P_2 – EDO – COMPUTAÇÃO

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QUESTÕES

ok! ~~1)~~ ENCONTRE UMA SOLUÇÃO PARTICULAR PARA

$$y'' - y = 8te^{t} + 2e^{t}$$

2) ENCONTRE UMA SOLUÇÃO GERAL PARA A EQUAÇÃO DIFERENCIAL

variação de parâmetros

$$y'' + 4y = tg\,2t$$

3) ENCONTRE UMA SOLUÇÃO GERAL PARA

$$y''' + y'' + 3y' - 5y = 0$$

ok ~~4)~~ UTILIZANDO TRANSFORMADAS DE LAPLACE, RESOLVA O PVI ABAIXO

$$\begin{cases} y^{(IV)} - y = 0 \\ y(0) = 0 \\ y'(0) = 1 \\ y''(0) = y'''(0) = 0 \end{cases}$$

OBS: – CADA QUESTÃO VALE 7/4 DE PONTOS.
– COLOQUE NOME NA PROVA.

GABARITO - P2

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1) FEITA EM SALA.

2) Solução da homogênea

$$y''+4y=0$$

Eq. característica associada à homogênea

$$r^2+4=0 \quad \therefore \quad r=0\pm 2i$$

$$y_h = e^{0t}\cdot(C_1\cos 2t + C_2 \operatorname{sen} 2t)$$

$$y_h = C_1 \cos 2t + C_2 \operatorname{sen} 2t$$

Pelo método da variação de parâmetros

$y_1 = \cos 2t$

$y_2 = \operatorname{sen} 2t$

$$\begin{cases} v_1' \cos 2t + v_2' \operatorname{sen} 2t = 0 \\ v_1' (\cos 2t)' + v_2' (\operatorname{sen} 2t)' = (\operatorname{tg} 2t)/1 \end{cases}$$

$$\begin{cases} v_1' \cos 2t + v_2' \operatorname{sen} 2t = 0 \\ -2v_1' \operatorname{sen} 2t + 2v_2' \cos 2t = \operatorname{tg} 2t \end{cases}$$

$$\begin{cases} v_1' \cos 2t + v_2' \operatorname{sen} 2t = 0 & \times(+2\operatorname{sen} 2t) \\ -2v_1' \operatorname{sen} 2t + 2v_2' \cos 2t = \operatorname{tg} 2t & \times(\cos 2t) \end{cases}$$

②

$$\begin{cases} +2v_1' \operatorname{sen}2t\cos 2t + 2v_2'(\operatorname{sen}2t)^2 = 0 \\ -2v_1' \operatorname{sen}2t\cos 2t + 2v_2'(\cos 2t)^2 = \operatorname{sen}2t \end{cases}$$

(+)

$$2v_2'((\cos 2t)^2 + (\operatorname{sen}2t)^2) = \operatorname{sen}2t$$

$$2v_2' = \operatorname{sen}2t$$

$$-4v_2' = -2\operatorname{sen}2t$$

Integrando

$$-4v_2 = \cos 2t$$

$$\boxed{v_2 = -\frac{\cos 2t}{4}}$$

Mas $v_1'\cos 2t + v_2'\operatorname{sen}2t = 0$

$$v_1'\cos 2t + \frac{(\operatorname{sen}2t)}{2}\operatorname{sen}2t = 0$$

$$v_1' = -\frac{(\operatorname{sen}2t)^2}{2(\cos 2t)}$$

$$v_1' = -\frac{(1-\cos^2 2t)}{2(\cos 2t)}$$

(3)

$$v_1' = -\frac{1}{2}\sec 2t + \frac{\cos 2t}{2}$$

Integrando

$$v_1(t) = -\frac{1}{2}\int \sec 2t\,dt + \frac{1}{2}\int \cos 2t\,dt$$

$$u = 2t$$
$$du = 2\,dt$$
$$dt = \frac{du}{2}$$

Logo

$$v_1(t) = -\frac{1}{2}\int \sec u\,\frac{du}{2} + \frac{1}{2}\int \cos u\,\frac{du}{2}$$

$$v_1(t) = -\frac{1}{4}\int \sec u\,du + \frac{1}{4}\int \cos u\,du$$

$$v_1(t) = -\frac{1}{4}\left(\ln|\sec u + \operatorname{tg} u|\right) + \frac{1}{4}\operatorname{sen} u$$

$$v_1(t) = -\frac{1}{4}\ln|\sec 2t + \operatorname{tg} 2t| + \frac{1}{4}\operatorname{sen} 2t$$

$$y_p = v_1(t)\,y_1(t) + v_2(t)\,y_2(t)$$

$$y_p = \left(-\frac{1}{4}\ln|\sec 2t + \operatorname{tg} 2t| + \frac{1}{4}\operatorname{sen} 2t\right)\cos 2t + \left(-\frac{\cos 2t}{4}\right)\cdot \operatorname{sen} 2t$$

(4)

