CMSC351 Spring 2025 (§0101,§0201,§0301) Homework 8

Due Thursday Apr 17, 2025 by 23:59 on Gradescope.

Directions:

- Homework must be done on printouts of these sheets and then scanned properly, or via latex, or by downloading, writing on the PDF, and uploading.
- Do not use your own blank paper!
- The reason for this is that gradescope will be following this template to locate the answers to the problems so if your answers are organized differently they will not be recognized.
- Tagging is automatic, do not manually tag.

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1. When applying our implementation of Karatsuba to each of the following A and B what would A_1, A_0, B_1, and B_0 be in each case? [10 pts]

A	B	A_1	A_0	B_1	B_0
54	19				
1268	2019				
831	196393				
8311	212				
4387342	1870023420				

2. Consider the product (35127)(1246). In what follows, SDM means single-digit multiplication.

   (a) Using schoolbook multiplication how many SDMs will be performed? [5 pts]

Schoolbook SDM count =	

   (b) Draw the corresponding Karatsuba tree and identify the number of SDMs which will be performed. [15 pts]

   Solution:

3. For asymptotic complexity we've only been using one variable but it's fine to have more than one variable inside Θ, $\mathcal{O}$, and Ω. The expression carries over to the definition and each variable gets its own bound. So if V and E are variables then we could have, for example:

$$f(V,E) = \mathcal{O}(V+E) \text{ if } \exists V_0, E_0, C > 0 \text{ such that } \forall V \geq V_0, \forall E \geq E_0\,,\ f(V,E) \leq C(V+E)$$

Note: These problems take about two lines each, you don't need much space!

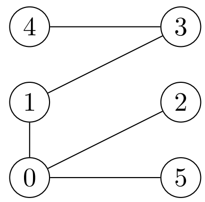
(a) Prove from the definition that $V + 2E = \mathcal{O}(V+E)$. [10 pts]

Solution:

(b) Prove from the definition that $VE + E + V = \mathcal{O}(VE)$. [10 pts]

Solution:

4. Given the following graph:



(a) Fill in the adjacency matrix for this graph below: [6 pts]

(b) Write down the adjacency list for this graph. Please put each sublist in increasing order to help us grade more easily: [4 pts]

5. Justin thinks that if a simple graph has V vertices and E edges then $V \leq 2E$. His proof goes like this: [10 pts]

   "Each edge requires the existence of two vertices, suggesting $V = 2E$, but vertices are allowed to be shared by edges, meaning we might have fewer vertices."

   What is wrong with his proof?

   Solution:

6. If a simple graph has V vertices and E edges, explain why $E \leq V^2$. [10 pts]

   Solution:

7. Suppose an unweighted graph with 5 vertices (indexed 0 to 4) has the following adjacency matrix. The values of x, y, and z are unknown, they are either 0 or 1. [10 pts]

$$\begin{bmatrix} 0 & 1 & x & 0 & z \\ 1 & 0 & 1 & 0 & 0 \\ x & 1 & 0 & 1 & y \\ 0 & 0 & 1 & 0 & 1 \\ z & 0 & y & 1 & 0 \end{bmatrix}$$

Suppose the shortest path algorithm is run on this graph starting at $source = 0$ and ending at $target = 4$. The length of that shortest path was found to be 3. We want to know the possible triples (x, y, z).

What are the possible triples (x, y, z)?

8. Suppose the shortest path algorithm is run a graph with six vertices labeled 0 to 5, starting at the vertex 0. No target is specified so it runs until the queue is empty. The progression of the queue is shown here: [10 pts]

Starting Queue	0
Then	3,4
Then	4,2,5
Then	2,5
Then	5,1
Then	1
Then	Empty

What will the predecessor list look like?

Index	0	1	2	3	4	5
Entry	NULL					