Discussion 3

Discussion 3 - Thursday, June 26th

Reminders

1. Project 3 due **Wednesday, July 2nd**
2. Your midterm is **Thursday, July 3rd** instead of discussion. The format will be the same as quizzes (i.e. on Gradescope, during Discussion time), but extend the entire allotted time rather than 25 minutes. Logistics post soon.

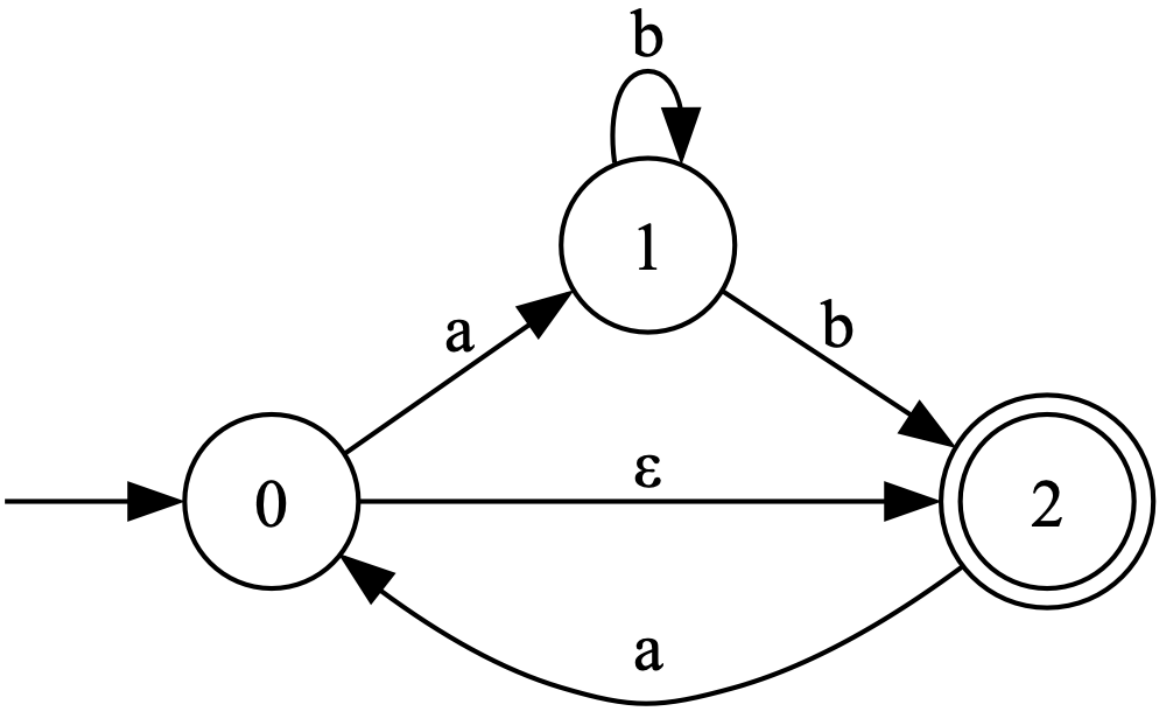
NFA/DFA Continued

Exercises

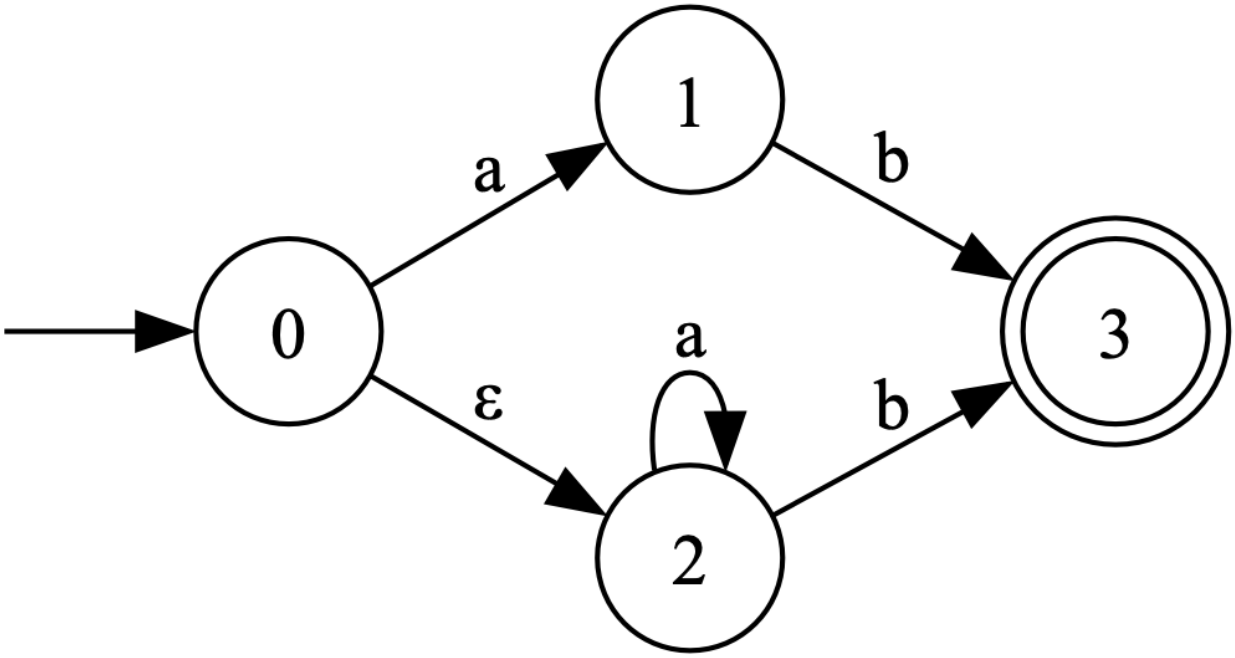
NFA -> DFA

1. Trace through the NFA -> DFA conversion algorithm using the table method for the following NFAs:

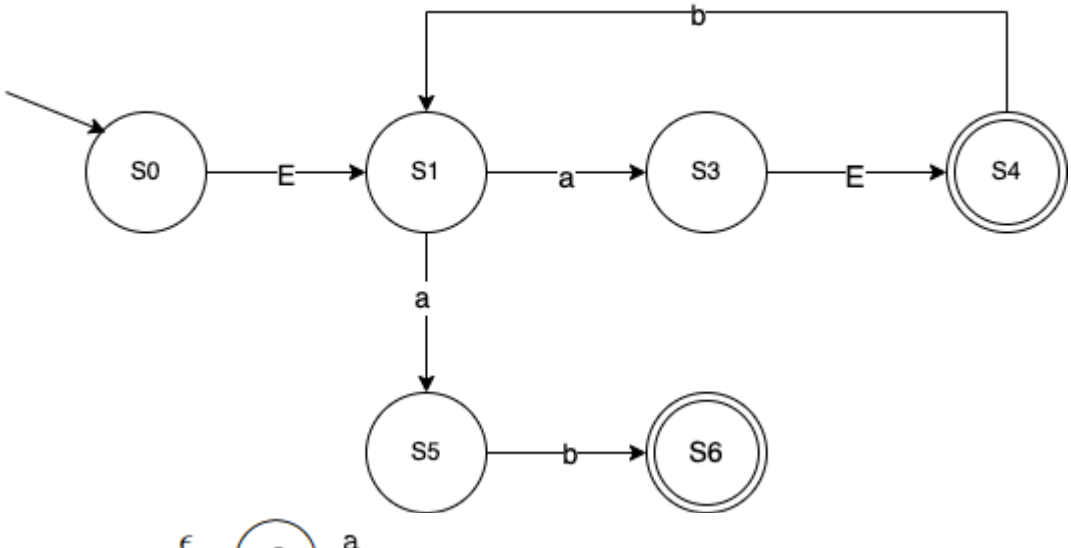
a)



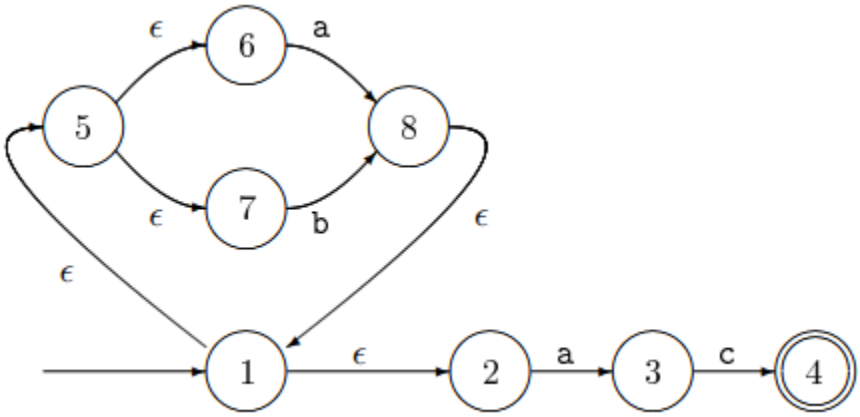
b)



c)

