MAA 4211 HOMEWORK 1 S2020

- DUE THURSDAY FEBRUARY 13 IN CLASS
- SUBMITTED WORK MUST BE NEAT, LEGIBLE, AND THE ANSWER SHEETS MUST BE STAPLED TOGETHER WITH NO RAGGED EDGES.
- 57 points. The score will be scaled to be score out of 20 points.

[1](21 points: 3 points apiece)

Write out the following definitions.

(a) The set $S \subseteq R$ is bounded above

(b) The set $S \subseteq R$ is bounded below

(c) The set $S \subseteq R$ has a maximum element

(d) The set $S \subseteq R$ has a minimum element

(e) The axiom of the least upper bound

(f) The real number r is a rational number.

(g) The rational numbers are dense in the reals.

[2](18 points)

(a) Let (a_n) be a sequence. DEFINE: The sequence (a_n) converges to the limit l as $n \to \infty$ **.(4 points)**

(b) It is required to prove that the sequence $a_n = \dfrac{6n^2 - 4n + 1}{2n^2 + n + 3}$ converges to the limit 3 as $n \to \infty$.

1. What must be shown (write in terms of definition)**(3 points)**
2. With all steps displayed show that $|a_n - 3| \le \dfrac{4}{n}$ for all $n \ge 8$. **(7 points)**
3. Using step 2 show that $\lim_{n\to\infty} a_n = 3$ (**4 points)**

[3](18=6+6+6 points. SHOW ALL CALCULATIONS)

(a) Compute the least positive integer k such that for some positive integer m, the rational number $\dfrac{m}{3^k}$ lies in the interval $(\sqrt{911}, \sqrt{913})$.

(b) Compute the least positive integer k such the EVERY interval of length 0.0331 contains a at least TWO rational numbers with denominator 3^k.

(c) For the value of k found in (b) determine three rationals with denominator 3^k in the interval $(\sqrt{911}, \sqrt{913})$