P3 – CÁLCULO A VÁRIAS VARIÁVEIS

1) Se $u = e^{a_1x+a_2y+a_3z}$ / $a_1x + a_2y + a_3z = 1$, mostre que $f_{xx} + f_{yy} + f_{zz} = u$.

$$\frac{\partial u}{\partial x} = \frac{\partial(a_1x + a_2y + a_3z)}{\partial x} * e^{a_1x+a_2y+a_3z}$$

$$\frac{\partial u}{\partial x} = a_1 * e^{a_1x+a_2y+a_3z}$$

$$\frac{\partial u^2}{\partial xx} = a_1^2 * e^{a_1x+a_2y+a_3z} \qquad \frac{\partial u^2}{\partial yy} = a_2^2 * e^{a_1x+a_2y+a_3z} \qquad \frac{\partial u^2}{\partial zz} = a_3^2 * e^{a_1x+a_2y+a_3z}$$

Sendo assim:

$$f_{xx} + f_{yy} + f_{zz} = (a_1^2 + a_2^2 + a_3^2) * e^{a_1x+a_2y+a_3z} = u = e^{a_1x+a_2y+a_3z}$$

2) Se $u = x^4y + y^2z^3$ / $x = r * s * e^t$ / $y = r * s * e^{-t}$ / $z = s * r^2 * sen(t)$, encontre u_s, onde $r = 2, s = 1, t = 0$.

$$u(r,s,t) = (r * s * e^t)^4 * (r * s * e^{-t}) + (r * s * e^{-t})^2 * \left(s * r^2 * sen(t)\right)^3$$

$$u(r,s,t) = \left(r^4 * s^4 * e^{4t}\right) * (r * s * e^{-t}) + \left(r^2 * s^2 * e^{-2t}\right) * \left(s^3 * r^6 * sen^3(t)\right)$$

$$u(r,s,t) = \left(r^5 * s^5 * e^{3t}\right) + \left(r^8 * s^5 * e^{-2t} * sen^3(t)\right)$$

$$u(r,s,t) = s^5 * \left(r^5 * e^{3t} + r^8 * e^{-2t} * sen^3(t)\right)$$

$$\frac{\partial u}{\partial s}(r,s,t) = 5 * s^4 \left(r^5 * e^{3t} + r^8 * e^{-2t} * sen^3(t)\right)$$

$$\frac{\partial u}{\partial s}(2,1,0) = 5 * 1^4 \left(2^5 * e^{3*0} + 2^8 * e^{-2*0} * sen^3(0)\right)$$

$$\frac{\partial u}{\partial s}(2,1,0) = 160$$

3) Considere a função $f(x, y) = x2 + y2 - 2y$ no ponto $P_0 = (2,2)$

a) Determine a taxa de variação de f em P_0 na direção do vetor (1,1):

$$\nabla f(x,y) = (f_x, f_y)$$

$$\nabla f(x,y) = (2x, 2y - 2)$$

$$\nabla f(2,2) = (2 * (2), 2 * (2) - 2)$$

$$\nabla f(2,2) = (4,2)$$

$$\nabla f(P_0) * \frac{u}{||u||}$$

$$\nabla f(2,2) * \frac{(1,1)}{|\left|\sqrt{1^2 + 1^2}\right||}$$

$$(4,2) * \frac{(1,1)}{||\sqrt{2}||}$$

$$(4,2) * \frac{(4+2)}{||\sqrt{2}||}$$

$$\frac{6}{\sqrt{2}} = 3\sqrt{2}$$

b) Direção e sentido de (4,2)

$$\sqrt{4^2 + 2^2} = 20$$

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4) Analise a natureza dos pontos críticos da função $f(x, y) = x\,sen(y)$:

$$\frac{\partial f}{\partial x} = sen(y) \qquad \frac{\partial f}{\partial y} = x\cos(y)$$

$$\frac{\partial f^2}{\partial xx} = 0 \qquad \frac{\partial f^2}{\partial yy} = -x\,sen(y)$$

$$\frac{\partial f^2}{\partial xy} = \cos(y)$$

$$\frac{\partial f}{\partial x} = sen(y) = 0 \rightarrow\rightarrow\rightarrow y = k\pi, k \in \mathbb{Z}$$

$$\frac{\partial f}{\partial y} = x\cos(y) = 0 \rightarrow\rightarrow\rightarrow x = 0$$