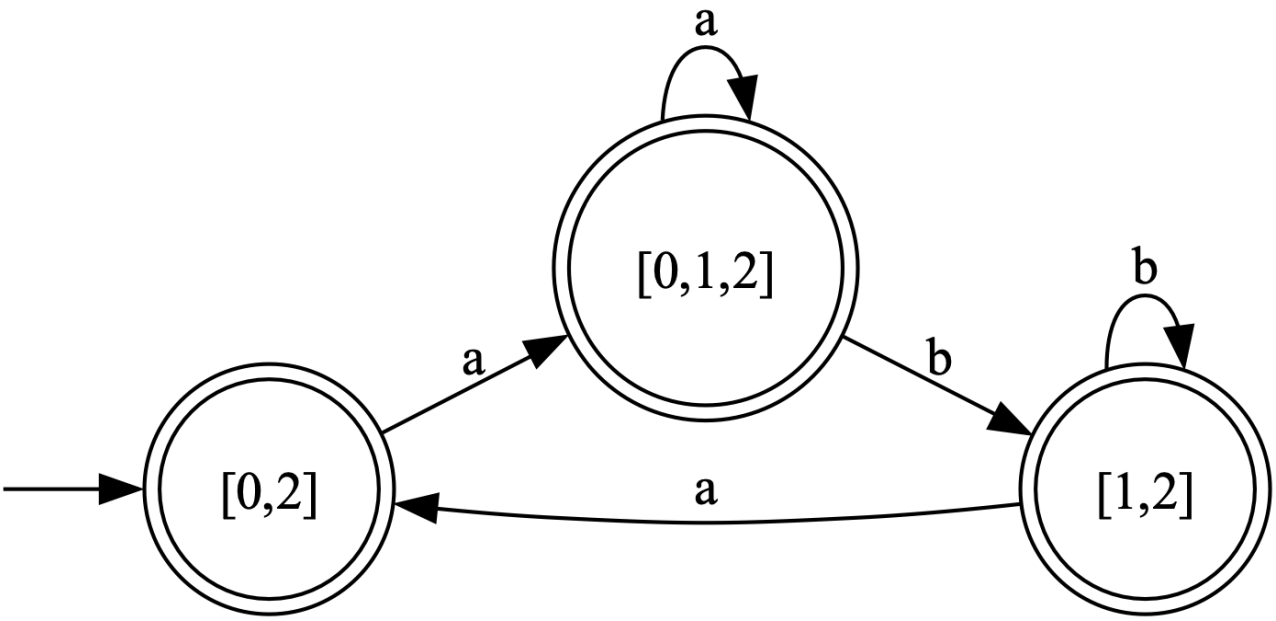


d)

