<u>**FINAL ASSIGNMENT MAA 4211 SPRING 2020**</u>

INSTRUCTIONS:

1. This assignment is in lieu of the final examination

2. The assignment must be done on your own (no discussing or copying from someone else)

3. **You may use the class notes and/or the text book but no other resources**

4. Show all work and all steps of your solutions in a coherent organized way. Partial credit can be given only if work is shown.

5. Calculators are permitted

6. Solutions must be written on writing sheets

7. **Once the assignment is completed, make a clear, legible PDF scan (ensure that the writing can be clearly seen, dark script on a clear background), make sure the answer sheets are in proper order and oriented properly. You may use a smart phone camera with a <u>cam-scan</u> application to make the PDF file.**

8. IMPORTANT: Please submit just ONE FILE (do not submit the separate answer sheets as separate files; do not submit multiple files).

9. Submit to the drop box in CANVAS.

10. You will have time from time of posting on Thursday April 23 to 3:00 PM on Saturday April 25, 2020 to complete the exam and submit to the drop box.

[1] **(22 points)**

(a) Let (a_n) be sequence of real numbers. DEFINE: The sequence (a_n) converges to the limit l as $n \to \infty$. **(4 points)**

(b) Let the sequence (a_n) converges to the limit l as $n \to \infty$. Prove the following (cite the relevant inequalities)

1. $|a_n| \to |l|$ as $n \to \infty$ **(5 points)**
2. There exists $n_0 \in \mathbb{N}$ such that $|a_n| < |l| + 1 \ \forall n \ge n_0$ **(5 points)**
3. $a_n^2 \to l^2$ as $n \to \infty$ **(8 points)**

[2](14 points)

Let $a_n = 4n^5 + 200n^4 - 1000n^2 + 30n + 10{,}000, n = 1, 2, 3.....$

(a) With careful attention to computations and estimates of details, compute an n_0 such that

$a_n > \frac{7}{2} n^5$ for all $n \ge n_0$ **(7 points)**

(b) With careful attention to computations and estimates of details, compute an m_0 such that

$a_n \le 6n^5 \ \forall n \ge m_0$ **(7 points)**

(The computed n_0 and m_0 must be in terms of the coefficients of the expression for a_n .)

[3](18 points)

Let $b_n = \dfrac{9n^4 - 8n^3 + 7n^2 - 300}{3n^4 + 3n^3 + 2n^2 + 100}, n = 1, 2, 3.....$

(a)Compute the limit of the above sequence. **(3 points)**

(b)Prove with careful attention to details that the limit is what is computed in (a). Structure your proof as follows:

1) State what must be proved (in terms of ε, n_0) **(5 points)**
2) With careful attention to details and estimates prove that the ε, n_0 condition is fulfilled. **(10 points)**

[4](18 points)

Let $a_n = \dfrac{(1.03)^n}{n^{10}}, n = 1, 2, 3...$.

(a)Compute the largest integer n_0 such that $a_n > a_{n+1} \ \forall 1 \le n < n_0$ **(8 points)**

(b) With carefully stated reasons prove that the sequence (a_n) is not bounded above and thereby conclude that (a_n) diverges to $+\infty$ as $n \to \infty$ **(10 points)**

[5](20 points)
Let (a_n) be a strictly increasing sequence of POSITIVE real numbers bounded above by an $M > 0$.

(a)Let $b_n = \dfrac{a_n^3}{2 + a_n^2}$ for $n = 1, 2, 3, ...$ Prove that (b_n) is strictly increasing and bounded above by M.
(10 points)

(b) Let $c_n = (a_1 a_2 ....a_n)^{\frac{1}{n}}$, for $n = 1, 2, 3...$. Prove that (c_n) is strictly increasing and bounded above by M.
(10 points)

[6](18 points)
The sequence (a_n) is defined as follows:

$$a_1 = 1, a_2 = 3, \text{ and } a_{n+2} = a_{n+1} + \frac{1}{a_n}, n \geq 1$$

(a) Compute the values of a_3, a_4, a_5 in rational arithmetic (that is, as rational numbers) **(6 points)**

(b) Prove that (a_n) is strictly increasing. (You may assume that the sequence is positive.)(**6 points)**

(c) Prove that the sequence is not bounded above and hence is divergent to $+\infty$ (HINT: Assume that the sequence is bounded above; cite the reason for its convergence and find what its limit must be**)** **(6 points)**

[7](18 points)
(a) DEFINE:The sequence (a_n) is a Cauchy sequence. **(4 points)**

(b) PROVE: If a sequence (a_n) is convergent, then it is a Cauchy sequence. **(7 points)**

(c) PROVE: If a sequence (a_n) is Cauchy, then it is bounded. **(7 points)**

[8](12 points)
A sequence is defined as follows:

$$a_1 = \sqrt{3}, a_2 = \sqrt{1+\sqrt{3}}, a_3 = \sqrt{3+\sqrt{1+\sqrt{3}}}, a_4 = \sqrt{1+\sqrt{3+\sqrt{1+\sqrt{3}}}}, ...$$

(a)Write out a recursive expression for a_n . That is express a_{n+1} in terms of a_n . (HINT: Consider the cases of n being odd and n being even separately) **(4 points)**

(b) Write out a recursive expression for a_{2n+1} in terms of a_{2n-1} **(4 points)**

(c) Write out a recursive expression for a_{2n+2} in terms of a_{2n} **(4 points)**

140 points