

# Discussion 1

## Discussion 1 - Thursday, June 12th

### Reminders

1. Quiz 1 next **Tuesday, June 17th**
  1. Quizzes will open **at 9:30** and you will have **25 minutes** to complete it.
  2. Topics will be announced soon on Piazza.
2. Project 2 released! Due **Wednesday, June 18th**
3. If you need to make up the quiz in some way, or you need an ADS accommodation, please **email your instructor** with valid documentation

### Exercises

#### OCaml Lists and Patern Matching



1. Write the following functions in OCaml **using recursion**:

`remove_all lst x`

- **Type:** `'a list -> 'a -> 'a list`
- **Description:** Takes in a list `lst` and returns the list `lst` without any instances of the element `x` in the same order.

```
remove_all [1;2;3;1] 1 = [2;3]
remove_all [1;2;3;1] 5 = [1;2;3;1]
remove_all [true; false; false] false = [true]
remove_all [] 42 = []
```

#### ▼ Solution

```
let rec remove_all lst x = match lst with
| [] -> []
| h::t -> if x = h then (remove_all t x) else h::(remove_all t x);;
```

`index_of lst x`

- **Type:** `'a list -> 'a -> int`
- **Description:** Takes in a list `lst` and returns the index of the first instance of element `x`.
- **Notes:**
  - If the element doesn't exist, you should return `-1`
  - You can write a helper function!

```

index_of [1;2;3;1] 1 = 0
index_of [4;2;3;1] 1 = 3
index_of [true; false; false] false = 1
index_of [] 42 = -1

```

### ▼ Solution

```

let index_of lst x =
  let rec helper lst x i = match lst with
  | [] -> -1
  | h::t -> if x = h then i else (helper t x (i + 1))
  in helper lst x 0;;

```

## 2. Give the type for each of the following OCaml expressions:

**NOTE:** Feel free to skip around, there are a lot of examples! 🤖

```

[2a] fun a b -> b < a
[2b] fun a b -> b + a > b - a
[2c] fun a b c -> (int_of_string c) * (b + a)
[2d] fun a b c -> (if c then a else a) * (b + a)
[2e] fun a b c -> [ a + b; if c then a else a + b ]
[2f] fun a b c -> if a b != a c then (a b) else (c < 2.0)
[2g] fun a b c d -> if a && b < c then d + 1 else b

```



### ▼ Solution

```

[2a] 'a -> 'a -> bool
[2b] int -> int -> bool
[2c] int -> int -> string -> int
[2d] int -> int -> bool -> int
[2e] int -> int -> bool -> int list
[2f] (float -> bool) -> float -> float -> bool
[2g] bool -> int -> int -> int -> int

```

## 3. Write an OCaml expression for each of the following types:

```

[3a] int * bool list
[3b] (int * float) -> int -> float -> bool list
[3c] float -> string -> int * bool

```

```
[3d] (int -> bool) -> int -> bool list
[3e] ('a -> 'b) -> 'a -> 'a * 'b list
[3f] ('a -> 'b) -> ('b -> 'c) -> 'a -> 'c
[3g] 'a -> 'b list -> 'a -> 'a * 'a
```

### ▼ Solution

```
[3a] (1, [true])
(* NOTE: same thing as `int * (bool list)` *)

[3b] fun (a, b) c d -> [a + 1 = c; b +. 1.0 = d]

[3c] fun a b -> (int_of_float a, b = "a")

[3d] fun f a -> [f a; a = 2]

[3e] fun f a -> (a, [f a])

[3f] fun f g a -> g (f a)

[3g] fun a b c -> if (a = c && b = []) then (a,a) else (c,c)
```



### 4. Give the type of the following OCaml function:

```
let rec f p x y =
  match x, y with
  | ([], []) -> []
  | ((a,b)::t1, c::t2) -> (p a c, p b c)::(f p t1 t2)
  | (_, _) -> failwith "error";;
```

### ▼ Solution

```
('a -> 'b -> 'c) -> ('a * 'a) list -> 'b list -> ('c * 'c) list
```

### 5. What values do **x**, **y**, and **z** bind to in the following code snippet?

```
let x = match ("lol", 7) with
  | ("haha", 5) -> "one"
  | ("lol", _) -> "two"
  | ("lol", 7) -> "three"
;;

let y = match (2, true) with
  | (1, _) -> "one"
  | (2, false) -> "two"
  | (_, _) -> "three"
  | (_, true) -> "four"
;;

let z = match [1;2;4] with
  | [] -> "one"
  | 2::_ -> "two"
```

```

| 1::2::t    -> "three"
| _          -> "four"
;;

```

### ▼ Solution

```

x: two
y: three
z: three

```

## Higher Order Functions (Map & Fold)

Consider the following higher order functions:

```

let rec map f xs =
  match xs with
  | [] -> []
  | h::t -> (f h)::(map f t)

let rec fold f a lst =
  match lst with
  | [] -> a
  | h::t -> fold f (f a h) t

let rec fold_right f lst a =
  match lst with
  | [] -> a
  | h::t -> f h (fold_right f t a)

```



### Map vs Fold

Both `map` and `fold` are higher-order functions, but are used in different scenarios.