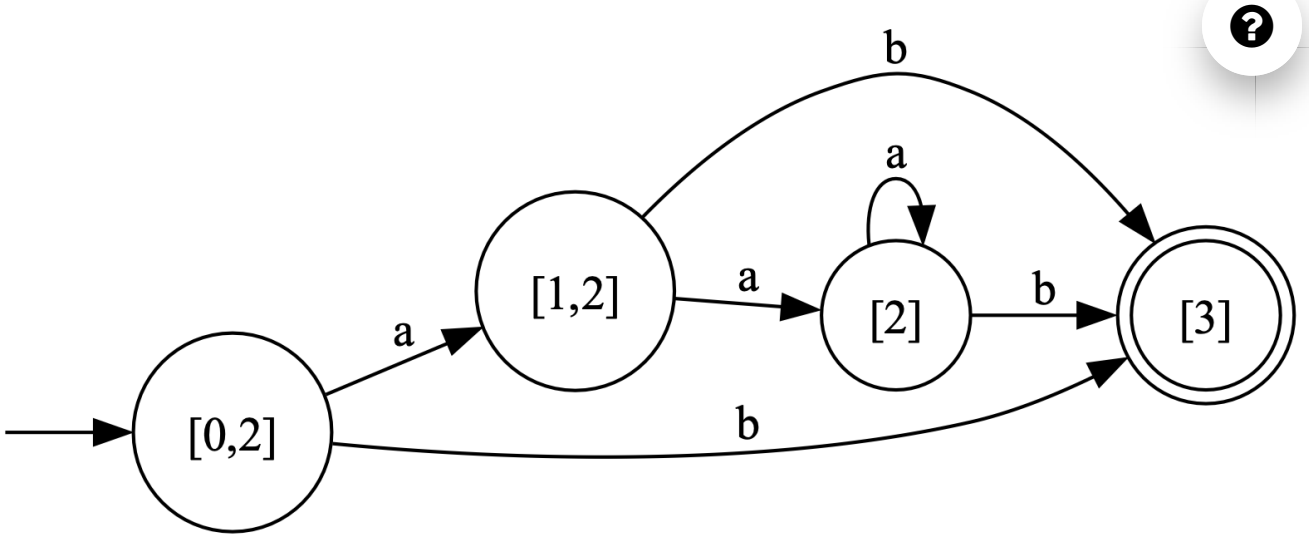


▼ Solutions!

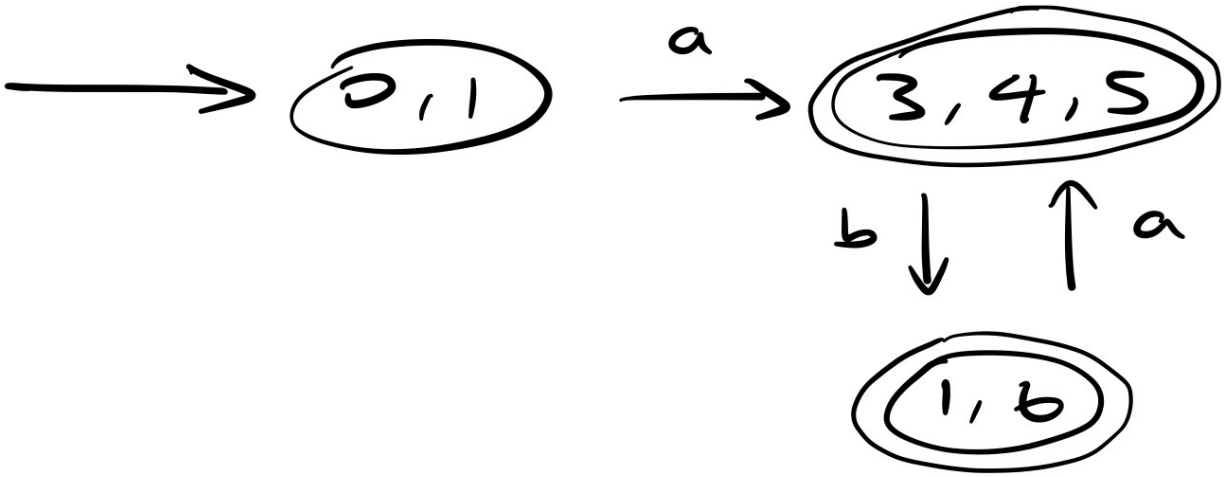
a)



