Topic-04-maximum-contiguous-sum-Part-3

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MCS - Kadane's Algorithm!

① Intro: This is a dynamic programming approach to the MCS.

② Preliminaries

(A) defn: The MCS ending at i (index) = MCS which goes up to, and must include, index i

ex A = [4, -8, 6, -1, 3, 1, 5, 9]

↳ i=4 MCS ending at i=4 is 6-1+3 = 8

(B) obs: The MCS ending at i is one of:

- Just A[i]
- A[i] + MCS ending at i-1

ex

	0	1	2	3	4	5	
A =	1	-10	5	2	-8	2	...

MCS end at i-1 is -1 ↳ MCS end. at i=5 is 2

A =	1	-10	4	-1	6	2

MCS end at i-1 = 8 ↳ MCS end. at i=5 is 8+2 = 10

to get MCS ending at some i just take

max { A[i], A[i] + MCS ending at i-1 }

(C) obs: MCS ending at i=0 is A[0] oops... (d) below

③ Ex Consider

	0	1	2	3	4	5	6	7
	4	-8	6	-1	3	1	5	9

MCS end at i=0 is 4

MCS end at i=1

MCS end at i=7 max{9, 9+14} = 23

MCS end at i=6

is 4

MCS end at i=1
max{-8, -8+4} = -4

MCS end at i=2
max{6, 6-4} = 6

MCS end at i=3
max{-1, -1+6} = 5

MCS end at i=4
max{3, 3+5} = 8

MCS end at i=5
max{1, 1+8} = 9

MCS end at i=6
max{5, 5+9} = 14

(d) overall MCS = Max. of all MCS ending at all i;
in our ex, it's 23

④ Pseudocode

```
\\ PRE: A is a list of length n.
maxoverall = A[0]
maxendingati = A[0]
for i = 1 to n-1
    maxendingati = max(maxendingati+A[i],A[i])
    maxoverall = max(maxoverall,maxendingati)
end
\\ POST: maxoverall is the maximum contiguous sum.
```

- maxoverall = A[0]: set something
- maxendingati = A[0]: think i=0 so this is MCS end at i=0
- maxendingati = max(maxendingati+A[i],A[i]): find new MCS ending at new i; ⊛ rev. order from above
- maxoverall = max(maxoverall,maxendingati): if better, update overall MCS

⑤ Time Complexity

$T(n) = \Theta(1)$ stuff before loop + $\Theta(1)$ stuff inside loop (n-1 times)

$T(n) = c_1 + c_2(n-1)$ for ex.

$T(n) = \Theta(n)$

this is better!

Summary:

Brute Force	:	$\Theta(n^3)$
DAC	:	$\Theta(n \lg n)$
Kadane's Alg.	:	$\Theta(n)$ ← BEST!

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