`map`

`map` is a structure-preserving operation, meaning that it applies a function to each element of a list and returns a new list of the same structure but with the new values.

`fold`

`fold` processes the structure from either the left (`fold_left`) or the right (`fold_right`), and uses an accumulator to combine the elements through a given function, ultimately reducing the structure to a single value. Note that `fold_left` is **tail-recursive**, but `fold_right` is not.

Write the following functions using either `fold`, `fold_right`, and / or `map`:

`list_square nums`

- **Type:** `int list -> int list`
- **Description:** Given a list of integers `nums`, return a list where each value is squared.
- **Examples:**

```
list_square [1; 2; 3; 4] = [1; 4; 9; 16]
list_square [0; 5; 6] = [0; 25; 36]
list_square [] = []
list_square [-3; -2; -1] = [9; 4; 1]
```

### ▼ Solution

```
let list_square nums = map (fun x -> x * x) nums
```

swap\_tuples tuples

- **Type:** `(int * int) list -> (int * int) list`
- **Description:** Given a list of two element tuples, swap the first and second elements of each tuple.
- **Examples:**

```
swap_tuples [(1, 2); (3, 4)] = [(2, 1); (4, 3)]
swap_tuples [(5, 10); (7, 8)] = [(10, 5); (8, 7)]
swap_tuples [(0, 0)] = [(0, 0)]
```

### ▼ Solution

```
let swap_tuples tups = map (fun (a,b) -> (b,a)) tups
```



list\_product nums

- **Type:** `int list -> int`
- **Description:** Given a list of `nums`, return the product of all elements in the list.
- **Examples:**

```
list_product [2; 5] = 10
list_product [3; 0; 2] = 0
list_product [] = 1
```

### ▼ Solution

```
let list_product nums = fold (fun acc elem -> acc * elem) 1 nums
```

list\_add x nums

- **Type:** `int -> int list -> int list`
- **Description:** Given a number `x` and a list of integers `nums`, return `nums` with all of its values incremented by `x`.
- **Examples:**

```
list_add 1 [1;2;3;4] = [2;3;4;5]
list_add 3 [1;2;3;4] = [4;5;6;7]
list_add 1 [] = []
list_add (-3) [7;10] = [4;7]
```

### ▼ Solution!

```
let list_add x nums = map (fun num -> num + x) nums
let list_add x nums = map ((+) x) nums (* sillier version *)
```

`mold f lst`

- **Type:** `('a -> 'b) -> 'a list -> 'b list`
- **Description:** Rewrite the `map` function using `fold`
- **Examples:**

```
mold (fun x -> x = 3) [1;2;3;4] = [false;false;true;false]
mold (fun x -> x - 1) [1;2;3;4] = [0;1;2;3]
mold (fun x -> 0) [1;2;3;4] = [0;0;0;0]
mold (string_of_int) [1;2;3;4] = ["1";"2";"3";"4"]
```

- **Addendum:** What happens if we use `fold_right` instead of `fold`? How does this affect the order of iteration?



### ▼ Solution!

```
let mold f lst = List.rev (fold (fun a x -> (f x)::a) [] lst)
let mold f lst = fold (fun a x -> a @ [(f x)]) [] lst

(* Notice how we don't have to reverse the list! *)
let mold f lst = fold_right (fun x a -> (f x)::a) lst []
```

If we append to the accumulator using `(f x) :: a`, we are adding elements to the front of the list. Since `fold` processes the list from left to right, the output list will be made in reverse order. However, `fold_right` processes the list from right to left, which preserves the original order without needing to reverse it at the end. The order of iteration matters here!

