CMSC 351 Summer 2025 Homework 11

Due Thursday 21 August 2025 by 23:59 EST on Gradescope.

Directions:

- Homework must be done on printouts of these sheets and then scanned properly, or via Latex, or by downloading, writing on the PDF, and uploading. If you use Latex please do not change the Latex formatting.
- Do not use your own blank paper!
- The reason for this is that Gradescope will be following this template to locate the answers to the problems so if your answers are organized differently they will not be recognized.
- Tagging is automatic, you will not be able to manually tag.

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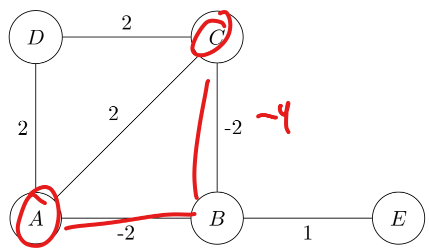
1. Complete the following Sudoku puzzle! [12 pts]

8	9	1	3	7	4	6	5	2
7	2	6	1	5	8	9	4	3
3	4	5	6	2	9	1	7	8
4	7	9	5	8	6	2	3	1
1	6	2	4	3	7	8	9	5
5	3	8	9	1	2	4	6	7
9	1	3	8	6	5	7	2	4
6	8	7	2	4	3	5	1	9
2	5	4	7	9	1	3	8	6

2. For each of the following decision problems and instances write YES or NO in each box stating whether the witness is valid or not.

(a) Q: Given a weighted graph, are there two vertices which have the property that there is a walk between them whose total weight is 0 ? [9 pts]

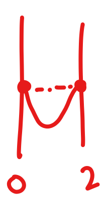
I: This graph:



Witness	Valid?
A and C	NO
A and B	YES
A and E	NO

(b) Q: Given a function, is there an interval on which it is increasing? [9 pts]

I: The function $f(x) = x^3 - 4x$



←

Witness	Valid?
$[-2, -1.9]$	YES
$[0, 2]$	NO
$[2, \infty)$	YES

$(-2)^3 - 4(-2)$

$= -8 + 8 = 0$

$(-1.9)^3 - 4(-1.9)$

3. Consider this decision problem: [20 pts]

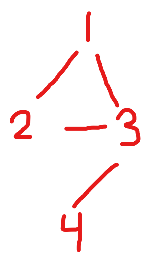
> Q: Given a simple, connected, unweighted, undirected graph with $n \geq 3$ vertices represented by an adjacency matrix A, is there a cycle of length exactly 3?

Explain why $Q \in P$.

Your explanation does not need to be a formal proof but should be clear enough to explain how the DTM runs in polynomial time and under what conditions YES and NO are output.

Solution:

BFS/DFS run in O(V+E) for checking if any cycle exists.
But for this example, in the adjacency matrix A, you can just brute force check every 3-pair of vertices in the adjacency matrix, checking if each edge is distinct, and that it starts/ends with the same vertex.
Checking the actual values would take linear time because you just have to check if they exist, so if they aren't infinity.
The actual amount of 3-pairs is a lot, but you can disregard duplicates because the cycle 1-2-3 == 2-3-1



here, n = 4 (n>=3).
Follow the steps. Check all 3-pairs
All possible (distinct) 3-pairs (I think):
(1-2-3)
(1-3-4)
(2-1-4)
(2-4-3)

This is $\binom{4}{3} = \frac{4\cdot(4-1)(4-2)}{6} = \frac{4\cdot3\cdot2}{6} = 4$

Which makes sense if you think about it because here, we shouldn't check every permutation, just the unique ones, so we choose distinct 3-pairs from the set n.
This is literally n choose 3 $\binom{n}{3}$

$$= \frac{n(n-1)(n-2)}{6} = \frac{(n^2-n)(n-2)}{6} = \frac{n^3-2n^2-2n+3n}{6} = \frac{\boxed{n^3}-2n^2+n}{6}$$

$$= \frac{n^3}{\cancel{6}} \text{ checks}$$

$= n^3$ (in time complexity because we can drop the coefficient of 1/6)

$= O(n^3) = O(n^k)$,

so $Q \in P$ ✓

YES will output when there are 3 distinct vertices and there exists an edge between all 3 of them (all edges are non-infinity)
NO will output if there is not, or all other cases I suppose

4. Suppose k is a positive integer. The Weight-Constrained Spanning Tree Problem asks:

> Q: Given a simple, connected, weighted, and undirected graph (represented as an adjacency matrix) and an integer $k \geq 0$, is it possible to find a spanning tree with total weight less than or equal to k?

For a graph I and a spanning tree x we wish to define a verifier $V(I, x)$ which satisfies the verifier definition of NP.

(a) What definition of $V(I, x)$ works? [5 pts]

Solution:

(b) Justify why $V(I, x)$ runs in polynomial time as a function of the number of vertices. [5 pts]

Solution:

(c) Show that $V(I, x)$ satisfies the verifier definition. [10 pts]

Solution:

5. The Nonempty Subset Target Sum Problem asks:

> Given a set S of integers and a target k, find and return a nonempty subset of S which adds up to k. If none exists, return NONE.

(a) Just for a warm-up, given $S = \{-1, 3, 7, 2\}$ and $k = 4$, what is a result of the Nonempty Subset Target Sum Problem? [5 pts]

Solution:

{-1,3,2}

(b) Just for a warm-up, given $S = \{-1, 3, 7, 2\}$ and $k = 100$, what is a result of the Nonempty Subset Target Sum Problem? [5 pts]

Solution:

NONE

(c) Suppose we have an oracle which works as follows: We give it a set of integers, an element x in the set, and a target k, it tells us whether or not there is a nonempty subset of the set which includes x and adds up to k. [20 pts]

Given a set S of integers, explain why the Nonempty Subset Target Sum Problem is polynomially reducible to the oracle.

You can treat the set as a list for anything algorithmic.

Solution: