Assignement 2

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Contents

1 Y possesses the following density function.

$f(y) = cy$ if $0 \le y \le 2$; 0 otherwise

- Find the c that makes f(y) a PDF

$$\int_0^2 (cy)dy = 1 => c\int_0^2 ydy = 1 => \frac{c}{2}[y^2]_0^2 = 1$$

$$=> \frac{c}{2}4 = 1 => 2c = 1 => c = \frac{1}{2}$$

- Find the CDF

$$P(Y \le y) = F(y) = \frac{1}{2}\int_0^y (y)dy => \frac{1}{4}[y^2]_0^y = \frac{y^2}{4}$$

F(y) = $\frac{y^2}{4}$ when $0 \le y \le 2$; 0 when $y < 0$; 1 when $y > 2$

- Use F(y) to find $P(1 \le Y \le 2)$

$$P(1 \le Y \le 2) = F(2) - F(1) = \frac{2^2}{4} - \frac{1^2}{4} = 1 - \frac{1}{4} = \frac{3}{4}$$

2 A Random Variable Y has the following CDF:

F(y) = $1 - e^{-y^2}$ if $y \ge 0$; 0 otherwise

- Find f(y)

$$f(y) = F‘(y) = \frac{d}{dy}(1 - e^{-y^2}) = 2ye^{-y^2}$$

- Find $P(Y \ge 2)$

$$P(Y \ge 2) = 1 - P(Y \le 2) = 1 - F(2) = 1 - (1 - e^{-2}) = 1 - 0.98 = 0.02$$

- Find $P(Y > 1|Y \le 2)$

$$P(Y > 1|Y \le 2) = \frac{P(Y > 1, Y \le 2)}{P(Y \le 2)}$$

$$\frac{P(1 \le Y \le 2)}{P(Y \le 2)} = \frac{F(2) - F(1)}{F(2)} = \frac{1 - e^{-2^2} - (1 - e^{-1^2})}{(1 - e)}$$

$$\frac{0.98 - 0.63}{0.98} = 0.357$$

3 Random Variable Y has the following PDF

- E(Y) $= \mu = \int_0^1 y(\frac{3}{2}y^2 + y)dy = \frac{3}{2}\int_0^1 y^3 dy + \int_0^1 y^2 dy$

  $\frac{3}{8}[y^4]_0^1 + \frac{1}{3}[y^3]_0^1 = \frac{3}{8} + \frac{1}{3} = 0.7083$

- V(Y) = E(Y^2) - μ^2
  E(Y^2) $= \int_0^1 y^2(\frac{3}{2}y^2 + y)dy = \frac{3}{10}[y^5]_0^1 + \frac{1}{4}[y^4]_0^1 = 0.55$
  V(Y) $= 0.55 - 0.7083^2 = 0.0483$

4 Random Variable Y has a Uniform Distribution

$Y \sim Uniform(50, 70)$
F(y) = 0 for $y \leq 50$; $\frac{y-50}{20}$ for $50 \leq y \leq 70$; 1 for $y \geq 70$

$$P(Y \geq 65|Y \geq 55) = \frac{P(Y \geq 65, Y \geq 55)}{P(Y \geq 55)} = \frac{P(Y \geq 65)}{P(Y \geq 55)}$$

$$=> \frac{1 - P(Y \leq 65)}{1 - P(\leq 55)} = \frac{1 - \frac{65-50}{20}}{1 - \frac{55-50}{20}} = \frac{1 - \frac{3}{4}}{1 - \frac{1}{4}} = \frac{1}{3}$$

5 Use Table to find:

- $P(0 < Z < 1.2) = \Phi(1.2) = 0.88493$
- $P(-0.9 < Z < 0) = \Phi(0) - (1 - \Phi(0.9)) = 0.5 - (1 - 0.81594) = 0.31594$
- $P(0.3 < Z < 1.56) = \Phi(1.56) - \Phi(0.3) = 0.94062 - 0.61791 = 0.32271$
- $P(-0.2 < Z < 0.2) = \Phi(0.2) - (1 - \Phi(0.2)) = 0.57926 - 0.42074 = 0.15852$
- $P(-1.56 < Z < -0.2) = \Phi(-0.2) - \Phi(-1.56)$
  $= (1 - \Phi(0.2)) - (1 - \Phi(1.56)) = 0.42074 - 0.05938 = 0.36136$

6 Random Variable $Y \sim Exponential(\lambda)$

- Find the PDF:
  $f(y) = \lambda e^{-y\lambda}$ for $y \geq 0$; 0 otherwise

- Find the CDF:

$$F(y) = \lambda \int_0^y e^{-y\lambda} = -\lambda^2 [e^{-y\lambda}]_0^y = -\lambda^2 e^{-y\lambda}$$