`list_sum_product lst`

- **Type:** `int list -> int * int * bool`
- **Description:** Write a function that takes in an `int list` and returns an `int * int * bool` tuple of the following form:
  - The first element is the **sum** of the **even** indexed elements
  - The second element is the **product** of the **odd** indexed elements.
  - The third element is a boolean that will be **true** if the sum and the product are equal, otherwise **false**.
- **Note:** The list is 0 indexed, and 0 is an even index.
- **Examples:**

```
list_sum_product [] = (0,1,false)
```

```
list_sum_product [1;2;3;4] = (4,8,false)
list_sum_product [1;5;4;1] = (5,5,true)
list_sum_product [1;-2;-3;4] = (-2,-8,false)
```

### ▼ Solution!

```
let list_sum_product lst =
  let (sum, product, index) = fold
    (fun (even, odd, i) num ->
      if i mod 2 = 0
      then (even + num, odd, i + 1)
      else (even, odd * num, i + 1))
    (0, 1, 0) lst
  in (sum, product, sum = product);;
```

## Records

Consider the following custom record type, which is similar to the return tuple of `list_sum_product`:

```
type results = {
  sum_even: int;
  product_odd: int;
  num_elements: int;
}
```

`record_sum_product lst`



- **Type:** `int list -> results`
- **Description:** Similar to the `list_sum_product` function above, but returns a `results` record with the following fields:
  - `sum_even` is the **sum** of the **even indexed** elements
  - `product_odd` is the **product** of the **odd indexed** elements.
  - `num_elements` is the number of elements in `lst`
- **Note:** The list is 0 indexed, and 0 is an even index.
- **Examples:**

```
record_sum_product [] = {sum_even = 0; product_odd = 1; num_elements = 0}
record_sum_product [1;2;3;4] = {sum_even = 4; product_odd = 8; num_elements = 4}
record_sum_product [1;5;4;1] = {sum_even = 5; product_odd = 5; num_elements = 4}
record_sum_product [1;-2;-3;4] = {sum_even = -2; product_odd = -8; num_elements = 4}
```

### ▼ Solution!

```
let record_sum_product lst =
  fold (fun {sum_even; product_odd; num_elements} num ->
    if num_elements mod 2 = 0
    then {
      sum_even = sum_even + num;
      product_odd;
      num_elements = num_elements + 1 }
    else {
      sum_even;
      product_odd = product_odd * num;
      num_elements = num_elements + 1 })
  {sum_even = 0; product_odd = 1; num_elements = 0} lst;;
```

Another exercise! Consider the following custom record types:

```
type weather_data = {
  temperature: float;
  precipitation: float;
  wind_speed: int;
}

type cp_weather_report = {
  days: weather_data list;
  num_of_days: float;
}
```

average\_temperature report



- **Type:** cp\_weather\_report -> float
- **Description:** This function takes a cp\_weather\_report record, containing a list of weather\_data records from College Park and returns the average temperature of College Park.
- **Note:** If the num\_of\_days within cp\_weather\_report is 0 then return 0.0
- **Examples:**

```
let ex1 = {
  days = [
    { temperature = 70.0; precipitation = 0.2; wind_speed = 10 };
    { temperature = 68.0; precipitation = 0.1; wind_speed = 12 };
    { temperature = 72.0; precipitation = 0.0; wind_speed = 8 };
    { temperature = 75.0; precipitation = 0.3; wind_speed = 15 }
  ];
  num_of_days = 4.0
}
average_temperature ex1 = 71.25

let ex2 = {
  days = [];
  num_of_days = 0.0
}
average_temperature ex2 = 0.0

let ex3 = {
  days = [
    { temperature = 30.0; precipitation = 0.0; wind_speed = 3 };
    { temperature = 35.0; precipitation = 0.0; wind_speed = 4 }
  ];
  num_of_days = 2.0
}
average_temperature ex3 = 32.5
```



## ▼ Solution!

```
let average_temperature reports =
  if reports.num_of_days = 0.0
  then 0.0
  else
    let total_temp =
      List.fold_left (fun sum day -> sum +. day.temperature) 0.0 reports.days
    in total_temp /. reports.num_of_days
```

## Variant Types

Let's build a custom binary `tree` data type in OCaml! First, we will define the `tree` type:

```
type 'a tree =
  | Leaf
  | Node of 'a tree * 'a * 'a tree
```

This recursively defines a `tree` to either be a:

- `Leaf`
- `Node` with a left sub-`tree`, a value, and a right sub-`tree`

`tree_add x tree`

- **Type:** `int -> int tree -> int tree`
- **Description:** Given an `int tree`, return a new `int tree` with the same values in the old tree incremented by `x`.
- **Examples:**

```
let tree_a = Node(Node(Leaf, 5, Leaf), 6, Leaf)
let tree_b = Node(Node(Leaf, 4, Leaf), 5, Node(Leaf, 2, Leaf))

tree_add 1 tree_a = Node(Node(Leaf, 6, Leaf), 7, Leaf)
tree_add 5 tree_b = Node(Node(Leaf, 9, Leaf), 10, Node(Leaf, 7, Leaf))
```

## ▼ Solution!

```
let rec tree_add x tree = match tree with
  | Leaf -> Leaf
  | Node(l, v, r) -> Node(tree_add x l, v + x, tree_add x r)
```