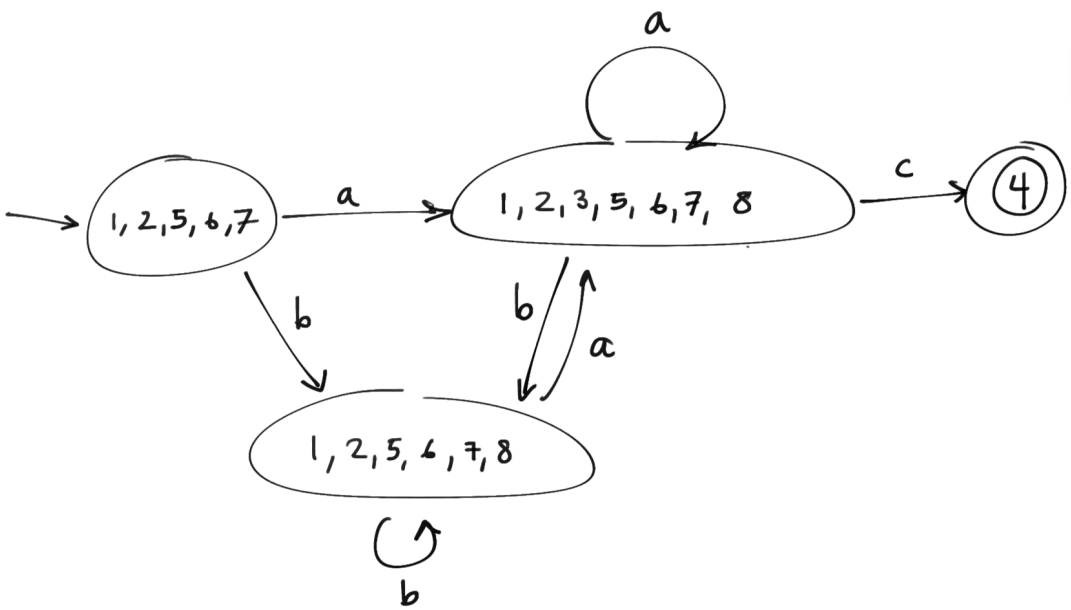
b)



c)



d)



Context Free Grammars

Exercises:

1. Consider the following grammar:

```
S -> S + S | 1 | 2 | 3
```

   - Write a leftmost derivation for the string: `1 + 2 + 3`

- Start with S and use the production rules on the LEFTMOST nonterminal ONE AT A TIME. (For a rightmost derivation, use the productions on the RIGHTMOST nonterminal.)
- ONE NONTERMINAL AT A TIME!!!! DON'T COMBINE STEPS!!!! (or you might lose credit)

- If there are 2 leftmost derivations or 2 rightmost for the same string in a grammar, what does that mean?

▼ Solution

Leftmost Derivation: S -> S + S -> S + S + S -> 1 + S + S -> 1 + 2 + S -> 1 + 2 + 3 OR S -> S + S -> 1 + S -> 1 + S + S -> 1 + 2 + S -> 1 + 2 + 3 There are two leftmost derivations for this string, which means the grammar is **ambiguous**.

1. Consider the following grammar:

```
S -> aS | T
T -> bT | U
U -> cU | ε
```

- Provide derivations for:
  - b
  - ac
  - bbc
- What language is accepted by this grammar?
- Create another grammar that accepts the same language.

▼ Solution

S -> T -> bT -> bU -> b S -> aS -> aT -> aU -> acU -> ac S -> T -> bT -> bbT -> bbU -> bbcU -> bbc This is the language of all strings equivalent to the regex `a*b*c*`. Another grammar that accepts this language is: `S -> Sc | T T -> Tb | U U -> Ua | ε`

1. Consider the expression and construct an equivalent CFG:

$a^xb^xc^y|a^x$ where x >= 0 and y >= 1

▼ Solution

`S -> A | B Union of two languages A -> CD Concatenation of two languages C -> aCb | ε Related number of 0 or more a's and b's D -> cD | c 1 or more c's B -> aB | ε 0 or more a's`

1. Consider the following grammars:

Grammar 1	Grammar 2	Grammar 3
S -> AB	S -> ASB|c	S -> Sc|AB
A -> aA|a	A -> aA|a	A -> aA|a
B -> bbB|ϵ	B -> bbB|ϵ	B -> bbB|ϵ

- Which grammar accepts both `"aaabb"` and `"aaabbcc"`?
- Which grammar is ambiguous?

▼ Solution

Grammar 3 accepts both `"aaabb"` and `"aaabbcc"`. Grammar 2 is ambiguous. For example, the string `"aac"` can be made in two ways.

1. Construct a CFG that generates strings for each of the following:

   - a^xb^y, where y = 2x.
   - a^xb^y, where y >= 3x.

   Can we represent strings of the form a^xb^xc^x, where x >= 0, with CFGs?

▼ Solution

1. `S -> aSbb | ε` 2. `S -> aSbbbB | B B -> bB | ε` We cannot accept strings of the form $a^xb^xc^x$, as there is no way for a CFG to have "memory" of multiple parts of the string, keeping the number of a's and c's equal.