CMSC 351 Summer 2025 Homework 6

Due Monday 4 August 2025 by 11:59pm on Gradescope.

Directions:

- Homework must be done on printouts of these sheets and then scanned properly, or via Latex, or by downloading, writing on the PDF, and uploading. If you use Latex please do not change the Latex formatting.
- Do not use your own blank paper!
- The reason for this is that Gradescope will be following this template to locate the answers to the problems so if your answers are organized differently they will not be recognized.
- Tagging is automatic, you will not be able to manually tag.

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1. Suppose we are using Counting Sort and we have the following non-cumulative `POS`:

`i`	0	1	2	3
Non-Cumulative `POS[i]`	1	2	3	1

   (a) How many elements are in the original list that we are sorting? [4 pts]

Number of elements:	

   (b) At the end, what is the sorted list? You will not need all the blanks below so only fill in as far as you need! [6 pts]

`i`	0	1	2	3	4	5	6	7	8	9
Sorted `A[i]`										

2. We are using Counting Sort to sort the following list of numbers: [6 pts]

`i`	0	1	2	3	4	5	6
Original `A[i]`	?	?	3	0	1	?	?

Fill in the blank entries in the following:

`i`	0	1	2	3
Non-Cumulative `POS[i]`			3	
Cumulative `POS[i]`		2		

3. Suppose we are using Counting Sort and we have the following cumulative `POS`:
Note: all the elements are integers.

`i`	0	1	2	3	4
Cumulative `POS[i]`	0	3	x	5	7

What value(s) could x be? [5 pts]

List all the possible values for x	

4. Suppose in the cumulative `POS` we have `POS[i]==POS[i+1]` for some `i`. Could the list contain both `i` and `i+1`? Explain briefly. [9 pts]

Solution:

5. Suppose we run Radix Sort on the following list where the digit `x` is unknown and the process runs as follows: [6 pts]

Start	`9x2`	`x62`	`7x6`	`152`	`2x4`
After Iteration 1	`9x2`	`x62`	`152`	`2x4`	`7x6`
After Iteration 2	`9x2`	`2x4`	`7x6`	`152`	`x62`
After Iteration 3	`152`	`2x4`	`x62`	`7x6`	`9x2`

What are the possible digit values for `x`?

Possible Values for `x`	

Scratch work; scratch work is not graded:

6. Your friend claims they ran Radix Sort on the following list where the digit `x` is unknown and the first two iterations ran as follows: [4 pts]

Start	`863`	`911`	`341`	`1x8`	`42x`
After Iteration 1	`911`	`341`	`42x`	`863`	`1x8`
After Iteration 2	`1x8`	`911`	`42x`	`341`	`863`

Why is this not possible?

Solution:

7. Suppose Radix Sort is used to sort the following list of strings. Show the state of the list after each iteration of the underlying sort. [10 pts]

Start	DIG	DIE	BID	DAD	BAD
After Iteration 1					
After Iteration 2					
After Iteration 3					

8. Suppose $b = 2$ is a fixed base and j is a fixed positive integer. Suppose that our lists of length n contain integers between 0 and n^j inclusive. What is the time complexity of using Radix Sort with underlying Counting Sort? Show work. [10 pts]

Solution:

9. Suppose A is an n-digit number and B is a k-digit number where k is a fixed constant. What is the Θ asymptotic time complexity of calculating AB via schoolbook multiplication? Explain. [5 pts]

Solution:

10. We are using Karatsuba's algorithm to calculate (2539)(12046). In what follows, SDM means single-digit multiplication.

(a) Using schoolbook multiplication how many SDMs will be performed? [5 pts]

Schoolbook SDM count =	

(b) Draw the corresponding tree and identify the number of SDMs which will be performed. [15 pts]

Solution:

11. Suppose we have two n-digit numbers A and B. Suppose an integer k divides n and suppose we break each of A and B into k blocks of n/k digits each as follows:

$$A = A_{k-1}...A_3A_2A_1A_0$$
$$B = B_{k-1}...B_3B_2B_1B_0$$

In the above, each A_i and each B_j is a block of n/k digits.

If we used some non-Karatsuba-like method we would need k^2 multiplications (each A_i times each B_j). Suppose then we reduced the number of multiplications to m by some Karatsuba-like approach.

(a) For which integers m would the Θ time complexity be faster than Karatsuba? Explain. [5 pts]
Solution:

(b) For which integers m would the Θ time complexity be slower than Karatsuba? Explain. [5 pts]
Solution:

(c) For which integers m would the Θ time complexity be the same as Karatsuba? Explain. [5 pts]
Solution: