

$$H = \begin{bmatrix} h_0 & h_1 & h_2 & h_3 & h_4 \\ h_1 & h_2 & h_3 & h_4 & h_5 \\ h_2 & h_3 & h_4 & h_5 & h_6 \\ h_3 & h_4 & h_5 & h_6 & h_7 \\ h_4 & h_5 & h_6 & h_7 & h_8 \end{bmatrix}$$

1 Short story

$$h \begin{bmatrix} 0 & 1 & 2 \\ 0 & j & i-1 \end{bmatrix} + h \begin{bmatrix} 0 & 1 & i \\ 0 & 1 & j \end{bmatrix} = h \begin{bmatrix} 0 & 1 & i-1 \\ 0 & 1 & j+1 \end{bmatrix} \quad (1)$$

$$h \begin{bmatrix} 0 & 1 & 3 \\ 0 & j & i-1 \end{bmatrix} + h \begin{bmatrix} 0 & 1 & i \\ 0 & 2 & j \end{bmatrix} = h \begin{bmatrix} 0 & 2 & i-1 \\ 0 & 1 & j+1 \end{bmatrix} \quad (2)$$

$$h \begin{bmatrix} 0 & 1 & l+1 \\ 0 & j & i-1 \end{bmatrix} + h \begin{bmatrix} 0 & 1 & i \\ 0 & l & j \end{bmatrix} = h \begin{bmatrix} 0 & l & i-1 \\ 0 & 1 & j+1 \end{bmatrix} \quad (3)$$

$$h \begin{bmatrix} k & 1 & l+1 \\ 0 & j & i-1 \end{bmatrix} + h \begin{bmatrix} k & 1 & i \\ 0 & l & j \end{bmatrix} = h \begin{bmatrix} 0 & l & i-1 \\ k & 1 & j+1 \end{bmatrix} \quad (4)$$

$$h \begin{bmatrix} 0 & 1 & 2 & 3 \\ 0 & 1 & j & i-1 \end{bmatrix} + h \begin{bmatrix} 0 & 1 & 2 & i \\ 0 & 1 & 2 & j \end{bmatrix} = h \begin{bmatrix} 0 & 1 & 2 & i-1 \\ 0 & 1 & 2 & j+1 \end{bmatrix} \quad (5)$$

$$h \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & j & i-1 \end{bmatrix} + h \begin{bmatrix} 0 & 1 & 2 & 3 & i \\ 0 & 1 & 2 & 3 & j \end{bmatrix} = h \begin{bmatrix} 0 & 1 & 2 & 3 & i-1 \\ 0 & 1 & 2 & 3 & j+1 \end{bmatrix} \quad (6)$$

$$h \begin{bmatrix} i+t & j \\ l & m \end{bmatrix} + h \begin{bmatrix} i & j+t \\ l & m \end{bmatrix} = h \begin{bmatrix} i & j \\ l+t & m \end{bmatrix} + h \begin{bmatrix} i & j \\ l & m+t \end{bmatrix} \quad (7)$$

$$h \begin{bmatrix} i+t & j & k \\ l & m & n \end{bmatrix} + h \begin{bmatrix} i & j+t & k \\ l & m & n \end{bmatrix} + h \begin{bmatrix} i & j & k+t \\ l & m & n \end{bmatrix} = h \begin{bmatrix} i & j & k \\ l+t & m & n \end{bmatrix} + h \begin{bmatrix} i & j & k \\ l & m+t & n \end{bmatrix} + h \begin{bmatrix} i & j & k \\ l & m & n+t \end{bmatrix} \quad (8)$$

$$\begin{aligned} & h \begin{bmatrix} i+t & j & k & l \\ m & n & p & q \end{bmatrix} + h \begin{bmatrix} i & j+t & k & l \\ m & n & p & q \end{bmatrix} + h \begin{bmatrix} i & j & k+t & l \\ m & n & p & q \end{bmatrix} + h \begin{bmatrix} i & j & k & l+t \\ m & n & p & q \end{bmatrix} \\ & = h \begin{bmatrix} i & j & k & l \\ m+t & n & p & q \end{bmatrix} + h \begin{bmatrix} i & j & k & l \\ m & n+t & p & q \end{bmatrix} + h \begin{bmatrix} i & j & k & l \\ m & n & p+t & q \end{bmatrix} + h \begin{bmatrix} i & j & k & l \\ m & n & p & q+t \end{bmatrix} \end{aligned} \quad (9)$$