`tree_preorder tree`

- **Type:** `string tree -> string`
- **Description:** Given a `string tree`, return the preorder concatenation of all the strings in the tree.
- **Examples:**



```
let tree_c = Node(Node(Leaf, " World", Leaf), "Hello", Node(Leaf, "!", Leaf))
let tree_d = Node(Node(Node(Leaf, " super", Leaf), " is", Node(Leaf, " easy!", Leaf)), "Recursion"
, Node(Leaf, " 🐼", Leaf))

tree_preorder tree_c = "Hello World!"
tree_preorder tree_d = "Recursion is super easy! 🐼"
```

### ▼ Solution!

```
let rec tree_add x tree = match tree with
| Leaf -> Leaf
| Node(l, v, r) -> Node(tree_add x l, v + x, tree_add x r)
```

```
tree_sum_product tree
```

- **Type:** `int tree -> int * int`
- **Description:** Given an `int tree`, return an `int * int` tuple of the following form:
  - The first element is the **sum** of **all** numbers in the tree
  - The second element is the **product** of **all** numbers in the tree
- **Examples:**

```
let tree_a = Node(Node(Leaf, 5, Leaf), 6, Leaf)
let tree_b = Node(Node(Leaf, 4, Leaf), 5, Node(Leaf, 2, Leaf))

tree_sum_product tree_a = (11, 30)
tree_sum_product tree_b = (11, 40)
```



### ▼ Solution!

```
let rec tree_sum_product tree =
  match tree with
  | Leaf -> (0, 1)
  | Node(l, v, r) ->
    let (lsum, lproduct) = tree_sum_product l in
    let (rsum, rproduct) = tree_sum_product r in
    (lsum + v + rsum, lproduct * v * rproduct)
```

## Review - Imperative OCaml

```
# let z = 3;;
   val z : int = 3
# let x = ref z;;
   val x : int ref = {contents = 3}
# let y = x;;
   val y : int ref = {contents = 3}
```

Here, `z` is bound to 3. It is immutable. `x` and `y` are bound to a reference. The `contents` of the reference is mutable.

```
x := 4;;
```

will update the `contents` to 4. `x` and `y` now point to the value 4.

```
!y;;
- : int = 4
```

Here, variables `y` and `x` are aliases. In `let y = x`, variable `x` evaluates to a location, and `y` is bound to the same location. So, changing the contents of that location will cause both `!x` and `!y` to change.

## Exercises

### Imperative OCaml Counter

**Recall:** The `unit` type means that no input parameters are required for the function.

Implement a counter called `counter`, which keeps track of a value starting at 0. Then, write a function called `next: unit -> int`, which returns a new integer every time it is called.

#### Example

```
(* First call of next () *)
# next ();;
: int = 1

(* First call of next () *)
# next ();;
: int = 2
```



#### Solution:

##### ▼ Click here!

```
# let counter = ref 0 ;;
val counter : int ref = { contents=0 }

# let next =
  fun () -> counter := !counter + 1; !counter ;;
val next : unit -> int = <fun>
```

### Function argument evaluation order

What happens when we run this code?

```
let x = ref 0;;
let f _ r = r;;
f (x:=2) (!x)
```

#### Solution:

##### ▼ Click here!

OCaml's order of argument evaluation is not defined. On some systems it's left to right on others it's right to left.

On my system, `f` evaluates to `0`, but on your system it may evaluate to `2`!

## Resources & Additional Readings

- [Spring 2023 OCaml HOF discussion](https://github.com/cm330-umd/spring23/tree/main/discussions/d4_hof) ↗ (https://github.com/cm330-umd/spring23/tree/main/discussions/d4\_hof)
- [Spring 2023 Project Review](https://github.com/cm330-umd/spring23/tree/main/discussions/d5_project_review) ↗ (https://github.com/cm330-umd/spring23/tree/main/discussions/d5\_project\_review)
- [Fall 2023 OCaml HOF discussion](https://github.com/cm330fall23/cm330fall23/tree/main/discussions/d6_ocaml_hof) ↗ (https://github.com/cm330fall23/cm330fall23/tree/main/discussions/d6\_ocaml\_hof)
- [Fall 2023 Python HOF + Regex discussion](https://github.com/cm330fall23/cm330fall23/tree/main/discussions/d2_hof_regex) ↗ (https://github.com/cm330fall23/cm330fall23/tree/main/discussions/d2\_hof\_regex)
- [Anwar's Imperative OCaml Slides](https://bakalian.cs.umd.edu/assets/slides/10-imperative.pdf) ↗ (https://bakalian.cs.umd.edu/assets/slides/10-imperative.pdf)

