Topic-04-maximum-contiguous-sum-Part-1

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MAXIMUM CONTIGUOUS SUM

① INTRO!

Sps we have a list of length n containing numbers

ex $A = [-3, 5, 1, -2, 8, -10, 1]$

If we look at all contiguous sums, what is the max?

ex For A, here are some CS:

$-3 + 5 + 1 = 3$

$5 + 1 - 2 + 8 = 12$

$-10 = -10$

not a CS:

$5 - 2 + 8$ (misses $+1$)

must use at least one elt.

Q: What's the max?

ex in ours, it's $5 + 1 - 2 + 8 = 12$ ← cannot get more!

② Application!

If list contains daily inc/dec in stock price, MCS = max contig. increase

③ Why are we looking at it?!

mostly b/c it introduces us to several (three!) different alg. approaches which we'll use a lot.

- Brute Force
- Divide and Conquer
- Dynamic Programming (saw in coin-changing!)

④ BRUTE FORCE!

Try EVERY possible contiguous sum!
Here's the pseudocode:

to start w/ something

```
\\ PRE: A is a list of length n.
max = A[0]                          c_1            ← Θ(1)
for i = 0 to n-1                    n times
    sum = 0                         c_1            ← Θ(1)
    for j = i to n-1                n - 1 - i + 1 times
        sum = sum + A[j]            c_1            ← Θ(1)
        if sum > max           ⎫
            max = sum          ⎬    c_2            ← Θ(1)
        end                    ⎭
    end
end
\\ POST: max is the maximum sum.
```

if larger, inc. overall MCS

for each i, this checks all MCS starting at index i

i, j iterate through all possible index pairs w/ $i \leq j$

How much time does this take?

$$T(n) = c_1 + \sum_{i=0}^{n-1}\left[c_1 + \sum_{j=i}^{n-1}(c_1+c_2)\right]$$

note: Each for loop creates a sum Σ.

SUMS TO KNOW!

$$\sum_{i=1}^{n} 1 = n \qquad ①$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad ②$$

now we need to simplify!

$$T(n) = c_1 + \sum_{i=0}^{n-1}\left[c_1 + \sum_{j=i}^{n-1}(c_1+c_2)\right]$$

var. on ①

$$= c_1 + \sum_{i=0}^{n-1}\left[c_1 + (c_1+c_2)(n-1-i+1)\right]$$

$$= c_1 + \sum_{i=0}^{n-1}\left[c_1 + (c_1+c_2)n - (c_1+c_2)i\right]$$

$$= c_1 + (c_1+(c_1+c_2)n)\sum_{i=0}^{n-1} 1 - (c_1+c_2)\sum_{i=0}^{n-1} i$$

① var on ②

$$= c_1 + (c_1+(c_1+c_2)n)(n) - (c_1+c_2)\frac{(n-1)(n)}{2}$$

$= \Theta(n^2)$ b/c n^2 is the highest power of n in

END $T(n) = \theta(n^2)$ → Loose Meaning → as n (list length) inc linearly the time req. inc. quadratically

⑤ Can we do better... faster?!