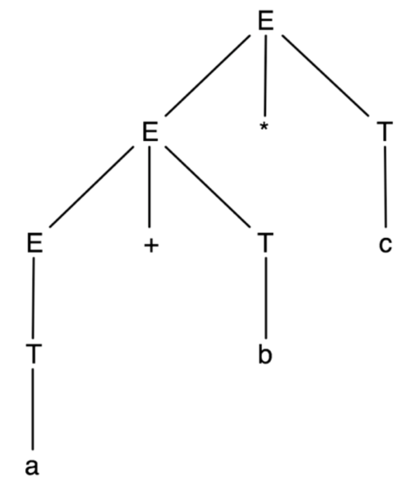
A Different Problem

- How about the string a+b*c ?

  E → E+T | E-T | E*T | T

  T → a | b | c | (E)

  E + T



- Doesn't have correct precedence for *
  - When a nonterminal has productions for several operators, they effectively have the same precedence

- Solution – Introduce new nonterminals

Final Expression Grammar

E → E+T | E-T | T	lowest precedence operators
T → T*P | P	higher precedence
P → a | b | c | (E)	highest precedence (parentheses)

Derivation of a+b*c:

E → E+T→T+T→P+T→a+T→a+T*P→a+P*P→a+b*P→a+b*c

Fixing the Expression Grammar

- Controlling precedence of operators
  - Introduce new nonterminals
  - Precedence increases closer to operands
- Controlling associativity of operators
  - Introduce new nonterminals
  - Assign associativity based on production form
    - $E \rightarrow E+T$ (left associative) vs. $E \rightarrow T+E$ (right associative)
      - But parsing method might limit form of rules

Conclusion

- Context Free Grammars (CFGs) can describe programming language syntax
  - They are a kind of formal language that is more powerful than regular expressions
- CFGs can also be used as the basis for programming language parsers (details later)
  - But the grammar should not be ambiguous
    - May need to change more natural grammar to make it so
  - Parsing often aims to produce abstract syntax trees
    - Data structure that records the key elements of program