2 Long story

$$\det \begin{bmatrix} h_0 & h_j & h_{i-1} \\ h_1 & h_{j+1} & h_i \\ h_2 & h_{j+2} & h_{i+1} \end{bmatrix} + \det \begin{bmatrix} h_0 & h_1 & h_j \\ h_1 & h_2 & h_{j+1} \\ h_i & h_{i+1} & h_{i+j} \end{bmatrix} = \det \begin{bmatrix} h_0 & h_1 & h_{j+1} \\ h_1 & h_2 & h_{j+2} \\ h_{i-1} & h_i & h_{i+j} \end{bmatrix} \quad (10)$$

$$\det \begin{bmatrix} h_0 & h_j & h_{i-1} \\ h_1 & h_{j+1} & h_i \\ h_3 & h_{j+3} & h_{i+2} \end{bmatrix} + \det \begin{bmatrix} h_0 & h_2 & h_j \\ h_1 & h_3 & h_{j+1} \\ h_i & h_{i+2} & h_{i+j} \end{bmatrix} = \det \begin{bmatrix} h_0 & h_1 & h_{j+1} \\ h_2 & h_3 & h_{j+3} \\ h_{i-1} & h_i & h_{i+j} \end{bmatrix} \quad (11)$$

$$\det \begin{bmatrix} h_0 & h_j & h_{i-1} \\ h_1 & h_{j+1} & h_i \\ h_{l+1} & h_{j+l+1} & h_{i+l} \end{bmatrix} + \det \begin{bmatrix} h_0 & h_l & h_j \\ h_1 & h_{l+1} & h_{j+1} \\ h_i & h_{i+l} & h_{i+j} \end{bmatrix} = \det \begin{bmatrix} h_0 & h_1 & h_{j+1} \\ h_l & h_{l+1} & h_{j+l+1} \\ h_{i-1} & h_i & h_{i+j} \end{bmatrix} \quad (12)$$

$$\det \begin{bmatrix} h_k & h_{j+k} & h_{i+k-1} \\ h_1 & h_{j+1} & h_i \\ h_{l+1} & h_{j+l+1} & h_{i+l} \end{bmatrix} + \det \begin{bmatrix} h_k & h_{k+l} & h_{j+k} \\ h_1 & h_{l+1} & h_{j+1} \\ h_i & h_{i+l} & h_{i+j} \end{bmatrix} = \det \begin{bmatrix} h_k & h_1 & h_{j+1} \\ h_{k+l} & h_{l+1} & h_{j+l+1} \\ h_{i+k-1} & h_i & h_{i+j} \end{bmatrix} \quad (13)$$

$$\det \begin{bmatrix} h_0 & h_1 & h_j & h_{i-1} \\ h_1 & h_2 & h_{j+1} & h_i \\ h_2 & h_3 & h_{j+2} & h_{i+1} \\ h_3 & h_4 & h_{j+3} & h_{i+2} \end{bmatrix} + \det \begin{bmatrix} h_0 & h_1 & h_2 & h_j \\ h_1 & h_2 & h_3 & h_{j+1} \\ h_2 & h_3 & h_4 & h_{j+2} \\ h_i & h_{i+1} & h_{i+2} & h_{i+j} \end{bmatrix} = \det \begin{bmatrix} h_0 & h_1 & h_2 & h_{j+1} \\ h_1 & h_2 & h_3 & h_{j+2} \\ h_2 & h_3 & h_4 & h_{j+3} \\ h_{i-1} & h_i & h_{i+1} & h_{i+j} \end{bmatrix} \quad (14)$$

$$\det \begin{bmatrix} h_0 & h_1 & h_2 & h_j & h_{i-1} \\ h_1 & h_2 & h_3 & h_{j+1} & h_i \\ h_2 & h_3 & h_4 & h_{j+2} & h_{i+1} \\ h_3 & h_4 & h_5 & h_{j+3} & h_{i+2} \\ h_4 & h_5 & h_6 & h_{j+4} & h_{i+3} \end{bmatrix} + \det \begin{bmatrix} h_0 & h_1 & h_2 & h_3 & h_j \\ h_1 & h_2 & h_3 & h_4 & h_{j+1} \\ h_2 & h_3 & h_4 & h_5 & h_{j+2} \\ h_3 & h_4 & h_5 & h_6 & h_{j+3} \\ h_i & h_{i+1} & h_{i+2} & h_{i+3} & h_{i+j} \end{bmatrix} = \det \begin{bmatrix} h_0 & h_1 & h_2 & h_3 & h_{j+1} \\ h_1 & h_2 & h_3 & h_4 & h_{j+2} \\ h_2 & h_3 & h_4 & h_5 & h_{j+3} \\ h_3 & h_4 & h_5 & h_6 & h_{j+4} \\ h_{i-1} & h_i & h_{i+1} & h_{i+2} & h_{i+j} \end{bmatrix} \quad (15)$$

$$\det \begin{bmatrix} h_{i+l+t} & h_{i+m+t} \\ h_{j+l} & h_{j+m} \end{bmatrix} + \det \begin{bmatrix} h_{i+l} & h_{i+m} \\ h_{j+l+t} & h_{j+m+t} \end{bmatrix} = \det \begin{bmatrix} h_{i+l+t} & h_{i+m} \\ h_{j+l+t} & h_{j+m} \end{bmatrix} + \det \begin{bmatrix} h_{i+l} & h_{i+m+t} \\ h_{j+l} & h_{j+m+t} \end{bmatrix} \quad (16)$$

