COT 4420: Homework 1
Math Rep and elementary Formal Languages

1. Let S and T be subsets of a set U. Defining $S' = U \setminus S$ and $T'' = U \setminus T$, show that

$$(S \cup T)' = S' \cap T''$$

2. T/F (circle)

If $A = \emptyset$ or $B = \emptyset$ then $A \times B = \emptyset$	T	F
If $A = \emptyset$ then $\mathbf{P}(A) = \emptyset$ (power set)	T	F
There is a bijection from $\emptyset$ into itself	T	F
If $|A| = n$, then A has n subsets of size $n-1$	T	F
If $|A| = 2$, then $|\mathbf{P}(A^2)| = 32$	T	F
If $A = \{a\}$ then A^2 has one element	T	F

3. For each of the three cases, give *infinite* sets A and B such that

   a) $A \cap B$ is finite
   b) $A \cap B$ is infinite
   c) $A \cap B$ is empty

4. a) Show by induction that $(n+1)^2 = n^2 + (2n + 1)$.
   b) Show the same without induction (I really hope you were tempted to do that!)

5. Let $\Sigma = \{0,1\}$ be an alphabet. Enumerate five elements of the following languages:

   a) Binary numbers that are even
   b) Binary numbers that contain as many 0 as 1 (not leading 0s, of course)