$y_p(t) = -\frac{1}{4} \ln|\sec 2t + tg 2t|$

Solução geral

$$\boxed{y_G = C_1 \cos 2t + C_2 \sin 2t - \frac{1}{4} \ln|\sec 2t + tg 2t|}$$

3) $y''' + y'' + 3y' - 5y = 0$

Equação característica

$r^3 + r^2 + 3r - 5 = 0$

Possíveis raízes racionais $\{-1, 1, -5, 5\}$

$(-1)^3 + (-1)^2 + 3(-1) - 5 \neq 0$

$(1)^3 + (-1)^2 + 3.(1) - 5 = 0$ (1ª raiz)

$(-5)^3 + (-5)^2 + 3.(-5) - 5 \neq 0$

$(5)^3 + (-5)^2 + 3.5 - 5 \neq 0$

$$\begin{array}{l|l} r^3 + r^2 + 3r - 5 & r - 1 \\ \cline{2-2} -r^3 + r^2 & r^2 + 2r + 5 \\ \hline \quad 2r^2 + 3r - 5 & \\ \quad -2r^2 + 2r & \\ \hline \quad\quad 5r - 5 & \\ \quad\quad -5r + 5 & \\ \hline \quad\quad\quad 0 & \end{array}$$

Logo, $r^3+r^2+3r-5=(r-1)(r^2+2r+5)$

$r_1=1$ e, para $r^2+2r+5=0$

$$r=\frac{-2\pm\sqrt{4-20}}{2}$$

$$r=\frac{-2\pm 4i}{2}$$

$$r=-1\pm 2i$$

$$r_2=-1+2i\ ,\ r_3=-1-2i$$

$$y_c=C_1e^{1\cdot t}+e^{-t}\cdot(C_2\cos 2t+C_3\,\text{sen}\,2t)$$

$$y_c=C_1e^{t}+C_2e^{-t}\cos 2t+C_3e^{-t}\,\text{sen}\,2t$$

4) $y^{(IV)}-y=0$

Aplicando transformadas de Laplace

$$\mathcal{L}\{y^{(IV)}-y\}=\mathcal{L}\{0\}$$

$$\mathcal{L}\{y^{(IV)}\}-\mathcal{L}\{y\}=0$$

$$s^4Y(s)-s^3y(0)-s^2y'(0)-sy''(0)-y'''(0)-Y(s)=0$$

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$$Y(s).(s^4-1)-s^2=0$$

$$Y(s)=\frac{s^2}{s^4-1}$$

$$Y(s)=\frac{s^2}{(s-1)(s+1)(s^2+1)}$$

$$\frac{s^2}{(s-1)(s+1)(s^2+1)}=\frac{A}{s-1}+\frac{B}{s+1}+\frac{Cs+D}{s^2+1}$$

$$s^2=A(s+1)(s^2+1)+B(s-1)(s^2+1)+(Cs+D)(s^2-1)$$

$$s^2=A(s^3+s+s^2+1)+B(s^3+s-s^2-1)+(Cs^3-Cs+Ds^2-D)$$

$$\begin{cases} A+B+C \quad\quad =0 \\ A-B+ \quad\quad D=1 \\ A+B-C \quad\quad =0 \\ A-B \quad\quad -D=0 \end{cases}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 1 & 1 \\ 1 & 1 & -1 & 0 & 0 \\ 1 & -1 & 0 & -1 & 0 \end{array}\right) \underset{i=2,3,4}{\xrightarrow{L_i \leftarrow L_i - L_1}} \left(\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 0 \\ 0 & -2 & -1 & 1 & 1 \\ 0 & 0 & -2 & 0 & 0 \\ 0 & -2 & -1 & -1 & 0 \end{array}\right)$$

$$L_4 \leftarrow L_4 - L_2$$

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$$\left(\begin{array}{cccc:c} 1 & 1 & 1 & 0 & 0 \\ 0 & -2 & -1 & 1 & 1 \\ 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & -2 & -1 \end{array}\right)$$

$-2D=-1 \therefore D=1/2$

$-2C=0 \therefore C=0$

$-2B-C+D=1 \therefore B=-1/4$

$A+B+C=0 \therefore A=+1/4$

$$Y(s)=+\frac{1/4}{s-1}-\frac{1/4}{s+1}+\frac{1/2}{s^2+1}$$

Aplicando transformada inversa de laplace

$$y(t)=+\frac{1}{4}\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\}-\frac{1}{4}\mathcal{L}^{-1}\left\{\frac{1}{s-(-1)}\right\}$$

$$+\frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{s^2+1^2}\right\}$$

Com auxílio da Tabela 8

$$y(t) = +\frac{1}{4}e^{t} - \frac{1}{4}e^{-t} + \frac{1}{2}\operatorname{sen} t$$

OU

$$y(t) = +\frac{1}{2}\left(\frac{e^{t} - e^{-t}}{2}\right) + \frac{1}{2}\operatorname{sen} t$$

$$y(t) = +\frac{1}{2}\operatorname{senh} t + \frac{1}{2}\operatorname{sen} t$$

$$y(t) = \frac{\operatorname{sen} t + \operatorname{senh} t}{2}$$