$$\begin{aligned} & \det \begin{bmatrix} h_{i+l+t} & h_{i+m+t} & h_{i+n+t} \\ h_{j+l} & h_{j+m} & h_{j+n} \\ h_{k+l} & h_{k+m} & h_{k+n} \end{bmatrix} + \det \begin{bmatrix} h_{i+l} & h_{i+m} & h_{i+n} \\ h_{j+l+t} & h_{j+m+t} & h_{j+n+t} \\ h_{k+l} & h_{k+m} & h_{k+n} \end{bmatrix} + \det \begin{bmatrix} h_{i+l} & h_{i+m} & h_{i+n} \\ h_{j+l} & h_{j+m} & h_{j+n} \\ h_{k+l+t} & h_{k+m+t} & h_{k+n+t} \end{bmatrix} \\ &= \det \begin{bmatrix} h_{i+l+t} & h_{i+m} & h_{i+n} \\ h_{j+l+t} & h_{j+m} & h_{j+n} \\ h_{k+l+t} & h_{k+m} & h_{k+n} \end{bmatrix} + \det \begin{bmatrix} h_{i+l} & h_{i+m+t} & h_{i+n} \\ h_{j+l} & h_{j+m+t} & h_{j+n} \\ h_{k+l} & h_{k+m+t} & h_{k+n} \end{bmatrix} + \det \begin{bmatrix} h_{i+l} & h_{i+m} & h_{i+n+t} \\ h_{j+l} & h_{j+m} & h_{j+n+t} \\ h_{k+l} & h_{k+m} & h_{k+n+t} \end{bmatrix} \end{aligned} \quad (17)$$

$$\begin{aligned} & \det \begin{bmatrix} h_{i+m+t} & h_{i+n+t} & h_{i+p+t} & h_{i+q+t} \\ h_{j+m} & h_{j+n} & h_{j+p} & h_{j+q} \\ h_{k+m} & h_{k+n} & h_{k+p} & h_{k+q} \\ h_{l+m} & h_{l+n} & h_{l+p} & h_{l+q} \end{bmatrix} + \det \begin{bmatrix} h_{i+m} & h_{i+n} & h_{i+p} & h_{i+q} \\ h_{j+m+t} & h_{j+n+t} & h_{j+p+t} & h_{j+q+t} \\ h_{k+m} & h_{k+n} & h_{k+p} & h_{k+q} \\ h_{l+m} & h_{l+n} & h_{l+p} & h_{l+q} \end{bmatrix} \\ &+ \det \begin{bmatrix} h_{i+m} & h_{i+n} & h_{i+p} & h_{i+q} \\ h_{j+m} & h_{j+n} & h_{j+p} & h_{j+q} \\ h_{k+m+t} & h_{k+n+t} & h_{k+p+t} & h_{k+q+t} \\ h_{l+m} & h_{l+n} & h_{l+p} & h_{l+q} \end{bmatrix} \\ &+ \det \begin{bmatrix} h_{i+m} & h_{i+n} & h_{i+p} & h_{i+q} \\ h_{j+m} & h_{j+n} & h_{j+p} & h_{j+q} \\ h_{k+m} & h_{k+n} & h_{k+p} & h_{k+q} \\ h_{l+m+t} & h_{l+n+t} & h_{l+p+t} & h_{l+q+t} \end{bmatrix} = \det \begin{bmatrix} h_{i+m+t} & h_{i+n} & h_{i+p} & h_{i+q} \\ h_{j+m+t} & h_{j+n} & h_{j+p} & h_{j+q} \\ h_{k+m+t} & h_{k+n} & h_{k+p} & h_{k+q} \\ h_{l+m+t} & h_{l+n} & h_{l+p} & h_{l+q} \end{bmatrix} \\ &+ \det \begin{bmatrix} h_{i+m} & h_{i+n+t} & h_{i+p} & h_{i+q} \\ h_{j+m} & h_{j+n+t} & h_{j+p} & h_{j+q} \\ h_{k+m} & h_{k+n+t} & h_{k+p} & h_{k+q} \\ h_{l+m} & h_{l+n+t} & h_{l+p} & h_{l+q} \end{bmatrix} + \det \begin{bmatrix} h_{i+m} & h_{i+n} & h_{i+p+t} & h_{i+q} \\ h_{j+m} & h_{j+n} & h_{j+p+t} & h_{j+q} \\ h_{k+m} & h_{k+n} & h_{k+p+t} & h_{k+q} \\ h_{l+m} & h_{l+n} & h_{l+p+t} & h_{l+q} \end{bmatrix} \\ &+ \det \begin{bmatrix} h_{i+m} & h_{i+n} & h_{i+p} & h_{i+q+t} \\ h_{j+m} & h_{j+n} & h_{j+p} & h_{j+q+t} \\ h_{k+m} & h_{k+n} & h_{k+p} & h_{k+q+t} \\ h_{l+m} & h_{l+n} & h_{l+p} & h_{l+q+t} \end{bmatrix} \end{aligned} \quad (18)$$