Assignment 2

STA4321 - Intro. Mathematical Statistics I

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Due by Tuesday July 9th at 11:59PM. Submission should only be online via Canvas

Solve all problems and submit via Canvas in a single PDF. A typed submission using LaTeX is preferred (10% bonus).

1. An advertising agency notices that approximately 1 in 50 potential buyers of a product sees a given magazine ad, and 1 in 5 sees a corresponding ad on television. One in 100 sees both. One in 3 actually purchases the product after seeing the ad, 1 in 10 without seeing it. What is the probability that a randomly selected potential customer will purchase the product? Define your events first!

2. The Bureau of the Census reports that the median family income for all families in the United States during the year 2003 was $\$43,318$. That is, half of all American families had incomes exceeding this amount, and half had incomes equal to or below this amount. Suppose that

four families are surveyed and that each one reveals whether its income exceeded $\$43,318$ in 2003.

(a) List the points in the sample space. Use counting methods.

(b) Identify the simple events in each of the following events:

- A: At least two had incomes exceeding $\$43,318$.
- B: Exactly two had incomes exceeding $\$43,318$.
- C: Exactly one had income less than or equal to $\$43,318$.

(c) Make use of the given interpretation for the median to assign probabilities to the simple events and find P(A), P(B), and P(C).

3. Three imported wines are to be ranked from lowest to highest by a purported wine expert. That is, one wine will be identified as best, another as second best, and the remaining wine as worst.

(a) Describe one sample point for this experiment.

(b) List the sample space.

(c) Assume that the expert really knows nothing about wine and randomly assigns ranks to the three wines. One of the wines is of much better quality than the others. What is the probability that the expert ranks the best wine as the "last/worst" wine?

4. The manager of a stockroom in a factory has constructed the following probability distribution for the daily demand (number of times used) for a particular tool.

y	0	1	2
p(y)	0.1	0.5	0.4

It costs the factory \$10 each time the tool is used. Find the mean and variance of the daily cost for use of the tool.

5. A multiple-choice examination has 15 questions, each with five possible answers, only one of which is correct. Suppose that one of the students who takes the examination answers each of the questions with an independent random guess. What is the probability that he answers at least ten questions correctly? define your random variable first!

6. A new surgical procedure is successful with a probability of p. Assume that the operation is performed five times and the results are independent of one another. What is the probability that

   - What is the distribution of the number of successful operations in this experiment. Write the probability function?

   - all five operations are successful if $p = 0.8$?

   - exactly four are successful if $p = 0.6$?

- less than two are successful if $p = 0.3$?

7. Suppose that 30% of the applicants for a certain industrial job possess advanced training in computer programming. Applicants are interviewed sequentially and are selected at random from the pool.

    - Define the random variable of interest and write down its probability function.

    - Find the probability that the first applicant with advanced training in programming is found on the fifth interview.

    - Find the expected number of interviews before having an applicant with advanced training in programming?

8. An oil prospector will drill a succession of holes in a given area to find a productive well. The probability that he is successful on a given trial is 0.2.

    - Define the random variable of interest and state its distribution?

- What is the probability that the third hole drilled is the first to yield a productive well?

- If the prospector can afford to drill at most ten wells, what is the probability that he will fail to find a productive well?

9. A warehouse contains ten printing machines, four of which are defective. A company selects five of the machines at random, thinking all are in working condition. What is the probability that all five of the machines are nondefective? (define your RV, state the distribution, write down its equation and then answer the question)

10. Let Y denote a random variable that has a Poisson distribution with mean $\lambda = 2$. Find

    - $P(Y = 4)$.

    - $P(Y \geq 4)$.

- $P(Y < 4)$.

- $V(Y)$.

11. Customers arrive at a checkout counter in a department store according to a Poisson distribution at an average of four per hour. During a given hour, what are the probabilities that

    (a) no more than 9 customers arrive?

    (b) at least 5 customers arrive?

    (c) exactly 9 customers arrive?

12. Let consider Y is a binomial random variable, $Y \sim Binom(n, p)$. We know that $E(Y) = np$ and $V(Y) = npq$. We will use mgf(t) to prove them again.

    (a) Show that the moment-generating function of a binomial distribution is

    $$mgf_Y(t) = (q + pe^t)^n$$

    (b) Find E(Y) and V(Y) using the derivatives of $mgf_Y(t)$.