

MCS - Kadane's Algorithm!

① Intro: This is a dynamic programming approach to the MCS.

② Preliminaries

(A) defn: The MCS ending at i ^(index) = MCS which goes up to, and must include, index i

ex $A = [4, -8, 6, -1, 3, 1, 5, 9]$
 $\hookrightarrow i=4$ MCS ending at $i=4$ is $6-1+3=8$

(B) obs: The MCS ending at i is one of:

- Just $A[i]$
- $A[i] + \text{MCS ending at } i-1$

ex

$A = [1, -10, 5, 2, -8, 2, \dots]$

MCS end at $i-1$ is -1 \uparrow MCS end. at $i=5$ is 2

$A = [1, -10, 4, -1, 6, 2]$

MCS end at $i-1$ is 8 \hookrightarrow MCS end. at $i=5$ is $8+2=10$

to get MCS ending at some i just take
 $\max \{ A[i], A[i] + \text{MCS ending at } i-1 \}$

(C) obs: MCS ending at $i=0$ is $A[0]$

oops... (d) below

③ ex

Consider

$A = [4, -8, 6, -1, 3, 1, 5, 9]$

MCS end at $i=0$ is 4

MCS end at $i=1$

\uparrow

\uparrow

\uparrow

\uparrow

\uparrow

\uparrow

\uparrow

\uparrow

\uparrow

\uparrow

\uparrow

\uparrow

\uparrow

\uparrow

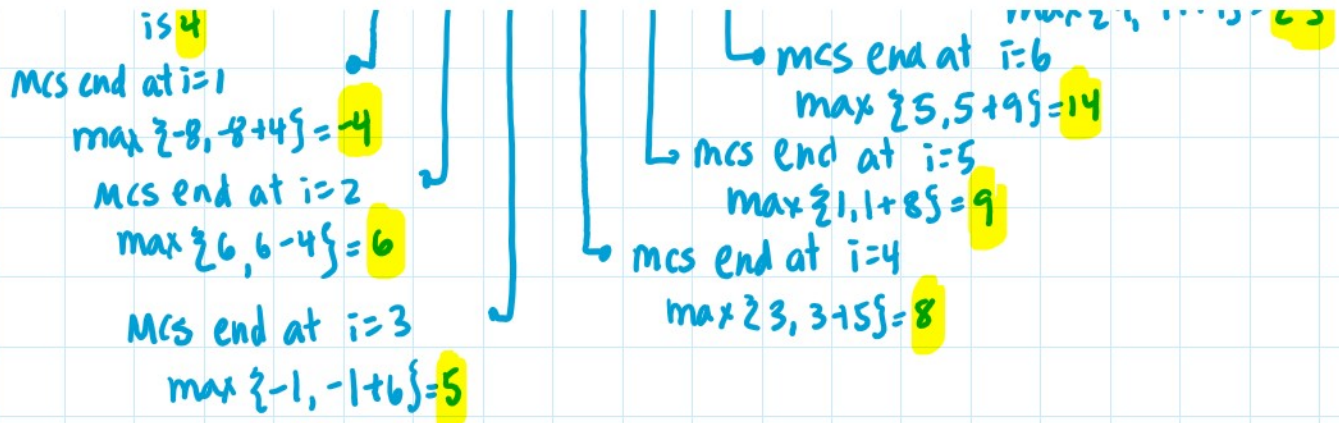
\uparrow

\uparrow

\uparrow

MCS end at $i=7$ $\max \{ 9, 9+14 \} = 23$

MCS end at $i=6$ $\max \{ 5, 5+14 \} = 19$



(d) Overall MCS = Max. of all MCS ending at all i ;
 In our ex, it's 23

④ Pseudocode

```

\\ PRE: A is a list of length n.
maxoverall = A[0]
maxendingati = A[0]
for i = 1 to n-1
    maxendingati = max(maxendingati+A[i], A[i])
    maxoverall = max(maxoverall, maxendingati)
end
\\ POST: maxoverall is the maximum contiguous sum.
  
```

set something

think $i=0$ so this is MCS end at $i=0$

find new MCS ending at new i

rev. order from above

if better, update overall MCS

⑤ Time Complexity

$T(n) = \theta(1)$ stuff before loop + $\theta(1)$ stuff inside loop $n-1$ times

$$T(n) = c_1 + c_2(n-1) \quad \text{for ex.}$$

$$T(n) = \theta(n)$$

this is better!

Summary:	Brute Force :	$\theta(n^3)$
	DAC :	$\theta(n \lg n)$
	Kadane's Alg.	$\theta(n)$ \rightarrow BEST!

DAC : $\Theta(n \lg n)$
Kadane's Alg: $\Theta(n)$ \rightarrow BEST!