Os pontos da forma $(0, k\pi)$ são pontos críticos.

$$D(x, y) = f_{xx} * f_{yy} - (f_{xy})^2$$

$$D(x, y) = 0 - (\cos(y))^2$$

$$D(0, k\pi) = -(\cos(k\pi))^2$$

$$D(0, k\pi) = -(\pm 1)^2$$

$$D(0, k\pi) = -1$$

5) Calcule $\iint_D (\cos(x))^2 e^y$ da: $0 \leq x \leq 1 \,/\, 0 \leq y \leq \frac{\pi}{4}$

$$\iint_D (\cos(x))^2 e^y\, da$$

$$\iint_0^{\frac{\pi}{4}} e^y * \int_0^1 (\cos(x))^2 dx$$

$$\left(e^{\frac{\pi}{4}} - 1\right) * \left(\frac{1}{4}(sen(2)) + \frac{1}{2}\right)$$

$$\left(e^{\frac{\pi}{4}} - 1\right) * \left(\frac{sen(2) + 2}{4}\right)$$

6) A energia cinética de um corpo com massa m e velocidade v é $k = \frac{mv^2}{2}$. Mostre que $\frac{\partial k}{\partial m} * \frac{\partial^2 k}{\partial v^2} = k$

$$k = \frac{mv^2}{2}$$

$$\frac{\partial k}{\partial m} = \frac{v^2}{2}$$

$$\frac{\partial k}{\partial v} = \frac{2mv}{2} = mv$$

$$\frac{\partial^2 k}{\partial v^2} = m$$

$$\text{Então: } \frac{v^2}{2} * m = \frac{mv^2}{2} = k$$

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7) Analise a natureza dos pontos críticos da função $f(x,y) = x^4 + y^4 - 4xy + 1$

$$\frac{\partial f}{\partial x} = 4x^3 - 4y = 0 \rightarrow\rightarrow\rightarrow y = x^3$$
$$\frac{\partial f}{\partial y} = 4y^3 - 4x = 0 \rightarrow\rightarrow\rightarrow x = y^3$$

Sendo assim:

$$x = y^3 = \left(x^3\right)^3 = x^9$$
$$x^9 - x = 0$$
$$x * \left(x^8 - 1\right) = 0 \rightarrow \begin{cases} x = 0 \rightarrow y = 0 \\ x = 1 \rightarrow y = 1 \\ x = -1 \rightarrow y = -1 \end{cases}$$

$$\frac{\partial f^2}{\partial xx} = 12x^2$$
$$\frac{\partial f^2}{\partial yy} = 12y^2$$
$$\frac{\partial f^2}{\partial xy} = -4$$

$$D(x,y) = f_{xx} * f_{yy} - \left(f_{xy}\right)^2$$
$$D(x,y) = 12x^2 * 12y^2 - (-4)^2$$
$$D(x,y) = 144(xy)^2 - 16$$
$$D(0,0) = -16$$
$$D < 0 \text{ logo, é um ponto de sela}$$

$$D(x,y) = 144(xy)^2 - 16$$
$$D(1,1) = 144 - 16$$
$$D(1,1) = 128$$
$$D > 0 \text{ e } \frac{\partial f^2(1,1)}{\partial xx} = 12 * (1)^2 = 12 > 0 \text{ logo, é um ponto de mínimo}$$

$$D(x,y) = 144(xy)^2 - 16$$
$$D(-1,-1) = 144 - 16$$
$$D(-1,-1) = 128$$
$$D > 0 \text{ e } \frac{\partial f^2(-1,-1)}{\partial xx} = 12 * (-1)^2 = 12 > 0 \text{ logo, é um ponto de mínimo}$$